Quiz4SP23

Monday, February 20, 2023 10:51 PM

Quiz4SP23

CS146: Quiz 4
Due Tuesday, February 21, at 7:00AM
10 points

This quiz will be a written quiz. Please submit your answers to these questions by uploading a file to Canvas. You may type or handwrite your solutions. You are free to use the textbook, slides, class notes, but **DO NOT** consult any other resources.

Question 1) [3 points]

The solution to the recurrence $T(n) = 4T(n/2) + n$ turns out to be $T(n) = \Theta(n^2)$. Show that a substitution proof with the assumption $T(n) \le cn^2$ fails. Then show how to subtract a lower-order term to make a substitution proof work.

let $T(n) \le cn^2$ for $n \ge n_0$

$\le 4c\left(\frac{n}{2}\right)^2 + n$

$= cn^2 + n \longrightarrow$ doesn't satisfy $T(n) \le cn^2$

$T(n) \le cn^2 - bn$ for $n \ge n_0$, $b > 0$

$\le 4\left(c\left(\frac{n}{2}\right)^2 - \frac{bn}{2}\right) + n$

$= 4\left(c\left(\frac{n}{2}\right)^2 - \frac{bn}{2}\right) + n$

$= 4\left(c\frac{n^2}{4} - \frac{bn}{2}\right) + n$

$= cn^2 - 2bn + n = cn^2 - n(2b-1)$

$\le cn^2 - n(2b-1)$

• let $b = 1 \rightarrow T(n) \le cn^2 - n \rightarrow$ satisfy $T(n) \le cn^2$

Question 2) [2 points]

Sketch the recursion tree to generate a good guess for the asymptotic upper bound on its solution. Then use the substitution method to verify your answer.

$T(n) = 2T(n/4) + n^3$

Sum Row

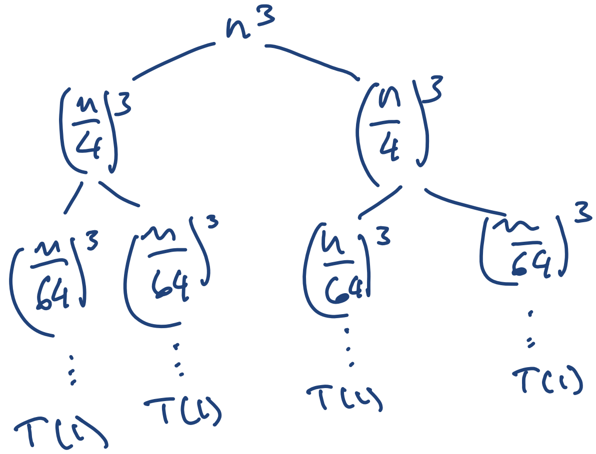
n^3

$\frac{1}{32} n^3$

$4 \times \left(\frac{n}{64}\right)^3$

$2^i \times \frac{n^3}{4^{3i}}$

Tree



$$T(n) = n^3 + \frac{1}{32} n^3 + 4\left(\frac{n}{64}\right)^3 + \cdots + 2^i \cdot \frac{n^3}{4^{3i}}$$

Question 3) [2 points]

Use the master theorem to give an asymptotic tight bound for the following recurrences. Tell me the values of a, b, the case from the master theorem that applies (and why), and the asymptotic tight bound.

3a) $T(n) = 2T(n/4) + n$ $a = 2,\ b = 4,\ f(n) = n$

3b) $T(n) = 16T(n/4) + (\sqrt{n})^3$ $a = 16,\ b = 4,\ f(n) = (\sqrt{n})^3$

a)
watershed func $n^{\log_4 2} = n^{1/2}$, driving func $= f(n) = n$

choose $\epsilon = 1$ $f(n) = n = O(n^{\log_4 2 - 1}) = O(n)$

case 1 applies $\Rightarrow T(n) = \Theta(n^2)$

b) watershed func $n^{\log_4 16} = n^2$, driving func $= f(n) = (\sqrt{n})^3$
$= n\sqrt{n}$

choose $\epsilon = 1$ $f(n) = \Omega(n^{\log_4 16 + 1})$
$= \Omega(n^3) \le \Omega(n\sqrt{n})$

$16(n/4) \le n\sqrt{n}$ $(a f(n/b) \le c f(n))$

Cas 3 applies

$T(n) = \Theta(n\sqrt{n})$

Question 4) [3 points]

$T(n) = 4T(n/2) + n^2 lgn$ $a = 4, b = 2$

a) Consider the above recurrence relation, explain why Case 1 and Case 3 of the Master Theorem do not apply to the recurrence.
b) Explain why our expanded definition of Case 2[1] of the Master Theorem does apply and use it to determine a tight bounder.

a) watershed func: $n^{\log_2 4} = n^2$
driving func: $f(n) = n^2 lg n$

Case 1. need to check $f(n) = O(n^{\log_b a - \epsilon})$, $\epsilon > 0$
Since there is no $\epsilon > 0$ that satisfy $n^2 lg n = n^2$,
case 1 doesn't apply

Case 3: need to check $a f(n/b) \le c f(n)$
Since there is no $\epsilon > 0$ that satisfies $4(n/2) < n^2 lg n$
case 3 doesn't apply

b) choose $k = 1$, $f(n) = \Theta(n^{\log_2 4} lg n) = n^2 lg n$
Then $T(n) = \Theta(n^2 lg^2 n)$

[1] If there exists a constant $k \ge 0$ such that $f(n) = \Theta(n^{\log_b a} lg^k n)$, then $T(n) = \Theta(n^{\log_b a} lg^{k+1} n)$