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RESEARCH ARTICLE

Fuzzy Linear Programming Formulation for Time Prediction in Product Delivery

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ABSTRACT The product delivery process is a set of steps to transport a product of an origin to a delivery point. Nowadays, there are different platforms or software based on classical algorithms for network optimization that help develop routes. Although these informatics systems provide vehicle routes, their prediction time has low accuracy compared to real times in the product distribution, since these systems do not consider the elements that affect route planning, i.e., despite providing vehicle routes, these software systems have low prediction accuracy. To address this problem, an alternative is to use artificial intelligence systems that consider the knowledge of the product delivery planning into route optimization models, thus increasing the time accuracy, but adding the challenge of having to interpret correctly the vehicle route ambiguities. Motivated by the latter, we propose a new fuzzy linear programming formulation to predict delivery times for products. Unlike previous studies, our methodology considers various parameters in the distribution process and offers an effective way to identify which parameters should be used. Our strategy combines the abstraction power of fuzzy logic and the result that provides a route optimization analysis, i.e., this work brings the best of the two worlds to address the difficult problem of shortest-route in product delivery. For that, our methodology has three steps. First, we introduce our formulation that incorporates a Fuzzy Inference System (FIS) into linear programming to achieve accurate time predictions in product delivery. Second, we propose a fuzzy adjustment coefficient to consider the uncertain factors in product distribution and the expertise of the delivery staff. Finally, we develop a Geographic Information System (GIS) to visualize the distribution route and its time. On the other hand, we evaluate this methodology in the routes of a soft drink company using statistical analyses. Experimental results are feasible and promising. For example, in real-world scenarios, our approach reduced the Mean Absolute Percentage Error (MAPE) by 56% compared to methods that utilize artificial intelligence.

INDEX TERMS Fuzzy logic, linear programming, product delivery, time prediction.

I. INTRODUCTION

The shortest path model determines the minimum distance between two points, namely the source and destination nodes in a network [4]. This mathematical formulation is highly adaptable [17] and can be customized to suit various situations and constraints [17]. Due to its versatility, this mathematical model is utilized in several fields, including robotics [7], autonomous navigation [8], electrical distribution [9], and product delivery [10], among others.

There are various methods for route planning. The simulation approach breaks down the vehicular environment into multiple routing parameters and a behavioral description for each parameter [18], [19], [20]. This approach depends

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on modeling algorithms, typically discrete-event or agent-based simulation techniques, to model specific aspects of the vehicular environment. Generally, simulation is effective for certain problems. For instance, it enables the identification of suboptimal solutions and the consideration of uncertainty in the network model. However, the vehicular environment comprises an infinite number of elements that influence route planning. Furthermore, these elements may alter their behavior due to factors that are difficult to simulate, such as rain or street deterioration. This limitation is significant because an ideal simulated shortest path model demands extensive parameters and high computational resources.

In a nonlinear shortest path model [21], [22], [23], [24], the objective function or constraints may contain nonlinear terms. This allows for greater flexibility in representing the mathematical model, as it can capture nonlinear effects with exponential, logarithmic, trigonometric functions, among others. For instance, the capacity constraint of vehicular flow can be expressed in exponential terms to eflect that capacity decreases exponentially as congestion increases. However, solving nonlinear equations is more computationally expensive than solving linear equations, which may require more computational resources and time to obtain a solution.

Heuristic shortest path models define the problem and establish a strategy based on the systematic exploration of possible solutions [25], [26], [27], [28]. In these works, the objective function is not always mathematically defined and is based on intuition and prior knowledge to minimize processing time. To generate initial solutions, we evaluate the quality of each solution based on the objective function by adjusting the path until a feasible and efficient solution is found. Heuristic shortest path models offer flexibility through systematic exploration of possible solutions using intuition and prior knowledge. However, they do not guarantee optimal solutions. This is because the heuristic strategy may not explore all possible routes, relying heavily on intuition and prior knowledge of the environment.

A linear shortest path model is a mathematical representation of an optimization problem that aims to find the shortest path with an objective function and linear constraints [29], [30], [31], [32]. This formulation can be solved using techniques such as the Simplex method to find the optimal solution. The ability to arrive at optimal solutions has the advantage of ensuring that the chosen route is the best possible one. This can significantly impact the efficiency and profitability of a transport operation or communications network. However, the prediction time of these systems has low accuracy compared to real product distribution times because they do not consider all the elements that affect route planning. The model assumes that all variables have a linear and direct relationship with the cost of the route, which may not be accurate in reality due to potential nonlinear relationships. It is important to consider this limitation when interpreting the results.

A current trend is the use of artificial intelligence systems in route optimization models that consider knowledge of product delivery planning [33], [34], [57], . This increases the accuracy of the times provided by representing uncertainty or back-feeding the input data. However, it also adds the challenge of correctly interpreting the ambiguities of the vehicle routes, specifically modeling the elements that affect the route planning. Otherwise, inaccurate, incomplete, or uncertain data may lead to suboptimal or incorrect solutions.

The primary contribution of this method is the integration of delivery personnel's expertise to model the factors affecting route planning through a Fuzzy Inference System (FIS) combined with a linear programming formulation. This integration allows us to make more realistic delivery-time predictions and efficient paths than traditional and current short route approaches. Our motivation stems from the success of artificial intelligence systems in route optimization models. We propose a methodology for predicting delivery times. Specifically, we focus on the product delivery process, as previous work in artificial intelligence has been limited to generalized shortest path models. In contrast to prior work, we do not simply add coefficients to account for uncertainty. Instead, we suggest incorporating the expertise of product delivery personnel to model the factors that impact route planning. This approach enables us to accurately constrain the variables to be included in our mathematical formulation. Furthermore, this proposal combines the abstraction power of fuzzy logic with the output provided by route optimization analysis. This work brings together the best of both worlds to address the challenging problem of finding the shortest route for product delivery.

In section II, we presented the related works that determine the location of the research. Section III presents the preliminary information used. Section IV contains our proposed method. Section V describes the experiments designed to evaluate the feasibility of the proposed method and the results achieved. Finally, the conclusions and future work are indicated in the last section.

II. RELATED WORK

This section presents a review of the current state of linear shortest path models that incorporate artificial intelligence techniques. The primary goal of these approaches is to enhance the accuracy of optimization models by representing uncertainties and providing feedback on input data. However, a challenge arises in properly interpreting the ambiguity inherent in vehicular routes. To address this challenge, genetic algorithms, neural algorithms, fuzzy algorithms, or combinations of these are commonly utilized.

Genetic algorithms with linear models can extensively explore the search space in route planning [37], [38], [39], [40]. However, when considering genetic processing for the pathway, the result may be suboptimal, even though the linear models converge to an optimal solution. Alternatively, genetic processing can be incorporated internally into the linear model variables to account for the ambiguity of the transport network. Modeling ambiguous variables using a genetic approach can significantly increase run time by involving the evaluation of multiple solutions. It is important to note that the more ambiguous elements are considered, the more complex the evaluation function may become in terms of time and resources.

Shortest path models using neural networks present a promising approach for route optimization [41], [42], [43], [44]. However, one of the challenges is collecting quality data to build a sufficiently large and representative set. This requires accurate information that considers various circumstances, such as the characteristics of the transport network, road capacity, speed restrictions, different schedules, and traffic congestion. These characteristics are crucial because if they are not accurately represented in the training, it can result in low accuracy when implementing the model. Additionally, processing and analyzing large datasets can be time and computationally intensive.

A neuro-fuzzy system is a hybrid system that combines fuzzy logic with neural networks to handle relevant parameters within vehicular routes and achieve higher performance [45], [46], [47], [48]. However, previous literature suggests that this approach is computationally more complex and difficult to implement. However, due to the reliance on neural networks, extensive data sets and significant computational resources are still required for training.

In the field of route planning, various fuzzy models with linear programming have been developed [49], [50], [51], [52]. Our research has focused on developing a set of models specifically for this topic. However, previous work has an important limitation as it only focuses on shortest path planning in a general way, without considering crucial aspects related to the distribution of product delivery. Product distribution requires careful consideration of specific aspects, such as loading and unloading at delivery points, which add complexity to the process. Product distribution requires careful consideration of specific aspects, such as loading and unloading at delivery points, which add complexity to the process. It is important not to overlook these factors.

The urban travel time estimation techniques presented in papers [53], [54], [55] address the challenge of estimating travel time in urban networks. However, they adopt different approaches. For instance, papers [53], [54] are based on fuzzy logic, where both incorporate fuzzy logic to account for uncertainties in travel time caused by factors such as traffic congestion. Paper [53] employs fuzzy linear programming to identify the optimal route with the minimum travel time, yielding superior results compared to classical shortest path methods. Paper [54] utilizes fuzzy logic with an adaptive neural network architecture, accounting for the state of the road, traffic zones, and rain intensity to estimate the travel time of each road segment. Finally, paper [6] is based on artificial neural networks (ANNs), proposing the direct use of ANNs to estimate the travel time of each path within a road network. In general, all three approaches aim to improve travel time estimation in urban environments by taking into account real-world complexities that are not captured by traditional methods. Fuzzy logic helps incorporate uncertaihttps://www.savethevideo.com/es/downloadern factors, while ANNs can learn from historical data to predict travel times.

The current methodologies have two main deficiencies compared to our proposal. First, these approaches represent uncertainty using coefficients that simplify operational complexity. Second, they overlook the tacit knowledge of delivery personnel, which means critical logistical factors remain unmodeled. In contrast to previous methods, our approach goes beyond simply adding coefficients to represent uncertainty. We propose incorporating the knowledge and experience of product delivery personnel, allowing us to accurately and effectively model the various elements that influence product delivery planning in a specific distribution area. Additionally, this approach enables us to properly discretize the relevant variables in our mathematical formulation.

III. PRELIMINARIES

In this section, some basic concepts are introduced, including linear programming formulation of the shortest-route problem and fuzzy sets theory.

A. LINEAR PROGRAMMING FORMULATION OF THE SHORTEST-ROUTE PROBLEM

Consider a directed network or digraph $G = (N, A)$, consisting of a finite set of nodes $N = \{1, 2, 3, \ldots, n\}$ and a set of directed edges $A = \{(i,j) : i, j \in N, i \neq j\}$ joining pairs of nodes in N. For all edges $(i,j) \in A$ let there be one nonnegative weight denoted by $c_{i,j}$, [11]. **Fig.** 1 shows a digraph $G = (N, A)$.

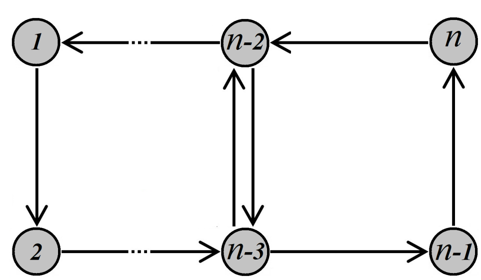


FIGURE 1. Digraph *G*.

A linear programming formulation of the shortest-route is defined by equations 1-3, where $x_{i,j}$ are indicator variables and $c_{i,j}$ are the times of the edge (i,j) for $i \neq j$.

$$min = \sum_{i=1}^{n} \sum_{j \neq i}^{n} c_{i,j} x_{i,j} \tag{1}$$

$$s.t. = \sum_{j \neq i}^{n} x_{i,j} - \sum_{j \neq i}^{n} x_{j,i} = b_i, \ i = 1, 2, 3, \ldots, n \quad (2)$$

$$x_{i,j} \geq 0 \, for \; all \; i, j \quad (3)$$

Based on the flow conservation equation (**equation** 2), $\sum_{j \neq i}^{n} x_{i,j}$ represents the total edge out of node i $\sum_{j \neq i}^{n} x_{j,i}$ indicates the total edge into node i. Where each node i in G is associated to one number b_i. Nodes with $b_i > 0$ are called sources, and nodes with $b_i < 0$ are called sinks. For $bi = 0$, node i is called a transshipment. In addition, non-negativity constraint in **equation** 3, indicates that the decision variable $x_{i,j}$ is set either 0 or a positive integer number.

B. FUZZY NUMBERS

Definition 1: Let $\widetilde{\mathrm{A}}$ be a fuzzy subset of a universe of discourse X. Where all $x \in X$, there is a number $\mu_{\widetilde{\mathrm{A}}}(x) \in [0, 1]$ which is assigned to represent the membership degree of x in $\widetilde{\mathrm{A}}$, and is called the membership function of $\widetilde{\mathrm{A}}$, [[13]].

Definition 2: A fuzzy number $\widetilde{\mathrm{A}}$ is a normal and convex fuzzy subset of X.

$$\widetilde{\mathrm{A}} = \{(x, \ \mu_{\widetilde{\mathrm{A}}}(x)), \ x \in X\} \quad (4)$$

where

$$0 \ \leq \ \mu_{\widetilde{\mathrm{A}}}(x) \ \leq \ 1 \quad (5)$$

Definition 3: A triangular fuzzy number $\widetilde{\mathrm{A}}$ can be defined as $\widetilde{\mathrm{A}} = (a_1, a_2, a_3)$, where $a_1, a_2, a_3 \in X$, the membership can be determined in **equation** 6 and shown in **Fig.** 2.

$$\mu_{\widetilde{\mathrm{A}}}(x) = \begin{cases} 0; x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}; a_1 < x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}; a_2 < x < a_3 \\ 0; a_3 \leq x \end{cases} \quad (6)$$

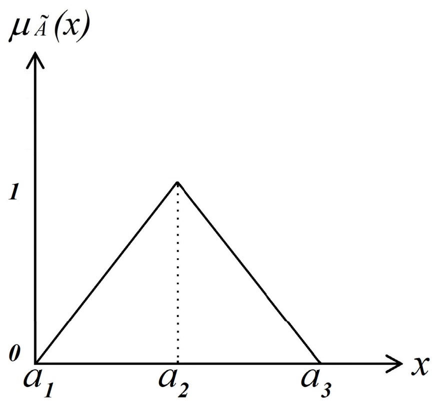


FIGURE 2. A triangular fuzzy number.

Definition 4: A trapezoidal fuzzy number $\widetilde{\mathrm{A}}$ can be defined as $\widetilde{\mathrm{A}} = (a'_1, a'_2, a'_3, a'_4)$, where $a'_1, a'_2, a'_3, a'_4 \in X$, the membership can be determined in **equation** 7 and shown in **Fig.** 3.

$$\mu_{\widetilde{\mathrm{A}}}(x) = \begin{cases} 0; x \leq a'_1 \\ \dfrac{x - a'_1}{a'_2 - a'_1}; a'_1 < x < a'_2 \\ 1; a'_2 \leq x \leq a'_3 \\ \dfrac{a'_4 - x}{a'_4 - a'_3}; a'_3 < x < a'_4 \\ 0; a'_4 \leq x \end{cases} \quad (7)$$

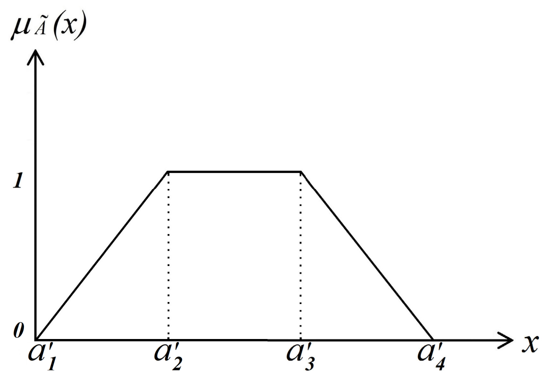


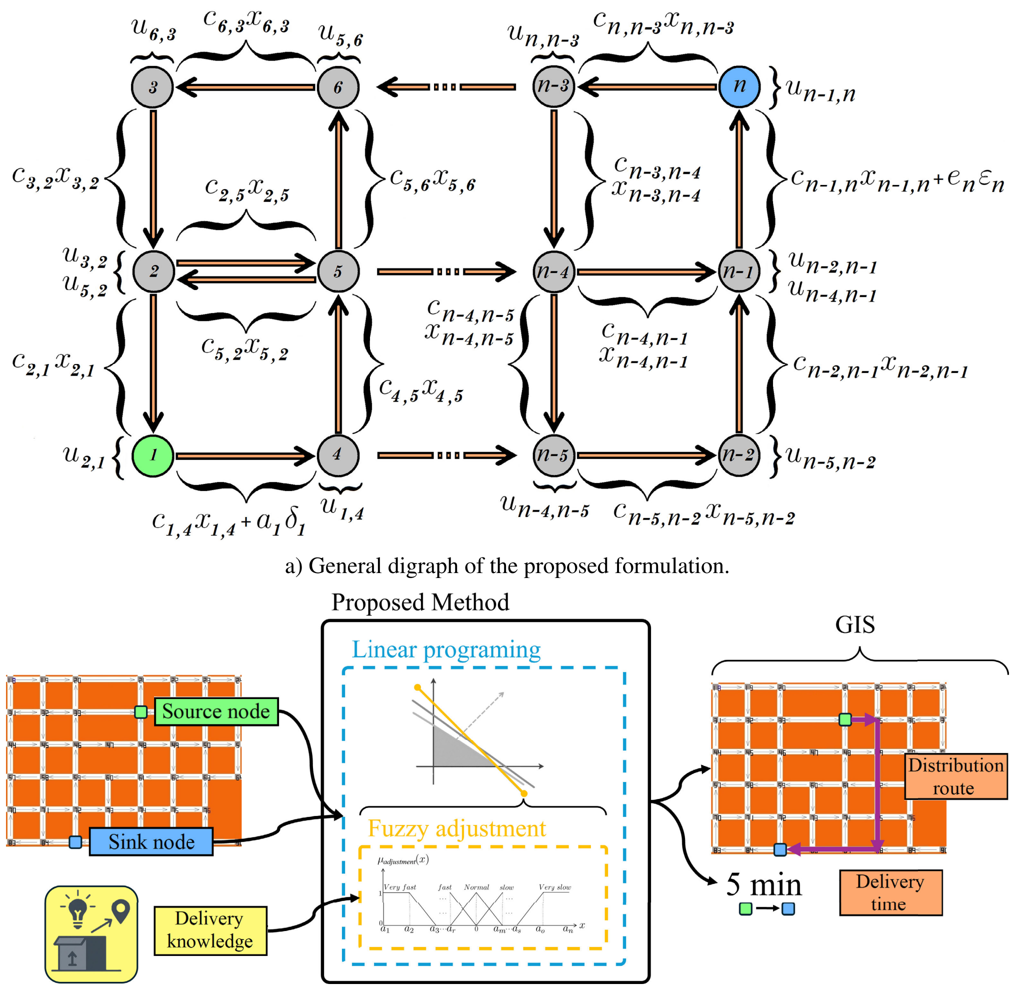
FIGURE 3. A trapezoidal fuzzy number.

IV. THE PROPOSED METHODOLOGY

In this section, we present the proposed methodology to obtain time prediction in product delivery using a fuzzy inference system integrated into a linear programming formulation. Our strategy is to use the knowledge of the product delivery staff in a linear programming formulation for the shortest route. This strategy combines the abstraction power of fuzzy logic and the result that provides a route optimization analysis, i.e., this work brings the best of the two worlds to address the difficult problem of shortest-route in product delivery. For that, this methodology simplifies the task of path planning in three steps. First, we introduce our formulation that incorporates a Fuzzy Inference System (FIS) into linear programming to achieve accurate time predictions in product delivery (**Subsection** IV-A). Second, we propose a fuzzy adjustment coefficient to consider the uncertain factors in product distribution and the expertise of the delivery staff (**Subsection** IV-B). Finally, we develop a Geographic Information System (GIS) to visualize the distribution route and its delivery time (**Subsection** IV-C). **Fig.** 4b shows the block diagram of the proposed methodology.

A. FUZZY LINEAR PROGRAMMING FORMULATION

The proposed formulation uses a digraph $G = (N, A)$ which consists of a finite set of nodes $N = \{ 1, 2, 3, \ldots, n \}$ and a set of directed edges $A = \{(i, j) : i, j \in N, i \neq j\}$ joining pairs of nodes in N. For all arcs $(i, j) \in A$ must be two nonnegative weights denoted by $c_{i,j}$ and $u_{i,j}$. We refer to $c_{i,j}$ and $u_{i,j}$ as the time-lapse to cross a vehicular flow and an intersection,



a) General digraph of the proposed formulation.

b) Block diagram of the proposed method.

FIGURE 4. **(a) General digraph of the formulation: orange arrows represent the edges, circles denote intermediate nodes, the green node is the source, and the purple node is the sink. (b) Block diagram of the proposed method, which combines linear programming and fuzzy adjustment to compute distribution routes in a GIS environment.**

respectively. Furthermore, for consider the times of loading and unloading for a delivery vehicle, we propose two binary variables $(\delta_k, \varepsilon_k)$ and two coefficients (a_k, e_k), $k \in N$. **Fig.** 4a shows the general digraph of the proposed formulation. This digraph represents a product delivery area with vehicular flows and streets of a city, i.e., in this representation, the edges are the vehicular flows, and nodes are the intersections. Where the orange arrows are the edges, the circles are the nodes, the green node is the source node, and the purple node is the sink.

The proposed fuzzy linear programming formulation for product delivery is presented in **equations** 8-10. The coefficient $\widetilde{d}_{i,j}$ is a fuzzy adjustment coefficient to consider the uncertain elements in product distribution and the knowledge of the product delivery staff, i.e., this coefficient adjust the values of the coefficient $c_{i,j}$.

$$min = \sum_{i=1}^{n} \sum_{j \neq i}^{n} [\widetilde{d}_{i,j}\, c_{i,j} + u_{i,j}] x_{i,j} + \sum_{k=1}^{n} [a_k \delta_k + e_k \varepsilon_k] \quad (8)$$

$$s.t. = \sum_{j \neq i}^{n} x_{i,j} - \sum_{j \neq i}^{n} x_{j,i} = b_i, \;\; i = 1, 2, 3, \ldots, n \quad (9)$$

$$x_{i,j} \geq 0 \, for \; all \; i, j \quad (10)$$

1) CORNER INTERSECTION $U_{I,J}$

The coefficient $u_{i,j}$ considers the time-lapse to cross an intersection by a delivery vehicle, i.e., the vehicle's time to cross a road corner with a traffic light, 1×1 signal, among others. This corner has different times according to its classification and spatial information. For that, we consider four different types of corners (traffic light, 1×1, corner crossing with preference, and without preference) and two geographical areas (downtown city and regular urban area). In our formulation, we consider a directed intersection since the time varies according to the edge $x_{i,j}$ used. For example, in the **Fig.** 4a the coefficients $u_{3,2}$ and $u_{5,2}$ can have different times; if $u_{3,2}$ and $u_{5,2}$ are corner crossing with preference, and without preference, respectively. **Table** 2 f) shows the different time-lapses to cross an intersection by a delivery vehicle.

2) BINARY VARIABLES

The binary variables (δ_k, ε_k) and their coefficients (a_k, e_k) consider the time-lapse to loading and unloading for a delivery vehicle. The binary variable δ_k assigns to the source node an average time of product loading, and the binary variable ε_k assigns to the sink node an average time of product unloading. These average times are of a soft drink company where the proposed formulation was evaluated. For that, we survey the drivers of the soft drink company, asking about their load and unload times. **Table** 3 d) shows the load and unload times.

The threshold function (S) is used to consider the average time of product loading. Subject to $k = \{\ 1, 2, 3, \ldots, \texttt{or}\ n\ \}$, $k \in N$. If the threshold function is 1, the binary variable δ_k is inside source node. The threshold function (S) is defined as:

$$\mathrm{S}(\delta_k) = \begin{cases} 1 & \texttt{if}\ \ k = \text{source node}, \\ 0 & \qquad\qquad \text{otherwise}, \end{cases} \tag{11}$$

The threshold function (S_2) is used to consider the average time of product unloading. Subject to $k = \{\ 1, 2, 3, \ldots, \texttt{or}\ n\ \}$, $k \in N$. If the threshold function is 1, the binary variable ε_k is inside sink node. The threshold function (S_2) is defined as:

$$\mathrm{S}_2(\varepsilon_k) = \begin{cases} 1 & \texttt{if}\ \ k = \text{sink node}, \\ 0 & \qquad\qquad \text{otherwise}, \end{cases} \tag{12}$$

B. FUZZY ADJUSTMENT COEFFICIENT $\widetilde{D}_{I,J}$

Our strategy combines the abstraction power of fuzzy logic and the result that provides a route optimization analysis. For that, we integrate a fuzzy adjustment coefficient $\widetilde{d}_{i,j}$ into our linear programming formulation (**Subsection** IV-A). This coefficient considers the uncertain elements in product distribution and the knowledge of the product delivery staff, i.e., we propose a Fuzzy Inference System (FIS) to consider the elements that affect route planning. This adjustment coefficient adjust every vehicular flow time $c_{i,j}$ into our formulation, thus increasing the time accuracy, but adding the challenge of modeling the vehicle route ambiguities. For that, in the fuzzy input step we consider the uncertain elements of product delivery (**Subsection** IV-B1). The inference process provides the knowledge of product delivery staff (**Subsection** IV-B2). Finally, the output is the adjustment of the vehicular route (**Subsection** IV-B3).

1) FUZZY INPUTS

The uncertain elements of product delivery (factors that affect route planning) are the fuzzy inputs of our fuzzy system. These inputs were obtained from the soft drink company where the proposed formulation was evaluated. For that, we survey the drivers of the soft drink company, asking about the factors that affect the route planning, i.e., to implement our methodology, we propose to evaluate the factors that affect route planning with the help of the product delivery staff where the proposed method is implemented. The fuzzy sets, membership functions, and inference rules used in this study are context-specific, i.e., derived from the knowledge of local delivery personnel. Thus, applying the model to a different sector or region would likely require adapting or redefining these components to reflect the new operational conditions and expert knowledge. In the experiments of this paper, our survey has 24 elements, and these elements are grouped inside six factors (Temporal factor, Spatial factor, Environmental factor, Driver factor, Company factor, and Social factor). **Table** 9 shows the factors that affect our route planning (**APPENDIX-B**).

We then asked drivers to rate the importance of the factors in their daily lives from 1 to 10. To reduce the number of factors, we apply the Pareto principle, also known as the 80/20 rule [1], considering the sum of the drivers' ratings. This analysis allows us to find a balance between performance and processing costs. We recommend using 20% of factors that the product delivery staff considers the greatest impact. We use this information to define our input membership functions. In this implementation, we focus on four functions (schedule, traffic, street deterioration, and precipitation) that represent approximately 20% of the factors considered (**Table** 9 **APPENDIX-B**). **Fig.** 8 a-d) show the input membership functions used at the soft drink company where the proposed formulation was evaluated (**APPENDIX-A**).

In our methodology, we propose to evaluate the factors that affect route planning with the help of delivery experts where the proposed approach is implemented, i.e., we suggest surveying your drivers to identify the relevant factors. Depending on where the methodology is applied, the number of fuzzy inputs to use and the sets to apply are determined. For example, in a desert area is not necessary to consider rainfall. Additionally, in areas where deliveries are not made at night, we can eliminate the nighttime fuzzy set from the schedule membership function. **Table** 3 a) shows our fuzzy inputs and the number of sets implemented.

2) FUZZY INFERENCE PROCESS

Our formulation uses the fuzzy inference process to integrate the knowledge of product delivery staff. This inference process requires fuzzy inputs to define the rule base. These rules were defined considering the knowledge of distribution staff as drivers, distribution managers, and logistics managers of the soft drink company. For that, we use the Max-Min inference method, rules IF-THEN type, and the operators AND, OR and NOT [14], [15]. The membership functions associated with each fuzzy input were produced directly by those experts so that the conditions and linguistic labels represent their operational experience. Consequently, if the methodology is deployed in a new geographic area environment, the input variables, fuzzy rules, and their membership functions should be studied and recalibrated to capture the local expertise and operating conditions. A generalization of the our fuzzy rules is defined as:

$$R_i: \text{ IF } z \text{ is } B, \text{ AND } w \text{ is } C, \ \ldots \text{Then } g \text{ is } D$$

where i denotes the i^{th} rule R_i, (z, w, g) are the linguistic variables, (B, C, D) are fuzzy sets, and AND is the operator used. These sets (B, C, D) are defined over the domain of the linguistic variables (z, w, g). To implement our methodology, we propose to define the fuzzy rule base with the help of the product delivery staff where the proposed method is implemented, i.e., we define fuzzy rules using the knowledge of the distribution staff. **Tables** 7, 8 show our fuzzy rule base.

3) FUZZY OUTPUT

In the proposed methodology, the fuzzy output is the adjustment coefficient $\widetilde{d}_{i,j}$ of our proposed formulation. This coefficient considers the uncertain elements in product distribution and the knowledge of the product delivery staff. For that, our output membership function $\mu_{\text{adjustment}}(x)$ is composed of three types of sets to decrease, increase or unchanged the time $c_{i,j}$ of every vehicular flow $x_{i,j}$. The decrease set ϑ has a non-integer domain less than or equal to zero $\vartheta \leq 0$. This set consists of fuzzy sets to reduce flow time $\vartheta = \{\texttt{slow}, \ldots, \texttt{very slow}\}$. Also, the increase set φ has a positive domain greater than or equal to zero $\varphi \geq 0$. This set consists of fuzzy sets to augment flow time $\varphi = \{\texttt{fast}, \ldots, \texttt{very fast}\}$. The unchanged set ψ is the intersection between the decrease and increase set. This set has a single fuzzy set with a domain equal to or close to zero $\psi = \{\texttt{Normal}\}$. On the other hand, the universe of discourse of the output function is the value of our adjustment coefficient $\widetilde{d}_{i,j}$. We refer to a_0 and a_n as the lower and upper range, respectively. **Fig.** 5 shows our generalized output membership function. **Fig.** 8 d) shows the output membership function used at the soft drink company where the proposed formulation was evaluated.

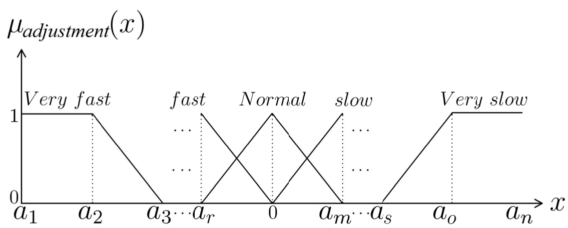


FIGURE 5. Our generalized output membership function.

We use the Center of Area (CoA) in the defuzzification step **equation** (13) [16]. Where g denotes the g^{th} rule, r is the number of rules, y_g is the centroid of activated rules and $\mathrm{M}(y_g)$ is the area of activated rules. On the other hand, $\widetilde{d}_{i,j}$ is the adjustment coefficient of our proposed formulation. This coefficient considers the uncertain elements in product distribution and the knowledge of the product delivery staff.

$$\widetilde{d}_{i,j} = \frac{\sum_{g=1}^{r} \mathrm{M}(y_g) y_g}{\sum_{g=1}^{r} \mathrm{M}(y_g)} \tag{13}$$

C. GEOGRAPHIC INFORMATION SYSTEM

Nowadays, there are different platforms or software based on classical algorithms for network optimization to help develop routes. These platforms are essential to provide delivery routes in a graphical form facilitating information to product distribution staff such as distribution managers, logistics managers, drivers, etc. However, in most cases, these platforms do not consider the elements that affect route planning. To address this problem, we develop a Geographic Information System (GIS) that use our fuzzy linear programming formulation (**Subsection** IV-B1). In this subsection, we present this computational system to facilitate its replication. For that, we present the equations to develop the graphical user interface for product distribution (**Subsection** IV-C1). The pseudocode of the proposed methodology to obtain time prediction in product delivery (**Subsection** IV-B2). Finally, our GIS block diagram (**Subsection** IV-B3).

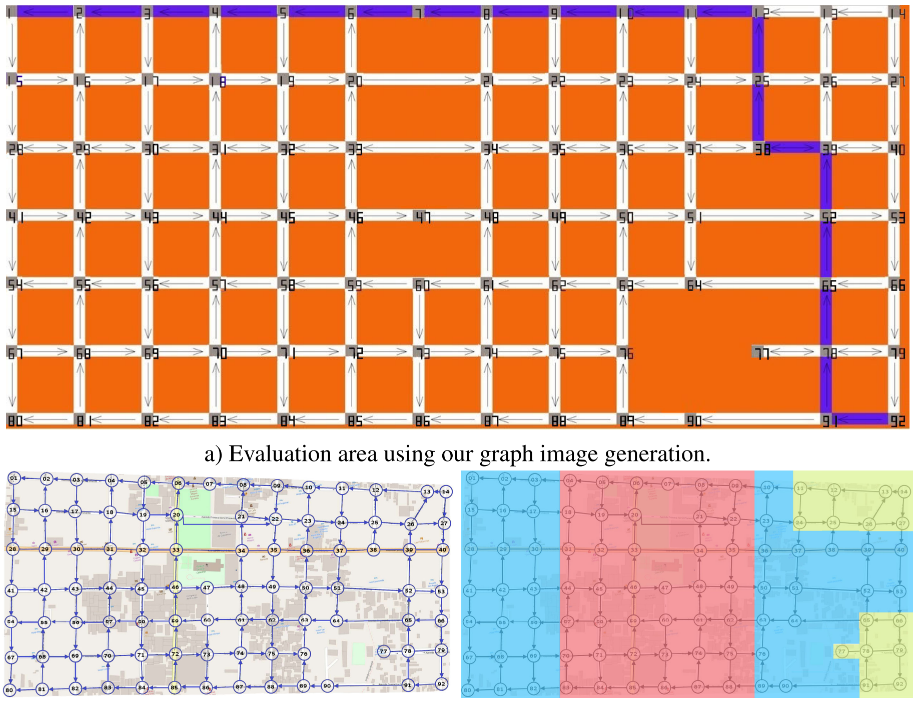
1) GRAPH IMAGE GENERATION

In the graphical user interface, we use a three-equation system to transform intersections and flows by image coordinates. These intersections and flows are the distances between the corners and sidewalks of city blocks. On the other hand, the image coordinates allow the creation of a virtual representation of a city region. For that, this system employs four distances $(p, q, u, v)_{a,b}$ of the corners and sidewalks. Also, we use three coordinate types: left upper intersection $(x, y)^1_{a,b}$, the right lower intersection of horizontal flow $(x, y)^2_{a,b}$, and its vertical flow $(x, y)^3_{a,b}$. Where a denotes the a^{th} vertical intersection, and b denotes the b^{th} horizontal intersection. The three-equation system is presented in **equations** 14-16. **Fig.** 7 a) shows the four distances $(p, q, u, v)_{a,b}$, and **Fig.** 7 b) shows the three coordinate types $(x, y)^1_{a,b}$, $(x, y)^2_{a,b}$, and $(x, y)^3_{a,b}$. We use these coordinates to plot blocks, vehicle flows, and the short route. **Fig.** 6 a) shows an example of our graph image generation. Where the orange sections are the blocks, the white sections are the vehicle flows, and the purple sections are the short route.

$$(x, y)^1_{a,b} = (\sum_{i=1}^{b}[p_{a,i-1} + u_{a,i-1}], \sum_{i=1}^{a}[q_{i-1,b} + v_{i-1,b}]) \tag{14}$$

$$(x, y)^2_{a,b} = (\sum_{i=1}^{b}[p_{a,i} + u_{a,i}], \sum_{i=1}^{a}[q_{i,b} + v_{i-1,b}]) \tag{15}$$

$$(x, y)^3_{a,b} = (\sum_{i=1}^{b}[p_{a,i} + u_{a,i-1}], \sum_{i=1}^{a}[q_{i,b} + v_{i,b}]) \tag{16}$$

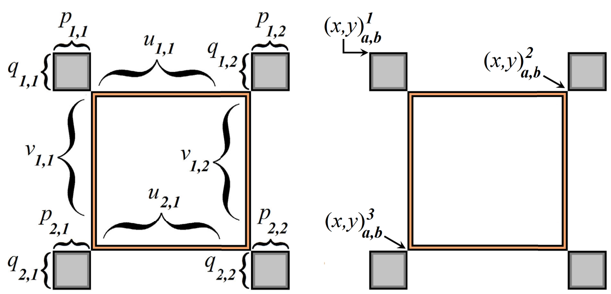


a) Evaluation area using our graph image generation.

b) Evaluation area

c) Traffic density

FIGURE 6. Evaluation area of the proposed model.



a) Distances

b) Coordinates

FIGURE 7. Distances and coordinates to plot block.

2) PSEUDOCODE

Table 1 shows the detailed pseudocode to obtain time prediction in product delivery. This pseudocode has two parameter sets to define. The digraph parameters present the evaluation area information (node number, edge, etc.). The fuzzy parameters are the inference process, inputs, output, etc. On the other hand, our code has three sequential operations. First, we compute the adjustment coefficient to consider the uncertain elements in product distribution and the knowledge of the product delivery staff. Second, we compute the binary variables and corner intersection to consider the time-lapse to loading and unloading a delivery vehicle and the time-lapse to crossing an intersection. Finally, we use our fuzzy linear programming model to obtain the time prediction in product delivery.

3) GIS BLOCK DIAGRAM

Our GIS block diagram consists of three main stages: the linear programming stage, the adjustment coefficient stage, and the graphical interface stage. In the first stage, we integrate the linear programming formulation to obtain time prediction in product delivery. For that, this phase has five inputs (source node, sink node, binary variables, vehicular flow times, and fuzzy adjustment coefficient), and two outputs (distribution route, and delivery time). On the other hand, the adjustment coefficient stage considers the uncertain elements in product distribution and the knowledge of the product delivery staff. For that, this phase has three steps, the fuzzy inputs, the inference process, and the outputs. Finally, the graphical interface stage allows the creation of a virtual representation of a city region. In this case, we use a three-equation system to transform intersections and flows by image coordinates using the distribution route and delivery time information.

V. DISCUSSION AND RESULTS

In this section, we present the evaluation area, evaluation metrics, the approaches analyzed, and their results. We use the Absolute Percentage Error in the quantitative evaluation. Also, we compare four different approaches (classic model [12], FLP [6], ANN [5], and our proposal). Finally, we discuss the results of the approaches using the evaluation metric.

TABLE 1. **Pseudo code for the proposed fuzzy linear programming formulation.**

Parameter definition:

A: Set of directed edges
p: Number of fuzzy inputs
r: Number of rules
z, w, g: Linguistic variables
B, C, D: Fuzzy sets
$\widetilde{d}_{i,j}$: Fuzzy adjustment coefficient
n: Number of nodes
δ_k, ε_k: Binary variables
$u_{i,j}$: Corner intersection
$x_{i,j}$: Edge

Compute fuzzy adjustment coefficient $\widetilde{d}_{i,j}$ for every vehicular flow time $c_{i,j}$

```
For e = 1 to A
    For o = 1 to p
        1: Fuzzy inputs, fig. 8
    End
    For o = 1 to r
        If (z = B ..., w = C)
            {2: Fuzzy inference process, g = D, table 7,8 }
    End
    3: Fuzzy output, compute d̃_{i,j}, equations 13
End
```

Compute binary variables (δ_k, ε_k) and corner intersection $u_{i,j}$

```
For k = 1 to n
    If (k = source node)
        { 4: δ_k = 1, equation 11 }
    If (k = sink node)
        { 5: ε_k = 1, equation 12 }
End
For e = 1 to A
    If (u_{i,j} = corner type)
        { 6: u_{i,j} = time of corner type, table 3 }
End
```

Compute fuzzy linear programming model

```
For i = 1 to n
    If (b_i = source node)
        { 7: b_i = 1 }
    Else if (b_i = sink node)
        { 8: b_i = −1 }
    Else { 9: b_i = 0 }
End
For i = 1 to n
    For j = 1 to n
        If (x_{i,j} = edge out of node i and j ≠ i )
            { 10: x_{i,j} = 1 }
        If (x_{j,i} = edge into node i and j ≠ i )
            { 11: x_{j,i} = −1 }
    End
End
12: Construct the fuzzy linear programming model, equations 8-10
13: Solve the equations 8-10 using the Big M method
```

A. EVALUATION AREA

We evaluate the proposed model in delivery routes of a soft drink company in Tuxtla Gutiérrez, Chiapas, Mexico. This evaluation area includes 68 blocks and 92 nodes from 2nd North and 4th South avenues and 5th West and 8th East Streets in Tuxtla City. **Fig.** 6 b) shows the evaluation area previously mentioned. **Fig.** 6 a) shows the result of our graph image generation (**Subsection** IV-C1) using the same delivery area. Finally, the evaluation area considers the three traffic densities (see **Fig.** 6 c) contemplated in the traffic membership function (see **Fig.** 8 c). Where the red section indicates high traffic, the purple section indicates average traffic, and the green section indicates low traffic.

B. PRECIPITATION

We evaluate the proposed model in different rainfall intervals (precipitation). We consider the intervals used by the National Water Commission in Mexico (CONAGUA) [58]. The period evaluated was May-November 2024 in Tuxtla Gutierrez Chiapas. We use the measurement unit millimeters (mm) to evaluate the different rainfall intervals. Finally, the precipitation intervals consider five fuzzy sets (no rain, rain, heavy rain, rainstorm, and heavy rainstorm) contemplated in the precipitation membership function (see **Fig.** 8 d).

C. ELEMENTS OF THE SOFT DRINK COMPANY

We consider specific elements of the soft drink company (**Table** 3). These elements can be considered for other delivery companies since variables such as traffic are usually relevant in almost all delivery companies. However, other variables are generally specific to certain companies. For example, in a desert region, pluvial precipitation is not a relevant consideration. We recommend that fuzzy sets, rules, times of binary variables, and corner intersection ($u_{i,j}$) be obtained during the implementation of the methodology.

We consider the time of product load to a delivery vehicle (coefficient a_i) with an average time of 6 minutes. Also, we consider the park time and the time of product unloading (e_i) with an average time of 7.5 minutes. We analyze the product delivery area to obtain the different times to cross an intersection ($u_{i,j}$). The **Table** 3 f) shows a list of 9 different types of intersections with their average time.

D. METRICS

In our evaluation, the metrics were calculated in seconds. In the short route evaluation, we compared the real-time (RT) with the route inferred time (IT) by some models (classical, fuzzy, neural, and our model). For that, we use the Mean Absolute Percentage Error (MAPE) or ($\bar{X}$). Where, N is the sample size, and X is $\frac{|\mathrm{RT}-\mathrm{IT}|}{\mathrm{RT}} * 100$. Also, we calculate the Absolute Percentage Error Variance (APEV).

$$\bar{X} = \frac{\sum \frac{|\mathrm{RT}-\mathrm{IT}|}{\mathrm{RT}} 100}{N} \tag{17}$$

TABLE 2. In this experiment, we consider different schedules for the same route starting from the source node 15. Additionally, we use two measures the mean absolute percentage error (MAPE) and the absolute percentage error variance (APEV). We compare four approaches (classic model [12], FLP [6], ANN [5], and our proposal).

Rou.	Times	Real time	Class. model	A. P. Error	Fuzzy model	A. P. Error	Neural model	A. P. Error	Our	A. P. Error
15-13	06:22:17	00:05:29	0:03:37	34.04	00:03:53	29.18	00:03:54	28.88	00:04:39	15.20
15-13	14:28:37	00:07:35	0:03:37	52.31	00:06:37	12.75	00:06:38	12.53	00:06:44	11.21
15-13	19:03:55	00:02:51	0:03:37	26.90	00:03:11	11.70	00:03:12	12.28	00:03:09	10.53
15-40	10:15:26	00:04:17	0:03:47	11.67	00:04:34	06.61	00:04:31	05.45	00:04:29	04.67
15-40	13:49:21	00:07:35	0:03:47	50.11	00:07:05	06.59	00:07:05	06.59	00:07:17	03.96
15-40	21:47:33	00:02:50	0:03:47	33.53	00:02:34	09.41	00:02:36	08.24	00:02:52	01.18
15-50	07:15:30	00:04:25	0:03:42	16.23	00:04:41	06.04	00:04:34	03.40	00:04:29	01.51
15-50	11:45:17	00:04:11	0:03:42	11.55	00:04:41	11.95	00:04:41	11.95	00:03:36	13.94
15-50	23:08:05	00:02:35	0:03:42	43.23	00:02:46	07.10	00:02:46	07.10	00:02:44	05.81
15-59	07:26:02	00:05:37	0:04:03	27.89	00:05:56	05.64	00:05:55	05.34	00:05:55	05.34
15-59	17:25:32	00:04:34	0:04:03	11.31	00:04:21	04.74	00:04:23	04.01	00:04:32	00.73
15-59	20:21:59	00:03:19	0:04:03	22.11	00:03:03	08.04	00:03:03	08.04	00:03:09	05.03
15-64	08:01:19	00:06:32	0:04:23	32.91	00:06:02	07.65	00:06:02	07.65	00:06:24	02.04
15-64	13:54:58	00:08:58	0:04:23	51.12	00:08:39	03.53	00:08:37	03.90	00:08:50	01.49
15-64	20:31:44	00:02:58	0:04:23	47.75	00:03:15	09.55	00:03:09	06.18	00:03:01	01.69
15-82	10:06:48	00:04:44	0:03:15	31.34	00:04:22	07.75	00:04:25	06.69	00:04:39	01.76
15-82	14:03:51	00:07:02	0:03:15	53.79	00:06:34	06.64	00:06:42	04.74	00:06:46	03.79
15-82	22:47:33	00:01:58	0:03:15	65.25	00:02:31	27.97	00:02:02	03.39	00:02:03	04.24
15-88	05:32:28	00:06:38	0:04:27	32.91	00:06:51	03.27	00:06:48	02.51	00:06:52	03.52
15-88	13:36:57	00:10:49	0:04:27	58.86	00:10:15	05.24	00:10:12	05.70	00:10:15	05.24
15-88	18:04:30	00:05:53	0:04:27	24.36	00:05:17	10.20	00:05:09	12.46	00:05:38	04.25
			MAPE	35.20		9.60		7.95		5.10
			APEV	264.1		46.57		32.64		17.36

TABLE 3. Elements of the soft drink company.

a)	**Fuzzy inputs**	**Components**	**Reference**
	Schedule	6 sets	**fig.** 8 (a)
	Traffic	3 sets	**fig.** 8 (b)
	Street deterioration	3 sets	**fig.** 8 (c)
	Precipitation	5 sets	**fig.** 8 (d)
b)	**Fuzzy rule bank**	**Components**	**Reference**
	Fuzzy rules	55 rules	**Tables** 7,8
c)	**Fuzzy output**	**Components**	**Reference**
	Adjustment	6 sets	**fig.** 8 (e)
d)	**Binary variables**	**Time (minutes)**	**Reference**
	Time of product loading (a_k)	5	**equation** 11
	Time of product unloading (e_k)	7.5	**equation** 12
f)	**Corner intersection ($u_{i,j}$)**	**Time (seconds)**	**Reference**
	Traffic light type 4	130	
	Traffic light type 3	80	
	Traffic light type 2	50	
	1x1 in downtown city	20	
	1x1	7.5	
	Without preference in downtown city	30	
	Without preference	10	
	With preference in downtown city	10	
	With preference	5	

$$APEV = \frac{\sum(X - \bar{X})^2}{N - 1} \tag{18}$$

E. APPROACH

We compare four different approaches to the short route. The first approach is the Linear Programming Formulation of the Shortest-Route Problem [12]. The second approach is a fuzzy model for arc optimization in urban roads [6]. The following approach uses artificial neuronal networks for travel time [5]. Finally, our methodology uses a fuzzy inference system integrated into a linear programming formulation. Unlike previous fuzzy approaches, we consider uncertain elements in product distribution and the knowledge of the product delivery staff. **Tables** 2-5 show quantitative results comparing four distinct approaches to the Shortest-Route.

F. RESULTS

Experiments were performed with different routes to evaluate the efficacy of four approaches to the short route (classic model [12], FLP [6], ANN [5], and our proposal). We examined experiments starting from the same source node (**Table** 4), random routes (**Table** 4), and precipitation environments (**Table** 5). We consider routes at three times of the day (morning, afternoon, and night).

There are several classical models for route optimization, such as Dijkstra's Algorithm [2], Floyd's Algorithm [3], and linear programming for finding the shortest route [12]. When the routes are identical, these models produce the same results for the same route. Therefore, we will only evaluate the linear programming model in this scenario. In this case, the classical route optimization approach provides the lowest performance (see **Table** 2-5). Although we contemplate different schedules and precipitation in the experiments, the results give the same time. The results remain constant because its mathematical formulation does not consider both uncertain elements in product distribution (**Subsection** IV-B1) and the expertise of the product delivery staff (**Subsection** IV-B2).

Fuzzy logic allows us to contemplate fuzzy variables within shortest-route models. Each area has different elements to consider. For example, considering precipitation in cities with desert climates would be irrelevant. However, previous work with fuzzy logic [6] considers limited or generalized variables. These variables reduce their performance in our experiments (see **Table** 2) and limit their implementation. For this reason, unlike previous approaches, we structure

TABLE 4. In this experiment, we consider routes with three schedules and random source nodes. Additionally, we use two measures the mean absolute percentage error (MAPE) and the absolute percentage error variance (APEV). We compare two approaches (classic model [12], and our proposal).

Route	Times	Real time	Classical model	A. P. Error	Our	A. P. Error
1-51	03:11 am	00:03:40	00:05:10	40.91	00:03:38	00.91
1-51	13:25 pm	00:09:47	00:05:10	47.19	00:10:10	03.92
1-51	21:23 pm	00:05:05	00:05:10	01.64	00:04:57	02.62
87-72	03:19 am	00:02:50	00:04:37	62.94	00:03:02	07.06
87-72	13:43 pm	00:09:00	00:04:37	48.70	00:09:41	07.59
87-72	21:41 pm	00:04:16	00:04:37	08.20	00:03:53	08.98
74-1	03:56 am	00:04:21	00:06:36	51.72	00:04:33	04.60
74-1	13:11 pm	00:09:34	00:06:36	31.01	00:11:12	17.07
74-1	21:15 pm	00:05:04	00:06:36	30.26	00:04:34	09.87
48-80	03:32 am	00:03:01	00:04:29	48.62	00:03:13	06.63
48-80	14:09 pm	00:08:32	00:04:29	47.46	00:09:05	06.45
48-80	21:58 pm	00:03:27	00:04:29	29.95	00:03:54	13.04
23-11	02:13 am	00:04:13	00:04:58	17.79	00:03:43	11.86
23-11	14:53 pm	00:10:15	00:04:58	51.54	00:09:41	05.53
23-11	21:32 pm	00:03:39	00:04:58	36.07	00:03:31	03.65
78-38	03:37 am	00:02:09	00:04:01	86.82	00:02:21	09.30
78-38	14:05 pm	00:08:57	00:04:01	55.12	00:08:24	06.15
78-38	22:09 pm	00:03:02	00:04:01	32.42	00:03:12	05.49
57-35	02:31 am	00:04:25	00:05:07	15.85	00:03:49	13.58
57-35	13:03 pm	00:09:32	00:05:07	46.33	00:10:15	07.52
57-35	23:24 pm	00:03:02	00:05:07	68.68	00:03:13	06.04
42-74	02:27 am	00:03:16	00:04:05	25.00	00:03:26	05.10
42-74	14:15 pm	00:07:21	00:04:05	44.44	00:08:11	11.34
42-74	21:50 pm	00:04:35	00:04:05	10.91	00:04:41	02.18
33-91	03:06 am	00:02:16	00:03:06	36.76	00:02:34	13.24
33-91	13:59 pm	00:08:51	00:03:06	64.97	00:07:49	11.68
33-91	20:30 pm	00:03:41	00:03:06	15.84	00:03:24	07.69
13-36	03:56 am	00:02:13	00:03:40	65.41	00:02:26	09.77
13-36	12:55 pm	00:06:03	00:03:40	39.39	00:06:57	14.88
13-36	22:45 pm	00:04:36	00:03:40	20.29	00:03:58	13.77
70-27	03:39 am	00:03:29	00:05:02	44.50	00:03:52	11.00
70-27	14:43 pm	00:08:36	00:05:02	41.47	00:08:49	02.52
70-27	20:55 pm	00:02:50	00:05:02	77.65	00:03:02	07.06
81-50	03:29 am	00:03:17	00:04:54	49.24	00:03:31	07.11
81-50	14:18 pm	00:09:23	00:04:54	47.78	00:09:54	05.51
81-50	21:13 pm	00:03:45	00:04:54	30.67	00:04:15	13.33
36-55	02:53 am	00:05:29	00:06:19	15.20	00:04:46	13.07
36-55	13:21 pm	00:11:51	00:06:19	46.69	00:12:15	03.38
36-55	23:57 pm	00:03:40	00:06:19	72.27	00:03:59	08.64
26-72	03:58 am	00:03:14	00:04:47	47.94	00:03:39	12.89
26-72	14:51 pm	00:08:05	00:04:47	40.82	00:08:27	04.54
26-72	20:11 pm	00:02:38	00:04:47	81.65	00:02:51	08.23
53-3	03:24 am	00:04:19	00:06:03	40.15	00:04:06	05.02
53-3	13:52 pm	00:11:29	00:06:03	47.31	00:11:57	04.06
53-3	21:15 pm	00:05:57	00:06:03	1.68	00:05:38	05.32
		MAPE		41.50		07.98
		APEV		413.29		15.46

fuzzy rules and variables for each case. Also, we propose a Pareto analysis (**Subsection** IV-B1) to find a balance between performance and processing cost. This analysis allows us to surpass the performance of the previous approach using fuzzy logic (see **Table** 2).

Neural models allow the generalization of tasks in fields such as computer vision or natural language processing. However, these approaches necessitate extensive training datasets for effective generalization and significant computational processing. In the case of the neural model for the short route [5], only 190 neurons are considered in the hidden layer, and 10,000 data elements are used for training. These limitations in both architecture and training limit its performance. In our case, we have surpassed this previous work because our methodology does not require the mentioned requirements (see **Table** 2). In this experiment, the proposed approach reduced the MAPE by 56% compared to the neural approach.

In contrast to prior studies, we have developed a methodology that considers various uncertain parameters in the distribution process and provides an effective way to determine which elements to utilize (**Subsection** IV-B1). We contemplate the expertise of the product delivery staff as a keystone in our mathematical formulation (**Subsection** IV-B2). This alternative provides a flexible approach that allows easy integration of variables that affect time prediction in product delivery.

We realize experiments using different source nodes to evaluate the efficacy of the proposed methodology (see **Table** 4). The experiments utilizing various source nodes enable us to assess the estimated times by considering different points within the evaluation area (**Fig.** 6 b). Additionally, it allows us to analyze fuzzy sets of traffic density. These experiments were performed using randomized routes. We examined different routes at three times of the day (morning, afternoon, and night). The **Table** 4 shows the

TABLE 5. In this experiment, we consider routes with three schedules and random source nodes in different rainfall intervals (precipitation). Additionally, we use two measures the Mean Absolute Percentage Error (MAPE) and the absolute percentage error variance (APEV). We compare two approaches (classic model [12], and our proposal).

Route	Times	Precipitation	Real Time	Classical model	A. P. Error	Our	A. P. Error
1-51	03:52 am	Rainstorm	00:07:42	00:05:10	32.90	00:07:33	01.85
1-51	13:28 pm	Rain	00:14:41	00:05:10	64.79	00:16:40	13.62
1-51	21:26 pm	Rain	00:07:07	00:05:10	27.40	00:06:32	08.19
87-72	03:44 am	Heavy rain	00:05:06	00:04:37	09.48	00:05:29	07.65
87-72	13:53 pm	Rain	00:12:36	00:04:37	63.36	00:14:49	17.58
87-72	21:36 pm	Rainstorm	00:08:32	00:04:37	45.90	00:08:05	05.34
74-1	03:29 am	Rain	00:06:31	00:06:36	01.15	00:06:25	01.68
74-1	13:05 pm	Rain	00:12:26	00:06:36	46.93	00:16:35	33.28
74-1	21:43 pm	Heavy rainstorm	00:12:40	00:06:36	47.89	00:11:44	07.34
48-80	03:54 am	Heavy rain	00:05:44	00:04:29	21.78	00:06:03	05.51
48-80	14:33 pm	Heavy rain	00:15:22	00:04:29	70.81	00:17:10	11.77
48-80	21:07 pm	Rain	00:04:50	00:04:29	07.18	00:05:56	22.73
23-11	02:06 am	Heavy rainstorm	00:10:58	00:04:58	54.70	00:10:00	08.81
23-11	14:23 pm	Rainstorm	00:23:34	00:04:58	78.93	00:21:41	07.99
23-11	21:50 pm	Heavy rain	00:06:56	00:04:58	28.38	00:07:15	04.46
78-38	03:38 am	Rain	00:02:48	00:04:01	43.71	00:02:53	03.42
78-38	14:34 pm	Rainstorm	00:19:41	00:04:01	79.60	00:17:43	09.98
78-38	22:14 pm	Rainstorm	00:06:59	00:04:01	42.43	00:07:31	07.79
57-35	02:10 am	Heavy rainstorm	00:11:56	00:05:07	57.09	00:10:05	15.51
57-35	13:50 pm	Rainstorm	00:20:01	00:05:07	74.44	00:21:19	06.49
57-35	23:03 pm	Heavy rain	00:05:09	00:05:07	00.78	00:05:32	07.29
42-74	02:32 am	Rain	00:04:54	00:04:05	16.67	00:05:03	03.00
42-74	14:04 pm	Rain	00:09:33	00:04:05	57.26	00:11:08	16.48
42-74	21:10 pm	Heavy rain	00:08:42	00:04:05	53.11	00:09:28	08.64
33-91	03:38 am	Heavy rain	00:04:05	00:03:06	24.02	00:04:59	22.04
33-91	13:22 pm	Rain	00:11:30	00:03:06	73.06	00:11:11	02.84
33-91	20:51 pm	Heavy rain	00:06:16	00:03:06	50.49	00:06:03	03.35
13-36	03:21 am	Heavy rainstorm	00:05:32	00:03:40	33.83	00:06:15	12.85
13-36	12:17 pm	Rainstorm	00:13:55	00:03:40	73.65	00:15:13	09.38
13-36	22:29 pm	Heavy rain	00:07:49	00:03:40	53.11	00:06:59	10.72
70-27	03:05 am	Heavy rainstorm	00:09:03	00:05:02	44.42	00:10:15	13.14
70-27	14:11 pm	Heavy rain	00:15:29	00:05:02	67.48	00:17:17	11.63
70-27	20:24 pm	Rain	00:03:58	00:05:02	26.89	00:04:02	01.71
81-50	03:23 am	Heavy rain	00:06:17	00:04:55	21.75	00:06:38	05.57
81-50	14:29 pm	Heavy rain	00:16:48	00:04:55	70.73	00:17:10	02.18
81-50	21:44 pm	Rain	00:05:56	00:04:55	17.13	00:06:29	09.27
36-55	02:35 am	Heavy rainstorm	00:13:41	00:06:07	55.30	00:12:02	12.06
36-55	13:44 pm	Rainstorm	00:22:51	00:06:07	73.23	00:23:43	03.79
36-55	23:52 pm	Heavy rain	00:05:53	00:06:07	03.97	00:06:37	12.46
26-72	03:41 am	Heavy rainstorm	00:08:34	00:04:46	44.36	00:09:38	12.45
26-72	14:57 pm	Heavy rain	00:14:41	00:04:46	67.54	00:16:21	11.35
26-72	20:09 pm	Rain	00:03:45	00:04:46	27.11	00:03:57	05.33
53-3	03:20 am	Rainstorm	00:09:04	00:05:58	34.19	00:08:50	02.57
53-3	13:31 pm	Rain	00:17:11	00:05:58	65.28	00:18:39	08.54
53-3	21:52 pm	Rain	00:08:21	00:05:58	28.54	00:07:40	08.18
			MAPE		44.06		09.29
			APEV		522.49		39.24

performance of our model compared to a classical model and real-time data. We can observe that our results are closer to the travel times compared to the classical model. In this experiment, we reduce 5.2 times the MAPE compared to classical approaches.

We analyze the same routes considering precipitation environments (see **Table** 5). It's important to note that when analyzing precipitation, we conducted the tests during times with a forecast for rain. Subsequently, we confirmed that the actual rainfall on the route matched the predicted range. The results obtained exhibit greater accuracy in approximating travel times when compared to those derived from the classical model. In this experiment, we reduced 4.7 times the MAPE compared to classical approaches.

However, due to their fuzzy nature, model performance is sensitive to the limitations established on fuzzy sets. Since these sets encode expert knowledge through membership functions, any significant change in their limits or shape can significantly alter the system output. Therefore, when the model is moved to a new environment or geographic context, it is essential to restructure and recalibrate the fuzzy sets to reflect the local conditions and accurately reflect the operational priorities of the environment. Fortunately, a key advantage of this approach is that recalibration can be easily

TABLE 6. In this experiment, we consider five samples with different route flows of the proposed methodology. The estimated time is provided in minutes, seconds, and hundredths of a second format.

Flows	Processing Time
48	00:05:18
65	00:06:24
105	00:06:47
142	00:06:54
181	00:07:12
Average	**00:06:31**

performed. Fuzzy sets are interpretable and can be adjusted based on expert knowledge or historical data from the new context.

The efficiency of the fuzzy adjustment in the proposed approach corresponds to the complexity of the Mamdani inference method. As the fuzzy tuning coefficient increases, a greater number of rules and membership functions are activated, which in turn increases processing time. However, this computational overhead remains acceptable on standard CPUs. **Table** 6 shows quantitative results of five samples with different route flows and their corresponding processing times using the proposed methodology. In this table, the processing average time of our GIS with the proposed fuzzy linear programming formulation is 6.5 seconds using an Aspire E1-421 computer. Furthermore, our proposal doesn't require large training sets, parallel computing, or specialized software, providing an affordable alternative for users.

We distinguish our proposed approach from other systems used for route planning by focusing on specific travel areas. We recommend this methodology for route inference in local companies that have an established distribution area. For example, it may be well suited for food delivery companies operating within a limited geographical area or for messenger services in specific delivery zones. In addition to performance improvements, we aim to enhance inference capabilities by employing fuzzy logic tailored to specific distribution areas through the creation of fuzzy rules. This targeted approach allows us to outperform various classical [12] and artificial intelligence methods [5], [6].

VI. CONCLUSION

This work introduced a new methodology to obtain time prediction in product delivery using a Fuzzy Inference System integrated into a linear programming formulation. Our strategy combined the abstraction power of fuzzy logic and the result that provides a route optimization analysis, i.e., this work brings the best of the two worlds to address the problem of shortest routes in product delivery. For that, this methodology had three phases. First, we proposed a fuzzy linear programming formulation for time prediction in product delivery. Second, we defined a fuzzy adjustment coefficient to consider the uncertain elements in product distribution and the knowledge of the product delivery staff. Finally, we developed a Geographic Information System (GIS) to visualize the distribution route and time.

We evaluate the proposed model in the delivery routes of a soft drink company. We examined experiments starting from the same source node, random routes, and precipitation environments. In these experiments, we analyzed the routes during three different times of the day: morning, afternoon, and night. The proposed approach reduced the MAPE by 56% compared to the neural approach in the experiment with the same source node. On the other hand, we reduce 5.2 times the MAPE compared to classical approaches considering random routes. Furthermore, we reduce 4.7 times the MAPE compared to classical approaches by considering random routes with precipitation environments.

Finally, we can conclude that our route planning methodology allowed delivery-time predictions and path selections that are substantially more exact than established baselines. By capturing the tacit knowledge of delivery personnel and translating it into fuzzy rules, we were able to integrate these rules into an optimization model. Our approach successfully reduced the Mean Absolute Percentage Error (MAPE) and Absolute Percentage Error Variance (APEV) across various experiments compared to state-of-the-art models. These improvements offer tangible operational benefits for the soft drink distribution network studied, where more accurate delivery time estimates improve vehicle utilization, customer satisfaction, and cost control.

We differentiate the proposed approach from other fuzzy inference system approaches for route planning estimation in the sense that we focus on particular areas of travel, i.e., the proposed method can be easily adapted to different regions and local conditions, making it suitable for small and medium-sized businesses. We recommend the proposed methodology for route inference in local companies with an established distribution area. Beyond performance gains, we aim to use fuzzy logic to work in particular distribution areas by creating fuzzy rules to increase inference performance. This particularization allows us to surpass different classical and artificial intelligence approaches. Also, it enables us to create a methodology that can be applied in any distribution area. Unlike deep learning models, our approach does not require large data sets, parallel computing, or specialized software, providing an affordable alternative for users.

Future work involves further investigation to consider automotive systems of data acquisition. These automotive systems can facilitate the acquisition of route prediction, trajectories, and delivery or travel times for individuals, freight cars, and public service vehicles. This data can facilitate the analysis of future road conditions, road infrastructure, traffic conditions, and other pertinent factors. Additionally, driving and vehicle conditions can be considered as potential variables in this analysis.

APPENDIX

A. INPUT AND OUTPUT MEMBERSHIP FUNCTIONS

See Figure 8.

B. FUZZY RULE BASE AND FACTORS OF ROUTE PLANNING

Tables 7–9.

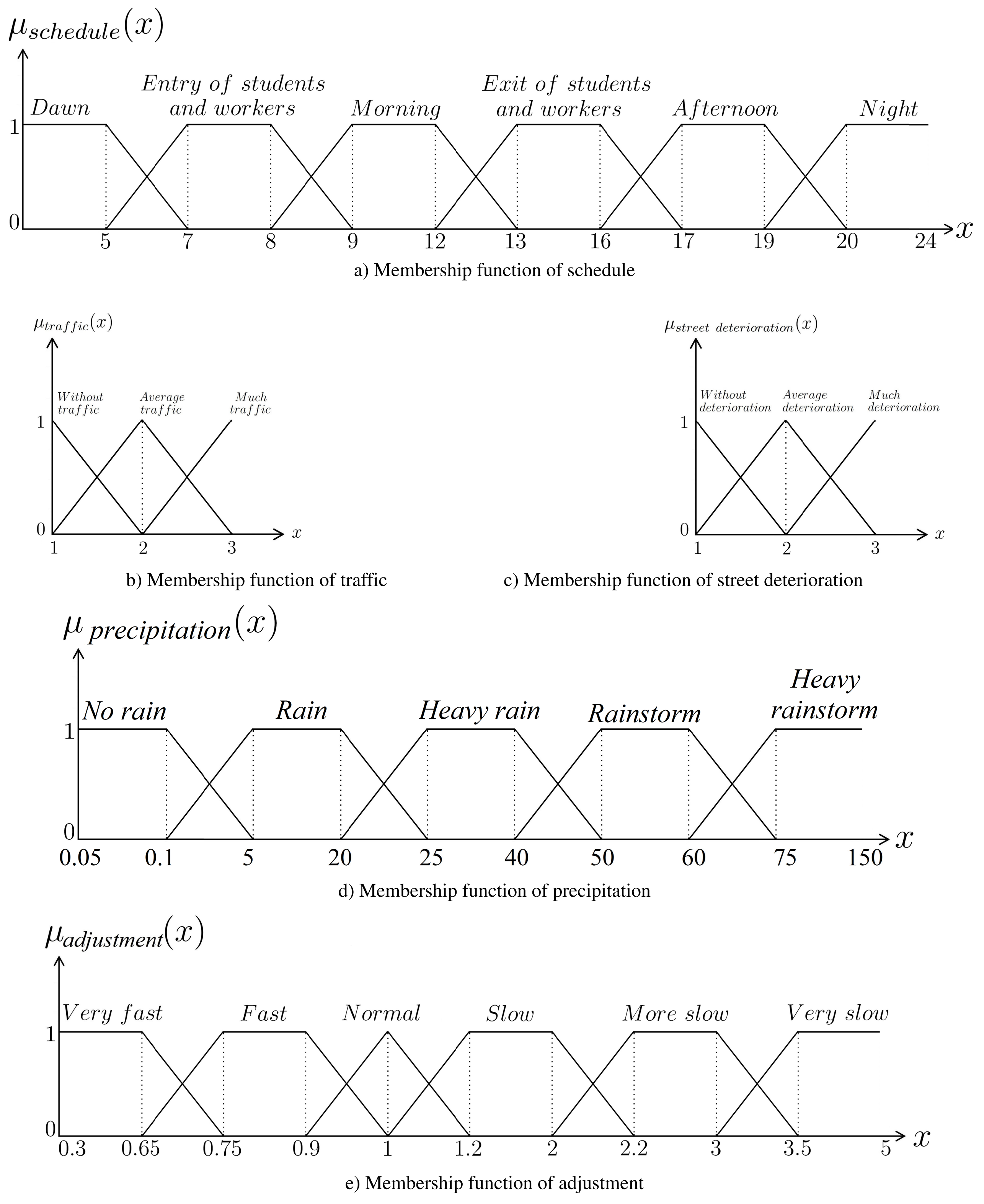


FIGURE 8. We apply these inputs and output membership functions to the delivery routes of a soft drink company in tuxtla gutiérrez, Chiapas, Mexico (Subsection V-A).

TABLE 7. Fuzzy rule base (a). These fuzzy rules were applied to the delivery routes of a soft drink company in Tuxtla Gutiérrez, Chiapas, Mexico (Subsection V-A). This table includes all rules in the membership functions of precipitation considering the sets of no rain and rain.

N.	Rule
	IF Precipitation **IS** No rain **AND** ...
	IF Traffic **IS** Without traffic **OR** Average traffic **OR** Much traffic **AND** ...
1	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Very fast
2	**IF** Schedule **IS** Dawn **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Very fast
3	**IF** Schedule **IS** Dawn **OR** Morning **OR** Afternoon **OR** Night **AND IF** Street deterioration **IS** Much d. **THEN** Adjustment **IS** More slow
4	**IF** Schedule **IS** Morning **AND IF** Street deterioration **IS** Average Determination **THEN** Adjustment **IS** Slow
5	**IF** Schedule **IS** Night **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Fast
6	**IF** Schedule **IS** Entry of students and workers **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
7	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
8	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Much traffic **IF** Street deterioration **IS** Without det. **AND** Average det. **AND** Much det. **THEN** Adjustment **IS** Very slow
9	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND** **IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Average det. **THEN** Adjustment **IS** Slow
10	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND** **IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Much det. **THEN** Adjustment **IS** More slow
11	**IF** Schedule **IS** Morning **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
12	**IF** Schedule **IS** Morning **AND IF** Traffic **IS** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
13	**IF** Schedule **IS** Exit of students and workers **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
14	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Fast
15	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
16	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Much traffic **AND IF** Street deterioration **IS** Without deterioration **OR** Average deterioration **THEN** Adjustment **IS** More slow
17	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND** **IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Slow
	IF Precipitation **IS** Rain **AND** ...
	IF Traffic **IS** Without traffic **OR** Average traffic **OR** Much traffic **AND** ...
18	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Very fast
19	**IF** Schedule **IS** Dawn **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Very fast
20	**IF** Schedule **IS** Dawn **OR** Morning **OR** Afternoon **OR** Night **AND IF** Street deterioration **IS** Much d. **THEN** Adjustment **IS** More slow
21	**IF** Schedule **IS** Morning **AND IF** Street deterioration **IS** Average Determination **THEN** Adjustment **IS** Slow
22	**IF** Schedule **IS** Night **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Fast
23	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
24	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
25	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Much traffic **IF** Street deterioration **IS** Without det. **AND** Average det. **AND** Much det. **THEN** Adjustment **IS** Very slow
26	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND** **IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Average det. **THEN** Adjustment **IS** Slow
27	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND** **IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Much det. **THEN** Adjustment **IS** More slow
28	**IF** Schedule **IS** Morning **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
29	**IF** Schedule **IS** Morning **AND IF** Traffic **IS** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
30	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Fast
31	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
32	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Much traffic **AND IF** Street deterioration **IS** Without deterioration **OR** Average deterioration **THEN** Adjustment **IS** More slow
33	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND** **IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Slow

TABLE 8. Fuzzy rule base (b). These fuzzy rules were applied to the delivery routes of a soft drink company in tuxtla gutiérrez, chiapas, Mexico (Subsection V-A). This table includes all rules in the membership functions of precipitation considering the sets of heavy rain, rainstorm and heavy rainstorm.

N.	Rule
	IF Precipitation **IS** Heavy rain **AND** ...
	IF Traffic **IS** Without traffic **OR** Average traffic **OR** Much traffic **AND** ...
34	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Normal
35	**IF** Schedule **IS** Morning **AND IF** Street deterioration **IS** Average det. **OR** Without det. **THEN** Adjustment **IS** Slow
36	**IF** Schedule **IS** Morning **OR** Afternoon **AND IF** Street deterioration **IS** Much deterioration **THEN** Adjustment **IS** More slow
37	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Street deterioration **IS** Much deterioration **THEN** Adjustment **IS** Slow

TABLE 8. *(Continued.)* **Fuzzy rule base (b). These fuzzy rules were applied to the delivery routes of a soft drink company in tuxtla gutiérrez, chiapas, Mexico (Subsection V-A). This table includes all rules in the membership functions of precipitation considering the sets of heavy rain, rainstorm and heavy rainstorm.**

38	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Fast
39	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Traffic **IS** Average traffic **OR** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
40	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Without deterioration **OR** Average deterioration **THEN** Adjustment **IS** Slow
41	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND IF** Street deterioration **IS** Much deterioration **THEN** Adjustment **IS** More slow
42	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Much traffic **AND** **IF** Street deterioration **IS** Without det. **OR** Average det. **OR** Much det. **THEN** Adjustment **IS** Very slow
43	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Normal
44	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Average traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
45	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **OR** Average traffic **AND** **IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** Slow
46	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **OR** Average deterioration **THEN** Adjustment **IS** More slow
	IF Precipitation **IS** Rainstorm **AND** ...
	IF Traffic **IS** Without traffic **OR** Average traffic **OR** Much traffic **AND** ...
47	**IF** Schedule **IS** Dawn **OR** Night **AND IF** Street deterioration **IS** Without det. **OR** Average det. **THEN** Adjustment **IS** Normal
48	**IF** Schedule **IS** Morning **AND IF** Street deterioration **IS** Without det. **OR** Average det. **THEN** Adjustment **IS** More slow
49	**IF** Schedule **IS** Afternoon **AND IF** Street deterioration **IS** Average deterioration **THEN** Adjustment **IS** More slow
50	**IF** Schedule **IS** Dawn **OR** Entry of students and workers **OR** Exit of students and workers **OR** Night **AND** **IF** Street deterioration **IS** Much deterioration **THEN** Adjustment **IS** More slow
	IF Street deterioration **IS** Without deterioration **OR** Average deterioration **OR** Much deterioration **AND** ...
51	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Without traffic **OR** Average traffic **THEN** Adjustment **IS** More slow
52	**IF** Schedule **IS** Entry of students and workers **OR** Exit of students and workers **AND IF** Traffic **IS** Much traffic **THEN** Adjustment **IS** Very slow
53	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Average traffic **OR** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** More slow
54	**IF** Schedule **IS** Afternoon **AND IF** Traffic **IS** Without traffic **AND** **IF** Street deterioration **IS** Without deterioration **THEN** Adjustment **IS** Slow
55	**IF** Precipitation **IS** Heavy rainstorm **AND IF** Traffic **IS** Without traffic **OR** Average traffic **OR** Much traffic **AND** **IF** Street deterioration **IS** Without deterioration **OR** Average deterioration **OR** Much deterioration **AND** **IF** Schedule **IS** Dawn **OR** Entry of students and workers **OR** Exit of students and workers **OR** Night **OR** Afternoon **AND** **THEN** Adjustment **IS** Very slow

TABLE 9. **Factors that affect route planning.**

Number	Factors
5	**Temporal factors** are variables structured in constant time intervals when distributing the product. Schedule, Day, Week, Month and Holidays.
4	**Spatial factors** represent the variables associated with the streets through which the products are distributed. Traffic, Street deterioration, Road signs and Paving.
5	**Environmental factors** are the variables that involve the environment in product distribution. Precipitation, Temperature, Dust, Litter, Blizzard and Mist.
5	**Driver factors** are the variables that influence the driver during the product distribution process. Experience, Age, Health status, Mood and Road education.
3	**Company factors** are variables generated by the company that influence product distribution. Type of vehicle, Vehicle condition and Lack of personnel.
2	**Social factors** are variables influenced by society that affect product distribution. Pedestrians and Drivers.
24	**Total factors**

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