**Game Theory - Assignment Report**

**COURNOT COMPETITION**

* Simultaneous moves (different from Stackelberg competition, where moves are played one after the other)
* We assume that there is no difference in product from firm 1 and firm 2
* Price is set by total quantity of goods out there (q),

q = q1+q2,

q1- goods produced by firm 1

q2- goods produced by firm 2

* Market price is decided by a function p(q)= a-bq —(for simplicity, let's assume b=1)
* Remember the function p(q) can be any type of function that can help us model the pricing.

a is constant

from here we can say that as supply increases price drops

q>a

* **MARGINAL COSTS**

c1 - marginal cost of firm 1

c2 - marginal cost of firm 2

* **OBJECTIVE**: Maximise the profit

p(q)\*qi - ci \* qi

**SOLVING STRATEGY**

* -firm 1’s best response to firm 2’ output decision
* -firm 2’s best response to firm 1’ output decision
* -find a pair of mutual best response

**DERIVING FIRM 1 BEST RESPONSE**

Maximise p(q)\*q1 - ci \* q1 = (a-q) \*q1 -c1\*q1

= (a-(q1 + q2)) \*q1 -c1\*q1

= (a q1 - q1^2- q1 q2 - c1 q1)

We plot the above fixing a and q2, maxima of the above function is when the derivative of above function is ZERO

q1= (a-q2-c1) / 2 (Here q1 must be positive) ---> BEST RESPONSE FUNCTION FIRM 1

What we observe is that q1 decreases with

1. increase in q2

* When firm 2 produces more the price of goods is going to decrease which causes the profits for firm 1 to decrease.
* Hence firm1 responds by decreasing production

2. increase in c1 (marginal cost)

The reason is that you will want to produce less of something which costs you a lot to manufacture.

**DERIVING FIRM 2 BEST RESPONSE**

Basically the same thing we did above here we just flip q1 and q2

q2= (a-q1-c2) / 2 (Here q1 must be positive) -> BEST RESPONSE FUNCTION FIRM 2

**DERIVING MUTUAL BEST RESPONSE**

Firms are in equilibrium when they don't want to change what they are doing given the other

firm’s strategy

A pair of q1 and a2 such that both the best response function hold simultaneously

->What we have is basically a system of variables with 2 equations and 2 unknown variables

A,c1,c2 – fixed q1,q2 – variables

q1 = (a+c1-2c2)/3

q2 = (a+c2-2c1)/3

**Game for Cournot Duopoly Model**

**Cupcake Delight**

Jim and Jessie are the workers at a renowned cupcake shop that’s always bustling with orders. Jim, the master of the vanilla counter, crafts an impressive **c1** number of cupcakes by the end of each day, while Jessie, the blueberry maestro, whips up **c2** delightful creations during her shift. A group of cupcakes with frosting on top

Description automatically generated

But this is where it gets interesting—The shop owner has a quirky method of calculating their earnings, considering not only the total cupcakes but also the amount of bread and icing used per cupcake. For Jim, these are **h1** and for Jessie, they’re **h2**.

Now here’s the twist: is it better for **h1** and **h2** to be high or low? Should Jim and Jessie focus on extravagance, layering their cupcakes with rich bread and thick icing? Or is efficiency the key, crafting simpler yet equally delicious cupcakes?

Price per cupcake, p = a-(c1+c2) ----[a is decided by owner and is fixed]

Payment received by Jim = p\*c1-h1\*c1-----(1)

Payment received by Jessie = p\*c2-h2\*c2------(2)

Help both Jim and Jessie maximize their pay while keeping the cupcakes as irresistible as ever. The shop’s reputation—and their wallets—are in your hands!

**Rules of game:**A display of cupcakes in a bakery

Description automatically generated

1. If the pay will be greater than or equal to zero.
2. Meaning, if the pay comes out to be negative by (1) or (2) then pay will be assigned as zero.
3. The values of c1, c2, h1 and h2 will be greater than zero.
4. Both players do not know the number of cupcakes they made and the bread and icing used.

**BERTRAND COMPETITION**

Firms produce a homogeneous good and compete on prices

Each firm chooses a price p1 and p2

We have a single consumer in this model and all the consumer does is decide whether to buy from a firm and which firm to buy from.

1. Prefers lower price than higher

2. The consumer has a reservation price v larger than marginal cost of production c

- A basic assumption so that we have a deal getting done

- Incase v<c the reservation cost is so low that no firm wants to produce at that

price

3. The consumer is indifferent between the firms i.e if the firms set same price for product the consumer can buy it from any of the firms

The firms have symmetric marginal cost of production c>0 (To keep things simple)

OBJECTIVE : Firms want to MAXIMISE their profit

1.EQUILIBRIUM PRICES : p1=p2=c

Hence no profit made.

- If a firm deviates to p<c -ve profit

- If a firm deviates to p>c no sale and no profit

-Firm 1 profit = (p1-c)D1(p1,p2)

Where D1(p1,p2) = demand faced by firm 1 if firm 2 charges price p2. It depends upon the price of both firms.

Equilibrium price can't be lower or greater than c

2.Suppose p1<p2

- Firm 2 makes nothing

-but choosing any value between c and p1 guarantees a profitable sale

- after firm 2 changes its price now firm 1 will also change and the companies will just continue doing this until equilibrium price is reached.

3.Suppose p1=p2

If both the firms charge the same price demand is split equally and firm makes a

profit of, profit =(p-c)1/2D(p).

Why equilibrium of prices is at marginal cost

- If p1>c then firm 2 will set a price between p1 and c then firm 2 will make all the

profit because firm 2 obtains the entire demand.

- Since firm 1 has no demand and makes a profit of zero firm 1 will lower the price

between c and p2.

- Now firm 1 makes a profit of zero firm 1 will lower the price between c and p2. Now

firm 1 obtains the entire demand.

- Both firms will repeat this process until charging a lower price does not increase

profits , which occurs at the price of marginal cost only.

Consider a deviation to price pd between p1 and c

-Its better if (pd - c) > 1/2(p1 - c)

We took 1⁄2 as the probability of the order going to firm 1

-So always one company will try to reduce its price so that it gets the sale done

**Game for Bertrand Duopoly Model**

**Concert Tickets**A pair of blue tickets

Description automatically generated

Two students (let’s refer them as student1 and student2) of a class bought tickets for a much-awaited concert in Mumbai during their winter vacations. However, as the winter vacations approached near student1’s family informed their plans of vacationing (at some places, not in Mumbai) and student2 had some important event to attend.

Now, both these students decided to sell their tickets to student3. Both want to sell out their tickets as soon as possible. Student3 is very diplomatic, he will buy the tickets of only that student who will sell them at a cheaper price. If the same offer is made, then he(student3) would take both the tickets and take his brother to the concert too.

**Rules of the game:**

1. Student1 and Student2 don’t disclose the offer to each other.
2. The student with a lower price offer will get the deal.
3. For the same offer, both will get the deal.The deal will be splitted equally.
4. The offer made is higher than the initial price of the ticket.