Modelling Real World Problems - 900393SCIY

Lab 8b – the Spatial Prisoners Dilemma

Introduction

The Prisoners' Dilemma is a simple game theoretical two-player game, where, if players act in their own interest, a sub-optimal solution will be reached. If they instead would "cooperate', a much better total score could be obtained. As such, the prisoners' dilemma is studied as a paradigm system for the evolution and maintenance of cooperation, in fields in Economics, Psychology, and Biology.

An example payoff matrix is provided in below table:

Score for Player A / Player B		**Player B**	
		Cooperate	Defect
Player A	Cooperate	3/3	0/4
	Defect	4/0	1/1

From the table it can be seen that the highest score of 6 can be obtained if both players cooperate, but from a selfish point of view an individual can always gain more by defecting, regardless of what the opponent does (gaining an extra point 3 vs 4, or 0 vs 1). Tragically, this would lead to both players defecting, which gives them both a score of only 1, and thus leads to a suboptimal solution.

A general setup for the prisoners' dilemma payoff matrix is:

Score for Player A / Player B		**Player B**	
		Cooperate	Defect
Player A	Cooperate	R/R	S/T
	Defect	T/S	P/P

Here "R" is the reward for cooperating, "T" is the temptation to defect, "S" is the suckers' payoff, and "P" is the punishment for both players defecting. For the matrix to be a prisoners' dilemma we should have $T>R>P\geq S$, and furthermore $2R>T+S$.

For the practical, we will use a specific choice for "R"=1, and "S"="P"=0. The temptation parameter "T" will be varied between $1<T<2$. The payoff matrix is for a player is thus:

Score for Player A		**Player B**	
		Cooperate	Defect
Player A	Cooperate	1	0
	Defect	T	0

In today's practical, you will study a spatial variant of the prisoners' dilemma, where individuals play against their 8 direct neighbours. This determines their score in the current timestep. Subsequently, for each cell the scores in the 8-cell neighbourhood (including the cell itself) will be evaluated, and the strategy of the cell with the highest score will be assigned to the cell in the next timestep. This is done simultaneously for all cells, and boundary conditions are toroidal.

The CA can be described by:
States: 0 = red = "D" = defecting individual ; 1 = blue = "C" = cooperating individual

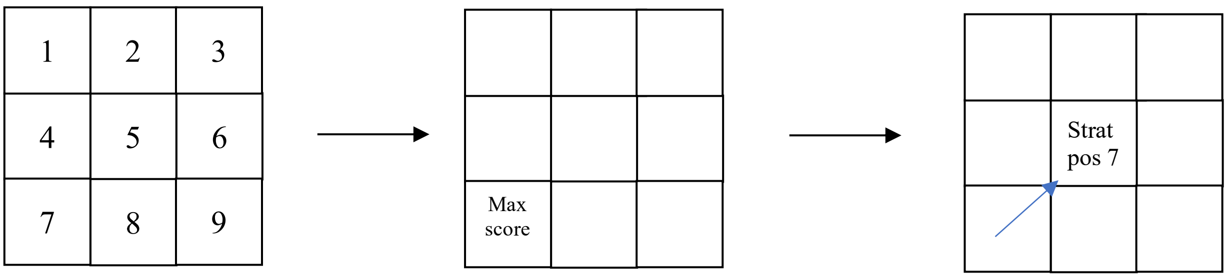
Determining the score of an individual (i.c. a cell):

sumC = number of cooperators around the cell (max 8)

If the cell is "D": $score = sumC.T$
If the cell is "C": $score = sumC$

Determining the next state of a cell:

For the next state of a cell the scores of all 9 cells around the cell (including the cell itself) will be evaluated, and the strategy of the cell with the highest score will be copied to the cell:



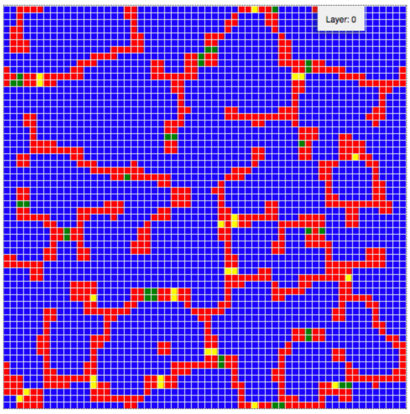
Extra plotting colours:

For the visualisation of the field two extra colours are used: "green" for cells that have just turned to cooperation, and "yellow" for cells that have just turned to defection. These additional colours are only used for plotting, and they do not affect the state of the cell.

Predicted mean field behaviour:

The mean field behaviour of the model is not so easy to simulate, as both the determination of the score, and the determination of the neighbour with the highest score are spatial processes. However, the model has a clear mean field prediction, as the average score of a "D" individual is expected to be larger than the average score of a "C" individual, because *T>1.* Consequently, in the mean field, the prisoners' dilemma should lead to quick extinction of the "C" strategy.

a) Run the spatial model (with T=1.59) until it reaches an attractor. Take a snapshot and describe the spatial patterns in the attractor.

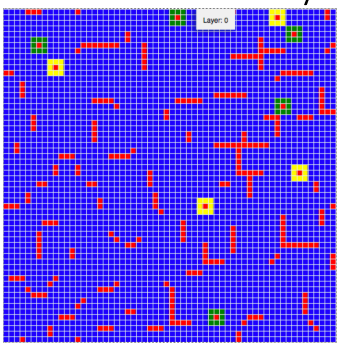


In the attractor the cooperators form large clusters with small ridges of defection between the clusters. There are a few locations where cooperation and defection alternates (in yellow and green).

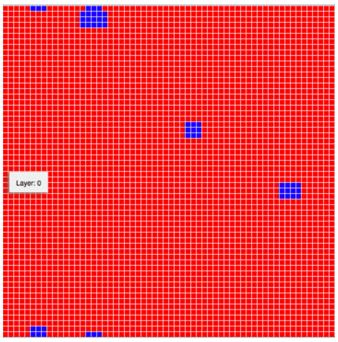
b) Rerun the model and carefully observe and describe what happens in the first few timesteps (use the "Step" button to advance one timestep at a time), both in terms of pattern formation, and in terms of the amount of observed cooperative behaviour in the system. Relate your observations to the mean field prediction of the system.

In the first few timesteps, the cooperator numbers sharply decrease. This is because we start with a random pattern, and in a random (mean field) situation, defectors have a higher score than cooperators. However, there are a few locations where cooperators initially formed a patch, and these patches can increase in size. This causes an increase in overall cooperation, until the edges of the patches start touching each other, after which the defectors in between get isolated. This is an ideal location for defection (getting help from two sides), and the defectors will persist in these locations.

c) Check the two extreme cases for T=1.01 and T=1.99. Describe how coexistence of cooperation and defection is reached and maintained. Include figures for both situations, and describe locally stable patterns.

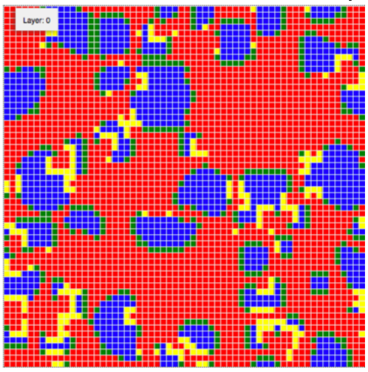


T=1.01 . Isolated defectors have maximal score (8.08) and can expand to a 3x3 block in the next timestep. However, this block will subsequently collapse to a single defector (yellow and green blinking objects). Single lines of defectors are also stable objects.



T=1.99. Defectors are now superior in almost all local configurations, except for a co-operator that is completely surrounded by other cooperators (score=8). Whether co-operator patches persist in the attractor depends on presence of 3x3 or larger co-operator blocks in the initial starting pattern.

d) Explore the dynamics for T=1.61. In which Wolfram Class should these dynamics be classified? How could you check whether your answer is correct?



Ongoing fluctuations throughout the field. Behaviour seems like Wolfram Class 3 (random), or maybe Class 4 (complex). You could e.g. check this by testing the effect size of a small local disturbance, and see whether you get a 1/f distribution in the effect size.

The change in dynamics between T=1.59 and T=1.61 is very large. We are now going to try to get a better understanding of what causes this change by studying the artificial setting of a 3x3 initial block of cooperators in a field of defectors. You can choose this initial setting by activating the "start with blocks" option on the left side of the field. In a way, we are now "conducting an experiment" within our model.

e) Study the development of the 3x3 block for T=1.61 , and compare this with T=1.59. Find the last timestep in which both simulations are identical, and find out why they differ onwards. What causes the transition to happen for these exact values of T ?

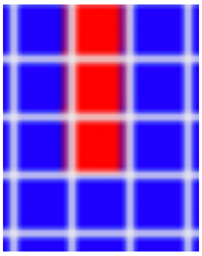
The critical situation happens at timestep t=12. Here, at 8 locations a defector is surrounded by 5 cooperators, giving a score of 5T. It competes with cooperators within the co-operator patch, so they have score 8. The critical point is at 5T = 8, which gives T_{crit}=1.6. Above this critical T value, the defector can enter into the co-operator patch.

f) Test the dynamics for the 3x3 block for T=1.60. What is the most striking difference with the observed patterns in question e) ? Find the line in the python code that "causes" this difference in behaviour.

At T=1.60, the score for the defector and cooperator in timestep 12 are equal. In this case they have an equal probability of occupying the cell that they compete for. In the python code, a tiny random value is added to each score, to make sure that ties in scores are impossible (and that each cell with the maximum score has an equal probability of being selected as winner).
The code is near the bottom of the fiole agent.py:

```
#add small noise to randomly break ties in score
fitness += self.random.random()*0.00001
```

g) Use the 3x3 block initial condition to find the first change in dynamics starting from T=1.01 and slowly increasing T. Develop and report and efficient procedure for finding this critical value for T. Provide a local drawing of the critical situation, and derive the exact condition for T where the dynamics change.
The first change happens at T=1.15 at timestep t=11, when the tip of a broken line of defectors starts "blinking".



At the tip, the defector is surrounded by 7 cooperators, which gives a score of 7T. The best competing cooperators have a score of 8. Therefore, the critical point is at 7T=8, which gives T_{crit}=8/7. T=1.15 is the first value above this critical point.

The finding that considering spatial interactions can enable persistence of cooperation has received considerable attention, in high ranking journals, starting with a Nature publication by Martin Nowak and Robert May in 1992 (https://www.nature.com/articles/359826a0.pdf) .

h) Several follow up papers have tested robustness of the initial result. For the following alterations of the model, give a prediction whether this will increase or decrease the amount of cooperation in the model:

- Adding movement of individuals
  Adding (random) movement is great for defectors, as this enables them to enter patches of cooperators. The amount of cooperation (strongly) decreases.
- Including the score for a cell playing against itself (this was done in the initial paper)
  Also including a score against yourself is further promoting cooperation. The simple reason is that each co-operator gets R added to its score, whereas defectors only get an addition P. This option was used in the original paper.
- Instead of selecting the maximum score, use a probabilistic rule where the probability of occupying a cell is proportional to the scores of the surrounding cells
  This option is bad for cooperation. Cooperation benefits from solid blocks of cooperators. Using a probabilistic rule for the next state allows defectors with an inferior score to occasionally occupy positions that are largely surrounded by cooperators. In the next timesteps, such defectors have a very high score, which allows them to locally increase and form a patch.