Analyzing Response Time Data (Part 2)

Statistical Modeling in Psychology

October 9-10 2024

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Outline

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Estimating Parameters

Fitting a model to data (in the frequentist sense) means finding the best point estimates of the model's parameters. There are a bunch of ways to do this.

- Method of moments
- Least squares
- Maximum likelihood
- Bayesian estimators
- Others...

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Method of Moments

- Central moments: $\overline{T}$, $\sum_{i=1}^{N}(T_i - \overline{T})^2$, $\sum_{i=1}^{N}(T_i - \overline{T})^3$, etc.
- Model parameters: μ, σ^2, τ, etc.

$$\begin{aligned}
\overline{T} &= f_1(\mu, \sigma^2, \ldots) \\
s^2 &= f_2(\mu, \sigma^2, \ldots) \\
\kappa_3 &= f_3(\mu, \sigma^2, \ldots)
\end{aligned}$$

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Ex-Gaussian Example

$$\begin{aligned}\overline{X} &\approx \mu + \tau \\ s^2 &\approx \sigma^2 + \tau^2 \\ \kappa_3 &\approx 2\tau^3\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}\hat{\tau} &= \left(\frac{\kappa_3}{2}\right)^{1/3} \\ \hat{\sigma^2} &= s^2 - \left(\frac{\kappa_3}{2}\right)^{2/3} \\ \hat{\mu} &= \overline{T} - \left(\frac{\kappa_3}{2}\right)^{1/3}\end{aligned}$$

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Exercises

EXERCISE: Consider a gamma distribution with shape k and rate lambda. Estimate these parameters for the Wagenmakers data set (or your own) using method of moments.
EXERCISE: Add a shift parameter to the gamma model you just fit using method of moments and fit it again.
Some things to think about:

- Does the method of moments return sensible estimates?
- Is the fit to the data "good"?
- Are the parameter constraints (e.g., $\hat{\sigma}^2 > 0$) guaranteed to be preserved?

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Nelder-Mead Simplex Algorithm

- Select starting values for n parameters
- Construct a “simplex” with $n + 1$ vertices
- “Crawl” the simplex through n-dimensional space by expanding, contracting and reflecting
- Stop when the vertices are all approximately equal

Nelder-Mead Simplex Algorithm

Methods of (Nonlinear) Least Squares

Example: For a function estimate $\hat{F}(t \mid h)$, minimize

$$\sum_{i=1}^{N} \left(\hat{F}(T_i \mid h) - F(T_i \mid \theta) \right)^2$$

R's optim Function

```
optim(par, function, method = "Nelder-Mead", control = list())
```

- par is a vector of parameter starting values
- function is the function to be optimized
- optim believes that “optimal” = “minimized”

Exercise: Fit a Wald or exGaussian distribution to Wagenmakers' data (or your own) by minimizing the sum of squares between the estimated and theoretical CDF of a model of your choice. Add a shift (non-decisional) parameter and do it again. Did adding the extra parameter improve the fit?

Maximum Likelihood

For a model pdf $f(t \mid \theta)$, maximize

$$L(\theta \mid T) = \prod_{i=1}^{N} f(T_i \mid \theta)$$

Exercise: Estimate the parameters of the Wald or Ex-Gaussian model by maximum likelihood, fitting the model to your own (or Wagenmakers') data. Consider the issues of

1. a product of a lot of tiny numbers;
2. outlier observations with likelihoods very close or equal to zero; and
3. keeping your parameter values in the range that they are supposed to be.

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Bayesian Estimators

(We'll talk about these tomorrow.)

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Some estimators are better than others

- An unbiased estimator on average is equal to the thing you're trying to estimate.
- An unbiased, minimum variance estimator also has the smallest possible variance.
- The mean squared error (MSE) of an estimator is

$$E(\hat{\theta} - \theta)^2 = Var(\hat{\theta}) + Bias(\hat{\theta})^2$$

- An estimator may maximize the likelihood. But MLEs don't necessarily exist, and they aren't guaranteed to be unique.
- Sometimes a biased estimator will have lower MSE than an unbiased estimator.

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Model Comparison Methods

- Frequentist:
  - Likelihood ratio tests (for nested models)
  - Root mean squared error of prediction
  - R^2
  - χ^2 goodness-of-fit
  - Kolmogorov-Smirnov test
- Bayesian:
  - Information criteria (AIC, BIC, WAIC, etc.)
  - Bayes factors
  - Minimum discription length
- Both:
  - Cross-validation
  - QQ-plots

Exercises

Fit two models to the data in mystery.Rdata. You can subset the data to a single stimulus condition or only correct responses, or you can try to fit the entire dataset.

Basic model comparison and goodness of fit evaluation procedures are described in today's R notebook. For the models you fit consider doing the following:

- If appropriate, perform a likelihood ratio test for nested models.
- Compute RMSE and R^2.
- Divide the data into training and test sets. Fit the models again to the training subset and evaluate the models by performing χ^2 goodness of fit tests on the test data.
- Perform a Kolmogorov-Smirnov test of the hypothesis that the test data follow the same distribution as the model that you fit to the training data.

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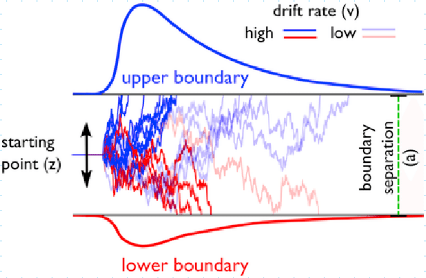
Drift Diffusion Model

a :	Response threshold for response A
z :	Starting point, often $a/2$
ν_A :	drift rate for responses to stimulus A
ν_B :	drift rate for responses to stimulus B
t_0 :	nondecision time (shift)



Drift Diffusion Model

EXERCISE: Fit the drift diffusion model to all the conditions in the mystery data set. There are 2 stimuli with correct and incorrect responses to each.