Exploring millennial responses of reefs to climatic forcing, insights from coupled wave and carbonate growth forward stratigraphic model

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Key Points.

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Abstract. (Type abstract here)

1. Introduction

Over more than two decades, forward modelling of carbonate platform evolution have been successfully able to replicate many aspects of reef stratigraphic architectures [*Bosence and Waltham*, 1990; *Parcell*, 2003; *Hasler et al.*, 2008; *Barrett and Webster*, 2012; *Seard et al.*, 2013; *Huang et al.*, 2015; *Kolodka et al.*, 2016]. These studies are used to understand and make predictions of subsurface stratal geometries and facies but also to quantify the influences of both allocyclic and autocyclic processes on carbonate systems (*Williams et al.* [2011] or *Burgess* [2013] to cite the most recent ones). Due to the broad range of spatial and temporal scales as well as the complex physical and biological feedbacks involved in the formation of coral reefs, stratigraphic carbonate numerical models have understandably relied on several assumptions in regards to carbonate production, oceanic forcing and coral communities interactions. Their mathematical representation of stratal construction and sedimentary processes can be divided in three main families.

The convential approach simulates 3D development of either pure carbonate or mixed carbonate-silicilastic stratigraphies by modelling the main sedimentary processes using deterministic algorithms with time step of several hundreds to thousands of years. In this type of approach, the mathematical strategies often rely on diffusion laws, carbonate in-situ and sediment production rates associated to different carbonate factories as defined by *James and Kendall* [1992] and use a mix of physical and empirical-based modelling methods. Complexity of the mathematical techniques used in these models usually depends on the problem to address [*Granjeon and Joseph*, 1999; *Warrlich et al.*, 2002; *Toomey et al.*, 2016]. The most used models in this space are *Dionisos* [*Granjeon and Joseph*, 1999] and *Carbonate3D* [*Warrlich et al.*, 2002]. Two notable other examples are *Simsafadim* [*Bitzer and Salas*, 2001] which incorporates *predator-prey* algorithm to simulate the effect of biological activity and *GPM* [*Tetzlaff*, 2005; *Hill*, 2006] which address the problem of supersaturation on carbonate platform depositional geometries.

The second family of mathematical models proposes to mitigate the lack of observational data and highly nonlinear relationships between the different components of a reef system by using a fuzzy logic approach [*Meesters et al.*, 1998]. Fuzzy-based numerical models have proven to be a good alternative to conventional approach which often suffers from a lack of quantitative knowledge about the system being modeled, and therefore limits the effective use of available dataset [*Parcell et al.*, 1998]. In addition to purely fuzzy-based stratigraphic forward modelling approaches (*e.g. FuzzyReef* from *Parcell* [2003] or *Fuzzim* from *Nordlund* [1999]), other models have combined conventional hydrodynamics flow model and associated sediment transport with fuzzy logic and fuzzy-set theory to predict facies distribution in carbonate environments (an example being *Sedsim* [*Griffiths et al.*, 2001; *Salles et al.*, 2011]).

The last family cellular automata

To our knowledge, the models discussed above are all lacking the details of the wave hydrodynamic conditions and associated ocean circulation which are known to control not only the sediment transport in reef-lagoonal systems but also the dispersal of reef larvae and, therefore, the biogeographical distribution of reef organisms [*Lowe and Falter*, 2015]. In this paper, we design a numerical model that address this issue and enable to ... (I will wait to get some results before going further)

2. Physical description

In this section, a new, deterministic three-dimensional carbonate forward model, pyReef is presented, which simulates reef growth, reef transport and lagoon development based on the coupling between four components: a wave transformation model, a long-term circulation model, a calcareous sand transport model and a carbonate production/disintegration fuzzy logic model (Fig. 1).

pyReef is an open-source and parallel model mainly written in Python and capable of simulating reef system evolution over time scale of hundreds to thousands of years and over 1 to 10's kilometres scale. The model source code, its associated documentation along with the input files for the examples discussed in this paper are available on Github (http://github.com/pyReef-model). The code is designed to be applied within a variety of environments, from fringing and barrier reefs to carbonate ramps and atolls.

Below we provide a detailed description of the physical algorithms implemented in pyReef as well as the key assumptions underlying our approach.

2.1. Extrinsic forcings

At basin-scale carbonate deposits are strongly controlled by large-scale forcings. Extrinsic processes will affect both

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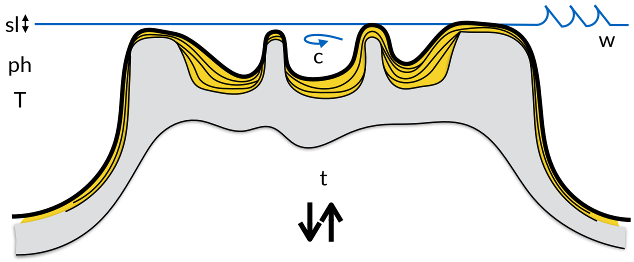

Figure 1. A schematic of 2D coral reef evolution model illustrating the main variables and forces simulated with *pyReef*, where **w** is wave forcing conditions, **c** is the long-term ocean circulation, **sl** is the sea-level, **t** is the tectonic and **ph** and **T** are the ocean's acidity and temperature respectively. The stratigraphic evolution and bed morphology are computes through time and are made of multiple coral assemblages.

the morphology of reef systems and stratigraphy of carbonate depositional sequences by modifying the carbonates production [*Hill*, 2006]. In pyReef, the following set of external forcing mechanisms could be considered: sea-level oscillations, subsidence/uplift rates, and regional oceanic conditions (*i.e.* sea temperature and acidity).
Accomodation space is a function of relative sea-level changes, which is the sum effect of eustatic sea-level changes, tectonic changes and sediment supply. Several studies have shown that the rates of accretion of coral reefs are largely constrained by changes in accomodation space (*Van Woesik et al.* [2015] or *Roff el al.* [2015] to cite a few). In our model, a sea-level curve can be imported from either a known eustatic curve (*e.g.* such as the ones from *Haq et al.* [1987] or *Miller et al.* [2005]) or directly defined by the user. The tectonic changes are provided as a series of temporal maps. Each map can have variable spatial cumulative displacements making it possible to simulate complex tectonic evolution with both uplift and subsidence conditions. These two forcing mechanisms will directly control the evolution of the hydrodynamic conditions and the associated sediment transport regime as well as the carbonate production described in the following subsections and illustrated in figure 1.
Changing sea surface temperatures and ocean acidification are known to have significant effects across reef systems by controlling the rate of coral reef growth [*Shaw et al.*, 2012; *Andersson and Gledhill*, 2013; *Zhang et al.*, 2013]. In pyReef, long-term regional scale evolution of either ocean's temperature or pH are set by the user as temporal-dependent functions. These functions are then used to control the carbonate production and disintegration as explained in subsection 2.5 (Fig. 1).

2.2. Wave transformation

SWAN, short for *Simulating WAves Nearshore*, is a third-generation, finite-difference, wave model used to predict wave propagation in coastal areas and estuaries. It is governed by the wave action balance equation [*Bretherton and Garrett*, 1968; *Hasselmann et al.*, 1973; *Holthuijsen et al.*, 1993; *Booij et al.*, 1999]:

$$\frac{\partial N}{\partial t} + \nabla_{\vec{x}} \cdot \left[\left(\vec{c_g} + \vec{U} \right) N \right] + \frac{\partial c_\theta N}{\partial \theta} + \frac{\partial c_\sigma N}{\partial \sigma} = \frac{S_{tot}}{\sigma} \quad (1)$$

where $N(\vec{x}, t, \sigma, \theta)$ is the wave action function of geographical space $\vec{x}$, time t, relative frequency σ and wave direction θ. $\nabla_{\vec{x}}$ is the gradient operator in space, $\vec{c_g}$ and $\vec{U}$ are the wave group velocity and ambient current vector respectively and c_θ, c_σ is the propagation velocity in θ and σ domain. Finally S_{tot} is the source term which can include wind, whitecapping, surf breaking and bottom friction [*Booij et al.*, 1999]. In our model, shoaling and refraction are accounted for from a series of deep-water wave conditions through time in the absence of wind forcing. Hence to compute wave field generation, the model requires bathymetric conditions and definitions of offshore significant wave height, characteristic period of the energy spectrum, wave direction and associated spreading angle (Fig. 2 panel a and Dir/T distribution plot). To evaluate reef responses over several hundreds of years, the approach taken here does not examine temporal evolving wave fields, such as those produced during storm events and relies on SWAN stationary mode. In pyReef, the wave transformation model is generally performed for time intervals varying from 0.5 to 10 years. Our aim is to simulate realistic wave fields by imposing a sequence of wave forcing conditions (*e.g.* series of fair-weather and/or storm events). At any given time interval, we define a percentage of activity for each deep-water wave conditions and the bathymetry is used to compute associated wave parameters. Possibility is given to derived these parameters for both low and high tides.

Combined with the climatic forcing described above, two additional wave factors could be adjusted in the model: the breaking parameter and the bottom friction.
In regions where wave height is close to water depth, wave breaking is an important source of energy dissipation on reefs [*Symonds et al.*, 1995; *Becker et al.*, 2014]. This effect is typically approximated with a constant breaking parameter γ_s [*Symonds et al.*, 1995; *Vetter et al.*, 2010] which values have been calibrated for different reef systems [*Apotsos et al.*, 2007; *Vetter et al.*, 2010; *Monismith et al.*, 2013; *Franklin et al.*, 2013; *Rogers el al.*, 2015].
The high rugosity of reefs plays a significant role on wave dynamics by increasing the frictional dissipation of wave energy flux [*Young*, 1989; *Lowe et al.*, 2005; *Lowe and Falter*, 2015]. This dissipation is usually approximated with a wave roughness friction factor f_w which values have been well constrained for sand grain [*Kamphuis*, 1975; *Grant and Madsen*, 1979; *Dean and Dalrymple*, 1991]. In phase-averaged wave action approach like SWAN, this bottom dissipation is parameterised as a function of wave excursion to bottom roughness scale with a maximum value of 0.3 for f_w [*Jonsson*, 1966; *Madsen et al.*, 1988]. Several studies [*Nelson*, 1996; *Lowe et al.*, 2005; *Lentz et al.*, 2015; *Rogers el al.*, 2015; *Monismith et al.*, 2015] indicates that this roughness factor can be much higher for reef systems (*i.e.* up to 5.0 for reef platform in the Red Sea [*Lentz et al.*, 2015]). To better estimate the impact of reef rugosity on frictional dissipation, we have modified the existing formulation for f_w in SWAN and used the proposed parameterisation from *Rogers el al.* [2015] based on *Swart* [1974]:

$$f_w = \begin{cases} exp\left[a_1 \left(A_b/k_N\right)^{a_2} + a_3\right], & A_b/k_N \geq 0.0369 \\ 50, & A_b/k_N < 0.0369 \end{cases} \quad (2)$$

where A_b is the wave excursion distance, k_N is the bottom roughness scale and the coefficients a_1 =5.213, $a_2 = -0.194$, and $a_3 = -5.977$ have been set based on *Rogers el al.* [2015] Palmyra study. Alternate coefficient from *Nielsen* [1992] can also be used (a_1 =5.5, $a_2 = -0.2$, and $a_3 = -6.3$). For large values of A_b/k_N, this formulation is similar to the one from Madsen et al. [1988] (implemented in SWAN), but extends the parameterisation for lower A_b/k_N. In pyReef, the bottom friction is based on the proposed formulation and requires the definition of the bottom roughness scale (k_N)

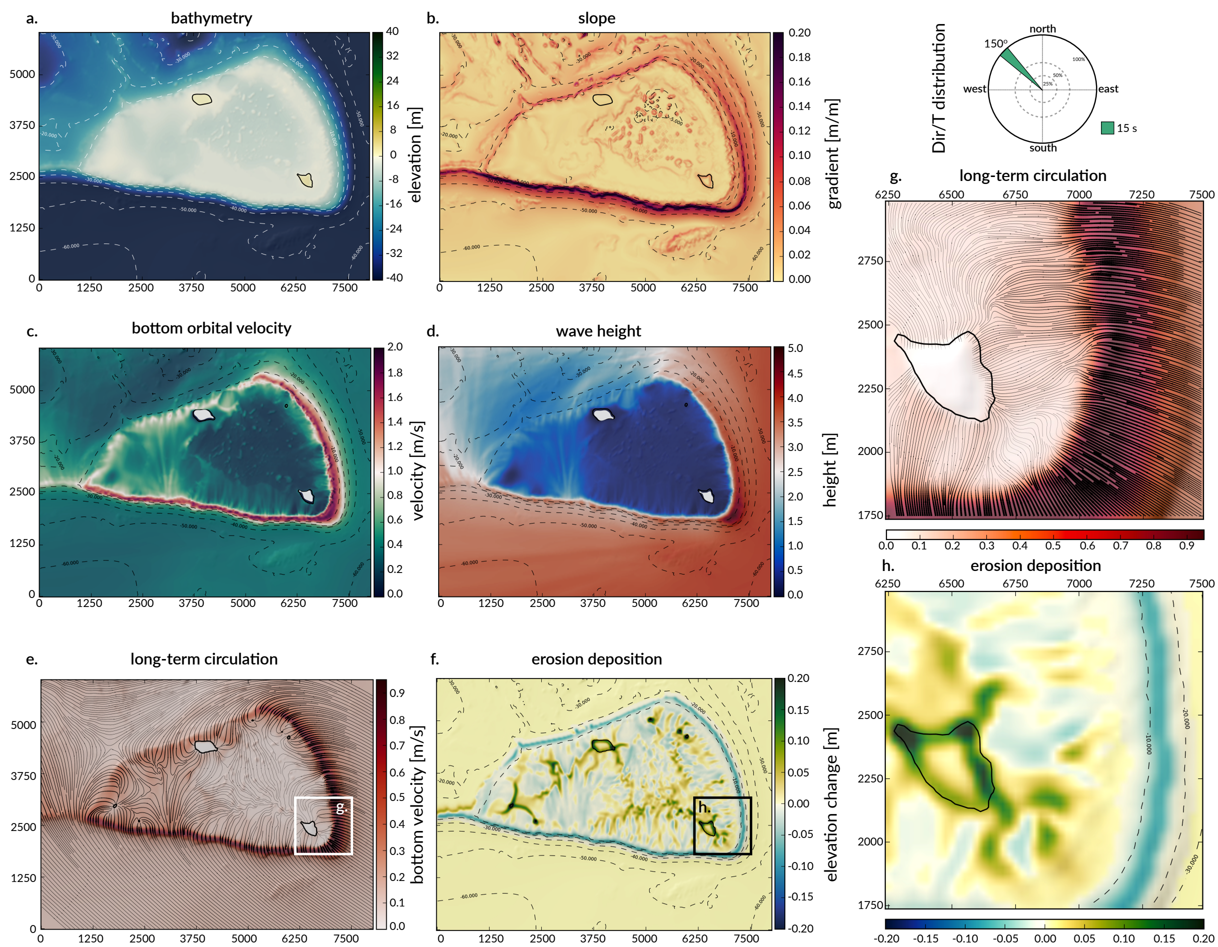

Figure 2. Illustration of sediment transport calculation in pyReef. At time step t, a gradient map (**b**) is first derived from the bathymetry (**a**). In this example, the circulation is forced by a boundary wave condition characterised by a significant wave height of 4 m, a characteristic period of the energy spectrum of 15 s, and a direction of 150 degrees. The wave transformation model is ran on the bathymetry provided at time t with a reef roughness constant of 0.1 and a wave breaking parameter of 0.66. From the considered boundary conditions, estimates of bottom orbital velocity (**c**) and wave height (**d**) are returned and used to compute wave-induced circulation in the area (**e** and **g**). The bottom velocity in panel **e** adds together the longshore and cross-shore components presented in section 2.3. The circulation is assumed to be representative of the wave conditions in the area for 5% of the wave forcing time interval (set to 10% of the year (roughly 1 month)). From the bottom velocity and main current and wave directions, the model then calculates sediment erosion based on the approach from *Soulsby* [1997]. The erosion and deposition apply only to loose carbonate sands (here only one type of coral assemblages is considered). Erosion takes place on the active layer which is set to 0.25 m and contains 50% of loose sediments. In addition to circulation-driven transport, a gravity-driven one is implemented and relies on the gradient map (**b**) and a diffusion coefficient set to 0.1 m^2/yr in this example. The resulting erosion deposition map is presented in panels **f** and **h**. From this map both the stratigraphic layers and surface are updated and either a new wave boundary forcing conditions active during the considered time interval is ran or the carbonate growth model is applied.

which values is generally set to 2-3 times the characteristic diameters of the studied region [*Nielsen*, 1992; *Lowe et al.*, 2005; *Rogers el al.*, 2015].

For each forcing conditions, the wave transformation model computes and returns the significant wave height, the mean wave direction and the root-mean-square value of the maxima of the orbital velocity near the bottom (panels c and d in figure 2 shows an example of bottom orbital velocity and wave height maps). These parameters are subsequently used to evaluate the long-term hydrodynamic forces active over the simulated region.

2.3. Long-term wave-driven circulation

Flow circulation in and around reef platforms depends on the complex interactions between the overlying water motion and the three-dimensional bottom roughness formed by reef organisms. Attempts to numerically simulate flow dynamics around individual coral colonies have been proposed [*Kaandorp et al.*, 2003; *Chang et al.*, 2009; *Chindapol et al.*, 2013] but such models require the flows to be solve down to few millimetres scale and are beyond the scope of our approach. Here we assume that the flow circulation in the reef platform is mainly driven by waves and regional-scale drivers of reefs hydrodynamics such as coastal upwelling or ocean currents are ignored. Numerical studies of wave-driven flow around reef systems commonly use depth-averaged Navier-

Stokes equations [*Raupach and Shaw*, 1982; *Symonds et al.*, 1995; *Lowe et al.*, 2005, 2009; *Pomeroy et al.*, 2012; *Taebi et al.*, 2011]. In the context of millennial scale reef platform evolution, these methods however are still computationally prohibitive. In pyReef, the proposed method consists in producing *snapshots* of wave-driven circulation distribution resulting from series of deep-water wave scenarios by computing time-averaged cross-shore and longshore currents (Fig. 2 panels e and g).

In nearshore environments, longshore current runs parallel to the shore and is generated by the radiation stresses associated with the breaking process for obliquely incoming waves and by the surplus water which is carried across the breaker zone towards the shoreline [*Longuet-Higgins*, 1970]. This current affect the transport and transfer of mass (loose carbonate sands, nutrients and carbon) in the nearshore reef waters [*Hamner and Wolanski*, 1988; *Monismith*, 2007; *Lowe and Falter*, 2015]. Many empirical formulation of longshore current have been proposed since the initial work from *Longuet-Higgins and Stewart* [1964] [*Komar and Inman*, 1970; *Komar and Miller*, 1975; *Galvin*, 1987; *Reniers and Battjes*, 1997; *Ruessink et al.*, 2001; *Grasmeijer and Ruessink*, 2003]. In pyReef, the approach from *Komar and Miller* [1975] is used to calculate the longshore current velocity ($\vec{v_l}$) in the middle of the breaking zone:

$$\vec{v_l} = \kappa_l u_b cos(\theta) sin(\theta) \vec{k} \tag{3}$$

where u_b is the maximum near-bed orbital velocity obtained from SWAN, θ the angle of incidence of the incoming waves, κ_l a scaling parameter and $\vec{k}$ the unit vector parallel to the breaking depth contour. For wave rays approaching the reef at on oblique angle, the component of wave energy flux parallel to the reef shore will drives this longshore velocity. The calculation of the angle of incidence in pyReef is quite straightforward and requires an estimate of wave breaking depth (defined for each wave scenario) and wave direction (obtained from the wave transformation model).

In addition to longshore current, two types of cross-shore velocities are simulated in pyReef. First we estimate the onshore velocity which is essential in predicting the shoreward transport of broken carbonate particles during fair-weather periods [*Elfrink et al.*, 1999; *Ruessink et al.*, 1998]. In our model, we assume a linear dependency between the near-bed orbital velocity and the intensity of this onshore current $\vec{v_o}$:

$$\vec{v_o} = \kappa_o u_b \vec{n} \tag{4}$$

with u_b the maximum near-bed orbital velocity, $\vec{n}$ the unit vector parallel to the incoming wave direction and κ_o a correction factor. κ_o can be derived from local shallow reef water conditions and reflects natural wave skewness and asymmetry which are known to play a central part in cross-shore velocity profile [*Grasmeijer and Ruessink*, 2003; *Crawford and Hay*, 2003]. The onshore velocity ($\vec{v_o}$) is usually associated to the wave crest and is stronger than the one due to the wave trough [*Isobe and Horikawa*, 1982; *Grasmeijer and Ruessink*, 2003].

During strong wave conditions, a second type of wave-induced cross-shore velocity is defined in pyReef and simulates an offshore-directed steady current referred to as undertow. The time-averaged and depth-averaged undertow velocity $\vec{v_u}$ is derived from the mass flux due to the wave motion and surface roller [*Svendsen et al.*, 1987]. Under normal incident periodic waves, *Longuet-Higgins* [1975] showed that the depth- and time-averaged undertow velocity is related to the wave's kinetic energy density and phase speed. Assuming equipartition of kinetic and potential wave energy, the total energy density of the wave is approximately [*Svendsen*, 1984]:

$$E_w = \frac{1}{8} \rho g H^2 \tag{5}$$

with ρ the ocean density and H the root-mean-square wave height returned by SWAN. Following *Cox and Kobayashi* [1998], the undertow velocity is assumed to have the following form in our model:

$$\vec{v_u} = -\kappa_u \frac{\sqrt{gh}}{8} \left(\frac{H}{h} \right)^2 \vec{n} \tag{6}$$

with κ_u an empirical coefficient [*Kobayashi et al.*, 1998], g the gravitational acceleration, h the water depth and $\vec{n}$ the unit vector parallel to the incoming wave direction.

2.4. Calcareous sand transport

A common feature of many reef systems is the presence of diverse marine communities, including seagrasses, coral reef organisms, sponges and mangroves. Simulating the evolution of these communities involves to resolve the details of their individual transport by waves and currents at fine scale and therefore requires very advanced and computationally expensive numerical models [*Lowe and Ghisalberti*, 2016]. The approach proposed here is based on a parametric transport model that consider the main hydrodynamic forces presented in previous section and relies on coral assemblages rather than individual communinities. Coral assemblages are derived from coral composition and comparison with modern coral zonation. In the Southern Great Barrier Reef, from 22 identified fossil coral species, *Dechnik et al.* [2015] was able to define four main coral assemblages and their palaeoenvironments. In pyReef, multiple coral assemblages can be defined and we assume that each assemblage is made of hard and loose corals. The hard coral (*e.g.* living part) can either grow in-situ or be disintegrated in loose particles. Only the loose particles are subject to transport and we assume that these particles can be represented by a unique diameter and density value.

To our knowledge, transport law for calcareous sand based on their hydraulic characteristics has not been proposed yet. It has been recognised [*Prager et al.*, 1996; *Dai*, 1997; *Smith and Cheung*, 2004] that variations in the hydraulic properties between siliceous and calcareous sand due to both their shapes and fall velocities lead to different transport behaviors. From flume experiments, *Smith and Cheung* [2005] shows that transport models designed for siliceous particles might be applied after a correction factor has been introduced. From their experiment, the model of *Engelund and Hansen* [1967] seems to provide the best fit with their dataset, however they conclude that these corrections are not universally applicable.

Therefore, in pyReef, we have chosen to build our transport model based on a classical approach. Assuming sediment transport induced by waves and currents, our method is derived from Soulsby and Van Rijn formulation [*Soulsby*, 1997]. We compute the total load transport rate resulting from the addition of bed load and suspended load for each type of loose carbonate sands initially set by the user. The model also assumes that each of sand particles are perfectly sorted. The method suits well with our approach as it can be applied in a quasi-steady form. An example of erosion deposition for a given long-term circulation representation is illustrated in figure 2 panels f and h. The transport rate is defined by:

$$q_t = (A_{sb} + A_{ss}) \bar{v} \left[\sqrt{\bar{v}^2 + \frac{0.018}{C_D} u_b} - \bar{u_{cr}} \right]^{2.4} (1 - 1.6 \nabla z) \tag{7}$$

where $\bar{v}$ is the depth-averaged current velocity obtained from the long-term wave driven circulation model defined in previous section, C_D is the drag coefficient (due to current alone) and ∇z the slope (Fig. 2 panel b). The bed load transport A_{sb} is given by:

$$A_{sb} = \frac{0.005h(d/h)^{1.2}}{(s-1)gd} \tag{8}$$

with s is the relative density of sediments and d is the median grain diameter. The suspended load transport A_{ss} is obtained from the following formula:

$$A_{ss} = \frac{0.012hd_\star^{-0.6}}{(s-1)gd} \tag{9}$$

where $d_\star$ refers to the dimensionless particle diameter. The threshold current velocity of motion ($\bar{u_{cr}}$) is defined by:

$$\bar{u_{cr}} = \begin{cases} 0.19\, d^{\,0.1}\, log_{10}(4h/d),\ 0.1 \geq d \geq 0.5\,\mathrm{mm} \\ 8.5\, d^{\,0.6}\, log_{10}(4h/d), \quad 0.5 \geq d \geq 2.\,\mathrm{mm} \end{cases} \tag{10}$$

In addition, to the above sediment transport model a multi-lithology non-linear diffusion model has been implemented to simulate secondary gravity-driven transport processes happening over longer temporal scale than wave and current induced ones. Based on *Rivenaes* [1997] and considering n different types of coral assemblages, the following set of nonlinear partial differential equations (PDEs) are used to calculate the proportion of each assemblage a_k:

$$\begin{aligned} &\textstyle\sum_{k=1}^n a_k = 1 \\ &\tfrac{\partial z}{\partial t} = \textstyle\sum_{k=1}^n \tfrac{1}{c_k} \nabla \cdot (\kappa_k a_k \nabla z) \\ &d_l \tfrac{\partial a_k}{\partial t} + a_k \tfrac{\partial z}{\partial t} = \tfrac{1}{C_k} \nabla \cdot (\kappa_k a_k \nabla z) \end{aligned} \tag{11}$$

where κ_k denotes the diffusion coefficient for sediment k, c_k is the compaction ratio and d_l is a constant representing the thickness of a prescribed top layer, in which sediments are transported. This set of PDEs is solved using a fully explicit schema following the approach from *Clark et al.* [2010].

To simulate bed morphology and stratigraphic evolution a classical multi-level bed framework tracks the distribution of every coral assemblage through time by layers (*Warner et al.* [2008] and reference therein). In pyReef, each layer stores the bulk properties including thickness and percentage of hard and loose coral assemblages. An active layer at the top of the stratigraphic pile is used to calculate the transport of the calcareous sand from both the wave-current driven and diffusion-based models presented above.

2.5. Carbonate growth and disintegration

The organisation of coral reef systems is known to be large and complex and we are still limited in our understanding of their temporal and spatial evolution [*Demicco*, 1998]. Additionally, most datasets of carbonate systems are often linguistic, context-dependent, and based on measurements with large uncertainties. Conventional deterministic techniques are often enable to address many of the significant variables which affect carbonate productivity [*Parcell et al.*, 1998].

Alternative approaches such as fuzzy logic, which is by definition able to cope with these imprecisions [*Demicco and Klir*, 2001; *Collin et al.*, 2015], have proven to be a viable alternative to simulate carbonate systems [*Salles et al.*, 2011; *Hattab et al.*, 2013]. Fuzzy logic method is able to create logical propositions from qualitative data by using linguistic logic rules and *fuzzy sets* [*Nordlund*, 1996]. These fuzzy sets are defined with continuous boundaries rather than *crisp* discontinuous ones usually used in conventional approaches [*Meesters et al.*, 1998]. In depth mathematical theory behind fuzzy logic method can be found in *Zadeh* [1965], *Zimmerman* [1991] and *Berkan and Trubatch* [1997].
Based on a fuzzy logic approach, carbonate system evolution in pyReef is driven entirely by a set of linguistic rules whose variables are fully adjustable. Therefore, the utility and effectiveness of the approach is mostly based on the user's understanding of the modelled carbonate system. The technique is specifically useful to understand how particular variable, in isolation or in combination with other factors, influences carbonate depositional geometries and reef adaptation (Fig. 3).

In its current form, pyReef employs five types of control variables: depth, wave energy (derived from ocean bottom orbital velocity), sedimentation rate, ocean's temperature and acidity. For each of these variables, one can define a range of fuzzy sets using membership functions [*Nordlund*, 1999]. A membership function is a curve showing the degree of truth (*i.e.* ranging between 0 and 1) of membership in a particular fuzzy set (Fig. 3). In pyReef, these curves can be simple triangles, trapezoids, bell-shaped curves, or have more complicated shapes as shown in Fig. 3. Production and/or disintegration of any specific coral assemblage is then computed from a series of fuzzy rules. A fuzzy rule is a logic *if-then* rule defined from the fuzzy sets [*Demicco and Klir*, 2001]. In our model, the combination of the fuzzy sets in each fuzzy rule is restricted to the *and* operator. The amalgamation of competing fuzzy rules is usually referred to as a fuzzy system. Summation of multiple rules from the fuzzy system by truncation of the membership functions produces a new fuzzy answer in the form of a new membership set (Fig. 3). The last step consists in computing a single number for this fuzzy set through *defuzzification* [*Zadeh*, 1965]. In pyReef, the centroid (center of gravity) for the area below the membership set is taken as the *defuzzified* output value.

Coupling of the wave transformation approach (SWAN) with the long-term circulation and sediment transport models and with the fuzzy logic technique presented above allow for numerical analysis of carbonate platforms evolution, stratigraphic architecture reconstruction and provides a numerical framework to quantitatively assess reef system responses to climatic forcing over millennial scales.

3. Models setup

4. Results

impact of storms on carbonate evolution (model with fair weather and model with 1 storm and after 2 storms)
impact of sea-level change and thermal subsidence
impact of change in pH and ocean temperature

5. Discussion

6. Conclusions

Acknowledgments. (Text here)

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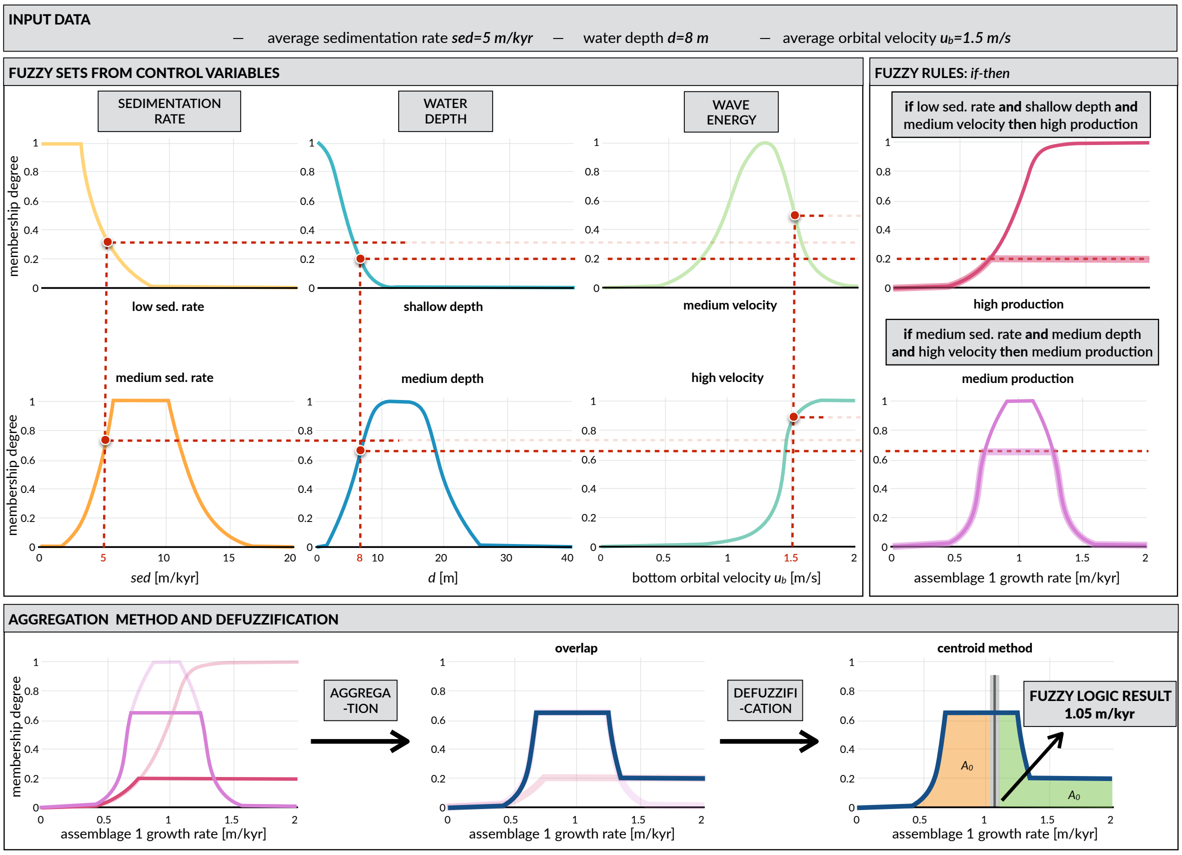

Figure 3. Diagram of fuzzy logic process used in pyReef to evaluate a specific coral assemblage (assemblage 1) growth rate. The approach is illustrated from a point on the carbonate platform with a water depth of 8 m, an average sedimentation rate of 5 m/kyr and an average bottom orbital velocity of 1.5 m/s. The production rate is related to three control variables in this example: the *sedimentation rate*, the *water depth* and the *wave energy*. For each of these variables two membership functions are defined (as an example the water depth is described using the *shallow depth* and *medium depth* functions). The combination of these functions forms a fuzzy set. Two fuzzy rules (based on *if-then* rules) control the production of coral assemblage 1. Each production membership function is then restricted by the minimum (*and* operator) of the membership degree values obtained from combination of the functions active in the considered rule. Aggregation of the truncated production membership functions is done by overlapping the curves and taking the maximum values. Finally the evaluation of the production rate is done through defuzzification by employing the centroid method which returns a *crisp* value of 1.05 m/kyr for the considered point.

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