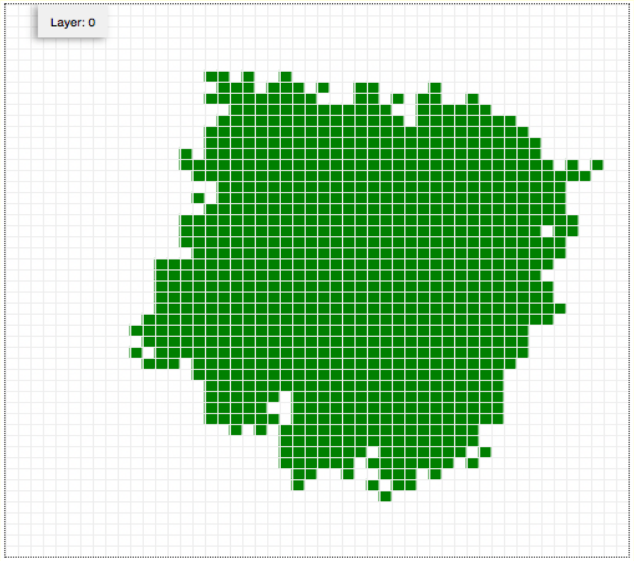
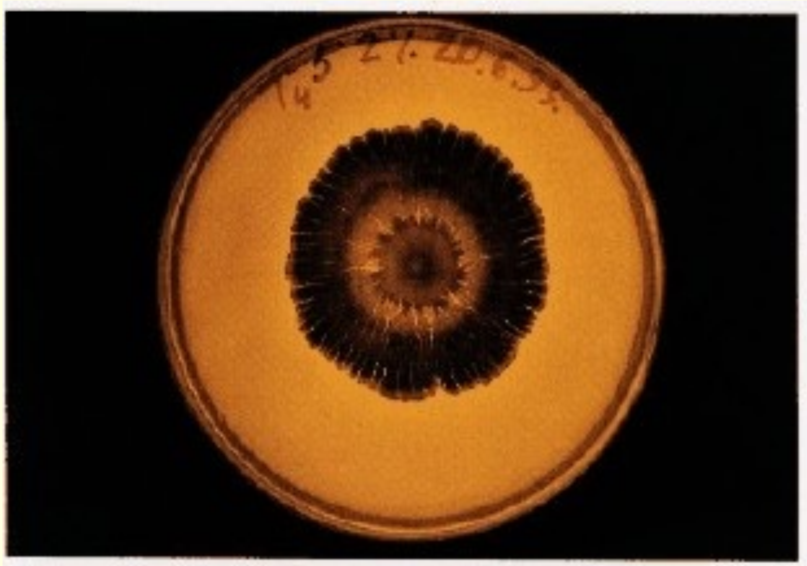
Spatial Patterns and Complexity

Population Growth
Diffusion Limited Aggregation
Game of Life

Spatial Population Growth



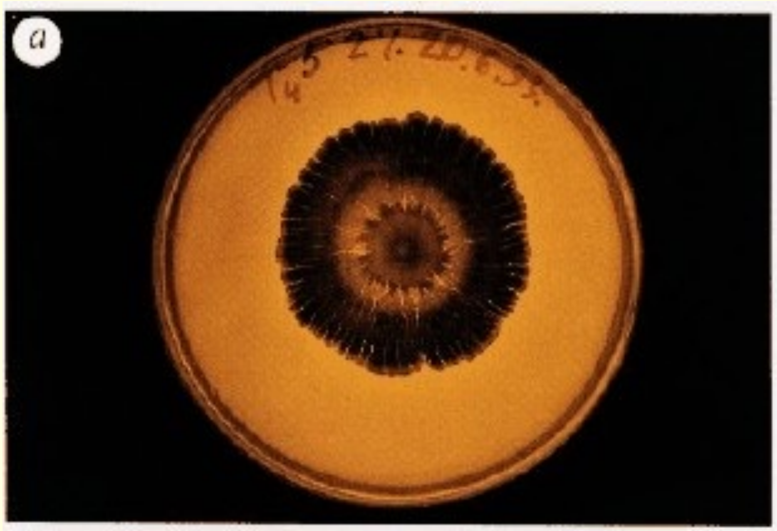
Population growth
in the CA model

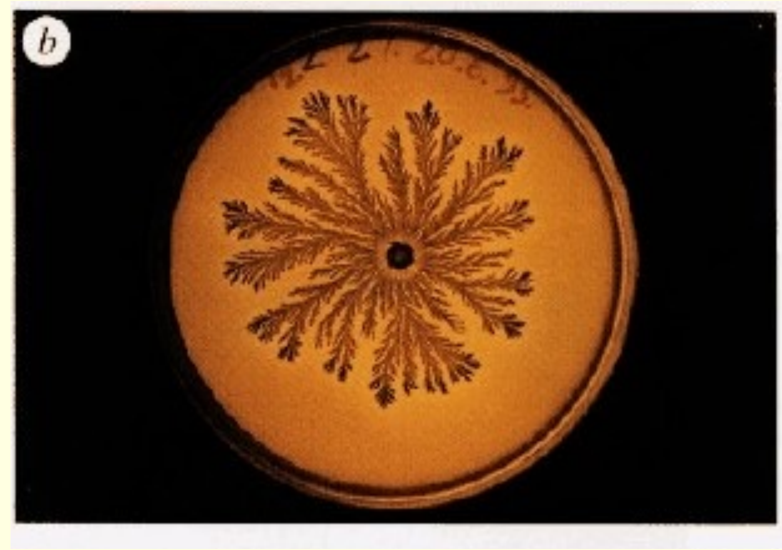


Circular colony growth
in a bacterial population

Circular versus fractal growth

Bacillus subtilis colony





sharp transition

normal food density

low food density

Question 1: Is this fractal pattern adaptive?

Question 2: How do the bacteria do it?

Pictures from: E. Ben-Jacob *et al.* , Nature 1994

Is the fractal pattern adaptive?

- The fractal growth pattern seems adaptive, as the bacterial colony can forage over a larger area
- Actually, the fractal structure might be quite optimal in reaching the food in an efficient way

How do the bacteria do it?

- How do they know when to switch to the alternative structure?
- How do they make sure they do this simultaneous?
- How do they plan the fractal structure?

The surprising answer to all these questions is:

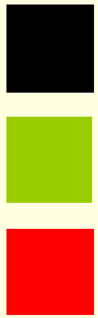
They don’t have to do anything!

The pattern formation is more a property of the food than of the bacteria

Diffusion Limited Aggregation (DLA)

The food particles that the bacteria use for their growth can be considered to randomly diffuse on the agar dish.

states:



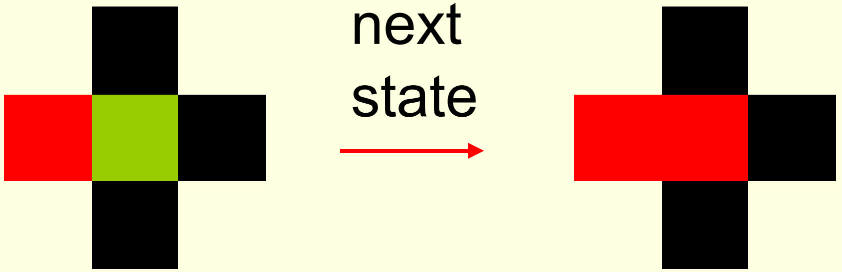
empty

food

bacterium

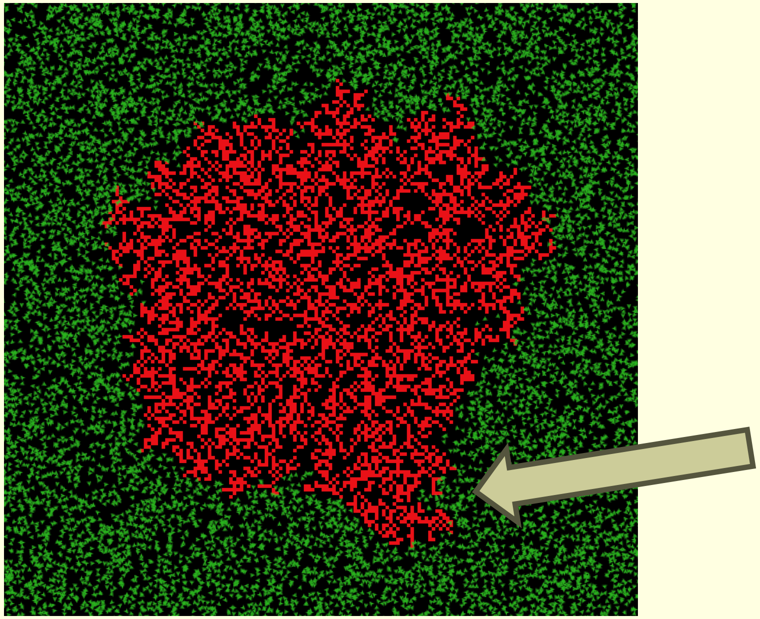
processes:

1. colony growth



2. diffusion of food

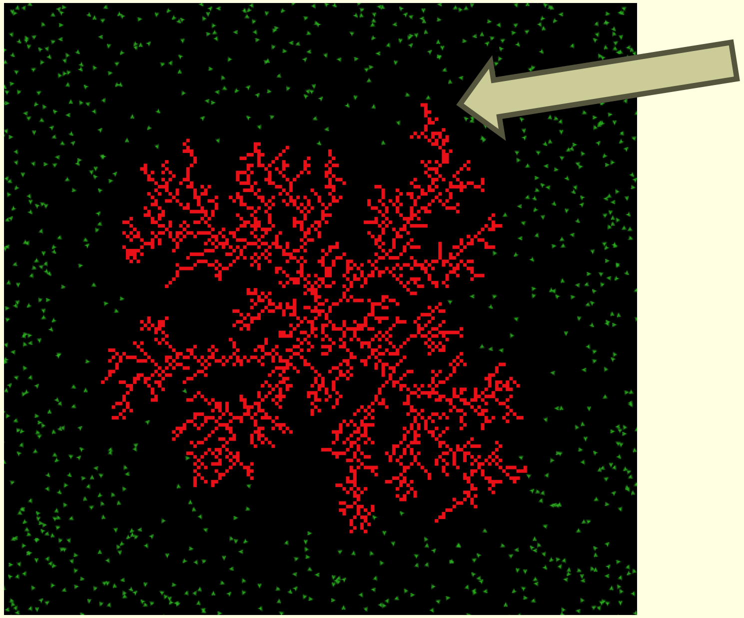
Nornal food conditions



Colony growth is approximately circular

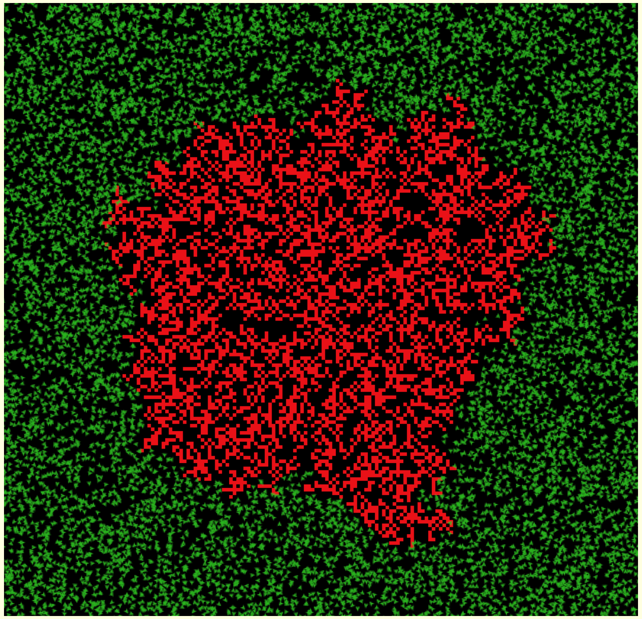
Local extensions (see arrow) will have reduced growth

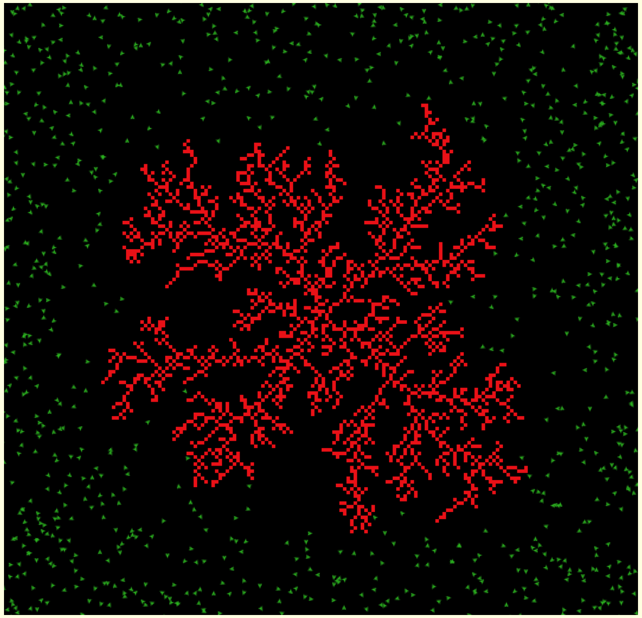
Low food conditions



At reduced food conditions the pattern shifts to fractal growth. Here, food conditions in between fractal arms are much lower than outside of the colony. This causes local extensions at the outside of the colony (see arrow) to actually accelerate in growth

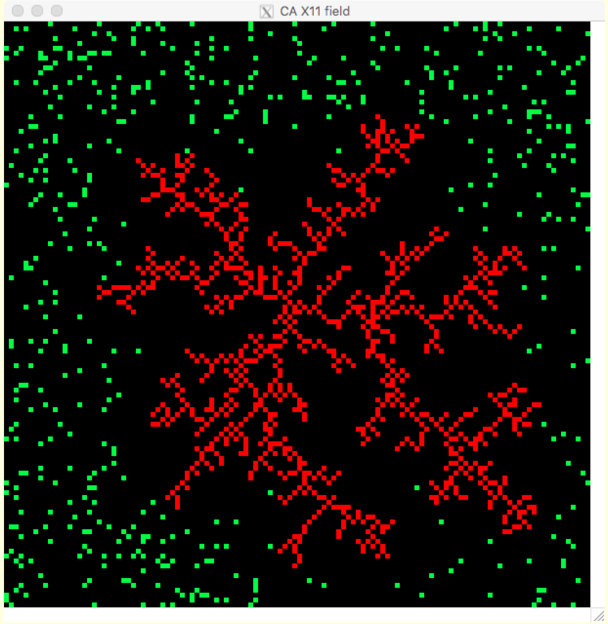
Phase transition in pattern formation



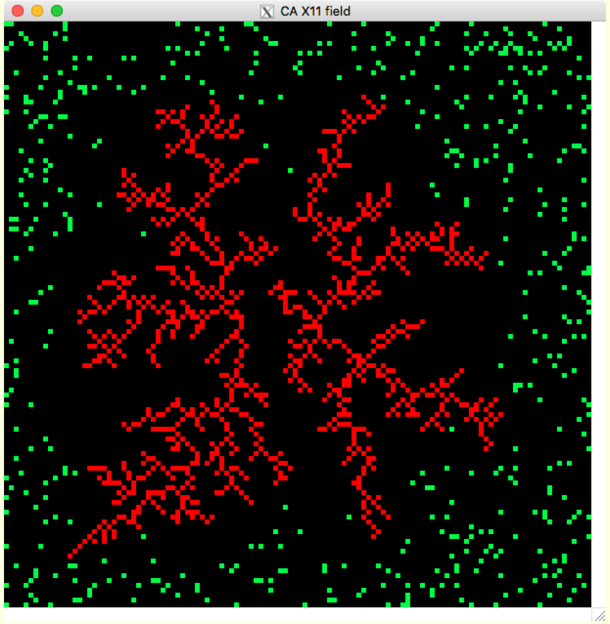


It turns out that there is a critical food density below which the pattern suddenly and abruptly changes from circular to fractal. This is because of the reversal of whether a local extension accelerates or reduces in growth rate. Also reducing the diffusion rate can trigger the fractal pattern.

Fractal growth patterns



1 colony



2 colonies

- Optimality of the pattern
- Competing colonies avoid each other, without the need to communicate

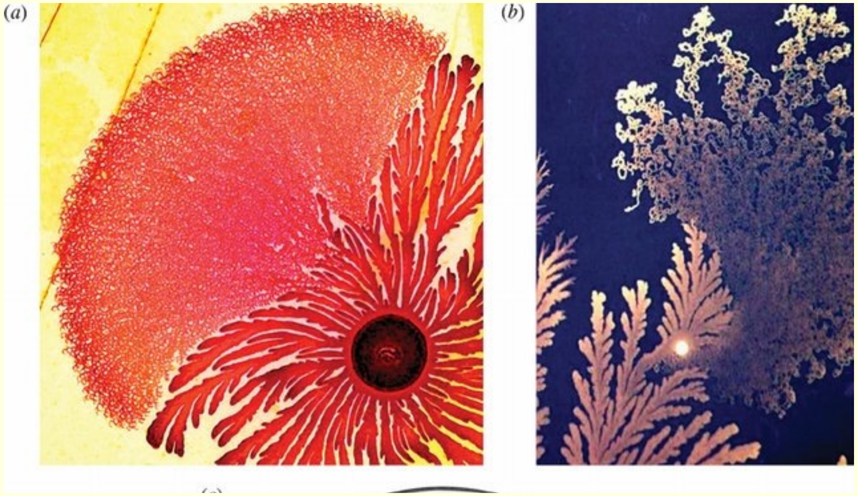
A role for evolution?

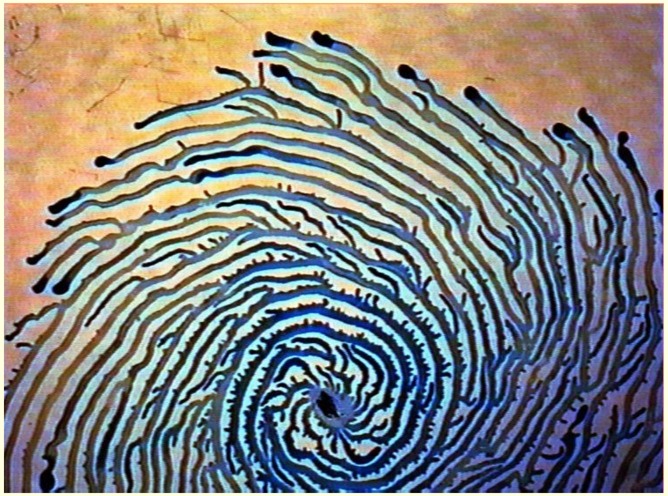
The fractal growth pattern seems adaptive, but the bacteria “get it for free”, that is, it is an emergent property of the interaction between the medium (the food) and the bacterial colony growth

What role could **evolution** play in the pattern formation of bacterial colonies at low density?

Patterns and Evolution

- Pattern formation affects colony efficiency
- Mutations can change and tune properties of the patterns





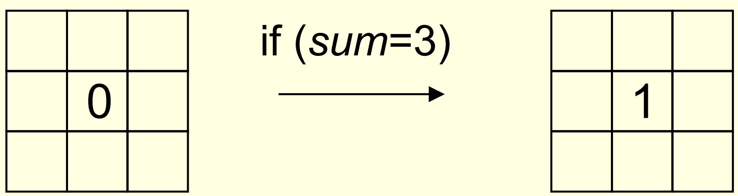
Colonies with mutant patterns, from: E. Ben-Jacob , H. Levine, R. Soc. Int. 2006

Game of Life

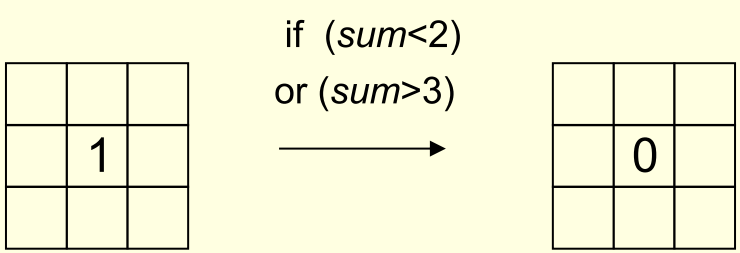
States: 0 = white = “E” = empty ; 1 = green = “X” = an individual

Processes:

Growth (if exactly 3 neighbours in state 1):

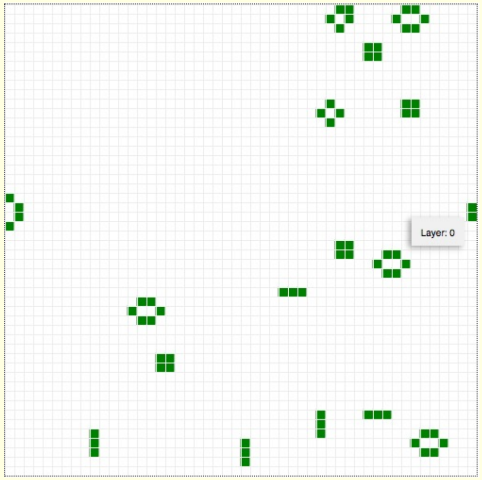


Death (if 1 or less, or 4 or more neighbours in state 1):



Practical Game of Life

(a) Run the model until it reaches a steady state or repeating state. Describe the patterns in this steady state. What is the density in this equilibrium? Could there potentially be an equilibrium state with a higher density

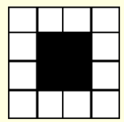


The equilibrium density for this figure is 0.028 (2.8% coverage). The equilibrium density can easily be increased by adding e.g. 2x2 blocks

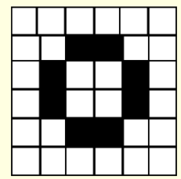
Theoretically, an equilibrium density with 50% coverage is possible. This equilibrium consists of alternating horizontal or vertical stripes. The equilibrium, however, is unattainable (i.e. has a zero domain of attraction), as any disturbance would lead to collapse of the pattern.

Some basic emergent patterns

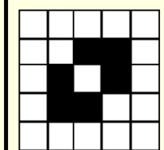
- Static:



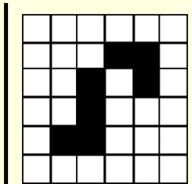
block



pond

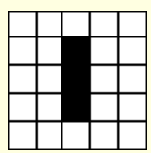


ship



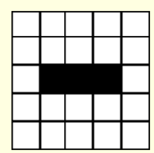
eater

- Cyclic stationary:



time = 1

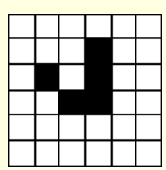
→



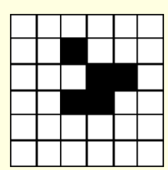
time = 2

- Cyclic moving:

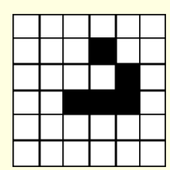
Glider (a “bit”)



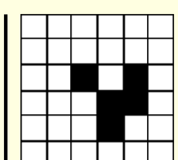
Time = 1



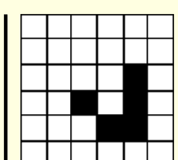
time = 2



time = 3



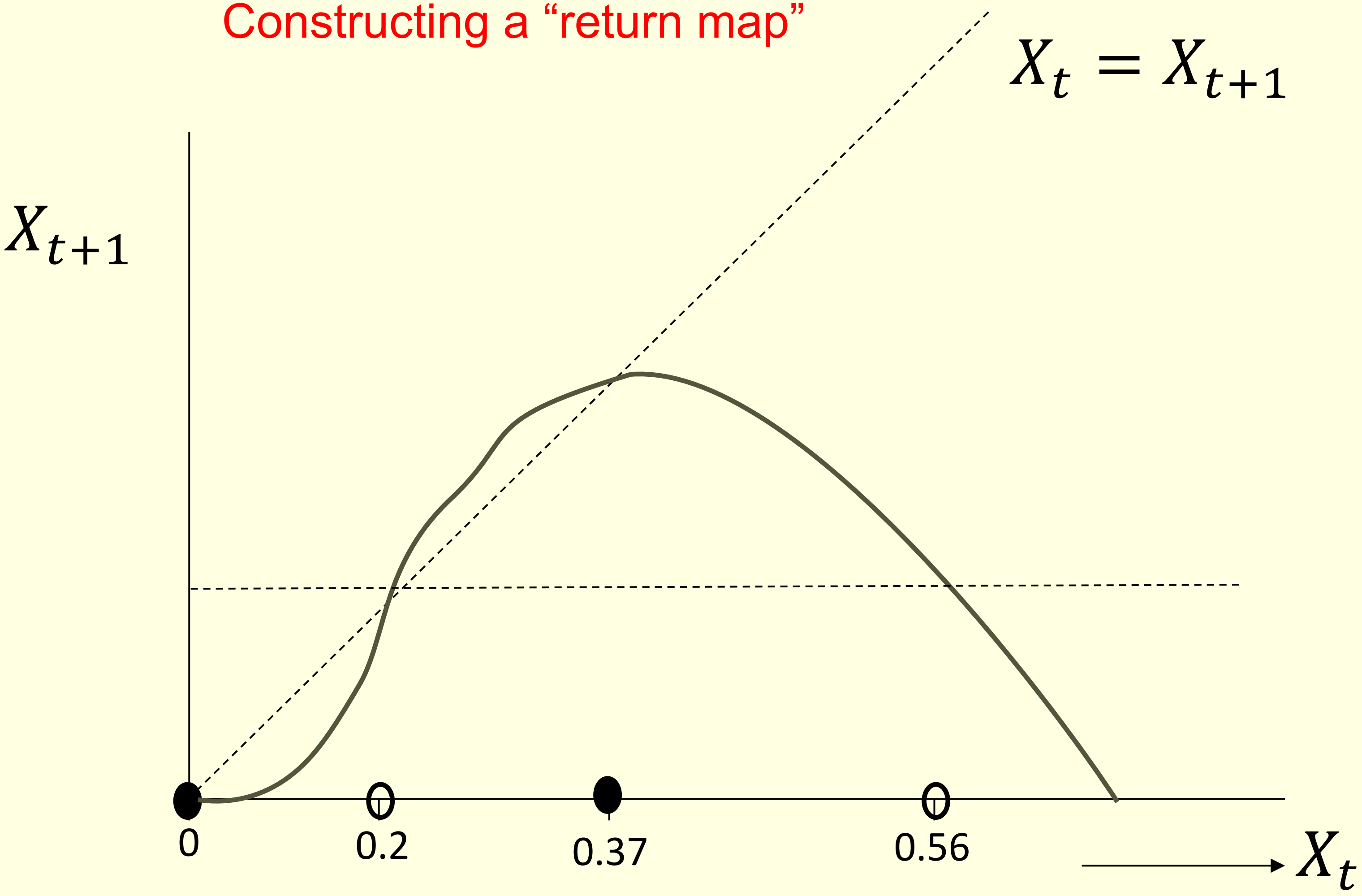
time = 4



time = 5

(b) Investigate the mean-field dynamics. Test the dynamics for different initial density. Try to assess equilibrium densities and indicate their stability.

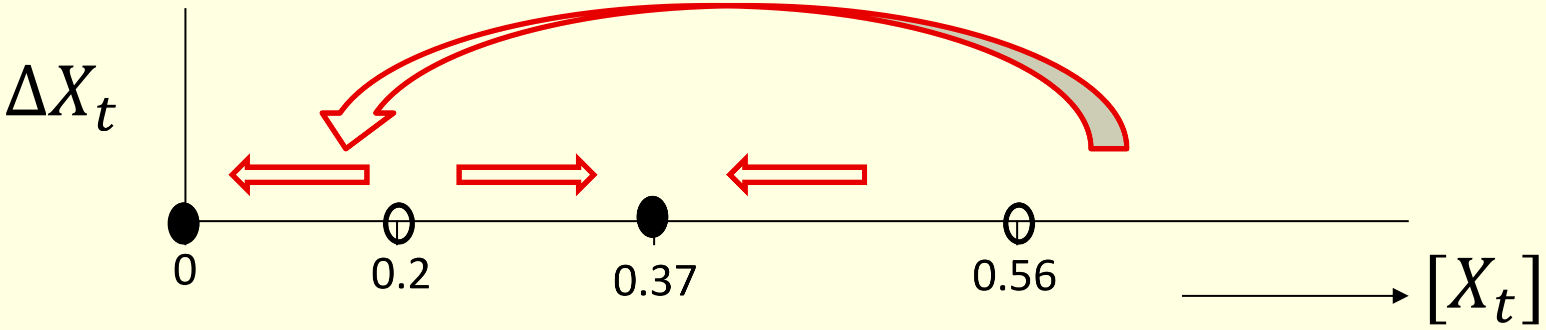
Constructing a “return map”



(b) Investigate the mean-field dynamics. Test the dynamics for different initial density. Try to assess equilibrium densities and indicate their stability.

The mean field model has stable equilibria at [X]=0 and [X]=0.37. In between these equilibria there is an unstable saddle node around [X]=0.2, below which you go to the [X]=0 equilibrium, and above which you go to the [X]=0.37 equilibrium.

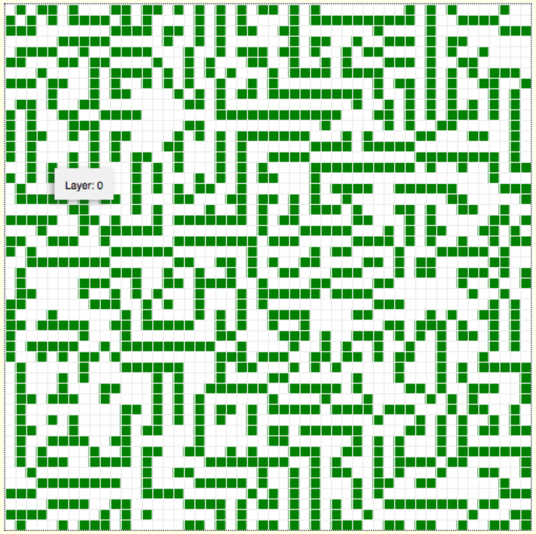
Furthermore, there is a second saddle node at around [X]=0.56. If you start above this density, in the next time step you will be below the first saddle node, and subsequently you will end up in the [X]=0 equilibrium. In summary, there is a region between 0.2 < [X] < 0.56 which is the domain of attraction of the [X]=0.37 equilibrium.



(c) What role does the mean field behaviour play in the local spatial dynamics of the Game of Life?

Actually, in the spatial Game of Life, the mean field dynamics play a role in the locally oscillating regions. In case locally the density is around 2/8 or 3/8, there will be an increase in the local population. However, because of the simultaneous updating, there is a possibility that in the next timestep the population is above the high saddle point of [X]=0.56, which will cause a local crash of the population. So, in a way, locally the population can oscillate around the [X]=0.37 equilibrium, but because of the large timestep (i.c. the simultaneous updating) it can oscillate towards the extinct equilibrium.

(d) What happens with “asynchronous” updating? Describe and try to explain the observed pattern and resulting equilibrium density.



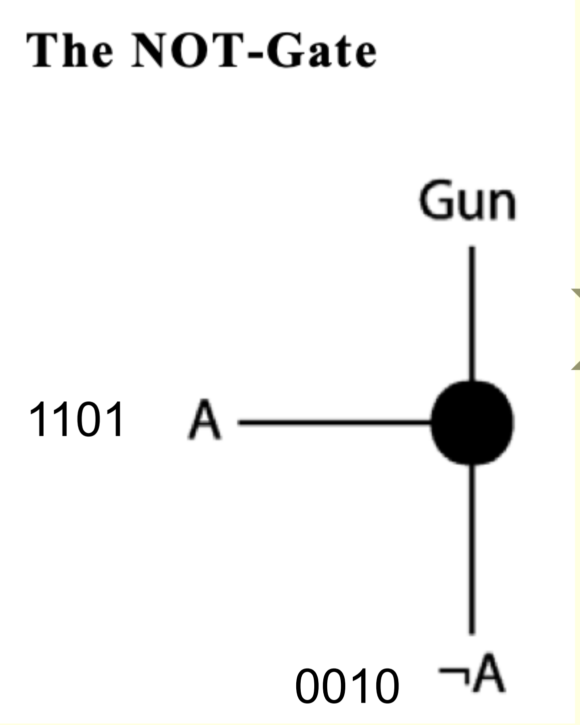
By applying updating of single cells, you strongly decrease the timestep in the model, therefore you no longer “overstep” the [X]=0.37 equilibrium. Actually, here something new happens, as the model discovers the striping pattern of question a.

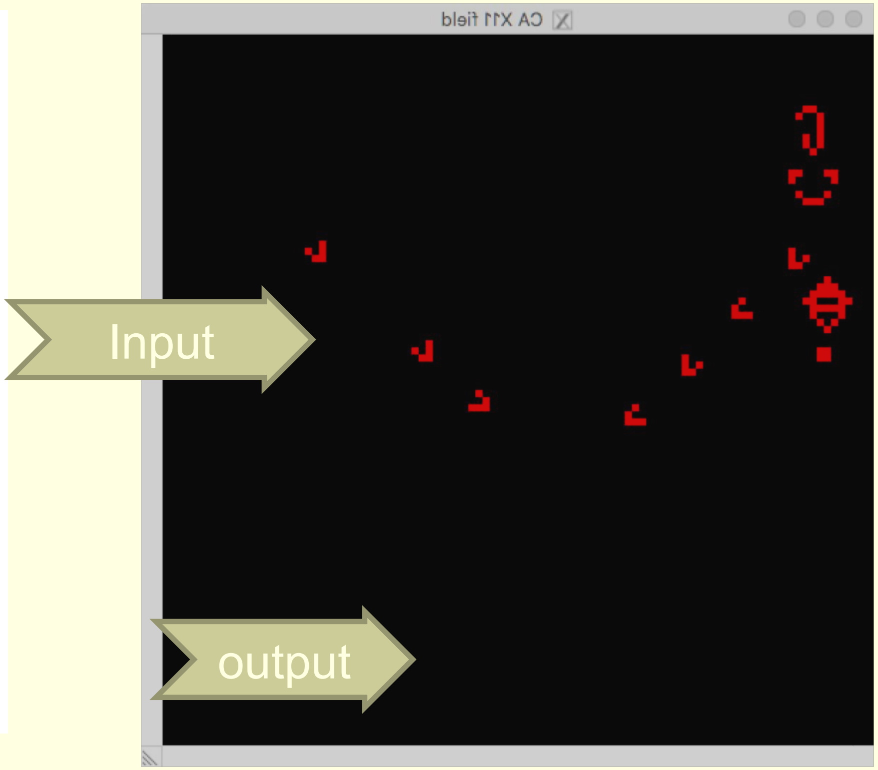
The equilibrium density is around [X]=0.42, which is below the theoretically possible [X]=0.5. This is because the system cannot decide between going to horizontal or vertical stripes. Interestingly, in a striping pattern the ‘1’s experience a very different local density than the ‘0’s, namely 2/8, versus 6/8.

Complexity in the Game of Life

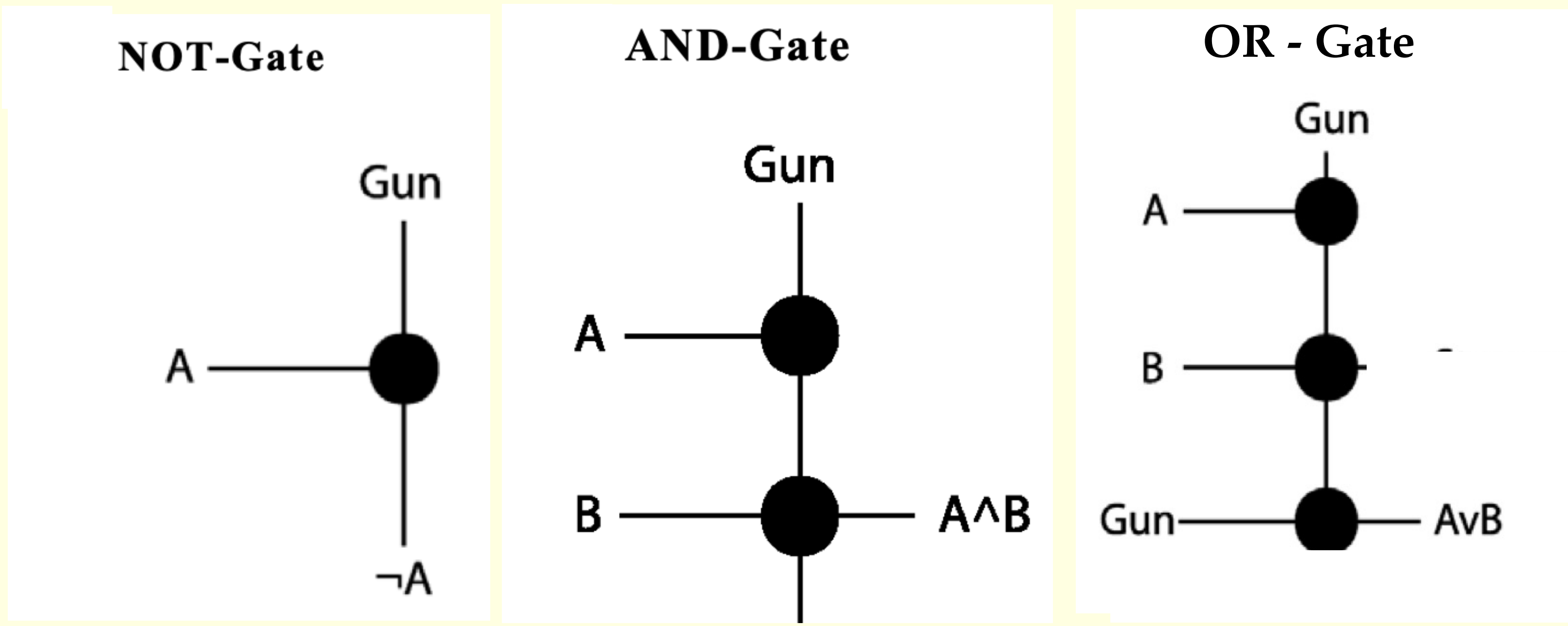
- A “glidergun” is a periodically repetitive pattern that produces “gliders” at a regular interval
- Each glider can be considered as a unit of information, and in this way the system can be used for computation

A simple logical gate





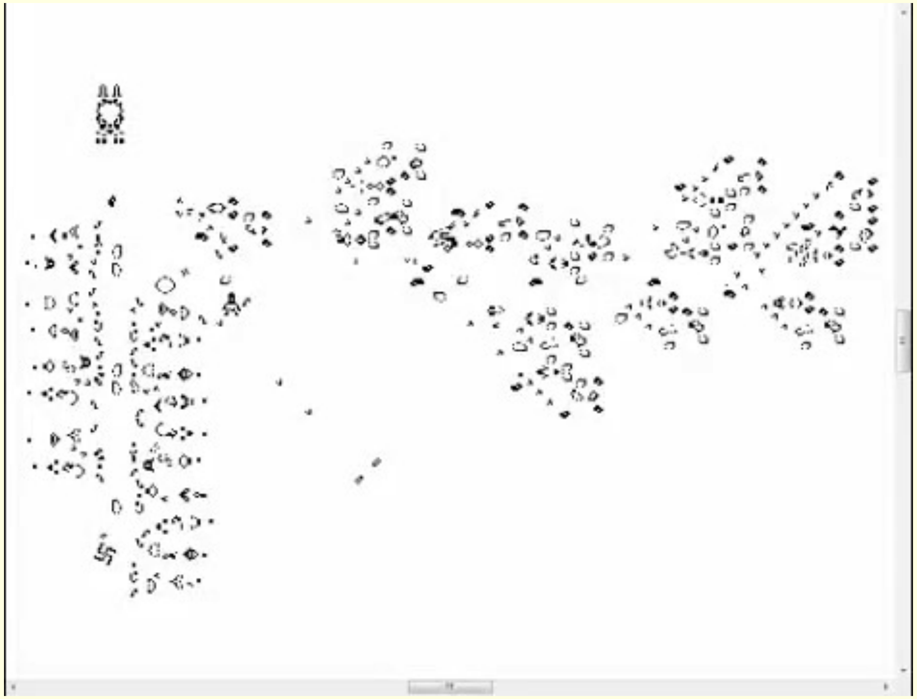
Basic logical operators



An amazing (proven!) theorem:

With a set of logical NOT, AND and OR operators you can build <u>any</u> computable function

GoL can do complex computations



e.g. producing gliders at prime number intervals:
https://www.youtube.com/watch?v=Bmaxrge3ANQ

Turing Machine

[Game of Life Turing Machine (Rendell 2010) and beyond ;-)]