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Progress in reentry trajectory planning for hypersonic vehicle

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Abstract: The reentry trajectory planning for hypersonic vehicles is critical and challenging in the presence of numerous nonlinear equations of motion and path constraints, as well as guaranteed satisfaction of accuracy in meeting all the specified boundary conditions. In the last ten years, many researchers have investigated various strategies to generate a feasible or optimal constrained reentry trajectory for hypersonic vehicles. This paper briefly reviews the new research efforts to promote the capability of reentry trajectory planning. The progress of the onboard reentry trajectory planning, reentry trajectory optimization, and landing footprint is summarized. The main challenges of reentry trajectory planning for hypersonic vehicles are analyzed, focusing on the rapid reentry trajectory optimization, complex geographic constraints, and cooperative strategies.

Keywords: hypersonic vehicle, reentry trajectory planning, onboard planning, reentry trajectory optimization, footprint.

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1. Introduction

Since human ushered in the space era, the space transportation and the global strike have stimulated a great interest in hypersonic vehicles for both civilian and military applications [1]. The need for a reliable and effective access to the space promotes a fast development of hypersonic vehicles. This progress has been witnessed by experimental success of the National Aeronautics and Space Administration (NASA)'s scramjet powered X-43A in 2004, the Air Force's X-51 in 2010, and the Defense Advanced Research Projects Agency (DARPA)'s Falcon HTV-2 in 2011. Although X-51 went through two serious test failures after its first flight, the recent test mission in 2013 succeeded in covering a large downrange more than 230 nm. Further, the X-37B orbital test vehicle also completed a successful flight test in 2012, which lasted for 469 days and demonstrated the great capability of flight inspection and data analysis.

The reentry reference trajectory is a core component of the reentry guidance for hypersonic vehicles. Hence, the reentry trajectory planning plays a key role in steering an efficient and safe flight of the hypersonic vehicles in the complex environment and meeting all kinds of mission requirements. Normally, hypersonic vehicles enter the atmosphere of the Earth at 70–120 km in altitude. The full reentry flight lasts from the entry interface of high orbital to a terminal area at an altitude of 20–30 km. In general, the reference trajectory is generated offline, and then, preloaded on hypersonic vehicles before launching. Further, it often requires correction of reference trajectories to track errors in reentry flights, and even requires a replan of reentry trajectories onboard for aborting or reaching a new target. However, it is known as a challenging task to plan or optimize a reentry reference trajectory for the hypersonic vehicles, because the entry dynamics is highly nonlinear with specific control authority. In addition, hypersonic vehicles must subject to many "hard" path constraints such as the heating rate, aerodynamic load, and dynamic pressure.

In the last ten years, many researchers have investigated new strategies to fast generate a feasible and complete three-degree-of-freedom (3DOF) reentry reference trajectory for hypersonic vehicles. Some methods are based on the previous idea that the planning schemes should focus on the aerodynamic acceleration, since it can be measured, and then, translated into the related velocity and position [2]. The reference profile of the drag acceleration is designed in consideration of both the downrange and the crossrange requirements simultaneously [2–4]. Many others make good use of the quasi-equilibrium glide phenomenon with lifting vehicles, seeking to satisfy all of common path constraints and terminal conditions in reentry flights [5–8]. The full trajectory of hypersonic vehicles is typically divided into the initial descent phase, the quasi-equilibrium glide phase, and the pre-terminal area energy management (TAEM) phase [5]. The overall objective of the onboard reentry trajectory planning is to create a feasi-

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ble constrained trajectory for hypersonic vehicles in a limited time.

In addition to onboard planning, numerous studies focus on the reentry trajectory optimization of hypersonic vehicles with different reentry flight missions. Of many numerical methods, pseudospectral methods have demonstrated one of the most convenient tools to generate the optimal reentry reference trajectory for a maneuvering hypersonic vehicle [9–12], which is previously complicated or unachievable with traditional numerical methods. Furthermore, many newly-developed intelligent methods also show great potential for reentry trajectory optimization, due to high accuracy and fast convergence rates in the presence of optimal control problems (OCP) [13–16].

The objective of this paper is to briefly review the recent research efforts of reentry trajectory planning for hypersonic vehicles. An outline of this paper is as follows. The methods of onboard reentry trajectory planning are presented in Section 2. It mainly focuses on the generation of constrained reentry trajectory using the quasi-equilibrium glide condition (QEGC) and evolved acceleration guidance logic for entry (EAGLE). Section 3 discusses the approaches of reentry trajectory optimization for hypersonic vehicles. The application of various pseudospectral methods is summarized, as well as some popular intelligent methods including the particle swarm optimization (PSO) and genetic algorithm (GA). The analysis of the landing footprint problems is presented in Section 4. The new challenges of reentry trajectory planning for hypersonic vehicles are discussed in Section 5, focusing on rapid reentry trajectory optimization, complex geographic constraints, and cooperative strategies. Finally, concluding remarks are made in Section 6.

2. Onboard planning

The planning of a feasible and complete reference trajectory is the basic component of reentry mission design for hypersonic vehicles. In the complex reentry environment, onboard trajectory planning should be fast and robust in the presence of all the path constraints, terminal conditions, and control authority. The typical path constraints include the constraints on the heating rate, dynamic pressure, and aerodynamic load as

$$\dot{Q} \leqslant \dot{Q}_{\max} \tag{1}$$

$$q \leqslant q_{\max} \tag{2}$$

$$|L\cos\alpha + D\sin\alpha| \leqslant n_{\max}. \tag{3}$$

Note that, these constraints are "hard" constraints, and they must be always within the maximum allowable values to keep a safe flight. The control profile, corresponding to the state history, must not exceed the system authority generally in terms of maximum magnitudes and rates. Terminal conditions are usually determined by different flight missions of hypersonic vehicles.

In this section, the onboard planning methods of the constrained reentry trajectory are discussed in detail, focusing on the applications of the QEGC and the EAGLE.

2.1 QEGC

A well known method for fast generation of the 3DOF reentry trajectory was proposed in [5,6], which makes good use of quasi-equilibrium glide phenomenon with lifting vehicles. The QEGC was introduced, and a theoretical framework for design and analysis of reentry trajectories was presented, subject to all common inequality and equality constraints.

For a lifting reentry vehicle, it is known that the dynamic pressure is sufficient enough at an altitude of less than 80 km. Therefore, the fight-path angle, as well as its rate, is very small in the region where the velocity of the hypersonic vehicle is higher than a cutoff value (normally 6 to 8 Mach). Setting the fight-path angle and its rate to zeros, the QEGC can be described as [5]

$$L\cos\sigma + (V^2 - 1/r)(1/r) = 0. \tag{4}$$

Using the 3DOF point-mass reentry dynamics of the reusable launch vehicle (RLV), Shen et al. [5] divided the full reentry trajectory into three phases, including the initial descent phase, quasi-equilibrium glide phase, and pre-TAEM phase. The tactical process of the onboard generation of the three-dimensional constrained trajectory is shown in Fig. 1. In the initial descent phase, the nominal angle of attack (AOA) and a constant bank angle are used to steer the vehicle into the reentry flight corridor that is an admissible region specified by "hard" constraints and the QEGC. An example of the reentry flight corridor is shown in Fig. 2. In the quasi-equilibrium glide phase, the range requirements are satisfied on the differential equation for velocity with range-to-go as the independent variable. The boundaries of possible bank angle are determined by the QEGC and the path constraints. The interface velocity, altitude, and flight-path angle conditions are achieved by searching for a "middle" value of the bank angle. An appropriate choice of bank reversal point may greatly reduce the heading error at the interface. Finally, a geometric curve of altitude versus velocity is used to estimate the trajectory in the pre-TAEM phase. More details of the onboard reentry trajectory planning using QEGC can be found in [5,6].

On the model of X-33 vehicle, the numerical examples demonstrated that using the QEGC, the generation of a feasible 3DOF reentry trajectory needs only 2–3 s on a desktop computer [5]. At this rate, the QEGC is an ideal tool for onboard design and analysis of constrained reentry trajectory.

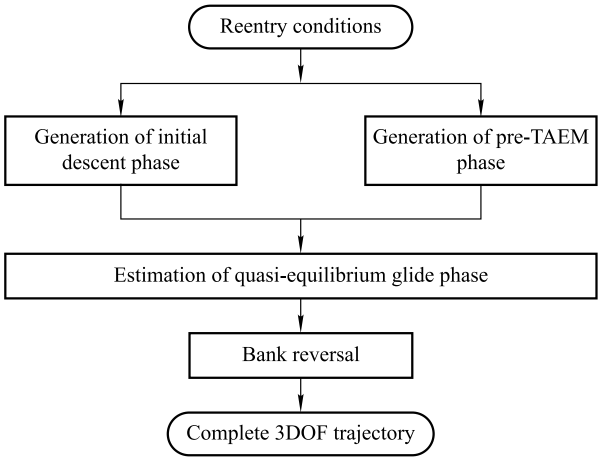

Fig. 1 Process of onboard reentry trajectory planning using QEGC

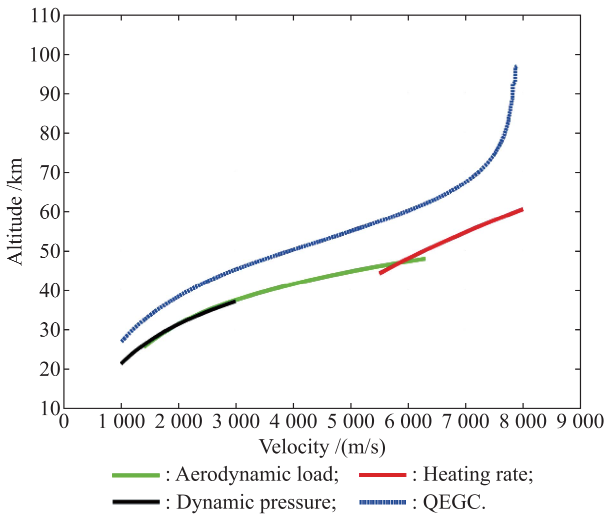

Fig. 2 Example of reentry flight corridor

However, it has a disadvantage that for vehicles with low L/D ratios, the QEGC is inapplicable as a result of the relative low speed. In order to fill this gap, Shen et al. [17,18] made an extension of the onboard reentry trajectory planning to the sub-orbital flight.

The successful use of the QEGC greatly promoted the reentry mission design. In 2004, Lu et al. [7] delineated an integrated approach for reentry trajectory planning, guidance, and control. They showed some preliminary examples of seamless integration on reentry missions in a simple simulation environment. Later, Lu et al. [8] presented an asymptotic analysis on quasi-equilibrium reentry flight for the purpose of providing a deep understanding of the QEGC. They focused on the quantitative estimates of the QEGC, promoting to describe the relationship between the quasi-equilibrium glide and exact trajectory.

In 2008, a new framework to design near-optimal reentry reference trajectory for hypersonic vehicle was proposed in [19]. Using the QEGC, a bank angle control law is developed in closed form. The numerical results showed that the difference between the optimal solutions and solutions of the proposed method are less than 1%. In addition, Xue et al. [20] performed an enforcement of inequality path constraints without increasing the complexity on the QEGC.

The very recent studies have updated the applications of the QEGC. Zhao et al. [21] proposed the reentry trajectory planning for the common aero vehicle (CAV) with geographic constraints. They demonstrated that using an improved QEGC, the threat avoidance can be achieved by geometry logic in the lateral profile design. Furthermore, the analysis and design of AOA was focused. Using the QEGC, Zhang et al. [22,23] introduced the feasible zone and optimal design of AOA. Xu et al. [24] also expatiated on the method of converting path constraints into the bounds of the AOA profiles.

2.2 EAGLE

The offline planning of a reference trajectory based on the drag acceleration profile was previously accepted by the reentry guidance design [25]. The experience in the space shuttle showed that the trajectory planning and reentry guidance should focus on the aerodynamic acceleration [26], since it can be accurately measured and simply translated into velocity and position by kinematic relations.

In 2002, Mease et al. [2] presented a direct extension of the drag acceleration planning method for seeking the large crossrange for reentry trajectory. It was the prototype of EAGLE. Using the total energy of hypersonic vehicle as independent variable, Mease employed reduced-order dynamics for the onboard planning. Both the heading angle and lateral acceleration are treated as variables to be specified. The constraints on heating rate, dynamic pressure, and acceleration are imposed as drag constraints. Then, the problem of reference trajectory design was divided into two sub-problems. One is the trajectory length subproblem for a consistent drag profile. The other is the trajectory curvature sub-problem for the lateral acceleration profile. The process of the onboard reentry trajectory planning using EAGLE is shown in Fig. 3.

On the previous research, Leavitt et al. [3,27] explicitly proposed the EAGLE with two integrated components: the trajectory planner and the trajectory tracking law. The trajectory planner was used to generate the reference drag acceleration and lateral acceleration profiles onboard. They introduced the higher-level logic of the drag acceleration planning including the successive approximation procedure, bank reversals management, and final heading logic. The test results showed that the EAGLE can achieve the specified target accuracy within allowable tolerances.

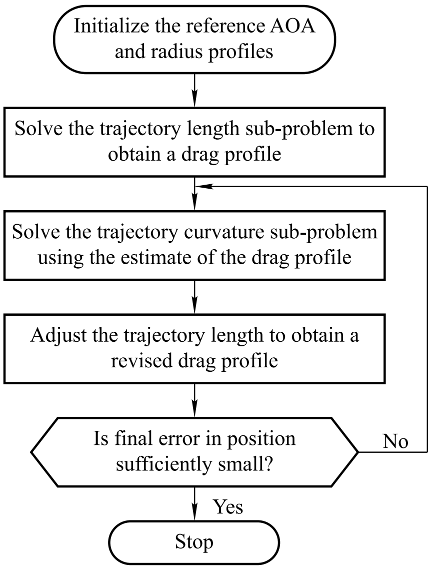

Fig. 3 Process of onboard reentry trajectory planning using EAGLE

Furthermore, Saraf et al. [4] presented a performance evaluation of the EAGLE in 2004. The evaluation was based on the performance criteria and scoring system from NASA Marshall Space Flight Center [28]. For each parameter, the scores were averaged from 100 dispersion cases. The EAGLE met with good results, finally scoring more than 0.97 (with a maximum of one) in all the tests.

Recently, Xie et al. [29,30] performed an extension of the EAGLE in consideration of geographic constraints. A key goal of the drag profile design is to obtain large lateral maneuverability. On a refined coordinate frame, they proposed to use a longitudinal subplanner to obtain the magnitude of bank angles and a lateral subplanner to determine the sign of bank angles. Similar to the original EAGLE, the subplanners are iteratively performed until all of the constraints are satisfied. On the common aero vehicle (CAV)-H model, several test cases show that the lateral control logic is capable of generating feasible reentry trajectories for hypersonic vehicles, meanwhile satisfying the geographic constraints.

3. Reentry trajectory optimization

Subject to the dynamic model, the purpose of reentry trajectory optimization for the hypersonic vehicles is to find the attack angle and the bank angle so that objective function is minimum, at the same time satisfying all the path constraints and boundary constraints.

Without loss of the generality, reentry trajectory optimization can be considered as an optimal control problem (OCP) in the following form. Determine the state $x(\tau) \in \mathbf{R}^n$ and the control $u(\tau) \in \mathbf{R}^m$ to minimize the objective function:

$$J = \varPhi(x(\tau_0), t_0, x(\tau_f), t_f) + \frac{t_f - t_0}{2} \int_{\tau_0}^{\tau_f} g(x(\tau), u(\tau), \tau; t_0, t_f) \mathrm{d}\tau \tag{5}$$

subject to the dynamic model, path constraints, and boundary conditions as

$$\dot{x}(\tau) = \frac{t_f - t_0}{2} f(x(\tau), u(\tau), \tau; t_0, t_f) \tag{6}$$

$$\varphi(x(\tau_0), t_0, x(\tau_f), t_f) = 0 \tag{7}$$

$$C(x(\tau), u(\tau), \tau; t_0, t_f) \leqslant 0 \tag{8}$$

Note that, the objective functions are usually different according to various reentry missions of the hypersonic vehicles, for example, the minimum time, minimum heat load, maximum crossrange, and maximum control margin.

The conventional indirect methods for solving the above trajectory optimization problem are based on Pontryagin's minimum principle, which leads to the Hamiltonian boundary value problem (HBVP). However, it is quite complicated to solve the HBVP. In this section, some new optimization methods for constrained reentry trajectory planning are discussed in detail.

3.1 Pseudospectral methods

3.1.1 Outline

Recent researches demonstrate that the pseudospectral methods (also called orthogonal collocation methods) have simple structures and provide faster convergence speed for optimal control problems with well-behaved solutions [12,31]. It helps to obtain the solution of large scale optimal control problems in an efficient manner computationally. In fact, the pseudospectral methods have already shown an outstanding performance to generate the optimal reentry trajectory for a maneuvering hypersonic vehicle, previously unachievable with traditional numerical methods.

The pseudospectral method is the state and control parameterization method. Reddien [32] first used pseudospectral methods in optimal control problems in 1979. The basic principle of pseudospectral methods is described briefly as follows. At a set of discretization points (DPs) and the collocation points (CPs), the state and control are approximated by a finite basis of interpolating polynomials. The derivative of each state is approximated by differentiating the global interpolating polynomial, which is equal to the vector field of the equations of motion. The distributions of DPs (used to discretize the state) and CPs (used to discretize the control) are selected, according to different pseudospectral methods. Like other direct methods, the pseudospectral methods transcribe the optimal control problem to the nonlinear programming problem (NLP). References [31,32] delineated a particular description of the pseudospectral methods.

The popular pseudospectral methods include the Chebyshev pseudospectral method (CPM) [33,34], Legendre pseudospectral method (LPM) [35–41], Radau pseudospectral method (RPM) [42–47], and Gauss pseudospectral method (GPM) [48–56]. The features of the four methods are shown in Table 1.

Table 1 Features of four pseudospectral methods

Method	Interpolating polynomial	Type of CP	Interval	Number of DP/CP
CPM	N Chebychev polynomial	Chebychev-Gauss-Lobatto (CGL)	$[-1, 1]$	N/N
LPM	N Legendre polynomial	Legendre-Gauss-Lobatto (LGL)	$[-1, 1]$	N/N
RPM	N Legendre polynomial	Legendre-Gauss-Radau (LGR)	$[-1, 1)$ or $(-1, 1]$	N/N–1
GPM	N–1 Legendre polynomial	Legendre-Gauss (LG)	$(-1, 1)$	N/N–2

Since the LPM, RPM, and GPM are presented with costate approximation procedures in detail, these three methods are generally used in complex hypersonic reentry applications. The recent development of the hp-adaptive pseudospectral method also shows a great performance in generating the optimal reentry trajectory. Fig. 4 shows a rough statistic of using pseudospectral methods in reentry trajectory optimization for hypersonic vehicles in the last ten years.

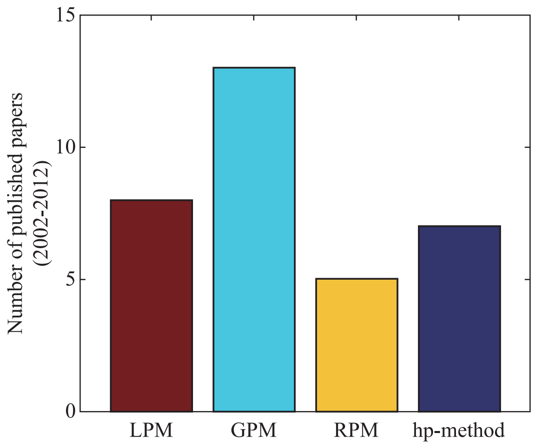

Fig. 4 Use of pseudospectral methods in reentry trajectory optimization

3.1.2 LPM

A high-quality costate estimate obtained from the LPM ensures its feasible use in reentry trajectory optimization. In 2002, Rao et al. [9] used LPM to design the reference reentry trajectory for a high L/D vehicle with specific control authority and path constraints. The numerical results showed that the trajectory designed by LPM is extremely similar to the optimal trajectory obtained by the first order necessary conditions. The ordinary differential equations (ODE) trajectory using the resulting AOA and the bank angle was also very close to the LPM trajectory with differences of 1.65 m in downrange and 18.08 m in crossrange. Later, Rao et al. [57] presented an extension of the LPM for non-sequential multiple-phase OCP using the continuity conditions on the state and control.

The successful application of LPM in reentry trajectory optimization advanced the update for an easy way to examine the optimality of direct methods. Gong et al. [37] demonstrated the convergence performance with combined state and control constraints. Williams et al. [38,39] presented several variants of the standard LPM, and provided common pseudospectral approaches to find the collocation points. Based on the above studies, recent work solved the reentry trajectory optimization problem for the maximum downrange. The feasible reference trajectory for hypersonic vehicles reached an accuracy under 10^{-3} [40,41].

However, the LPM has a disadvantage that it is intractable to formulate an exact mapping between the Karush-Kuhn-Tucker (KKT) conditions and the HBVP conditions [31]. In addition, it is found that using the LPM, the costate approximation probably has an error oscillating about true solutions [57].

3.1.3 GPM

The GPM is one of the convenient pseudospectral methods for the constrained reentry trajectory optimization of hypersonic vehicles. Using collocation at the LG points, GPM can be expressed equivalently in either differential or integral form. Numerical example has shown that using the GPM, the state and control can converge at a faster rate comparing with the LPM [57]. In addition, the GPM is capable of producing costate solutions with higher accuracy than the LPM on many problems [31].

In 2005, Benson et al. [48,49] presented the differential and integral GPM, first formulating a mapping between KKT conditions and first-order necessary conditions. Huntington [50] then improved the GPM by revising the pseudospectral transcription for the problem with dynamic constraints and path constraints. He also proposed a method to estimate the control at boundaries.

Recent work shows that the GPM performs well in reentry trajectory optimization for hypersonic vehicles. It provides an easy way to solve the path constraints and the control authority with satisfied accuracy and efficiency [51–55]. As shown in Fig. 5, the difference of the ODE and GPM trajectories generally demonstrates a monotonic decrease as the number of LG points increases. It is found that with more than sixty LG points [54], the GPM can basically lead to an error less than 5% between the ODE trajectory and approximated trajectory. Based on the multiphase technique, some extensions of the standard GPM were also proposed to optimize the reference trajectory of hypersonic vehicles with complex geographic constraints [58,59].

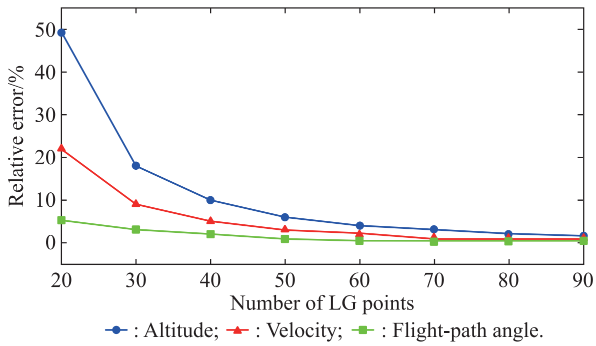

Fig. 5 Examples of relative errors between ODE and GPM trajectoies

3.1.4 RPM

Using collocation at LGR points [45], the RPM fast migrates from theory to many flight applications in the last years. One of the important advantages is that the RPM can accurately build a complete mapping between the costates of the OCP and the KKT multipliers of NLP [42,45]. Unlike the LPM, the approximation of the costate converges exponentially in the RPM. For the accuracy and efficiency in computation, the RPM is comparable to GPM, only leading to a final costate with less accuracy [31].

To solve the infinite-horizon nonlinear OCP, the RPM was introduced by Fahroo et al. [60], followed by a study of the direct trajectory optimization for the finite-horizon problem [42]. Numerical examples showed when the RPM would fail [61] and when it was proper to use the RPM for the OCP [31,56]. Thus, the RPM increasingly becomes a good choice for reentry trajectory optimization of hypersonic vehicles. The RPM has been applied to the design of the optimal reference trajectory for the suborbital launch vehicle (SLV) and RLV that steers the vehicles to a specified destination [46,47].

3.1.5 The hp-adaptive pseudospectral method

Recently, the novel use of hp-adaptive pseudospectral methods comes into focusing on numerically solving optimal control problems. The traditional pseudospectral method uses a single mesh interval and generally increases the degree of the polynomial for convergence. On many trajectory optimization problems, a fairly large-degree global polynomial is used to obtain an accurate approximation, resulting in an intractable or inefficient computation due to the global differentiation matrix [12]. However, the h-method [62] that requires a large number of mesh intervals and low-degree polynomial is more tractable than the traditional pseudospectral method. For the purpose of producing higher accuracy solutions with less computational effort, the so-called hp-adaptive pseudospectral method was derived by combining the best features of both the h-method and p-method [12]. Examples of DPs and CPs for both h-method and p-method are shown in Fig. 6.

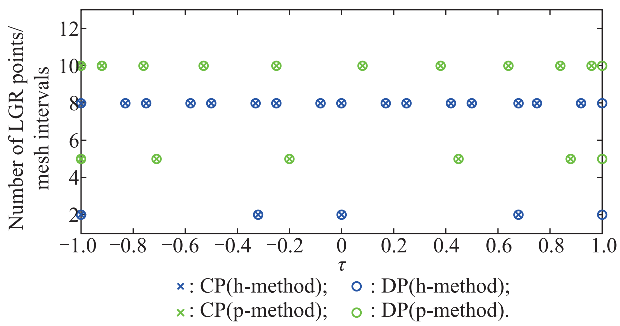

Fig. 6 Comparisons of DPs and CPs for the h-method and p-method

Darby et al. [11,62] first used the hp-adaptive pseudospectral method for direct trajectory optimization. They maximized the crossrange in the atmospheric reentry of the RLV. The dynamic constraints and boundary conditions of the reentry trajectory were involved. In the above studies, the total number of meshes, lengths of mesh intervals, and degree of polynomial in each mesh can be adjusted algorithmically until a specified error tolerance is satisfied. The hp-adaptive pseudospectral method uses as many low-degree meshes as possible in the trajectory segments of relatively high curvature, while the larger-degree polynomials are in the regions of the smooth trajectory. With three or four collocation points in each of the twenty uniform mesh intervals [11], the hp-method uses fewer collocation points than h-method, also showing slightly more computationally efficient than the p-method.

Several extensions of the hp-adaptive pseudospectral method have been used in the reentry trajectory optimization for CAVs and RLVs [46,63–65]. The costate estimation in optimal control problems also uses hp-method [66]. The stopping criterion, new polynomial degree, and locations of the new mesh points are improved for robustness and fast convergence.

3.1.6 Software packages

The numerous well-developed software packages are used to solve the NLP. Some popular software programs for complex hypersonic reentry applications include the SNOPT [67], NPSOL [68], IPOPT [69], OTIS [70], GPOP [71], POST-II [72], SOCS [73], GESOP [74], DIDO [75], etc. The novel use of these tools results in a much easier way to solve the reentry trajectory optimization in the form of the large-scale NLP than that of the HBVP.

3.2 Intelligent methods

In recent years, many studies have been performed on intelligent methods for trajectory planning. The PSO [76–82], GA [83–91], and ant colony optimization (ACO) [92] are the typical stochastic optimal approaches modeled on the concept of evolutionary approach [93]. These intelligent methods are becoming more popular due to their accuracy and speed qualities, which can be used in constrained reentry trajectory optimization of hypersonic vehicles.

3.2.1 PSO

The PSO is one of the swarm intelligence based methods that originally takes its inspiration from the natural phenomena like the motion of a flock of birds searching for a food source. The idea of the PSO method was first proposed in 1995 [76], and then modified [77]. As a population-based optimization tool, the PSO has the main strength that each particle uses the experience of the whole particles in the search space rather than only the experience of its own, resulting in a fast convergence [78].

Fig. 7 shows the basic process of the PSO. At a given iteration, each particle has a position vector, a velocity vector, and a vector of its previous best position [13]. The initial set of particles is typically randomly distributed in the searching space. Each particle in the swarm represents a possible solution and corresponds to a specific value of the fitness function [79]. Both the position and velocity vectors are updated using the following information: the distance between its current position and the best position so far of its own; the distance between its current position and the best position so far in the group. At the end of iteration, the best particle in the swarm is selected. References [78,79] described the PSO method in detail.

The PSO is convenient to obtain the optimal or near-optimal results of trajectory optimization problems since it typically avoids the use of Hamiltonian function and avoids computing its derivatives. Mapping the solution of the OCP into the PSO particles, studies have increasingly focused on the use of the PSO in constrained reentry trajectory optimization for hypersonic vehicles.

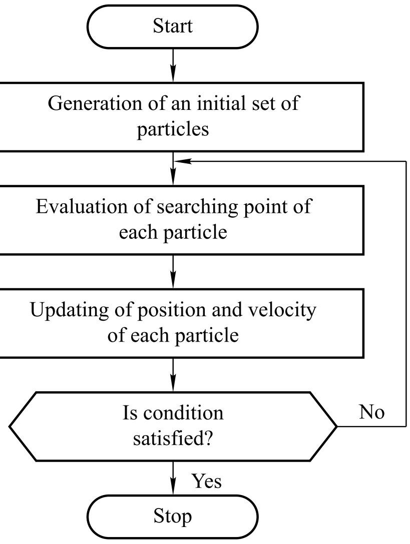

Fig. 7 Basic process of the PSO

Xie et al. [80,81] employed the PSO method to produce the magnitudes of bank angle in the longitudinal profile with free terminal time. The bank angle was parameterized, and a set of adaptive weights was added in the PSO. The initial condition of the particles was determined by the dynamic features of the vehicle. All of the path constraints and terminal conditions were combined with the fitness function. The statistical results showed that most of the longitudinal difference was less than 5 km. Xie et al. [14] also presented an upgrade to demonstrate the efficiency of the PSO via a comparison of the PSO method and the sequential quadratic programming (SQP) algorithm.

Recent verification of the accuracy for the PSO was conducted in 2013 [13]. Using sixty particles, the optimization was completed in sixty iterations, and all desired trajectory constraints were satisfied. It is found that the trade-off between number of particles and iterations in the PSO method can save the running time notably. Li et al. [82] also focused on the reduction of the optimization time and found that the PSO is a reliable and efficient method for reentry trajectory optimization.

3.2.2 GA

The GA is one of the stochastic search algorithms based on the mechanism of natural selection. The applications of advanced genetic operators are becoming widespread in the field of search, optimization, and machine learning [83].

The basic principle of the GA is survival of the fittest [84]. The GA has three key genetic operators, including the selection, crossover, and mutation. This method is typically viewed as a two-stage process. At the current population, the selection is used to create an intermediate population. Then, the crossover and mutation are used to create the next population [79,83]. As shown in Fig. 8, an implementation of a simple GA begins with a random population of chromosomes. The GA iteration starts by evaluating the vector of objective function associated with chromosomes. The good chromosomes are selected to improve objective function. Crossover is the main tool of GA to hybridize design traits for useful genetic information. Finally, the mutation creates a new population as the start of the next generation.

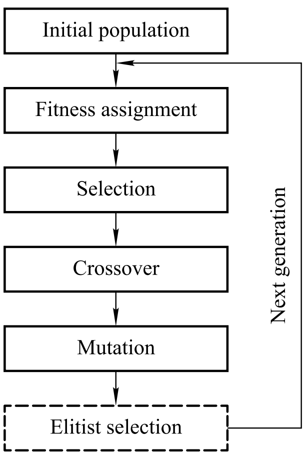

Fig. 8 Basic process of GA

The GA is on the rise in trajectory planning due to its versatility to optimize in complex search spaces. It has a rapid convergence on large-scale optimization problems, since the GA does not work on a single chromosome at a time, but on the whole population [83]. Another advantage which makes GA a reliable and robust method is that it requires no gradient information and it is not sensitive to the initial values for the computation [85].

The idea of using the GA in reentry trajectory optimization problem can date back to 1999. Desidei et al. [86] addressed a complex heat protection problem for hypersonic reentry vehicles. He made use of the GA to calculate the heat flux along the reentry flight. Yokoyama et al. [15,87] then proposed a new selection method using a fixed-parameter penalty function, in which the selection of surviving individuals is carried out on multiple criteria. The resulting reentry trajectory approached the near-optimal solution. They demonstrated that the GA was reliable in finding an initial solution. Further, the multi-objective trajectory optimization for the RLV was studied [88,89]. It was demonstrated that the non-dominated sorting GA [90] can obtain satisfactory solutions by balancing the heat load and crossrange.

More recently, Zhang et al. [85] found that simply applying GA to the reentry trajectory optimization may lower the efficiency of local search and reduce the accuracy of final solution. They proposed an improved GA with the SQP method to optimize the reentry trajectory for the RLV. The numerical example showed that the hybrid method seldom leads to a local optimum and can faster converge to the near-optimal solution. The reentry trajectory optimization was also completed by using a similar hybrid method based on the GA and LPM [91].

4. Landing footprint

The landing footprint for a hypersonic reentry vehicle provides critical boundary information of all the reachable landing locations on the Earth from a given reentry interface. The region of footprint mainly depends on the reentry state, path constraints, and the capability of lifting vehicles. It plays an important role in mission planning that influences the nominal reentry flight and abort situations.

The landing footprint is traditionally a two-dimensional set in terms of the longitude and latitude. It is described in two typical forms as shown in Fig. 9. One is shaped like a fan and the other is a polygon. E_1 and E_2 are the reentry points. In the first type, E_1C_1 represents the maximum downrange. The outer boundaries A_1C_1 and B_1C_1 are sets of end points with the maximum crossrange. In the second type, the footprint consists of four basic edges. The sides A_2B_2 and C_2D_2 relate to the minimum and maximum downranges, respectively. The sides A_2D_2 and B_2C_2 relate to the maximum crossrange. Early description of landing footprint can be found in [94,95].

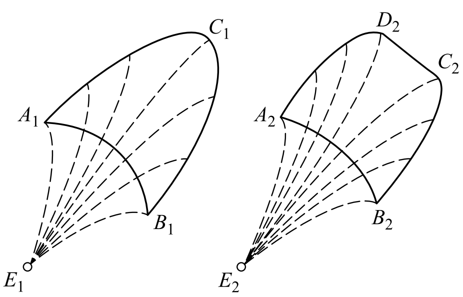

Fig. 9 Illustrations of footprints

One class of approaches for computing the landing footprint is based on the existing algorithm EAGLE. In 2004, Saraf et al. constructed the footprint boundaries from the end points of feasible reentry trajectories [96]. The method is built around the developed trajectory planner in EAGLE that can generate the footprints with fixed and varying AOA profiles, respectively. Leavitt et al. also improved the previous EAGLE planner such that it could generate near-maximum downrange and crossrange trajectories [97]. The result showed that the EAGLE footprint achieved slightly shorter downrange than the optimal landing footprint.

Another type of methods finds the landing footprint on the basis of the closest-approach problem, since it can be easily solved as a single parameter problem. In 2010, Lu et al. proposed a closed-form bank angle control law to fast generate the landing footprint for reentry vehicles [98]. They found that at some specified downrange, the problem to find the maximum crossrange is equal to the closest approach problem. They built the footprint boundary with the end points on the closest approach trajectory. Later, Li et al. made an improvement on the work of Lu [99]. They converted typical inequality constraints to the boundaries of AOA, and analyzed the impact of angle of attack on the landing footprint.

For finding the envelope of reentry trajectories, Benito et al. presented the concept of the reachable set from a given reentry state [100]. In a sense, the landing footprint is a specified horizontal plane in the reachable set. The computation and analysis of the reachable set can be used to evaluate the performance of hypersonic reentry vehicles and to select a nominal reentry state. Many other studies also focused on the footprint inner boundary problem [101] and the impact of controller failures to the landing footprint [102,103].

5. Challenging issues

As hypersonic vehicles continue to have a key role in the current and future civilian and military operations, many new challenges need to be addressed, as well as new techniques to be developed, for the effective and practical management of hypersonic vehicles. In this section, some typical challenging issues of reentry trajectory planning for hypersonic vehicles are discussed, focusing on the rapid trajectory optimization, complex geographic constraints, and cooperative strategies.

5.1 Rapid trajectory optimization

A key performance to evaluate the reentry trajectory planning methods is the computation time. An excellent onboard trajectory planning method is capable of generating a feasible and full reentry trajectory for hypersonic vehicles within several seconds [5]. In contrast, the reentry trajectory optimization typically costs much time to obtain a solution of the complex optimal control problem. Despite the fact that optimization approaches make a significant progress with the help of various software tools, there is still room for improvement.

The initial guess is generally time-consuming for optimization approaches. With a poor initial guess, the solution of the problem would probably get trapped into a local optimum or even not converge. However, some intelligent methods are not sensitive to the initial values of iteration computation [15,81]. They are reliable and convenient in finding an initial solution to the optimal trajectory of hypersonic vehicles.

Recent work also showed that the hp-adaptive method demonstrates faster computation time than the traditional pseudospectral methods [46]. A tradeoff between the number of collocation points, grid iterations, and mesh intervals may notably save the computation time. It is ideal to generate a full optimal reentry trajectory for hypersonic vehicles within half a minute.

5.2 Complex geographic constraints

Since the hypersonic vehicle is being designed with versatility, some complex geographic constraints are inevitably involved in the reentry trajectory planning. As shown in Fig. 10, the waypoints (for reconnaissance) and no-fly zones (for threat avoidance and geopolitical restrictions) are two kinds of newly-focused geographic constraints [58,104]. In a way, conventional reentry trajectory planning methods are powerless in the presence of these complex constraints. Many difficulties are arising such as the uncertainty of the passage time for each waypoint and the approximation of the trajectory near the no-fly zones.

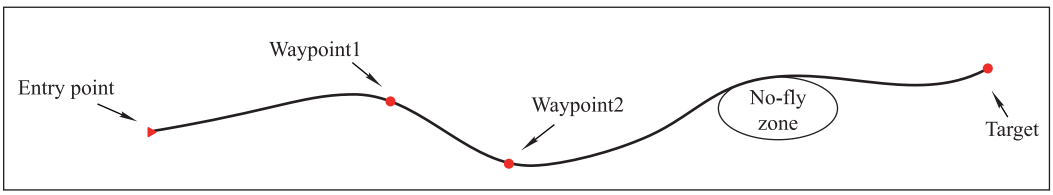

Fig. 10 Example of ground trajectory of hypersonic vehicles in reentry flight

A typical solution to the waypoint constraints may be the multiple phase technique. By dividing the reentry trajectory at waypoints, each of them turns to be the terminal point of the previous phase as well as the start point the next phase. The design of continuous conditions for multiple reentry phases is a key problem. As to no-fly zone constraints, a kind of lateral control logic has already been integrated into a rapid reentry trajectory planning method [104]. However, it has to take several minutes to generate a feasible reentry trajectory. An analytical geographic technique was also tested to solve trajectory optimization of hypersonic vehicles with no-fly zone constraints [58]. The future researches are likely to focus on the simple and reliable approaches of reentry trajectory planning including different practical geographic constraints.

5.3 Cooperative trajectory planning

Although numerous advances have been achieved in reentry trajectory planning, guidance, and control of hypersonic vehicles, there has been few researches in developing the algorithms and strategies for hypersonic vehicles in a cooperative fashion. As the improvements of fundamental researches are giving rise to the experimental success, it is foreseen that in the future, cooperative hypersonic vehicles will replace single ones for more complicated flight missions in different environments. The ability of trajectory design for a group of hypersonic vehicles in a cooperative mission is of great importance to a wide variety of civilian and military applications.

A significant issue is the level of complexity involved in the trajectory planning for a team of vehicles with competing interests. The reentry trajectories provided by the cooperative planner must be within the dynamic capabilities of all the individual hypersonic vehicles, meanwhile achieving the team objective in a satisfactory or optimal manner. Therefore, an external and comprehensive performance evaluation of single hypersonic vehicle is very essential to the cooperative trajectory planning.

In addition, the analysis of civilian and military demands for multiple hypersonic vehicles is also necessary, since the complexity of cooperative trajectory design is also increased by the uncertain and changing environment. The stable and robust trajectory planner is for certain a key component of the cooperative strategy design.

6. Conclusions

The reentry trajectory planning is expected to play an increasingly important role in the future development of hypersonic vehicles. The main problems focus on the onboard reentry trajectory planning, reentry trajectory optimization, and landing footprint. Due to the numerous nonlinear equations of motion, path constraints, terminal conditions, and limits on control authority, the reentry trajectory planning of hypersonic vehicles is much more complex than that of low-speed aircraft. Many difficult problems in the field have been solved successfully, while some others are in process. This paper herein briefly reviews the research efforts of reentry trajectory planning for hypersonic vehicles in the last ten years. Several challenging issues are also discussed to provide a potential source for the researchers in this new field. The rapid progress of reentry trajectory planning will provide a succession of novel breakthroughs for the fundamental researches and experimental advancement of hypersonic vehicles in the near future.

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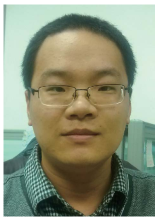
Biographies



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