Transit delay management

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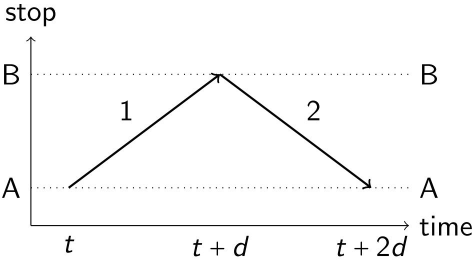
Outline

Motivation

Consider two trips, each with planned travel time d.



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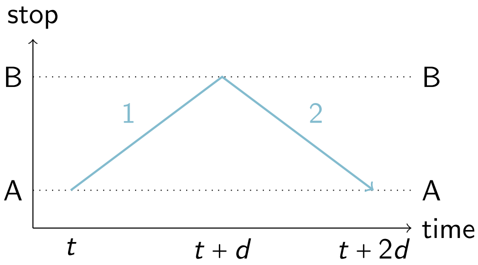
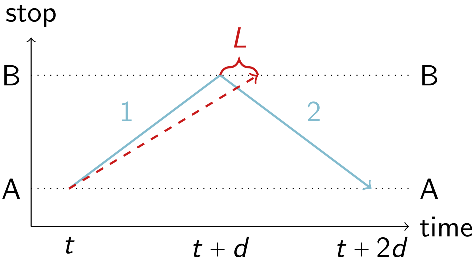


- Both may be operated by a single vehicle.

Motivation

Let L be a random variable representing the *primary delay* experienced on trip 1.



Definition: primary delay

Primary delay is the delay experienced throughout a trip's operation.

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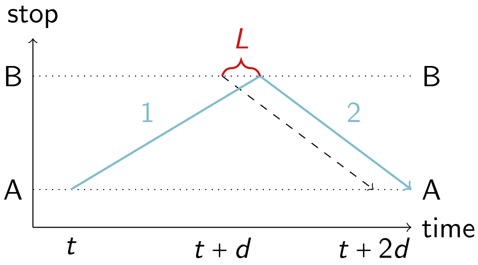
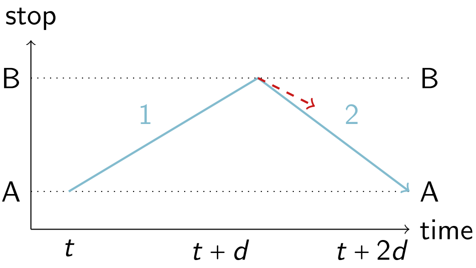


- If $L > 0$, then trip 2 will experience a *secondary delay* of L.

Definition: secondary delay

Secondary delay is the amount of time by which a trip is delayed from departing at its planned time.

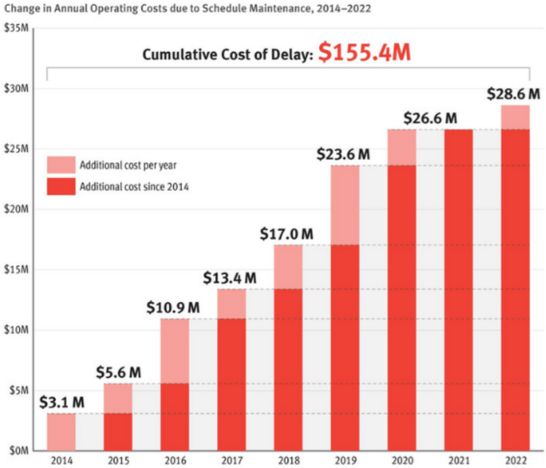
Motivation



Of course, trip 2 may also experience some primary delay ...

Motivation

In 2021, 34% of TransLink's service costs came from delays.



Motivation

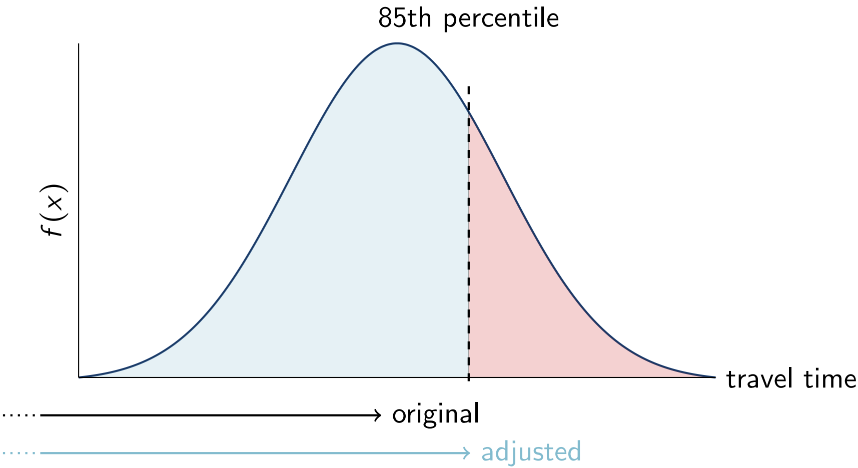
Key insights

- With many trips, delays can compound significantly.
- Vehicle scheduling influences delay propagation.

Outline

Run time analysis (RTA)

A common approach among transit agencies to address this problem is called run time analysis (RTA).



RTA*

Instead, find a *critical* percentile following the classical newsvendor model:

$$(\mathrm{RTA}^*) \qquad \min_{\Delta \geq \mathbb{E}[L]} \quad c^{\mathrm{srv}}\Delta + rh\mathbb{E}[\max\{L - \Delta, 0\}]. \tag{1}$$

- c^{srv} : the cost per hour of service operation
- h : the cost per rider-hour of passenger delay
- L : a random variable representing the trip's primary delay
- r : the trip's ridership
- Δ : the amount of time by which the trip's travel time is to be adjusted

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$$(\text{RTA}^*) \qquad \min_{\Delta \geq \mathbb{E}[L]} \quad c^{\text{srv}}\Delta + rh\mathbb{E}[\max\{L - \Delta, 0\}]. \tag{1}$$

The solution to (1) is

$$\Delta^* = \max\left\{\mathbb{E}[L], F^{-1}\left(1 - \frac{c^{\text{srv}}}{rh}\right)\right\}, \tag{2}$$

where $F(\cdot)$ is the cumulative density function of L and $F^{-1}(\cdot)$ is its inverse.

RTA*

$$\Delta^* = \max\left\{\mathbb{E}[L], F^{-1}\left(1 - \frac{c^{\text{srv}}}{rh}\right)\right\}$$

Example:

- L : symmetric around mean 0.
- c^{srv} : \$160/hour[1]
- h : \$37/rider · hour[2]

Then, $\Delta^* > 0$ if

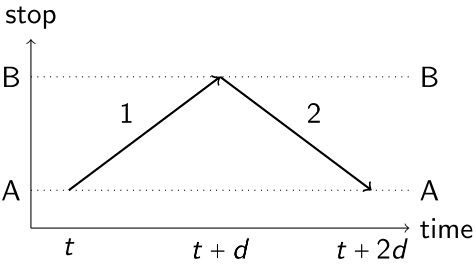
$$r \geq \frac{2c^{\text{srv}}}{h} \approx 9 \text{ riders.}$$

[1] From the BC Transit service plan.

[2] The average hourly wage (Litman, BC Stats).

RTA*

Let trips 1 and 2 be subject to IID primary delays L_1 and L_2.



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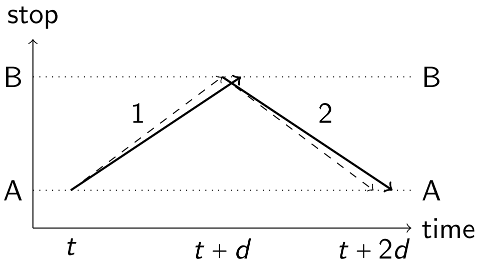


Under RTA*, any increase to the trips' planned travel time would require using two vehicles.

RTA*

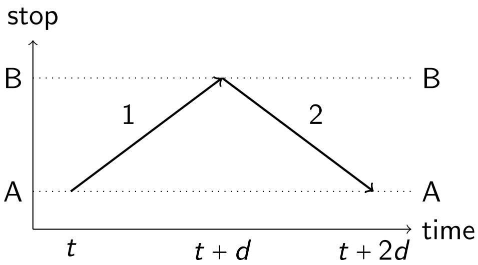
Key insights

- RTA* balances operational and delay costs at the trip level.
- Higher ridership necessitates additional travel time.
- If vehicles are expensive, RTA* can have costly downstream impacts.

Outline

Vehicle scheduling

By contrast, consider an approach that does not adjust travel times, but instead manages delay through vehicle scheduling.



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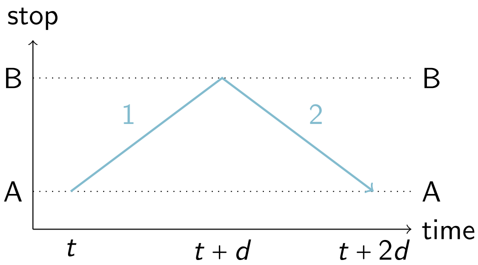

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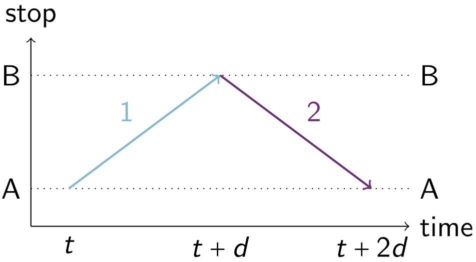

We can

- schedule both trips with one vehicle: $\{(1,2)\}$; or
- schedule each trip with its own vehicle: $\{(1),(2)\}$.

Vehicle scheduling

To choose the best vehicle schedule, we would like to solve

$$\text{Schedule Cost} = \min\left\{\text{Cost}_{\{(1,2)\}}, \text{Cost}_{\{(1),(2)\}}\right\},$$

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and the cost of scheduling with two vehicles is

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Because L_1 and L_2 are IID, it can be shown that

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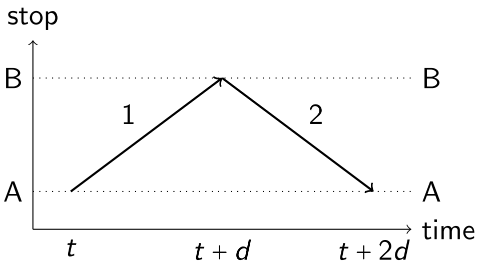
Therefore, the one-vehicle schedule dominates if

$$0 < \text{Delay}_{\{(1,2)\}} - \text{Delay}_{\{(1),(2)\}} < \frac{c^{\text{veh}}}{rh}.$$

As $r \to \infty$ we will eventually prefer the two-vehicle schedule.

Stochasticity



Example:

Trip	**Day 1** (ℓ_i^1)	**Day 2** (ℓ_i^2)	**Mean** ($\bar{\ell}_i$)
1	1	-1	0
2	1	-1	0

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By considering only mean delays,

$$\text{Delay}^{\text{mean}}_{\{(1,2)\}} = \lceil \bar{\ell}_1 \rceil^+ + \left\lceil \lceil \bar{\ell}_1 \rceil^+ + \bar{\ell}_2 \right\rceil^+ = 0,$$

and

$$\text{Delay}^{\text{mean}}_{\{(1),(2)\}} = \lceil \bar{\ell}_1 \rceil^+ + \lceil \bar{\ell}_2 \rceil^+ = 0.$$

Note: $\lceil x \rceil^+ = \max\{x, 0\}$.

Stochasticity

Trip	**Day 1 (ℓ_i^1)**	**Day 2 (ℓ_i^2)**	**Mean ($\bar{\ell}_i$)**
1	1	-1	0
2	1	-1	0

By considering stochastic delays,

$$\text{Delay}^{\text{stochastic}}_{\{(1,2)\}} = \frac{1}{2}\left(\sum_{s=1}^{2}\lceil \ell_1^s \rceil^+ + \sum_{s=1}^{2}\lceil\lceil \ell_1^s \rceil^+ + \ell_2^s \rceil^+\right) = 3/2,$$

and

$$\text{Delay}^{\text{stochastic}}_{\{(1),(2)\}} = \frac{1}{2}\left(\sum_{s=1}^{2}\lceil \ell_1^s \rceil^+ + \sum_{s=1}^{2}\lceil \ell_2^s \rceil^+\right) = 1.$$

Stochasticity

Thus,

$$\text{Schedule}^{\text{mean}} = \{(1,2)\},$$

and

$$\text{Schedule}^{\text{stochastic}} = \begin{cases} \{(1,2)\} & \text{if } \frac{rh}{2} < c^{\text{veh}}, \\ \{(1),(2)\} & \text{otherwise.} \end{cases}$$

Vehicle scheduling

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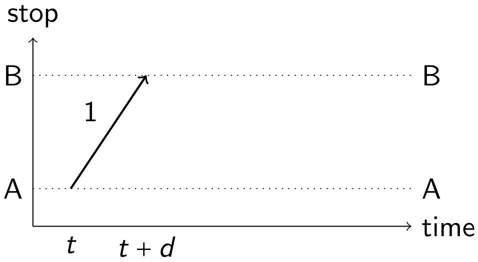
Key insights

- Higher ridership favors additional vehicles.
- Ignoring delay stochasticity can be costly.

Outline

Schedule-aware RTA*

Consider a single trip with planned travel time d. Let L be a random variable representing its primary delay.



The optimal schedule is $\{(1)\}$ with

$$\text{Cost}_{\{(1)\}} = c^{\text{veh}} + c^{\text{srv}} d + rh\text{Delay}_{\{(1)\}}.$$

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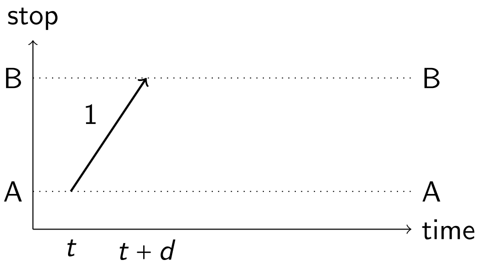

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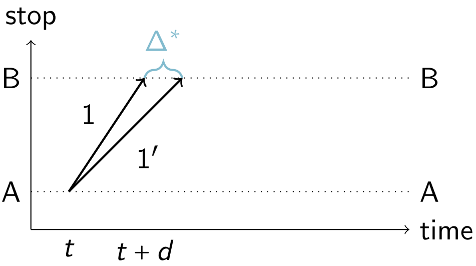
Can we improve this cost?

Schedule-aware RTA*



Let 1′ be the resulting trip after applying RTA* to 1 with

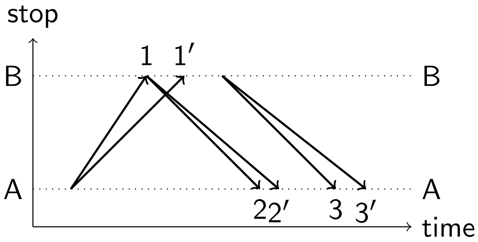
$$\text{Cost}_{\{(1')\}} = c^{\text{veh}} + c^{\text{srv}}(d + \Delta^*) + rh\text{Delay}_{\{(1')\}}.$$

Then we may potentially obtain a less costly schedule by solving

$$\text{Schedule Cost} = \min\left\{\text{Cost}_{\{(1)\}}, \text{Cost}_{\{(1')\}}\right\}.$$

Schedule-aware RTA*

With more trips, we can create a copy of each trip and apply RTA*. We then optimize over all possible vehicle schedules to obtain the optimal solution.



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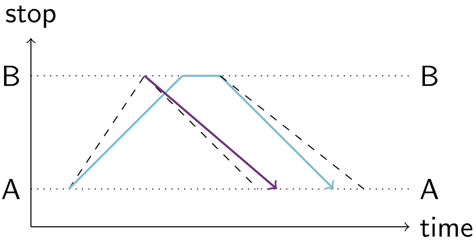

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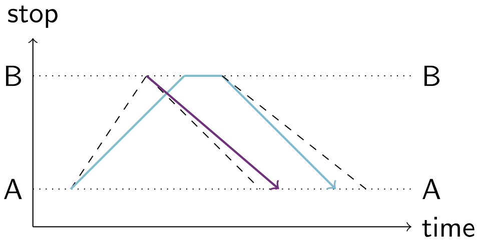


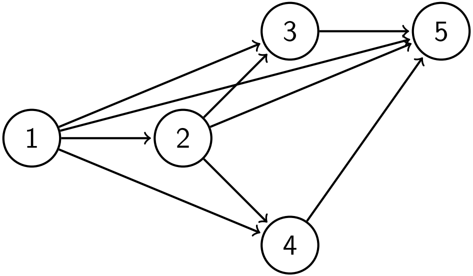
Key insight

- We may be able to reduce schedule costs by considering both original and RTA* adjusted versions of each trip.

Outline

Building a schedule

Consider a network of 5 trips.



Note: Depot omitted for conciseness

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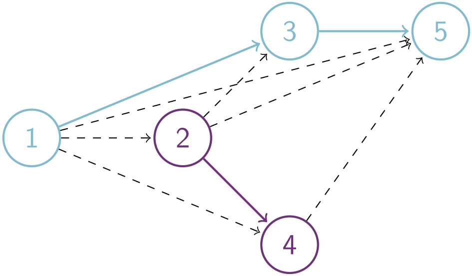
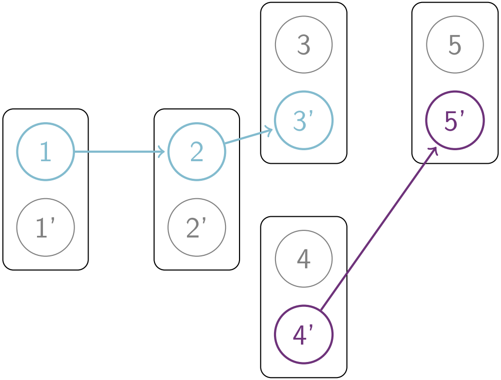


This is an example of a *feasible* vehicle schedule.

Building a schedule

Consider a network of 5 trips and their RTA* adjusted versions.



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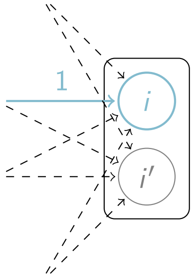
Building a schedule

- Let $\mathcal{T}$ ($\mathcal{T}'$) be the set of all unadjusted (RTA* adjusted) trips;
- i' be the RTA* adjusted version of trip i; and
- $x_{ij} = \begin{cases} 1 & \text{if } (i,j) \text{ included in the solution,} \\ 0 & \text{otherwise.} \end{cases}$

Then, a feasible vehicle schedule obeys

$$\sum_{j \in \mathtt{In}(i)} x_{ji} + \sum_{j \in \mathtt{In}(i')} x_{ji'} = 1 \quad \forall i \in \mathcal{T}.$$



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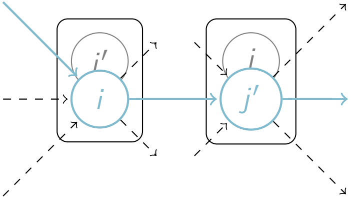
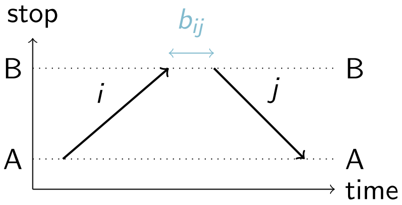


Secondary delays

- Let ℓ_i be the primary delay experienced on trip i; and
- b_{ij} be the buffer time between trips i and j.

Then y_i, the secondary delay experienced on trip i, is

$$y_j = \max\left\{\sum_{i \in \mathtt{In}(j)} (y_i + \ell_i - b_{ij}) x_{ij}, 0\right\} \quad \forall j \in \mathcal{T} \cup \mathcal{T}'.$$



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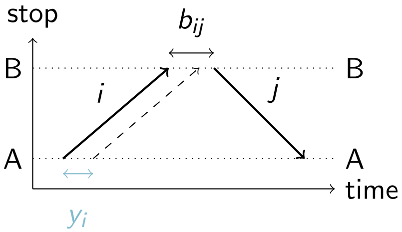

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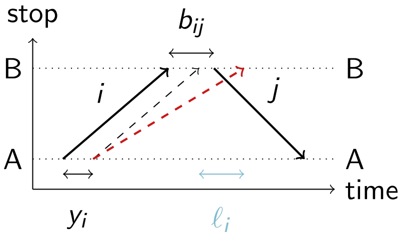

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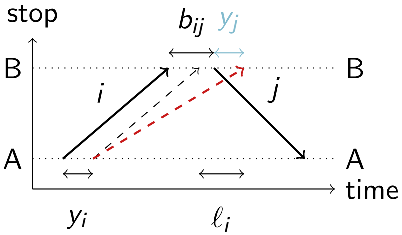

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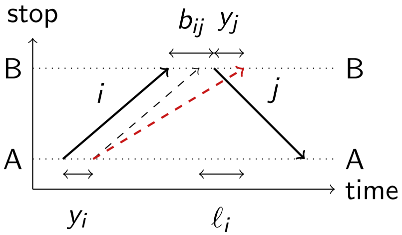


$$\vdots$$

$$y_i = \max\{y_k + \ell_k - b_{ki}, 0\}$$

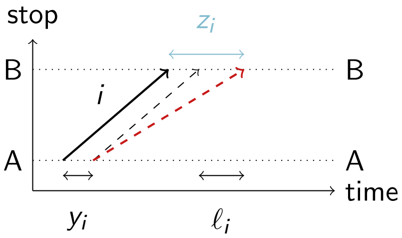
$$y_j = \max\{y_i + \ell_i - b_{ij}, 0\}$$

$$\vdots$$

End-of-trip delays

Similarly, we may calculate the end-of-trip delay, z_i, as

$$z_i = \max\{y_i + \ell_i, 0\} \quad \forall i \in \mathcal{T} \cup \mathcal{T}'.$$



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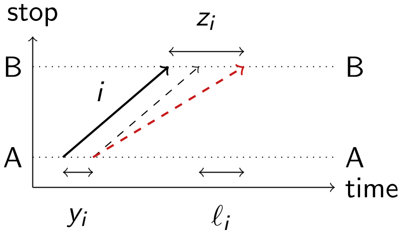

$$\vdots$$

$$z_i = \max\{y_i + \ell_i, 0\}$$

$$z_j = \max\{y_j + \ell_j, 0\}$$

$$\vdots$$

MIP

Thus, we propose the following stochastic mixed-integer program:

$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{y},\mathbf{z}} \quad & \sum_{(i,j)\in\mathcal{E}} c_{ij}x_{ij} + \frac{h}{S}\sum_{s=1}^{S}\sum_{i\in\mathcal{T}} r_i(z_i^s + z_{i'}^s) && && \text{(3a)}\\
\text{s.t.} \quad & \sum_{j\in\mathtt{In}(i)} x_{ji} + \sum_{j\in\mathtt{In}(i')} x_{ji'} = 1 && \forall i \in \mathcal{T}, && \text{(3b)}\\
& \sum_{j\in\mathtt{In}(i)} x_{ji} - \sum_{j\in\mathtt{Out}(i)} x_{ij} = 0 && \forall i \in \mathcal{T}\cup\mathcal{T}', && \text{(3c)}\\
& y_i^s \geq \sum_{j\in\mathtt{In}(i)} (y_j^s + \ell_j^s - b_{ji})x_{ji} && \forall i \in \mathcal{T}\cup\mathcal{T}', s\in\mathcal{S}, && \text{(3d)}\\
& z_i^s \geq y_i^s + \ell_i^s \sum_{j\in\mathtt{In}(i)} x_{ji} && \forall i \in \mathcal{T}\cup\mathcal{T}', s\in\mathcal{S}, && \text{(3e)}\\
& x_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{E}, && \text{(3f)}\\
& y_i^s \geq 0 && \forall i \in \mathcal{T}\cup\mathcal{T}', s\in\mathcal{S}, && \text{(3g)}\\
& z_i^s \geq 0 && \forall i \in \mathcal{T}\cup\mathcal{T}', s\in\mathcal{S}. && \text{(3h)}
\end{aligned}
$$

MIP

The objective,

$$\min_{\mathbf{x},\mathbf{y},\mathbf{z}} \quad \underbrace{\sum_{(i,j)\in\mathcal{E}} c_{ij} x_{ij}}_{\text{operating cost}} + \underbrace{\frac{h}{S} \sum_{s=1}^{S} \sum_{i\in\mathcal{T}} r_i (z_i^s + z_{i'}^s)}_{\text{delay cost}}, \tag{3a}$$

minimizes both operational and delay costs.

- c_{ij} : the cost of adding edge (i,j) to the solution
- $\mathcal{E}$: the edge set of possible trip connections
- h : the cost per rider-hour of passenger delay
- r_i : the ridership for trip i
- $\mathcal{S}$: the set of delay scenarios, with size $S = |\mathcal{S}|$

Outline

Sources

We consider the Victoria Regional Transit System (VRTS) in Victoria, British Columbia.

- Trip-level information: General Transit Feed Specification (GTFS)
- Travel time information: BC Transit[3]
- Ridership information: BC Transit
- Time period: September 2023 - December 2023

[3] BC Transit expressly disclaims any representations and warranties of any kind with respect to the information being released. In particular, BC Transit and any additional third parties do not represent or warrant that the information provided is accurate, complete, or current. Neither BC Transit nor any third parties, employees, or other representatives will be liable for damages of any kind, including, without limitation, direct, special, indirect, consequential, punitive, or exemplary damages for loss of income, profit, or savings, and claims of third parties, arising out of or in connection with the use of the information provided. All information is released "as is" and cannot be redistributed without permission.
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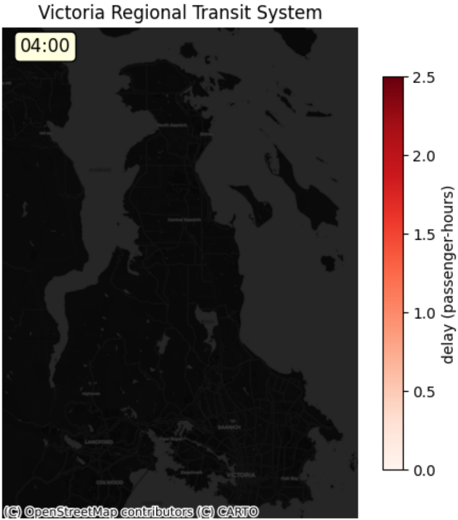
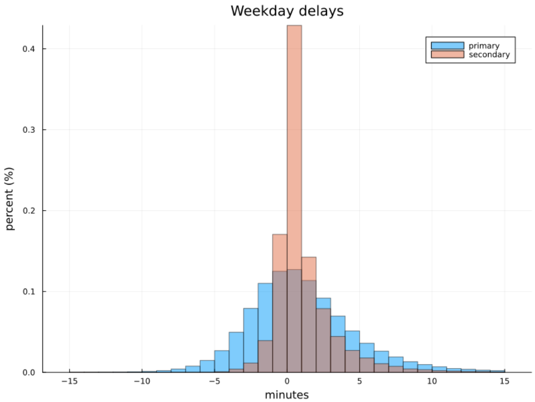
VRTS



Victoria Regional Transit System
04:00
2.5
2.0
1.5
1.0
0.5
0.0
delay (passenger-hours)
(C) OpenStreetMap contributors (C) CARTO

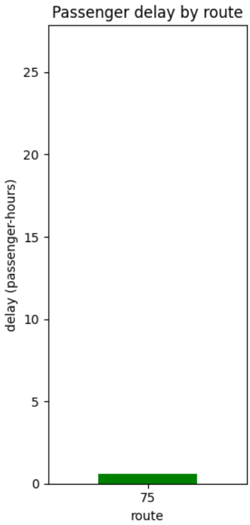
VRTS



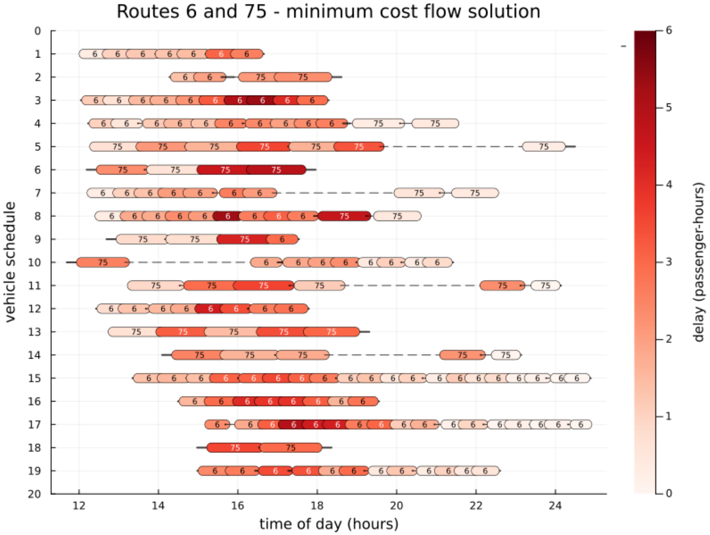
Delay (minutes)	μ	σ	P_{50}	P_{85}
primary	1.12	4.11	0.63	8.07
secondary	1.06	2.47	0.43	5.13

Outline

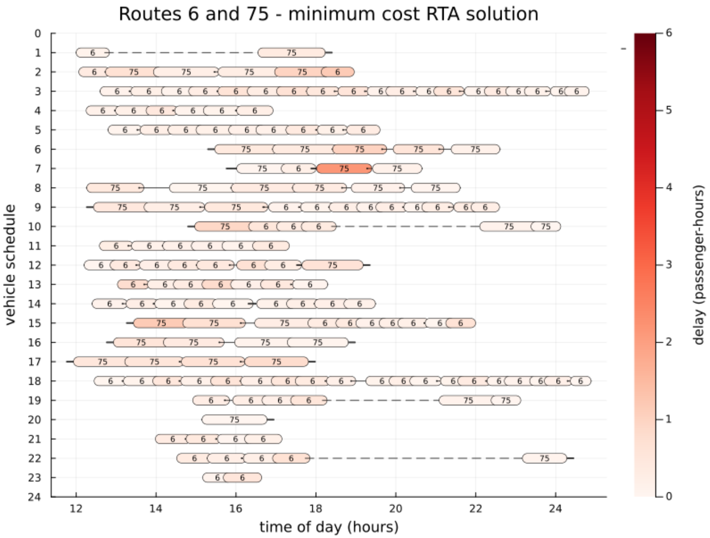
Routes 6 and 75



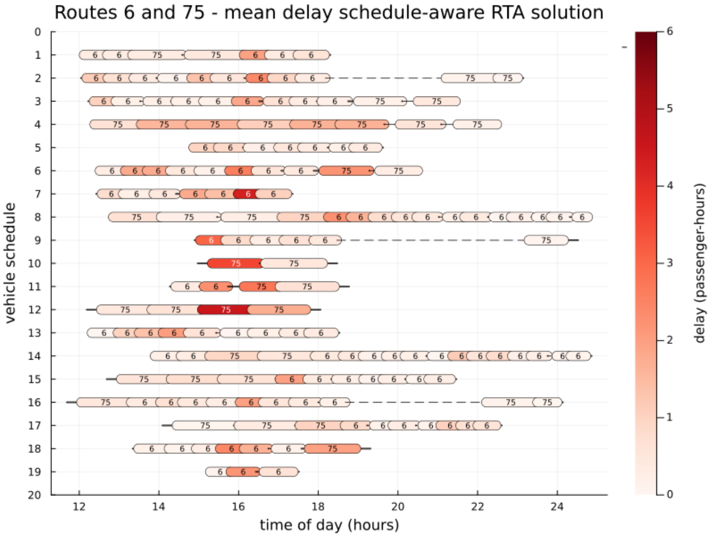
Routes 6 and 75



Routes 6 and 75



Routes 6 and 75



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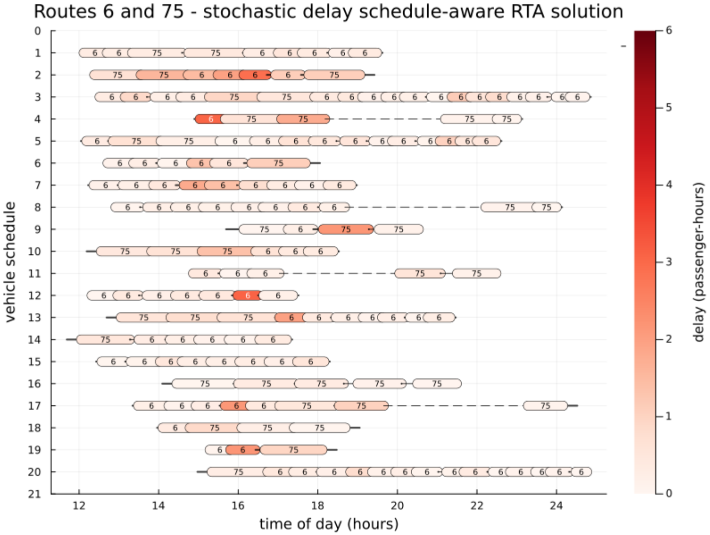

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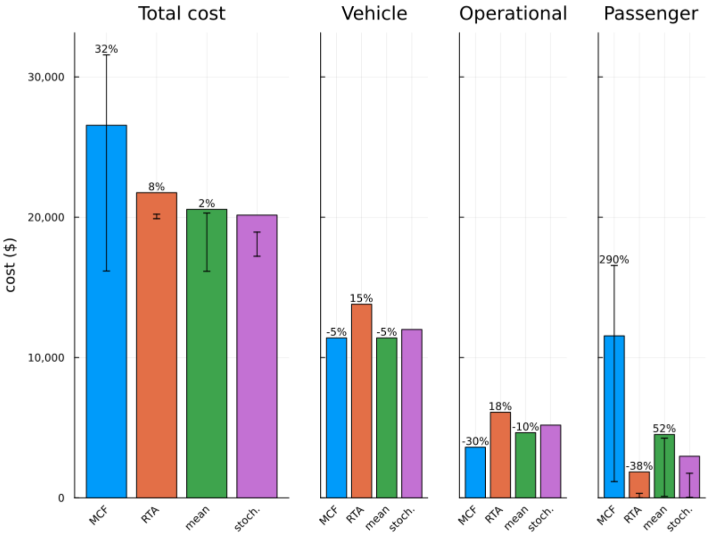

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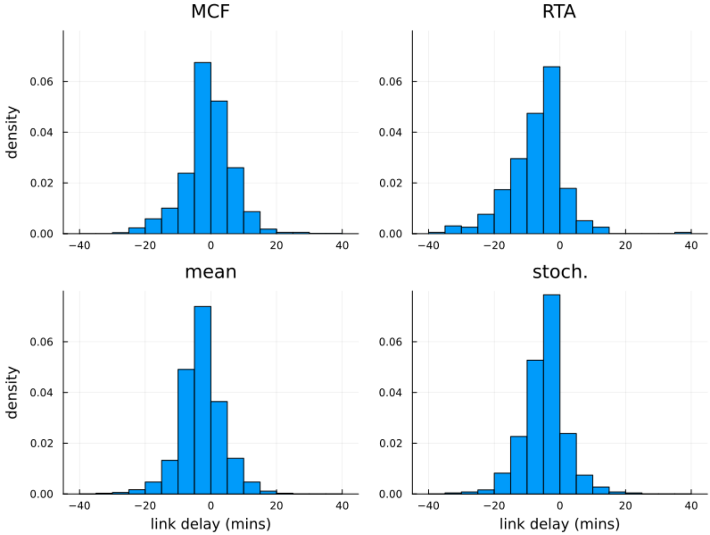
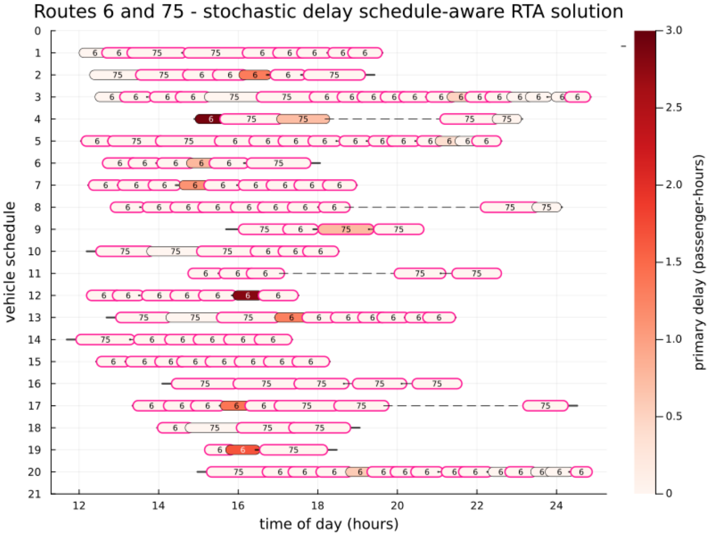


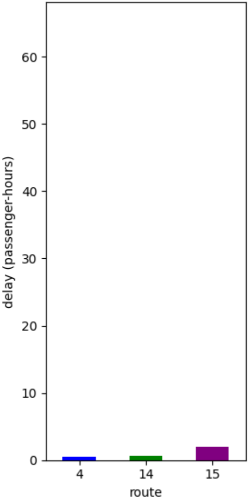
Routes 6 and 75



Routes 4, 14, and 15

Victoria Regional Transit System

Passenger delay by route



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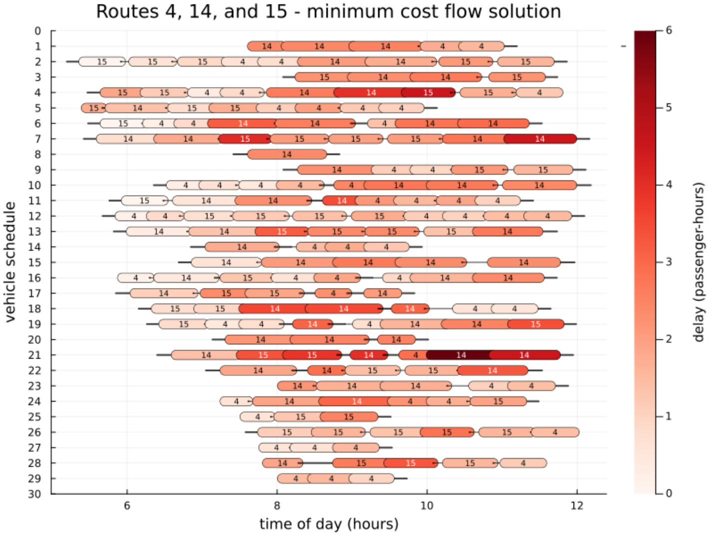

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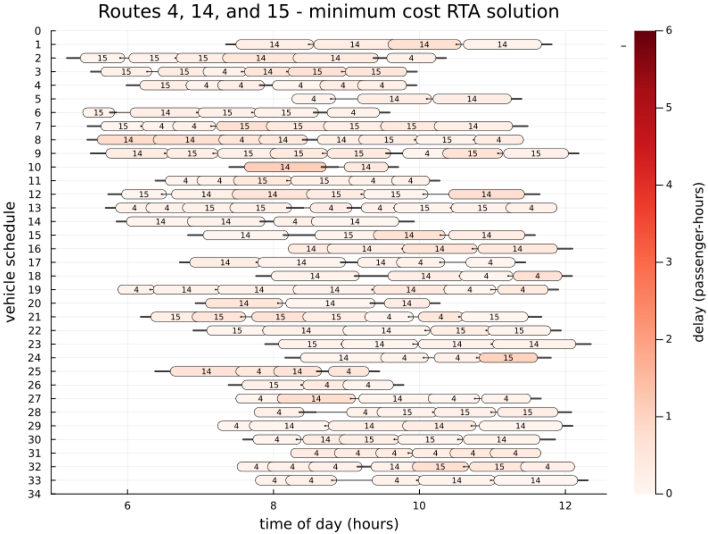

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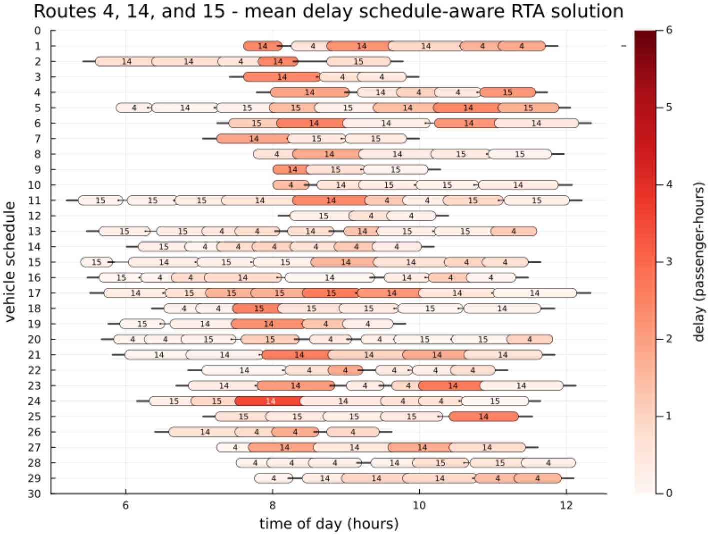

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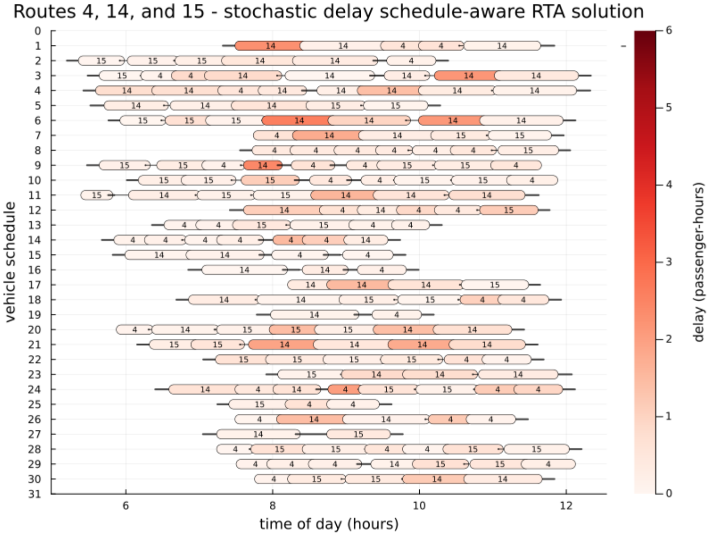
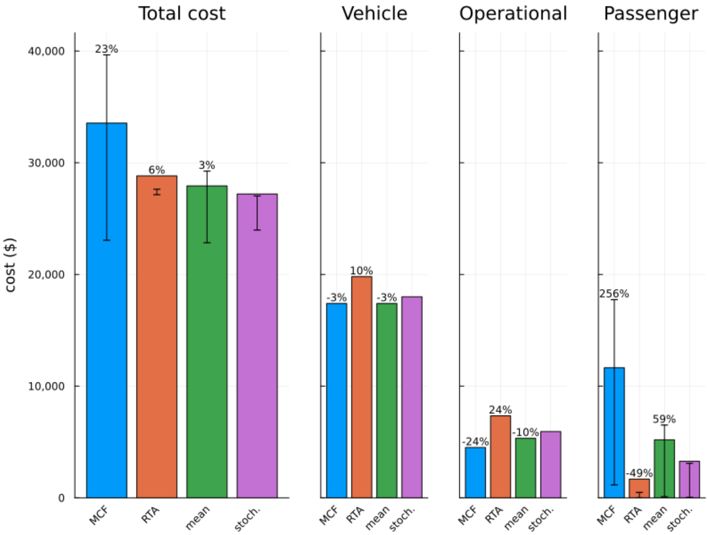
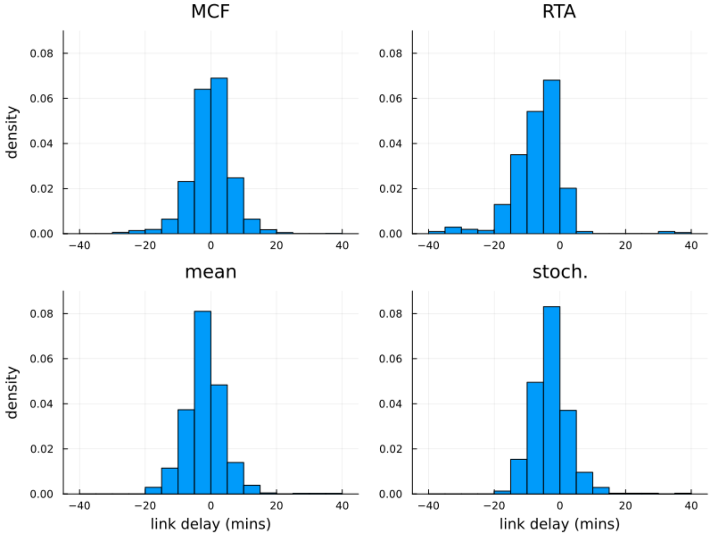


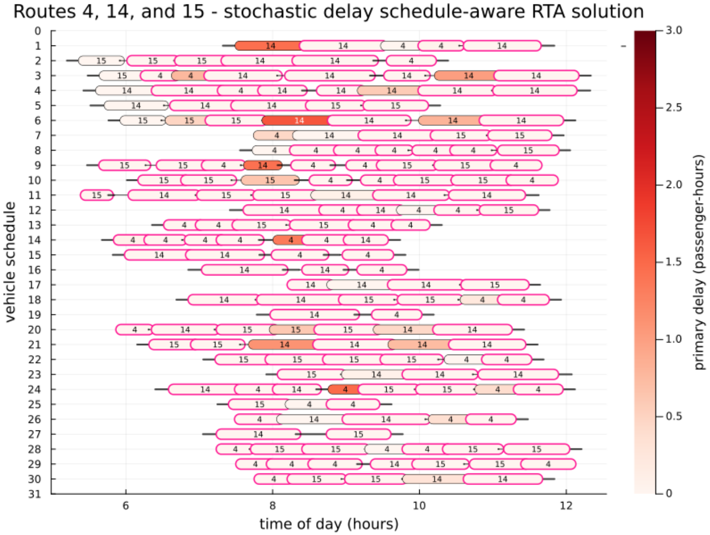
Routes 4, 14, and 15



Routes 4, 14, and 15



Routes 4, 14, and 15



Outline

Conclusion

Delays are inherent in our transportation networks, and have many negative effects.

- Driver/passenger frustration
- Operational staff and vehicles
- Limited expansion resources

Conclusion

But we can plan ahead to mitigate their effects.

- Sacrifice higher planned operational costs for reliability
  - Lower total costs
  - Less variance in total costs
  - Customer and staff satisfaction
- Simple tactics can yield delay improvements:
  - Pair routes with anticorrelated delay profiles
  - Apply targeted run time analysis
- Software solutions
- Combine with infrastructure investment

What next?

Next steps:

- Scale the model to handle larger instances
- Continue exploring simple tactics to improve scheduling in practice

Future work:

- Equity considerations
- Trip shifting
- Real-world implementation