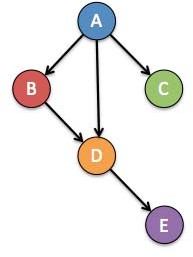
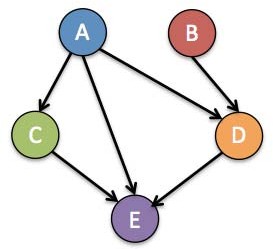
### Q1 - Bayesian Networks (7 points)

You are given two different Bayesian network structures 1 and 2, each consisting of 5 binary random variables A, B, C, D, E. Each variable corresponds to a gene, whose expression can be either “ON” or “OFF”.



### Network 1 Network 2

**(A – 2 points)** In class, we covered the chain rule of probability for Bayes Nets, which allows us to factor the joint probability over all the variables into terms of conditional probabilities.

For each of the following cases, factor P(A,B,C,D,E) according to the independencies specified and give the **minimum** number of parameters required to fully specify the distribution.

1. A,B,C,D,E are all mutually independent P(A,B,C,D,E) = P(A)P(B)P(C)P(D)P(E)

5 parameters (probability that each of the 5 genes is ON, independent of others)

1. A,B,C,D,E follow the independence assumptions of **Network #1** above P(A,B,C,D,E) = P(A)P(B)P(C|A)P(D|A,B)P(E|A,C,D)

16 parameters: 1 for P(A), 1 for P(B), 2 for P(C|A), 4 for P(D|A,B), and 8 for P(E|A,C,D)

1. A,B,C,D,E follow the independence assumptions of **Network #2** above P(A,B,C,D,E) = P(A)P(B|A)P(C|A)P(D|A,B)P(E|D)

11 parameters: 1 for P(A), 2 for P(B|A), 2 for P(C|A), 4 for P(D|A,B), 2 for P(E|D)

1. no independencies

P(A,B,C,D,E) cannot be simplified

25 - 1= 31 parameters (there are 32 combinations of A,B,C,D,E, must sum to 1)



|  |  |  |
| --- | --- | --- |
|  |  |  |

1. P(A=ON, B=ON, C=ON, D=ON, E=ON) P(A=ON, B=ON, C=ON, D=ON, E=ON)

= P(A=ON)P(B=ON|A=ON)P(C=ON|A=ON)P(D=ON|A=ON,B=ON)P(E=ON|D=ON)

= (0.6)(0.95)(0.5)(0.95)(0.1)

= 0.0271

1. P(E = ON | A = ON)

B, D, and E are conditionally independent of C given A, so C drops out. Therefore, we sum over the 4 {B, D} possibilities:

**P(E**  **ON | A  ON) **

****

**B,D{ON,OFF}**

**P(E  ON | D)P(D | A  ON,B)P(B | A  ON)**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| B | D | P(B|A=ON) | P(D|A=ON,B) | P(E=ON|D) | P(E=ON, B, D|A=ON) |
| ON | ON | 0.95 | 0.95 | 0.1 | 0.09025 |
| ON | OFF | 0.95 | 0.05 | 0.8 | 0.038 |
| OFF | ON | 0.05 | 0.9 | 0.1 | 0.0045 |
| OFF | OFF | 0.05 | 0.1 | 0.8 | 0.004 |

Summing over the last column, we obtain P(E=ON | A = ON) = 0.13675.

1. P(A = ON | E = ON) By Bayes’ rule,

*P*(*A*  *ON* | *E*  *ON* )  *P*(*E*  *ON* | *A*  *ON* )*P*(*A*  *ON* )

*P*(*E*  *ON* )

*P*(*E*  *ON* | *A*  *ON* )*P*(*A*  *ON* )



*P*(*E*  *ON* | *A*  *ON* )*P*(*A*  *ON* )  *P*(*E*  *ON* | *A*  *OFF*)*P*(*A*  *OFF*)

We already have P(E=ON | A=ON) from (ii), so we just need P(E=ON | A=OFF):

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| B | D | P(B|A=OFF) | P(D|A=OFF,B) | P(E=ON|D) | P(E=ON, B, D|A=OFF) |
| ON | ON | 0.1 | 0.3 | 0.1 | 0.003 |
| ON | OFF | 0.1 | 0.7 | 0.8 | 0.056 |
| OFF | ON | 0.9 | 0.1 | 0.1 | 0.009 |
| OFF | OFF | 0.9 | 0.9 | 0.8 | 0.648 |

Summing over the last column, we obtain P(E=ON | A=OFF) = 0.716. Therefore

**P(A  ON | E  ON) **

**(0.13675)(0.6)**

**(0.13675)(0.6)  (0.716)(0.4)**

** 0.2227**