Q1: Define Euler's Totient Function of a positive integer n with words.

A1: The number of non-negative integers less than n which are coprime to n.

Q2: Precicely describe how to calculate Euler's Totient Function of a positive integer n.

A2: First find the prime factorisation of n, call it $n = {p_1}^{e_1} \cdot {p_2}^{e_2} \cdots {p_k}^{e_k}$. Next calculate the following to find $\phi(n)$:

$$\prod_{i=1}^{k} \left({p_i}^{e_i - 1} \cdot (p_i - 1) \right)$$

Q3: State Euler's Theorem.

A3: If $\gcd(a, n) = 1$, then $a^{\phi(n)} \equiv 1 \pmod{n}$.

Q4: State Fermat's Little Theorem.

A4: If a is a positive integer and p is prime, then $a^p \equiv 1 \pmod{p}$.

Q5: What does it mean for a sett of integers to be pairwise coprime?

A5: The greatest common divisor of all the elements is 1.

Q6: State the Chinese Remainder Theorem.

A6: if $m_1, m_2, \cdots, m_r$ are pairwise coprime positive integers and $a_1, a_2, \cdots, a_r$ are integers, then the system of congruences

$$\begin{aligned} x \equiv & a_1 \pmod{m_1} \\ x \equiv & a_2 \pmod{m_2} \\ & \vdots \\ x \equiv & a_r \pmod{m_r} \end{aligned}$$

has a unique solution modulo $M := m1 \cdot m_2 \cdots m_r$ which is given by

$$x = \sum_{i=1}^{r} a_i M_i y_i \pmod{M}$$

where $M_i := M/m_i$ and $y_i :\equiv M_i^{-1} \pmod{m_i}$ for $1 \leq i \leq r$.

Q7:

A7:

Q8:

A8:

Q9:

A9:

Q10:

A10:

Q11:

A11:

Q12:

A12:

Q13:

A13:

Q14:

A14:

Q15:

A15:

Q16:

A16: