Hedging Mortgage Variable Rates: A Feasibility Analysis for Mortgage Holders in the United Kingdom

Quantitative Backtesting

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Executive Summary

This paper aims to compare various hedging strategies designed to mitigate the risk associated with mortgage interest payments among households in the United Kingdom (UK). It proposes a cross-hedging approach, aligning the standard variable rate with a position in the sterling London Interbank Offered Rate (LIBOR) future. To account for the loan structure, a tailored volatility model is introduced as input for the hedge ratios. Three distinct hedging strategies are explored: the Dynamic Conditional Correlation (DCC) Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model, the Constant Conditional Correlation (CCC) GARCH model, and the Ordinary Least Squares (OLS) model. Bivariate GARCH models are employed to ensure positive definite covariance and derive dynamic hedge ratios. The OLS model serves as a benchmark for comparing dynamic versus static hedging performance. Variance minimisation is the primary metric for assessing hedging performance, simulated based on loan repayments calculated from the average UK house price at a 75% loan-to-value ratio. The study introduces a unique hedging strategy, the 'DCCO GARCH model,' developed from the foundation of the DCC GARCH model. The paper finds that static hedging generally outperforms most dynamic strategies, with the exception of the DCCO model, which demonstrates comparable performance. Notably, the DCCO model outperforms, particularly under conditions of increased correlation and volatility. The paper concludes by suggesting avenues for further research in this specialised field, emphasising the importance of expanding knowledge in this domain.

Introduction

Within the present complex economic environment, a significant cause for apprehension stems from the considerable escalation in inflation rates, which not only affects the home economy but also has repercussions on worldwide markets. The United Kingdom (UK) serves as an illustrative example of the extensive implications of this global phenomena, as seen by its consumer price index (CPI) exceeding the global average (OECD, 2023). The Bank of England has taken immediate action in response to this prevailing trend, increasing its base rate to 5.25%, a level that has not been observed since 2008. This action serves to emphasise the seriousness of the issue (Giles, 2023).

The significance of this position is additionally emphasised by the statistics provided by the Office for National Statistics (2023), which illuminates the enduring presence of inflationary forces. The Consumer Prices Index (CPI) had a slight decrease from 7.9% in June to 6.8% in the twelve-month period preceding July 2023. However, with closer analysis, it becomes evident that there is a persistent inflationary trend. The Core CPIH, which includes energy, food, alcohol, and tobacco, exhibits a consistent growth rate of 6.4% in the 12-month period ending in July 2023, a figure that aligns with the growth rate observed in the preceding 12 months through June 2023.

The ramifications of enforcing more stringent monetary regulations have implications that reach beyond the boundaries of corporate financial institutions, exerting their impact on households throughout the entire nation. According to the Office for Budget Responsibility (OBR) (2023), the combination of increased inflation and rising borrowing costs presents a discouraging outlook for real household disposable income (RHDI). The RHDI had a notable decrease of 2.5% in the year 2022, and it is anticipated that this trend will further deteriorate in the upcoming year of 2023. The anticipated cumulative decline of 6% in the Relative Human Development Index (RHDI) from 2021 to 2024 serves as a clear indicator of the significant degradation in domestic welfare.

Mortgage rates play a crucial role in the transmission of monetary policy effects throughout the economy, serving as a significant channel within the complex framework of finance. The notable increase in the base rate leads to a similar rise in mortgage rates, exacerbating the challenges experienced by persons who are dealing with a decrease in their available income.

Indeed, with regards to the mortgage market, the implications of increasing borrowing expenses are having a significant impact on the housing sector, revealing a substantial shift in consumer behaviour and financial decision-making. The recent findings from the Bank of England shed light on the significant impact of rising interest rates on the housing market and household financial dynamics.

The Bank of England's disclosure of a significant decrease of around 10% in the number of approved mortgages, declining from 54,600 in June to 49,400 in July, is a distinct and impactful indication that increased borrowing expenses are influencing the preferences of prospective homebuyers. The significant decrease observed in approvals deviated from the economists' forecast of 51,000 approvals in a Reuters poll and also contradicted a brief period of positive growth observed between May and June (Financial Times, 2023).

The magnitude of this decline becomes further apparent when seen in comparison to the corresponding time in the previous year. In contrast to the data from July 2022, the approvals seen in July 2023 demonstrate a notable decline of 22%. This decline serves as a concerning indication of the impact that rising borrowing costs have on the aspirations of individuals in their pursuit of homeownership (Financial Times, 2023).

In light of this context, households across the United Kingdom (UK) that have variable mortgage rate contracts or are about to have their mortgage terms renewed find themselves embroiled in a twin difficulty among the complex tapestry of the current economic landscape. One prominent issue arises, namely the impending burden of financial strain. This tension manifests as a result of the continuously increasing mortgage payments that correspondingly climb with the escalating bank interest rates. Nevertheless, the level of intricacy becomes more profound with the advent of a subsequent peril, commonly referred to as the 'interest rate opportunity risk.' This potential danger arises when households choose the perceived security of a fixed interest rate, which may leave them susceptible to missing out on potential savings if interest rates decrease in the future.

The concept of individuals potentially sacrificing advantageous financial opportunities due to their choice of fixed interest rates, particularly in the context of decreasing interest rates, is emphasised by The Financial Times (2023).

Overall, “The average mortgage rate across all product types is 4.56% in Q2 2023, up from 3.38% in Q4 2022” (EMF, 2023, p. 4), indicating the growing burden of mortgage payments on disposable income among UK households. Furthermore, 18% of new mortgages in the first quarter of 2023 were based on variable rates, marking the highest proportion since Q3 2013. This highlights the increasing exposure to interest rate risks and the imperative need to manage this risk (EMF, 2023).

Aims and Objectives

The objective of this paper is to determine the feasibility for UK mortgage owners to hedge their interest rate risk with interest rate future using different hedging approach and to determine the most effective approach. In other words, this study specifically targets UK mortgage holders with a variable rate, aiming to decrease the variability in their mortgage repayments throughout the duration of their mortgage.

Mortgage holders with a variable rate are the most expose to change in their monthly mortgage payment and hence can support high mortgage debt burdens. The Bank of England (2023) rise concern about households ability to repay their mortgage and estimate that around four millions of household are currently expose to mortgage rate rise. In this context, hedging with futures means using futures contracts to reduce or minimise the risk linked to rate changes in an loan. This approach is often used to safeguard against potential losses caused by unfavourable rate movements. Hedging with futures requires determining the optimal contract amount to enter a position that offsets fluctuations in interest payments. This determination relies on establishing the optimal hedge ratio or minimum variance hedge ratio. The optimal hedge ratio essentially determines the number of units of the hedge instrument required to cover one unit of exposure. It is derived from the assumed variance-covariance matrix shared between the future price and the variable rate.

Therefore, this study will delve into three hedging approaches, seeking to address the diverse levels of variability in the optimal hedge ratio. Specifically, it will encompass both static and dynamic hedging techniques to manage the interest rate risk. Furthermore, we assess the effectiveness of hedging based on each approach's ability to reduce the variance in interest payments using its optimal hedge ratio. The first hedging approach imply a static variance-covariance matrix between the future price and the standard variable rate and hence a static minimum-variance hedge ratio because the hedging approach is based on the Ordinary Least-Squares (OLS) model. This regression-based model represents the simplest approach, both technically and computationally, to elucidate the relationship between two variables, making it indispensable for evaluation in this study. The other two approaches enable the modelling of a time-varying covariance-variance structure using bivariate Generalised Autoregressive Conditional Heteroskedasticity models (bivariate GARCH). However, the Constant Conditional Correlation (CCC-) GARCH model (Bollerslev, 1990) assume a constant correlation matrix whereas the Dynamic Conditional Correlation (DCC-) GARCH model introduced by Engle and Sheppard in 2001, enable the correlation matrix to be time-varying. Collectively, these three models enable the computation of varying degrees of variability in the variance-covariance structure each involving increased computational complexity and variability.

However, it is important to define what is the most appropriate underlying rate for the future contract to hedge against the variable mortgage rate. The primary factor determining interest rates within an economy is the central bank monetary policy. Therefore, the Bank of England base rate holds significant importance in the UK as it sets the cost of borrowing within the economy, consequently influencing the mortgage rates charged by banks to mortgage holders (Bank of England, 2023). However, the risk-free rate is the ultimate benchmark for short-term borrowing cost as the risk-free rate compare to the bank rate, can reflect the balance of supply and demand for short-term borrowing. The risk-free rate is theoretically the return on an investment with no risk of financial loss, often represented by government bonds. It is ultimately closely tied to the central bank rate, such as the Bank of England's base rate or the Federal Reserve's federal funds rate, as these rates influence short-term rates through monetary policy decisions. Typically, short-term government bonds are considered risk-free or nearly risk-free investments. Nonetheless, we will consider in this paper the overnight rates as the risk-free rate instead of the UK gilts rate as there is tax and regulatory factors that lead to gilts rates being inaccurate risk-free rates (Hull, 2021). In the UK, the overnight rate is the Sterling Overnight Index Average (SONIA) and represent the weighted average of brokered overnight rates (Hull, 2021). The SONIA rate is use as a reference rate in the UK economy which make it a point of reference for setting interest rates in various financial transactions, such as loans, derivatives, mortgages. It is possible to gain exposure to the SONIA rate fluctuations through the One Month SONIA Index Futures available on the Intercontinental Exchange (ICE). The futures quote on the settlement day equals 100 minus the arithmetic average of the SONIA one-day rates over the course of the month. However, the one-month SONIA contract is designed to hedge a £250,000 position, making it impractical for the average individual, particularly for those seeking hedging solutions for mortgage debt burdens. As a result, there are few, if any, instruments available for UK mortgage owners to manually hedge their interest payments. To facilitate this study, we will assume that the contract unit of trading is based on the index rate (which is 100 minus the arithmetic average of the SONIA one-day rates over the course of the month), intended to hedge a £100 position, instead of the index rate multiplied by £2,500 as stipulated in the contract. Although the current absence of financial instruments for UK households to access the risk-free rate prevents them from manually hedging their interest payments, this limitation doesn’t necessarily diminish the scope of this study. The study's primary focus is to propose various hedging strategies and evaluate their effectiveness. The primary obstacles to their implementation revolve around the contract's unit of trading and its accessibility to retail customers. However, these barriers might evolve, given the dynamic nature of the financial domain, indicating a flexible landscape. In response to ample demand, the financial sector has the capability to develop specialised products to meet these needs. Furthermore, even if we consider this barrier to be rigid despite the increasing demand from UK households for exposure to the risk-free rate, a business could act as an intermediary. It could match a large contract position with numerous customers seeking this exposure. With enough customers, the business might potentially match the significant contract unit of trading with a larger group of customers. Hence, the primary barrier to the application of these hedging strategies can be attributed to the potential lack of demand for their implementation. Therefore, the assumption made does not significantly undermine the aim of this research paper.

In this research paper, the reference rate utilised for hedging purposes is not the SONIA rate but rather the Sterling London Interbank Offered Rate (LIBOR). SONIA has been a reference rate only recently, whereas previously, the Sterling LIBOR held the position as the reference rate in the economy. Given the retrospective nature of this study, it's more fitting to utilise the reference rate that was predominant at that time. This choice aims to maintain relevance in establishing the relationship between the mortgage variable rate and the reference rate. Indeed, the sterling LIBOR rate has historically been an important reference rate and was considered as an estimate of unsecured borrowing rates for creditworthy banks (Hull, 2021).

LIBOR rates in generals “have served as reference rates for hundreds of trillions of dollars transactions throughout the world” (Hull, 2021, p. 100), However, LIBOR rates have been phased out due to their lack of complete risk-free status and inclusion of a credit-risk spread. Furthermore, their dependence on bank quotations involves a degree of subjectivity and susceptibility to manipulation, as evidenced during the 2008 financial crisis when LIBOR rates did not accurately reflect the underlying borrowing rates (Hull, 2021). It is worth noting that although we utilise the sterling LIBOR rate as the reference rate in this study, it's important to note that our backtesting spans from 1995 to 2023, encompassing the transition to SONIA as the new reference rate around 2021. This might raise controversy regarding the use of the sterling LIBOR rate as a reference after this transition period. However, official sterling LIBOR quotes continued during this time to accommodate existing contracts dependent on LIBOR, thus retaining relevance despite no longer being the official reference rate. Therefore, to maintain consistency, the sterling LIBOR rate will be used throughout the entire duration of the backtesting. However, applying the proposed hedging strategies outlined in this paper will necessitate some adjustments to accommodate the SONIA rate, utilising the One Month SONIA Index Futures as a hedging mechanism. This adaptation is not supposed to affect the foundation of this study or its conclusions.

Now that we've defined the hedging approaches and the reference rate for hedging purposes, we need to establish the loan type to model the interest payments we aim to hedge. The type of mortgage loan considered for this research is the amortising loan in which interest is repaid over time. This loan structure is the most common type of mortgage loan and requires the borrower to make consistent principal and interest payments, ensuring the loan's full repayment base on the current standard variable rate. Given the variability of this mortgage rate, this loan structure enables us to model fluctuations in interest payments. The variable rate chosen for the mortgage loan is the Revert-to-Rate (RTR) mortgage rate, sourced from the Bank of England. This metric monitors the standard variable rate that mortgage borrowers are subject to.

Therefore the objective of this paper is to analyse the relationship between the mortgage interest payments (which are based on the current amortising loan and the prevailing RTR rate) and the pricing of the one-month sterling LIBOR future using different models: OLS, CCC GARCH and DCC GARCH. Then, the objective is to determine which model constitute the best hedging strategy based on their ability to minimise the interest payments variance.

Literature Review

The mortgage literature is extensive but review focusing on how individuals can hedge their monthly interest payment risk is notably limited, if not entirely absent. This gap in research might be attributed to several potential reasons:

- As mentioned earlier, the absence of accessible financial instruments for retail customers makes manual hedging impractical in the real world. This significant barrier for the average population remains high, a factor we've overlooked in this study, assuming that substantial demand could potentially alter the financial regulatory landscape concerning these instruments. However, it's crucial to acknowledge that this statement is inherently speculative in nature.

- Additionally, these hedging approaches demand financial and economic literacy from users, which might be unlikely for the average UK household given the diverse backgrounds of individuals. However, this statement only holds partially true. As previously discussed, a business or institution could act as an intermediary, offering to hedge on behalf of UK households.
- An essential factor contributing to this is that many countries, including the UK, offer mortgages with fixed interest rates. This availability diminishes the necessity of hedging against variable rates, as this risk can be promptly eliminated by opting for a fixed interest rate. However, opting for a fixed interest rate entails a long-term commitment for mortgage owners, offering less flexibility. Conversely, choosing a variable rate and hedging when deemed necessary provides owners the flexibility of variable rates while mitigating associated risks. More importantly, the UK exhibits a population tendency favouring variable rates, as highlighted by the European Mortgage Federation (2023) data. Nearly 13% of outstanding mortgage loans in the United Kingdom are on variable rates, indicating a notable preference among borrowers for this type of mortgage. Additionally, this study could serve as a foundational reference for countries like Denmark (38%), Finland (95%), or Sweden (56%), where the European Mortgage Federation (EMF, 2023) notes a significant portion of outstanding mortgage loans tied to variable rates. It could provide valuable insights into managing variable-rate mortgage markets.

It is critical to emphasise that Koblyakova, Hutchison, and Tiwari (2013) found, in their paper 'Regional Differences in Mortgage Demand and Mortgage Instrument Choice in the UK,' that empirical evidence suggests households with lower incomes tend to choose variable-rate mortgages. This highlights the necessity for a risk-mitigating mechanism tailored to this demographic, given their preference for the flexibility of variable rates. A notable study by Zivney and Luft (1999) examines the practicality and effectiveness of individual mortgage hedging through the use of futures and options. However, it's important to note that the study's focus was to hedge interest rate risk when planning to take out a mortgage at a future point in time, rather than hedging interest rate risk on an outstanding loan, representing significantly different objectives. Nevertheless, this investigation is still useful for the objective of our current research endeavour, which is to develop a mortgage-owner-specific hedging strategy within the current economic environment. The study employs simulated market scenarios to assess the effect of various hedging strategies on mortgage capacity and downside risk. As a result, the research highlights empirical evidence that supports the potential benefits of utilising futures and options to hedge against potential mortgage rate increases. However, according to the research conducted by Zivney and Luft (1999), futures are superior to options as hedging instruments. This assertion is predicated primarily on the notion that options can impose substantial costs on those wishing to hedge their positions.

Numerous studies have evaluated the relative performance of dynamic hedge strategies compared to static hedges for financial assets. Koutmos, Kroner and Pericli (1998) demonstrates that hedge ratios derived from the dynamic joint distribution of financial assets are superior to traditional static hedge ratios. The study demonstrates a significant improvement in the effectiveness of hedging. Importantly, the study indicates that these gains are substantial even when realistic transaction costs are accounted for. The research demonstrates, within the framework of mean-variance analysis, the economic benefits of dynamic hedging regardless of the frequency of hedge updates. The dynamic nature of hedge ratios throughout the sample period, which can even turn negative in certain cases, is another crucial finding. This behaviour may be attributable to factors such as the negative convexity of the hedged assets. In a second study, Koutmos and Pericli (1999) demonstrate the superiority of a time-varying hedge ratio over static ones when cross-hedging mortgage-backed securities with the 10-year Treasury-note future contract, even when transaction costs are considered. The average improvement is 7% with the GARCH model when the alternative is the OLS model. However, the average improvement using the same model drops to 2.5% when applied to hedging foreign currencies with futures (Kroner, Sultan, 1993). Yet, this decline is understandable when considering direct hedging. In there paper, the authors consider a bivariate GARCH model suggested by Engle and Kroner (1995) knows as the BEKK model with an error-correction (EC) term which emphasise the long-term relationship between the two assets. Overall the EC GARCH model doesn’t produce any different result from the GARCH model and even slightly underperform. Yet, our GARCH model doesn’t include any correction term and also assumes a Gaussian distribution, similar to their study. It's important to exercise caution when comparing the average variance improvement of our GARCH model with their research because they estimate the parameters of their model simultaneously, whereas our model involves a two-step estimation process, which is computationally more accessible. Instead of straightforwardly modelling the conditional covariance matrix, our approach requires a two-step estimation.

The DCC GARCH model used in this study is the one introduced by Engle and Sheppard in 2001. Instead of modelling the conditional covariance matrix, this model decompose the covariance matrix into conditional standard deviations matrix and a correlation matrix which are both designed to be time-varying (Regnesentral, Orskaug, 2009). It is composed of two primary parts: calculating the dynamic conditional correlation between assets and estimating the individual volatilities of assets using GARCH models. The overview of the model based on Regnesentral and Orskaug (2009) paper is:

Modelling Individual Volatilities (Univariate GARCH models):

For each asset in the portfolio i, the univariate GARCH(1;1) model is used to model the conditional variance, sigma_{t,i}^2, of asset (sigma being the volatility and sigma^2 the variance):

sigma_{t,i}^2 = Ω_i + ∂_i*a_{t-1,i}^2 + ß_i*sigma_{t-1,i}^2

where:

- Ω_i is the constant term.

- ∂_i and ß_i are the coefficients of the GARCH process.

-a_{t-1,i}^2 is the squared residual (or error term) from the previous period.

Modelling Dynamic Conditional Correlation (DCC):

After estimating the univariate GARCH(1;1) models for each asset, the next step is to model the dynamic conditional correlation matrix, R_t, of the standardised disturbances ʒ_t:

ʒ_t = D_t^(-1)*a_t ~ N(0, R_t)

Where:

- D_t is the n*n diagonal matrix of conditional standard deviations of a_t at time t of the univariate GARCH models (and n is the number of asset).

H_t is the n*n variance-covariance matrix that we are attempting to modelled in this model and it has to be positive definite.

Because:

H_t = D_t*R_t*D_t

And D_t is positive definite since all the diagonal elements are positive, we need to ensure that R_t is positive definite and that all the element of the correlation matrix R_t is between -1 and 1 by definition. To ensure this:

R_t = Q_t_star^(-1)*Q_t*Q_t_star^(-1)
Q_t = (1-a-b)Q_bar + a*ɛ_{t-1}*transpose(ɛ_{t-1}) + b*Q_{t-1}

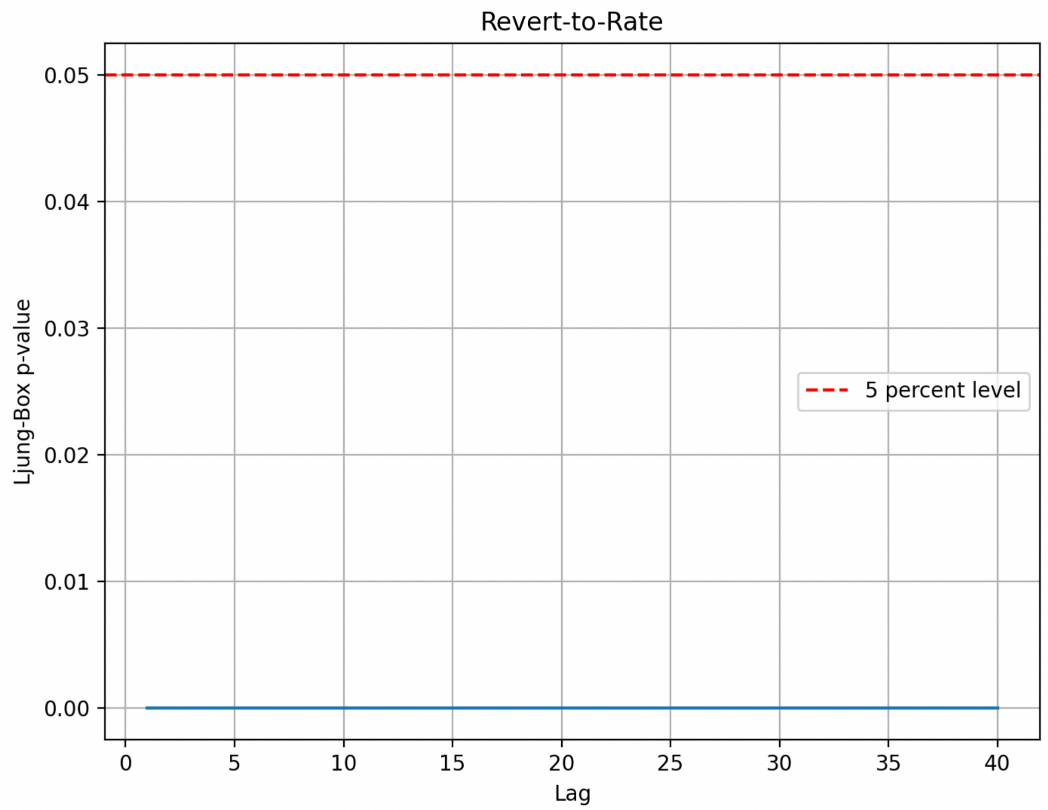
Where:
- Q_bar = cov(ɛ_t*transpose(ɛ_t)) is the unconditional covariance matrix of the standardised errors ɛ_t and can be estimated as the arithmetic average of ɛ_t*transpose(ɛ_t).
- a and b are scalars
- Q_t_star is a diagonal matrix with the square root of the diagonal elements of Q_t at the diagonal.

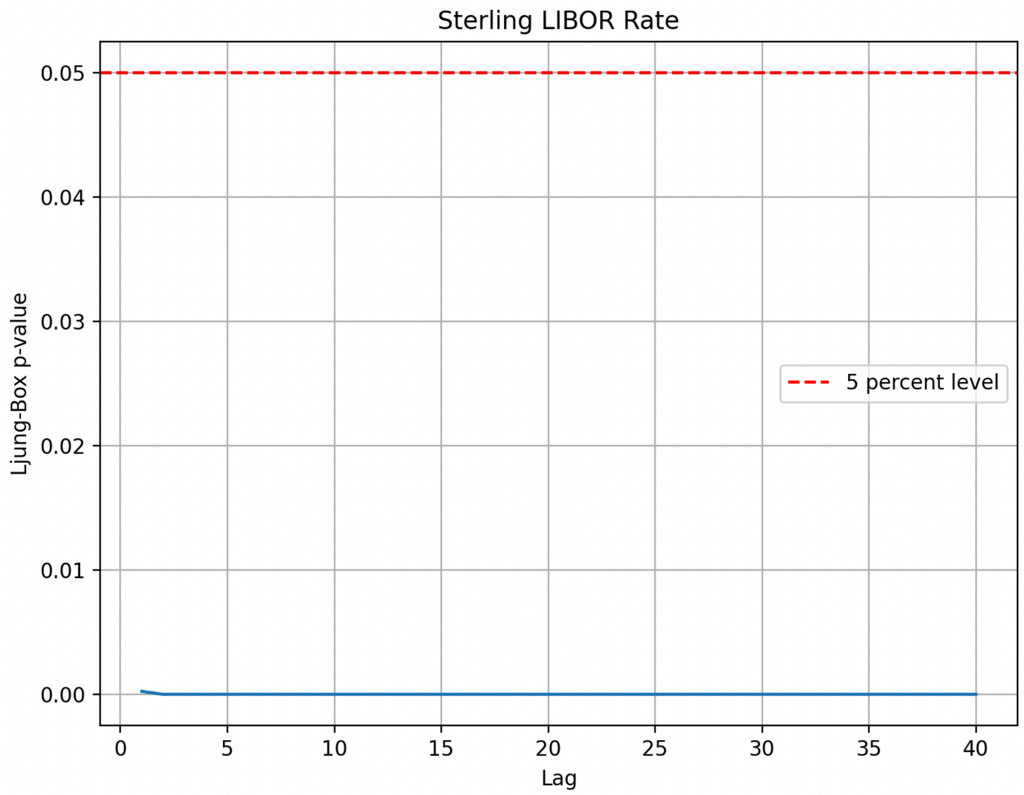
Furthermore, Q_t and Q_0 (the starting value of Q_t) as to also be positive definite to ensure R_t to be positive definite. Hence, the parameters a and b have to be superior or equal to 0, and aggregately inferior at 1, to guarantee H_t to be positive definite.

The DCC-GARCH model allows for more confident parameter estimation in multivariate GARCH models. Estimating all parameters simultaneously encounters convergence issues, primarily because the likelihood function becomes less informative as the number of parameters increases, leading to flat likelihood surfaces. The DCC approach mitigates this by sequentially estimating variance and correlations separately, thereby aiding in more reliable parameter estimation (Regnesentral, Orskaug, 2009). In this paper, the second multivariate GARCH model employed is the one introduced by Bollerslev in 1990. This model is notably simpler as it assumes a constant correlation matrix over time, focusing solely on modelling the conditional variance of each asset through univariate GARCH models. The model's variability adjustment serves as a link between the DCC-GARCH model and the fully static OLS model, creating a step-down transition in complexity and variability. Lastly, the regression model, the most popular one due to its simplicity and effectiveness. The OLS aims to find the best-fitting line through observed data points by minimising the sum of the squared residuals between observed and predicted values. OLS can be employed to estimate relationships between variables, such as determining the relationship between two assets (Koutmos, Pericli, 1999). The OLS model can also be used for hedging purpose. The hedge ratio is estimated by the slope coefficient of the model.

Modelling Hedging Approaches

In this study, our objective is to apply three models to our data, the DCC GARCH model, CCC GARCH model and The OLS model. These models are designed to capture and depict the relationship between the hedging instruments and the instruments we intend to hedge. The one-month sterling LIBOR rate used for hedging purpose is published by Thomson Reuters once





calculated by the Intercontinental Exchange (ICE) and retrieved on iborate.com. However, in this paper, the intended hedging instrument is a future contract based on the one-month sterling LIBOR. Unfortunately, there's no accessible data for the one-month sterling LIBOR futures. Consequently, we'll manually calculate the quote for the one-month sterling LIBOR futures contract using a methodology similar to ICE's calculation for its one-month SONIA index future. However, instead of doing the arithmetic average of the LIBOR one-day rates over the course of the month, we'll employ the end-of-month LIBOR rate as the settlement rate. Consequently, the settlement future quote will be 100 minus this settlement rate. This slight modification in the calculation methodology for the future quote allows us to acquire adequate data for conducting this study. Given the absence of sterling LIBOR one-day rates data. The end-of-month variable mortgage rate data are retrieved from the Bank of England website for a loan-to-value (LTV) of 75%. The retrieved data for the Revert-to-rate and LIBOR rate spans the period from January 31, 1995, to March 31, 2023 (338 observations per data set). The range from January 31, 1995 to 31 December, 2004 is the in-sample period, where the models' parameters will be estimated on and the rest will be the out-sample period where models accuracy are tested on. The returns for the future contract and the variable rate are modelled the following way:

r_t = (q_t - q_{t-1})/q_{t-1}

Where:
r_t is the return at time t.
q_t and q_{t-1} is the instrument quote at time t and t-1.

The average monthly house price in the United Kingdom is sourced from the HM Land Registry portal (2023), enabling us to manually calculate the average 75% loan-to-value (LTV) loan for each month during the backtesting period.

The choice of a 75% loan-to-value ratio is attributed to the substantial availability of variable mortgage rate data for this specific value. Ideally, our study aims to focus on individuals facing potential financial strain, suggesting a higher loan-to-value might have been more coherent. Unfortunately, the absence of historical variable mortgage rate data for higher loan-to-value ratios restricts our ability to opt for an alternate loan-to-value in this analysis.

Preliminary Statistics

Table 1: Preliminary Statistics

	Revert-to-Rate	**Sterling LIBOR Rate**
Observations	338	338
Mean	-0.00007	0.01266
Standard deviation	0.02575	0.04050
T-test	-0.048	1.156
Normal test	102.457*	440.463*
Skew test	-6.815*	17.299*
Kurtosis test	7.484*	11.884*
Kolmogorov-Smirnov test	0.466*	0.410*
Ljung-Box Test(20) for r_{i,t}	226.115*	153.379*
Ljung-Box Test(20) for r_{i,t}^2	158.022*	125.271*

*Significant at the 5% level.

The returns from the mortgage standard variable rate demonstrate a mean value very close to zero (-6.69e-05), signifying minimal deviation from a zero mean, accompanied by a variance of 0.0007, implying the spread of returns around this mean with a standard deviation of roughly 0.0258. The t-test, showing a t-statistic of -0.048 and a p-value of 0.9619, suggests no significant difference from zero, lacking adequate evidence to reject the null hypothesis. However, across various normality tests (Normal test, Skew test, and Kurtosis test), all yielding a p-value of 0.0000, the data significantly deviates from normality, exhibiting substantial skewness and kurtosis. Additionally, the Kolmogorov-Smirnov tests (KS-statistic= 0.466, p-value= 0.0000) reinforce strong evidence against normal distribution assumptions. The Ljung-Box test for autocorrelation at lag 20 displays significant autocorrelation (p-value = 7e-37), indicating dependency in the returns at that lag. Conversely, the sterling LIBOR rate returns exhibit a mean of 0.01265 and a variance of 0.04049 suggesting a broader spread around the mean with a standard deviation of approximately 0.2012. The t-test (t-stat = 1.156, p-value = 0.2483) shows no significant deviation from zero, similarly lacking substantial evidence to reject the null hypothesis. However, akin to the mortgage standard variable rate, the normality tests showcase a stark deviation from normal distribution assumptions, emphasising significant skewness and kurtosis. Moreover, the Kolmogorov-Smirnov tests (KS-statistic = 0.410, p-value = 0.0000, and KS-statistic D = 0.338, p-value = 0.0000) reinforce non-normality in the returns. The Ljung-Box test for autocorrelation indicates significant autocorrelation at lag 20, suggesting dependency in the returns at that specific lag.

In summary, these statistical tests collectively indicate that the return data exhibit non-normality, significant skewness and kurtosis, and their distribution significantly deviates from a normal distribution. The mean return is not significantly different from zero, but the datas' distribution differs significantly from a normal distribution.

Models Specifications

Despite clear deviations revealed in statistical tests, we adopt the normal distribution assumption for our return data, acknowledging its widespread use and simplicity in financial modelling. Hence, we assume that the conditional distributions of returns at each time point follow a normal distribution:

$$r_t|I_{t-1} \sim N(0, \sigma_t^2).$$

Where:

I_{t-1} is the information set available at time t-1

These equations describe the utilisation of univariate normal GARCH(1,1) models for estimating conditional variance. They assume that the dynamic behaviour of the conditional variance is defined by:

$$\sigma_{t,i}^2 = \Omega_i + \partial_i * r_{t-1,i}^2 + \text{ß}_i * \sigma_{t-1,i}^2,$$
$$r_{t,i}|I_{t-1,i} \sim N(0, \sigma_{t,i}^2).$$

Where:

- $r_{t-1,i}$ is the return vector at time t-1 for each asset i = 1,2, it is supposed to follow a conditional normal process with zero expected value and time varying conditional variance.
- $\sigma_{t,i}$ is the conditional variance at time t for the asset i.
- Ω_i, ∂_i, ß_i are the parameters of the model for the asset i.
- i = 1,2

We estimate the parameters by maximising the quasi-likelihood function:

$$Ln(L) = \sum_{t=1}^{T}\left(\ln(\sigma_{t,i}^2) + (r_{t,i}^2)/(\sigma_{t,i}^2)\right)$$

Once the parameters of the different univariate GARCH(1;1) model have been estimated (separately), we can modelled the variance-covariance matrix with the multivariate GARCH model. The CCC-GARCH model define the 2*2 variance-covariance matrix between the future and the variable rate as:

$$H_t = D_t * R_t * D_t$$

Were $R_t = R$ is 2*2 the time invariant correlation matrix, D_t is the 2*2 diagonal matrix of conditional standard deviation of r_t at time t and H_t is the 2*2 variance-covariance matrix at time t.

In this case, the correlation matrix is simply empirically estimated using the in-sample data. The correlation between the two instruments is computed using the Pearson correlation coefficient formula. This approach ensures an unbiased estimation of the correlation.

In the DCC-GARCH model, the conditional correlation matrix is derived from the standardised disturbances ε_t:

$$\varepsilon_t = D_t^{(-1)} * r_t \sim N(0,R_t).$$

And R_t is define as:

R_t = Q_t_star^(-1)*Q_t*Q_t_star^(-1)

Q_t = (1-a-b)Q_bar + a*ɜ_{t-1}*transpose(ɜ_{t-1}) + b*Q_{t-1}

Where:

- Q_bar = cov(ɜ_t*transpose(ɜ_t)) is the unconditional covariance matrix of the standardised errors ɜ_t and can be estimated as the arithmetic average of ɜ_t*transpose(ɜ_t).
- a and b are scalars
- Q_t_star is a diagonal matrix with the square root of the diagonal elements of Q_t at the diagonal.

The 2*2 conditional variance-covariance matrix is modelled by:

H_t = D_t*R_t*D_t

Where D_t is positive definite since all the diagonal elements are positive and R_t is positive definite and all its elements are between -1 and 1.

The quasi-likelihood function to maximise is:

Ln(L) = ∑{t=1, T}(ln(|R_t|) + transpose(ɜ_t)*R_t^(-1)*ɜ_t).

Before proceeding to establish the minimum variance hedge ratio, an overlooked yet crucial aspect from the outset needs attention. The models presented allow us to derive hedge ratios in relation to the volatility of the Revert-to-Rate and the volatility of the one-month LIBOR future contract. However, it's imperative to note that this paper doesn't aim to hedge interest rate volatilities with futures contracts. Instead, the focus lies distinctly on addressing the volatility of interest payments, marking a crucial distinction in our study's objective.

Simulating Loan Repayment

The type of mortgage loan considered for this research is the amortising loan in which interest is repaid over time. This loan structure requires the borrower to make consistent principal and interest payments, ensuring the loan's full repayment. In the early years of an amortising loan, a greater portion of the monthly payment goes towards interest and a smaller portion goes towards principal reduction. However, as the loan term progresses, the interest-to-principal ratio changes. This means that as the loan matures, the interest component becomes less important and the principal repayment becomes more important. The gradual reduction of the outstanding balance over time is the defining feature of amortising loans, which ultimately result in the loan being paid off in full by the end of the loan term.

Utilising the house price data available for each period, we employ the subsequent formula to simulate monthly mortgage payments, considering the Revert-to-Rate over the loan's maturity, under the assumption of a 30-year mortgage term and a 75% loan-to-value (LTV) ratio:

Monthly payment[t]= (Loan_Amount[t]*(R[t]**Loan_Term[t])*(1-R[t]))/(1-R[t]**Loan_Term[t]).

Where:

- R[t] = the Revert-to-rate[t]/12
- Loan_term = 12*30 = 360 months
- Loan_Amount[t] = Loan_Amount[t-1] - Monthly payment[t-1] + Interest payement[t-1]

- Loan_Amount[1] = house price (1 - 0.25)
- Interest payment[t] = Loan_amount[t]*R[t].

This formula enables us to simulate the repayment of each loan. The simulation logic begins with the initiation of the first loan in the initial months. Subsequently, for each following month, a new loan is initiated, and all previous loans are in the repayment process. Consequently, every month, the repayment process encompasses a growing number of loans initiated in the preceding months. At the end of the process, we generate a 338x338 diagonal matrix. Rows denote various data points including the date, monthly variable rate, future quote for the month, interest cash flow, hedge cash flow for each different deployed models, and more. Columns correspond to individual loans.

Interest Payments Volatility Model

Therefore, we arrive at the most important and subtle point of this research: the distinction between the volatility of the variable rate and the volatility of interest payments necessitates their individual modelling. The forthcoming demonstration will outline the reasons for this crucial differentiation. However, prior to delving deeper, we must establish some notation:

I[n] represents the nth interest payment over the course of the loan maturity.

L[n] represents the nth loan amount; thus, L(1) is the initial mortgage loan amount.

R[n] represents the nth monthly interest rate,

and M[n] represents the nth monthly payment.

The sum from 1 to n of x[i] is denoted as sum{1,n}(x[I]).

<u>Mathematical proof:</u>

The nth interest payment is calculated as:

I[n] = L[n] * R[n]

However, L[n] can be expressed as:

L[n] = L[1] - sum{1,[n-1]}(M[i]) + sum{1,[n-1]}(I[i])

This equation indicates that the nth loan amount is equal to the initial mortgage loan amount minus the sum of previous reimbursements (which is the monthly payment minus the interest paid).

Therefore, the nth interest payment can be written as:

I[n] = (L[1] - sum{1,[n-1]}(M[i]) + sum{1,[n-1]}(I[i])) * R[n]

The difference between the nth interest payment and the previous one is given by:

I[n] - I[n-1] = (L[1] - sum{1,[n-1]}(M[i]) + sum{1,[n-1]}(I[i])) * R[n] - (L[1] - sum{1,[n-2]}(M[i]) + sum{1,[n-2]}(I[i])) * R[n-1]

Using the identity: sum{1,[n-2]}(x[i]) = sum{1,[n-1]}(x[i]) - x[n-1])

After thorough development and subsequent refinement, we obtain:

$$I[n] - I[n-1] = L[1] * (R[n] - R[n-1]) - sum\{1,[n-1]\}(M[i]) * (R[n] - R[n-1]) + sum\{1,[n-1]\}(I[i])) * (R[n] - R[n-1]) - R[n-1] * (M[n-1] - I[n-1]).$$

If we denote the difference between I[n] and I[n-1] as 'ΔI', and the difference between R[n] and R[n-1] as 'ΔR', this notation brings clarity to the discussion.

Hence,

$$\Delta I = \Delta R * (L[1] - sum\{1,[n-1]\}(M[i]) + sum\{1,[n-1]\}(I[i])) - R[n-1] * (M[n-1] - I[n-1])$$

$$\Delta I = \Delta R * Ln - R[n-1] * (M[n-1] - I[n-1])$$

$$\Delta I/I[n-1] = (\Delta R * Ln - R[n-1] * (M[n-1] - I[n-1])) / I[n-1]$$

Where $I[n-1] = L[n-1] * R[n-1]$

$$\Delta I/I[n-1] = (\Delta R / R[n-1]) * (Ln / L[n-1]) - (M[n-1] - I[n-1]) / L[n-1]$$

The ratio $\Delta I/I[n-1]$ can be interpreted as the interest payment return. The term $(\Delta R / R[n-1])$ represents the Revert-to-Rate return. The ratio $Ln / L[n-1]$ denotes the outstanding loan ratio between the nth period and the previous one. Lastly, $(M[n-1] - I[n-1]) / L[n-1]$ is the reimbursement ratio from the previous monthly payment. This equation is a mathematical relationship that links the variable rate return with the interest payments return.

Therefore, from the equation:

$\Delta I/I[n-1] = (\Delta R / R[n-1]) * (Ln / L[n-1]) - (M[n-1] - I[n-1]) / L[n-1]$

We can determine the volatility of interest payments by taking the absolute value of this equation. To determine the precise value of this equation, however, we must know the direction of the return R[n]. Unfortunately, the GARCH model is only capable of predicting the amplitude of R[n] using [n-1] information, but not its direction.

Assuming all other values except the Revert-to-Rate return are given, and considering the Revert-to-Rate return follows a GARCH(1;1) process, given that the sole uncertain value in the interest payment equation is the variable rate, it can be concluded that the interest payment also follows the same GARCH(1;1) process. By substituting the variable rate returns with their volatilities in the preceding equations and computing their absolute values, we derive a downward biased estimate of the interest payment volatility:

sigma_I_t = abs(sigma_R_t* (Ln / L[n-1]) - (M[n-1] - I[n-1]) / L[n-1])

Where:

- sigma_I_t is the downward biased estimate of the interest payments volatility at time t.
- abs() computes the absolute value of the expressions enclosed within its parentheses.
- sigma_R_t is the variable rate volatility at time t.

The variable rate volatility at time t is forecasted via the multivariate GARCH model. Hence, we can now forecast the interest payment volatility at time t using the information available at time t-1 based on this equation.

Despite the bias in this estimate, it is still extremely valuable to mortgage holders. This estimate's precision increases in proportion to the monthly payment's volatility. In simpler terms, the optimal hedge becomes more precise as risk increases. This connection results from the following equation:

$$\Delta I/I[n-1] = (\Delta R/R[n-1]).*(Ln / L[n-1]) - (M(n-1) - I(n-1)) / L(n-1)$$

The repayment ratio, represented by the left-hand side element '(M[n-1] - I[n-1]) / L[n-1', tends to approach zero in an amortising loan structure when the monthly payment consists primarily of interest payments. Thus, when monthly payments are most expose to volatility (as a result of being comprised primarily of interest payments), the left-hand side shrinks, resulting in a more accurate estimation of interest payment volatility.

Now that we've specified the computation for interest payment volatility on the basis of variable rate volatility, we can incorporate it into our dynamic hedge ratio.

Minimum-Variance Hedge Ratios

We derive the dynamic hedge ratios from our multivariate GARCH models and the interest payment volatility model:

b_t = correlation_t*(sigma_I_t/sigma_F_t).

Where:

- b_t is the dynamic minimum-variance hedge ratio at time t.
- correlation_t is the correlation forecasted at time t.
- sigma_I_t is the interest payments volatility forecasted at time t.
- sigma_F_t is the future contract volatility forecasted at time t.

Notably, in the case of the CCC GARCH model, the forecasted correlation at a given time will be the invariant correlation empirically calculated from the in-sample data.

Therefore, the optimal number of future contracts to enter in a position with is:

$N_t = b_t * (Q_{I,t}/Q_{F,t})$

Where:

- N is the calculated number of contract that should be used in hedging at time t.
- b_t is the minimum-variance hedge ratio at time t.
- $Q_{I,t}$ represents the size of the most recent interest payment made, known at time t.
- $Q_{F,t}$ is the size of one future contract at time t.

The static hedge ratio is given by the estimated slope coefficient of the regression line. The regression-based hedge ratio is the most simple to estimate and it is estimate from the regression:

$\Delta R = a + b\Delta F + \varepsilon$

Where:

-F represents the future price at time t.

-R represents the variable rate at time t.

-a is the intercept coefficient.

-b is the slope coefficient.

-ε represents the error term, capturing unobserved factors affecting the relationship between spot and future prices.

The slope coefficient b represents the relationship between fluctuations in the variable rate and fluctuations in the future price. This coefficient represents the optimal proportion of an asset's exposure that must be hedged with futures contracts to minimise risk in the context of hedging. This method assumes that the volatility and covariance between the two time series remain constant over time. It is essential to note, however, that Koutmos and Pericli (1999) caution against such assumptions and highlight the potential disadvantages. They highlight the importance of employing time-varying hedge ratios, emphasising their superiority over fixed assumptions due to their ability to adapt to fluctuating market conditions. More importantly, the regression-based hedging ignore the fundamental difference between the interest payments volatility and the variable rate volatility demonstrated in the Interest Payment Volatility section, resulting in a biased estimate.

This is an important finding as it reveals that the dynamic hedge ratio is solely downward biased when the variable rate return is negative. In essence, the Interest Payment Volatility model misrepresents the interest payment volatility solely when the actual variable rate return is negative at time t. However, this downward bias works in favour of the user, as the dynamic hedge ratio recommends fewer contracts when the variable rate drops. Hence, the user employing the dynamic hedging approach is adequately protected against increases in interest rates but less shielded when the rates actually drop. This enable him to benefit from decrease in interest rates despite being hedged, whereas regression-based hedging tends to recommend entering into a futures position with too many contracts.

However, I use the term 'tends' because static hedging isn't consistently upward biased. It can also be downward biased, especially toward the end of the loan term, when payments primarily consist of repayments rather than interest. However, overall, the static hedging ratio remains biased.

Hedged Monthly Payments Specification

In the hedging literature, the hedged portfolio with futures is mostly expressed in terms of returns:

$$r_{h,t} = r_{s,t} - b_{t-1} * r_{f,t}$$

Where:

- $r_{h,t}$ is the hedged return
- $r_{a,t}$ is the return on the asset we attempt to hedge
- $r_{f,t}$ is the return on the future contract
- b_{t-1} is the minimum-variance hedge ratio

This method is efficient for evaluating hedging effectiveness and modelling the hedged return. However, in our case, the interest payments return doesn't depend solely on the current variable rate return but also on all the preceding variable rate returns. This is due to the interest payments at time 't' being determined by the monthly variable rate at the same time, multiplied by the outstanding loan at that moment. And the outstanding loan itself is influenced by the previous repayments, which are contingent on the preceding monthly variable rates. This relationship is elucidated in both the 'Simulating Loan Repayment' and the 'Interest Payments Volatility Model' sections. Hence, we modelled the hedged interests payments fluctuation the following way:

$$\Delta h_t = \Delta I_t + N_{t-1} * \Delta F_t$$

Where:

- Δh_t is the change in the hedged interest payments at time t, if perfectly hedged, it is equal zero.

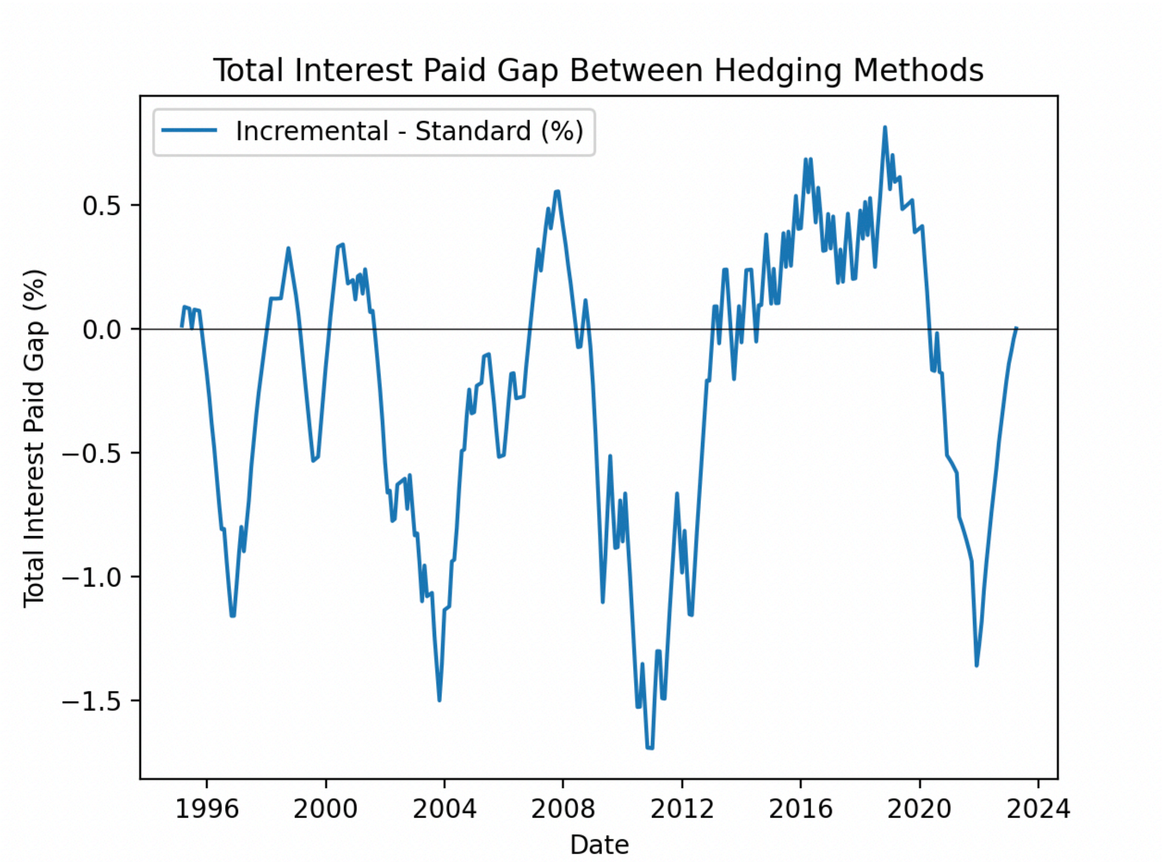
- ΔI_t is the change in interest payments at time t.

- ΔF_t is the change in future price at time t.

- $N\_\{t-1\}$ is the optimal number of future contracts to enter into a position at time t-1, is value is mostly negative.

If the interest payments are perfectly hedged, the mortgage owners should observe their monthly interest payments remain invariant, thereby

Figure 1: Comparative Analysis of Interest Cost Savings across Hedging Methods



eliminating uncertainty. The equation above captures the change in the

Figure 2: Interest Reduction by Hedging Method in Variable Rate Increases

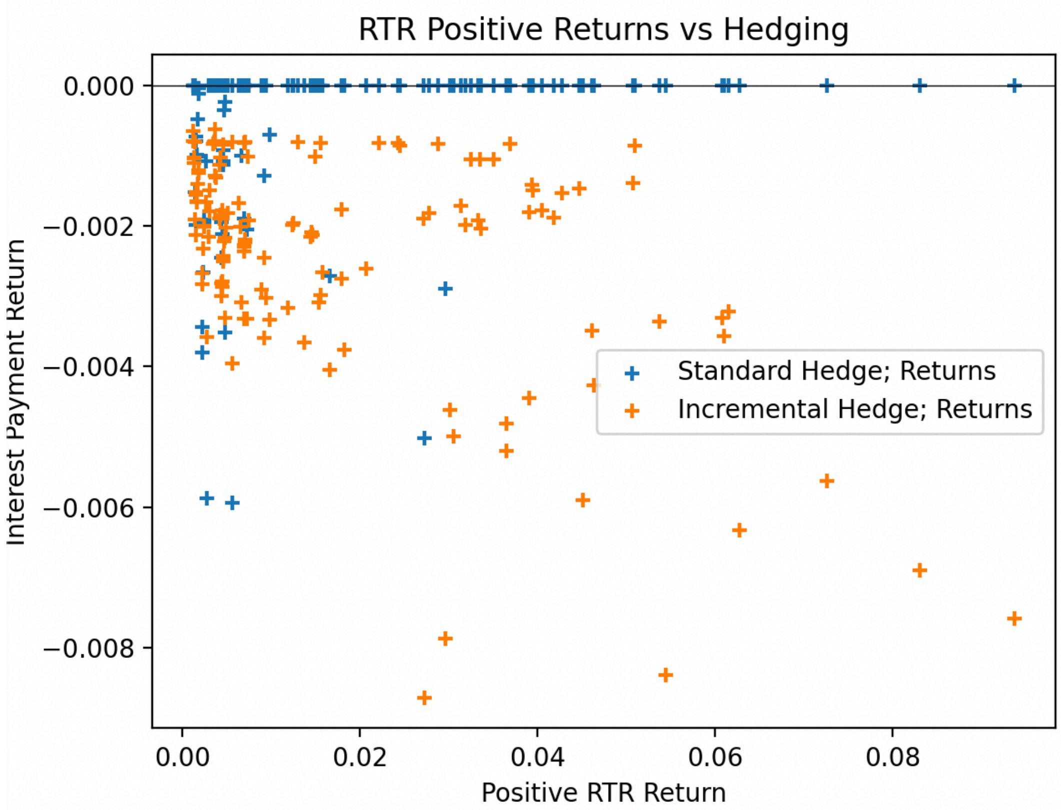
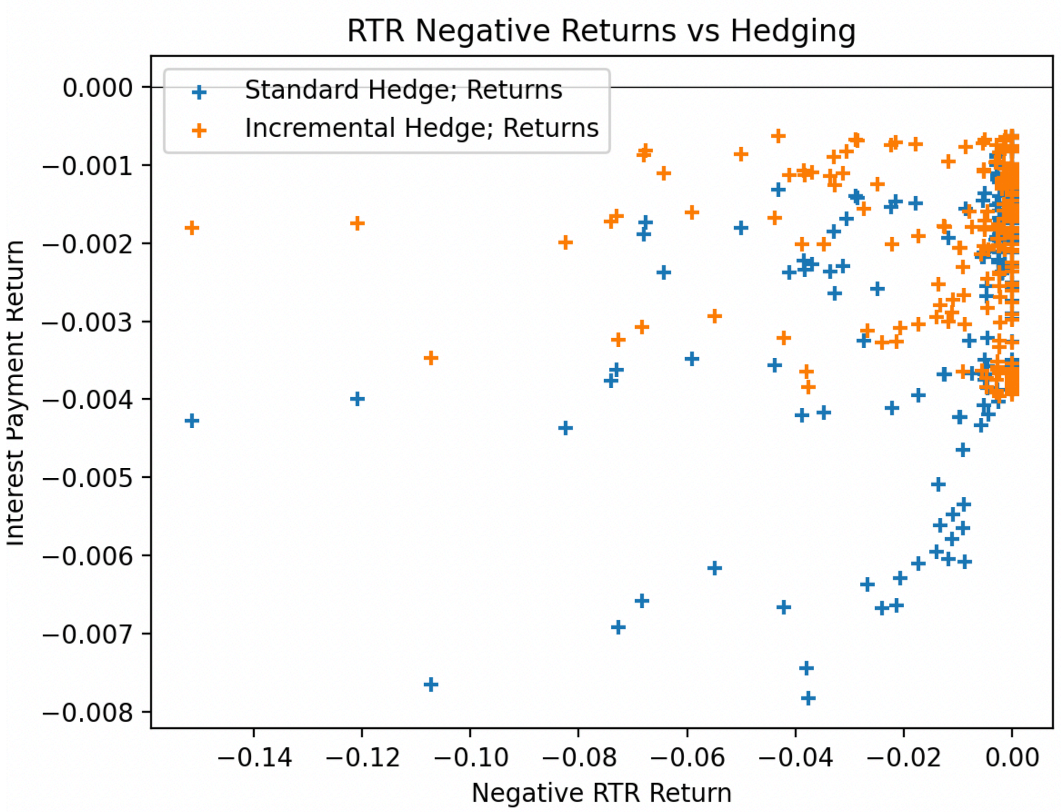


Figure 3: Interest Reduction by Hedging Method in Variable Rate Decreases



monthly interest payments when hedged, thereby illustrating the hedge effectiveness.

Incremental Volatility and Hedging

You have probably already noticed that something is off with the Hedged Monthly Payments Specification or with the Interest Payment Volatility Model. In fact, the specification of the interest payment volatility in the Interest Payment Volatility Model section seem to include an unnecessary component:

sigma_I_t = abs(sigma_R_t* (Ln / L[n-1]) - (M[n-1] - I[n-1]) / L[n-1])

Where:

- sigma_I_t is the downward biased estimate of the interest payments volatility at time t.
- abs() computes the absolute value of the expressions enclosed within its parentheses.
- sigma_R_t is the variable rate volatility at time t.

This equation calculate the volatility of interest payments within the framework of amortised loans, accounting for the variability in the variable rate and the characteristic decrease in interest payments as the mortgage loan is repaid. However, there is no economic rationale to hedge against the inherent decline in mortgage interest payments as the outstanding loan amount decreases. This natural decline is encapsulated in the equations by the ratio 'Ln / L[n-1],' representing the outstanding loan ratio between the nth period and the previous one, and by '-(M[n-1] - I[n-1]) / L[n-1],' signifying the repayment ratio from the preceding monthly payment.

if our objective were solely to hedge the additional volatility associated with the variable rate, excluding the hedging of interest payment declines resulting from the loan structure. In other words, hedging the incremental interest payments volatility. The incremental interest payments volatility is purely equal to the variable rate volatility:

sigma_Inc_t = sigma_R_t

Where:

- sigma_Inc_t is the additional volatility associated with the variable rate at time t.
- sigma_R_t is the variable rate volatility at time t.

And the hedging effectiveness could be expressed in return terms, similar to the current literature. However, there does exist an economic rationale for utilising our Interest Payments Volatility model. The reason is that the model is capable of accurately hedging against upward movements in the variable rate, albeit becoming downward biased when the variable rate declines.

To illustrate this, consider a scenario where the volatility at time 't' is entirely known at time 't-1,' yet the direction of the return remains unknown. Let's also assume a direct hedge against the variable rate risk using the variable rate itself. Consequently, the correlation remains constant and equals 1. At time t-1, we have certainty regarding the incremental interest payment volatility, but certainty regarding the interest payment volatility occurs only if the movement in the variable rate is upward, as outlined in the Interest Payment Volatility model. To accurately ascertain this volatility, we require knowledge of the variable rate return, an assumption being that such knowledge is unavailable. Let's conduct a simulation involving the repayment of our 338 mortgage loans while employing both the incremental approach and our proposed approach, which defines the interest payments volatility through our Interest Payments Volatility model. Figure 1 visualises whether a particular method effectively hedges monthly interest payments while capitalising on the natural decline attributed to the amortised loan structure. When comparing both approaches, it's unclear which method leads to greater savings in total interest paid by the end of the loan. The difference remains minimal, not exceeding 1.6%. Although the spikes appear larger, seemingly in favour of the Incremental Approach in Figure 1, there is no discernible pattern; the outcomes fluctuate, hovering around a neutral stance. The reason behind this lies in the reduction of monthly interest payments. The incremental approach outperforms our approach when rates increase, while our approach fares better during rate declines. As depicted in Figure 2 and Figure 3, the incremental approach, shown in Figure 2, leverages rising variable rates, offsetting increased interest payments and even reducing them further due to its higher exposure. The returns are widely distributed across the graph. Conversely, our approach precisely counters any rise in variable rates but doesn't capitalise on them by increasing exposure. The returns are concentrated and skewed toward zero. However, in figure 3, our approach is shown to be downward biased when returns are negative, being under-exposed to the variable rate. Its returns are widely spread across the graph, showcasing its ability to take advantage of declining rates and further reduce the interest payments, whereas the incremental approach is concentrated near the zero axis.

Insights into the Best Volatility Model

In the previous section, we delved into the fundamental disparity between the incremental and our approaches. The incremental method focuses on hedging the supplementary volatility arising from variable rates, neglecting the inherent volatility stemming from the amortised loans leading to a natural decline in interest payments over the loan's duration. This approach models volatility as follows:

sigma_Inc_t = sigma_R_t

Where:

- sigma_Inc_t is the additional volatility associated with the variable rate at time t.
- sigma_R_t is the variable rate volatility at time t.

Contrarily, our approach primarily aims to prevent any escalation in interest payments while capitalising on declining rates to reduce these payments further. This approach demonstrates a propensity for downward bias, allowing it to remain underexposed when the variable rate declines. The definition of interest payment volatility is as follows:

sigma_I_t = abs(sigma_R_t* (Ln / L[n-1]) - (M[n-1] - I[n-1]) / L[n-1])

Where:

- sigma_I_t is the downward biased estimate of the interest payments volatility at time t.
- abs() computes the absolute value of the expressions enclosed within its parentheses.
- sigma_R_t is the variable rate volatility at time t.
- Ln / L[n-1] representing the outstanding loan ratio between the nth period and the previous one

-(M[n-1] - I[n-1]) / L[n-1] representing the repayment ratio from the preceding monthly payment.

The preceding figure provided crucial insights into the characteristics of each volatility model. However, it didn't offer a definitive perspective on the superior model. The comparison between the two models revealed minimal differences, with savings on total interest not surpassing 1.6% and fluctuating considerably around zero. Moreover, each model seems more adept at capitalising on only one direction of the variable rate return. Nevertheless, Figure 4 contributes partially to addressing these queries by offering supplementary insights. In Figure 4, each data point represents the discrepancy between the variances of monthly interest payments across an entire loan. The comparison is made between the volatility modelled by the incremental approach and our proposed approach. As shown, our volatility model tends to generate less variance within the monthly interest payments of the loans. Specifically, the incremental volatility model produces more variance, ranging between 12% to 5% higher than our volatility model from 1995 to 2010. Post-2010, the gap narrows, with the incremental volatility model generating approximately 10% less variance. However, this outcome becomes less pertinent in light of the diminishing loan interest payment observations after 2010 for instance. As time progresses, each loan holds less information, impacting the relevance and accuracy of the resulting variance or other outcomes in our analysis. To emphasise this point, I've plotted the raw variance gap between the two models on the left axis. Although the percentage gap appears significant, in absolute terms, the variance gap reached a peak of 250 and a minimum of -50, showcasing the disparity between the incremental volatility model and our approach. However, it is important to remains cautious with this conclusion as it need further research.

Overall, both volatility models have their strengths and weaknesses and offer comparable performance in reducing the overall interest paid on loans while maintaining hedging strategies, with a difference not exceeding 1.6%. However, there are indications that our model tends to generate less variance in these monthly interest payments.

Findings

Models Estimations and Fit

The Models, described by the equations in the sub-section "Model Specification" are estimated using the Revert-to-rate (RTR) returns and the sterling LIBOR future returns. The estimations of the OLS model are displayed in Table 2. The parameters are estimated using the method of least squares. The estimations of the univariate gaussian GARCH model parameters are estimated independently and through maximum likelihood estimations. The CCC GARCH parameters are based on the univariate gaussian GARCH models estimations, the unconditional correlation is calculated empirically. The estimates are displayed in table 3. The DCC GARCH parameters are estimated through maximum likelihood estimations and are displayed in table 4. The univariate GARCH parameters reveal distinct characteristics between the Revert-to-rate and Sterling LIBOR Future. The long-term average variance (Omega) shows notably higher unconditional volatility for the Revert-to-rate compared to the Sterling LIBOR Future. Both rates exhibit a moderate level of persistence in volatility (Alpha), emphasising past shocks' weight in forecasting future volatility. In contrast, the low Beta suggests that past variances don't directly influence future variances in this model. When considering unconditional volatility, the Revert-to-rate demonstrates a substantially higher level compared to Sterling LIBOR Future, indicating greater volatility irrespective of conditions. The estimated parameters from the second step of the DCC-GARCH models, with 'a' being 0.9999 and 'b' equaling 0, suggest that prior correlation strongly influences the current correlation, emphasising the model's sensitivity to correlation patterns over time. However, the estimate for 'b' indicates that the model doesn't incorporate lagged conditional correlation for predicting future correlation trends. This implies that the model is highly responsive to immediate changes or new information in the underlying data. The Gaussian-distributed errors resulting from the univariate GARCH models, both in-sample and out-of-sample, are plotted across Charts 1-4. The standardised errors appear to have large negative and positive values across all charts and do not behave like white noise.

Chart 1: Future Standardised Errors, In Sample

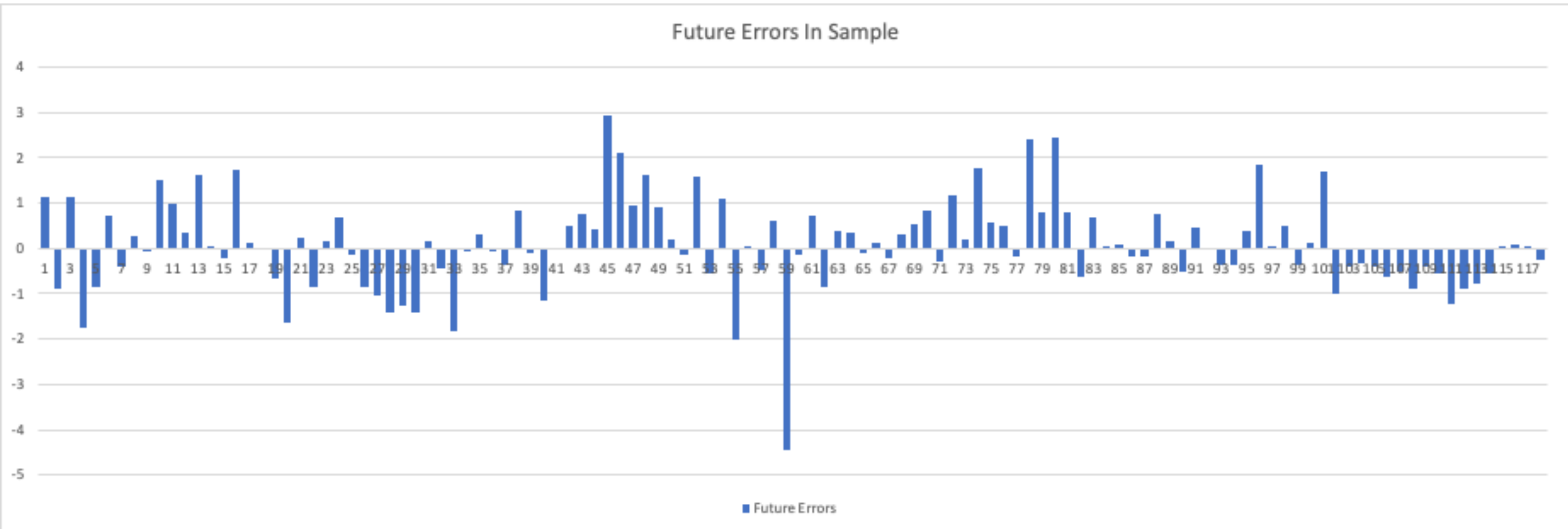


Chart 2: Future Standardised Errors, Out Sample

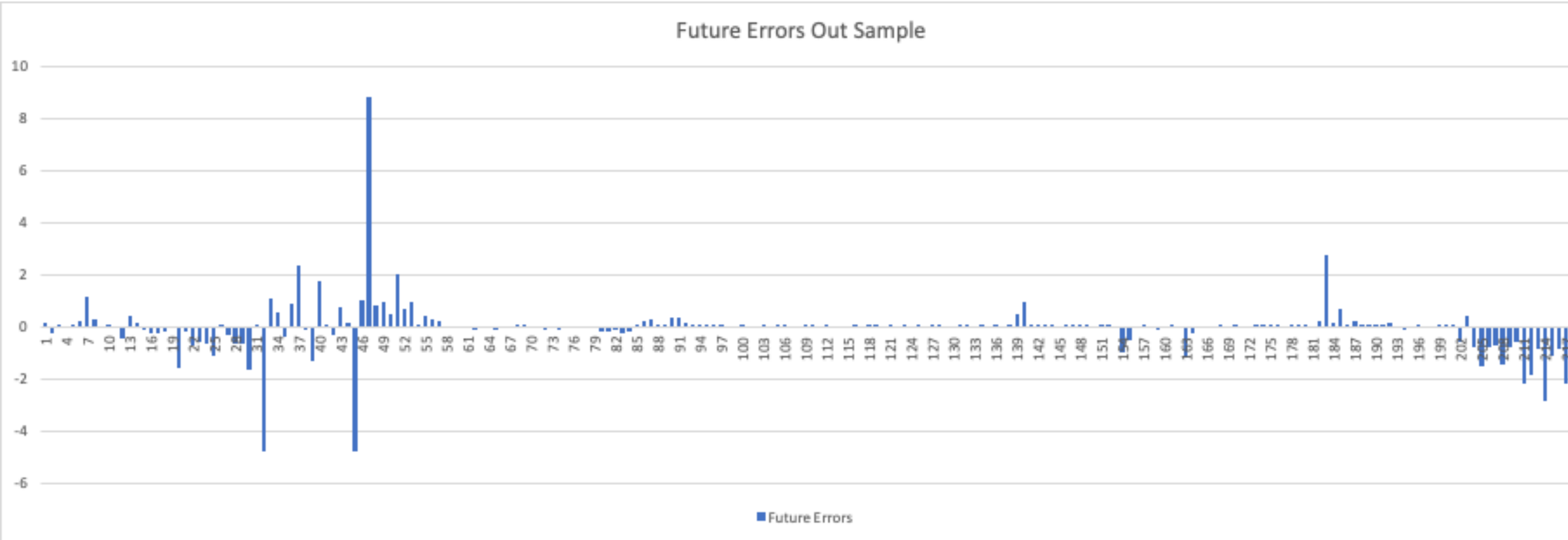


Table 2: OLS Regression Model Parameters

	Coefficients
Intercept	-0.0009
Slope coefficient	-4.8750*

*Significant at the 5% level.

Chart 3: Revert-to-rate Standardised Errors, In Sample

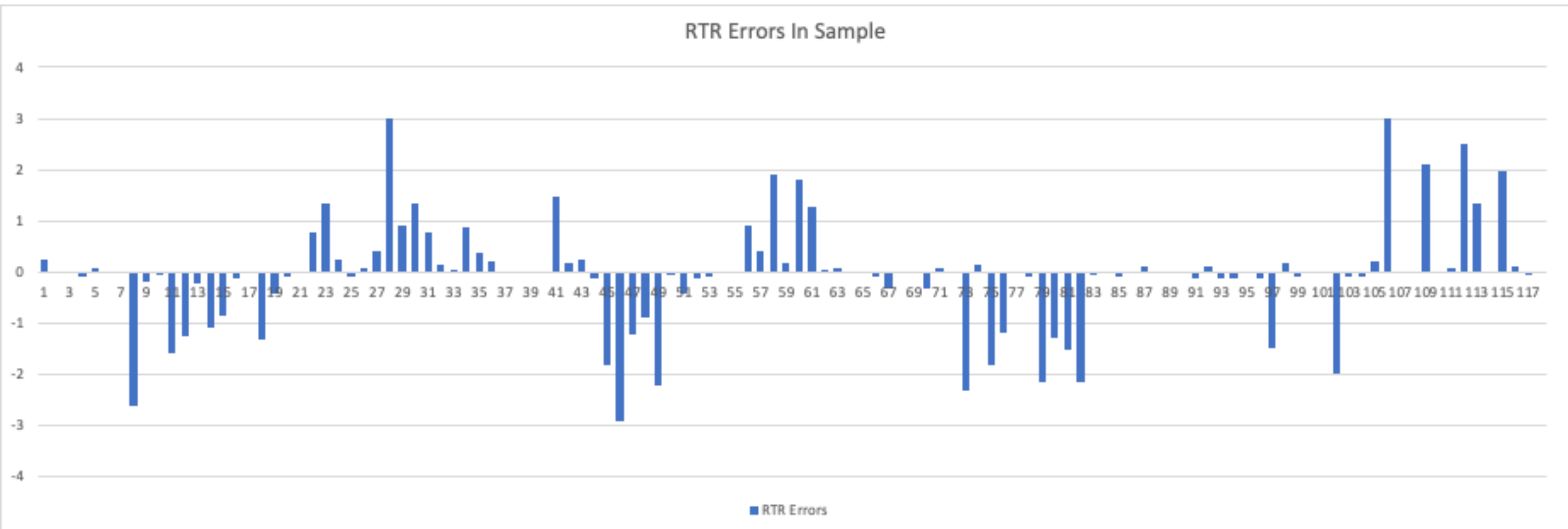


Table 3: CCC GARCH Model Parameters

	Revert-to-rate	**Sterling LIBOR Future**
Omega Ω	0.0001869	0.0000025
Alpha ∂	0.28	0.29
Beta ß	0	0
Unconditional volatility	0.0216149	0.0018649
Unconditional correlation	-0.4216396	-0.4216396

Table 4: DCC GARCH Model Parameters

	Revert-to-rate	**Sterling LIBOR Future**
Omega Ω	0.0001869	0.0000025
Alpha ∂	0.28	0.29
Beta ß	0	0
Unconditional volatility	0.0216149	0.0018649
a	0.9999	0.9999
b	0	0

Table 5: Univariate GARCH Models Diagnostics

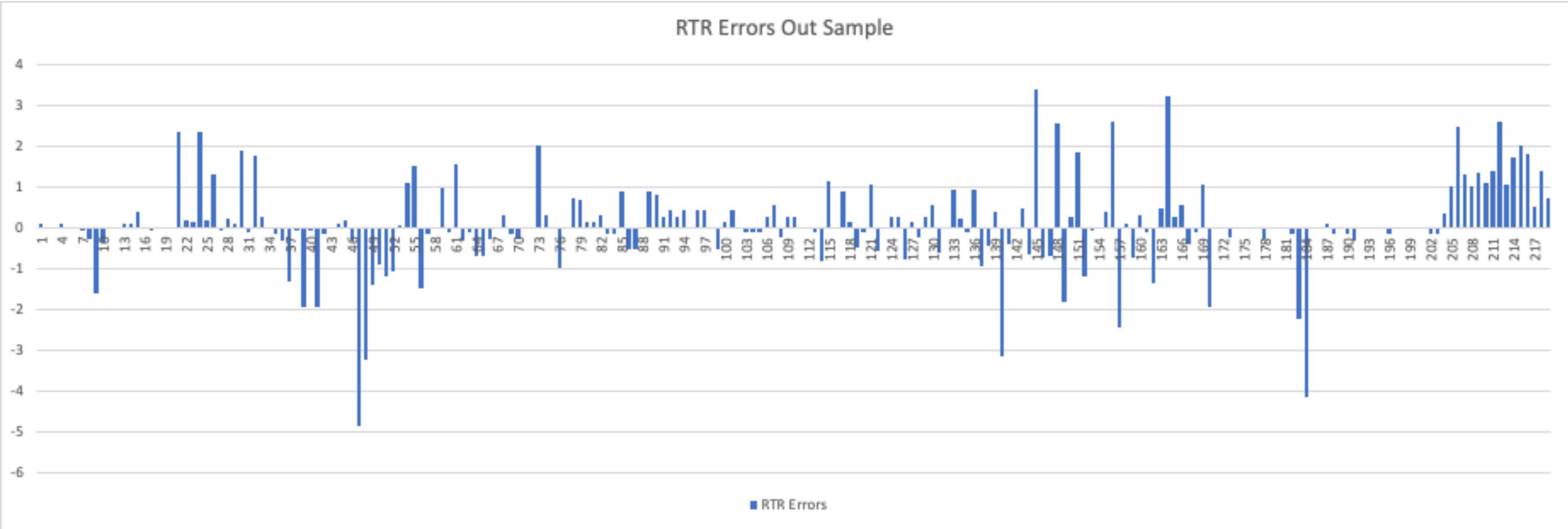
	Revert-to-rate In Sample	**Revert-to-rate Out Sample**	**Sterling LIBOR Future Out Sample**	**Sterling LIBOR Future Out Sample**
Observations	119	219	119	219
Mean	-0.045933	0.069031	-0.00007	-0.05095
Standard deviation	0.99528	1.02343	0.99952	0.95973
T-test	-0.503	0.998	0.714	-0.786
Normal test	8.238*	42.115*	15.993*	180.693*
Skew test	0.092*	-3.392*	-1.548	9.538*
Kurtosis test	2.869*	5.533*	3.687	9.472*
Kolmogorov-Smirnov test	0.227*	0.205*	0.093*	0.292*

	Revert-to-rate In Sample	Revert-to-rate Out Sample	Sterling LIBOR Future Out Sample	Sterling LIBOR Future Out Sample
Ljung-Box Test(20) for z_{i,t}	111.145*	60.353*	64.780*	74.155*
Ljung-Box Test(20) for z_{i,t} ^2	18.993	17.734	13.736	29.951

*Significant at the 5% level.

The provided data in table 5 represents statistical summaries and tests conducted on the standardise errors of the two financial variables: Revert-to-rate and Sterling LIBOR Future, in both In Sample and Out Sample scenarios. The key points observed in the means of these variables across different samples indicate subtle differences. The tests conducted on these variables shed light on the data's characteristics and model assumptions. Notably, in both the In Sample and Out Sample for both variables, certain statistical tests such as normality tests, skewness, kurtosis, Kolmogorov-Smirnov, and Ljung-Box tests, reveal significant departures from normality and autocorrelation assumptions. These significant departures might indicate underlying complexities or patterns in the data that the model may not fully capture. However, the means themselves do not exhibit significant differences from zero, implying stability in the central tendencies of the variables. Overall the statistical tests and the plotted standardised errors point to potential deviations from normality and autocorrelation assumptions in the data. The statistical assessments and the patterns observed in the standardised errors suggest potential deviations from the expected norms of normality and autocorrelation within the data. These deviations might affect the model's dependability in capturing specific aspects of the underlying financial dynamics.

Chart 4: Revert-to-rate Standardised Errors, Out Sample



DCC estimation tend toward 1 as warn in p.9 (Regnesentral, Orskaug, 2009), however even after removing all possible outliers and changing the starting value, problem of convergence or model highly sensible to correlation breakdown due to the relationship nature of the rates ? Because massively outperform when correlation breakdown, it is likely due to nature relationship assets. Like us improvement is small.

Comparative Analysis of Hedging Strategies

The DCC GARCH model displayed insufficient variance improvement, leading to its exclusion from the analysis. Despite a marginal enhancement in variance, it notably underperforms compared to other models considered for potential hedging methods. This setback prompts a deeper investigation into the underlying reasons for the dynamic hedge ratio's inadequacy, a topic explored further in this section. Instead of the DCC GARCH model, the analysis incorporates the DCCO GARCH model, a custom-built synthetic model tailored specifically for this study. The DCCO GARCH model mirrors the structure of the DCC GARCH model with one significant difference: the derived minimum-variance hedge ratio is set to 0 when the forecasted correlation at time t-1 for time t is positive. Put simply, when the forecasted

Figure 4: Variance Gap in Absolute and Percentage Terms Among Hedging Methods

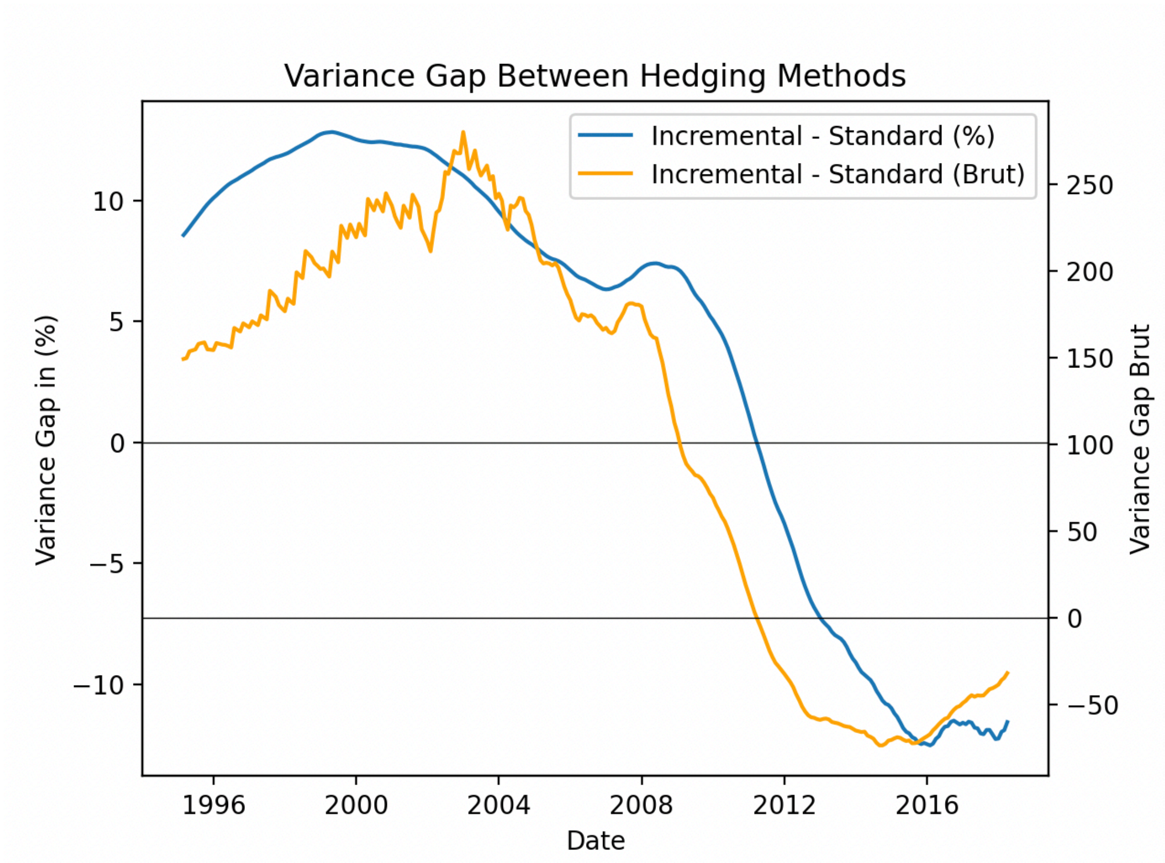
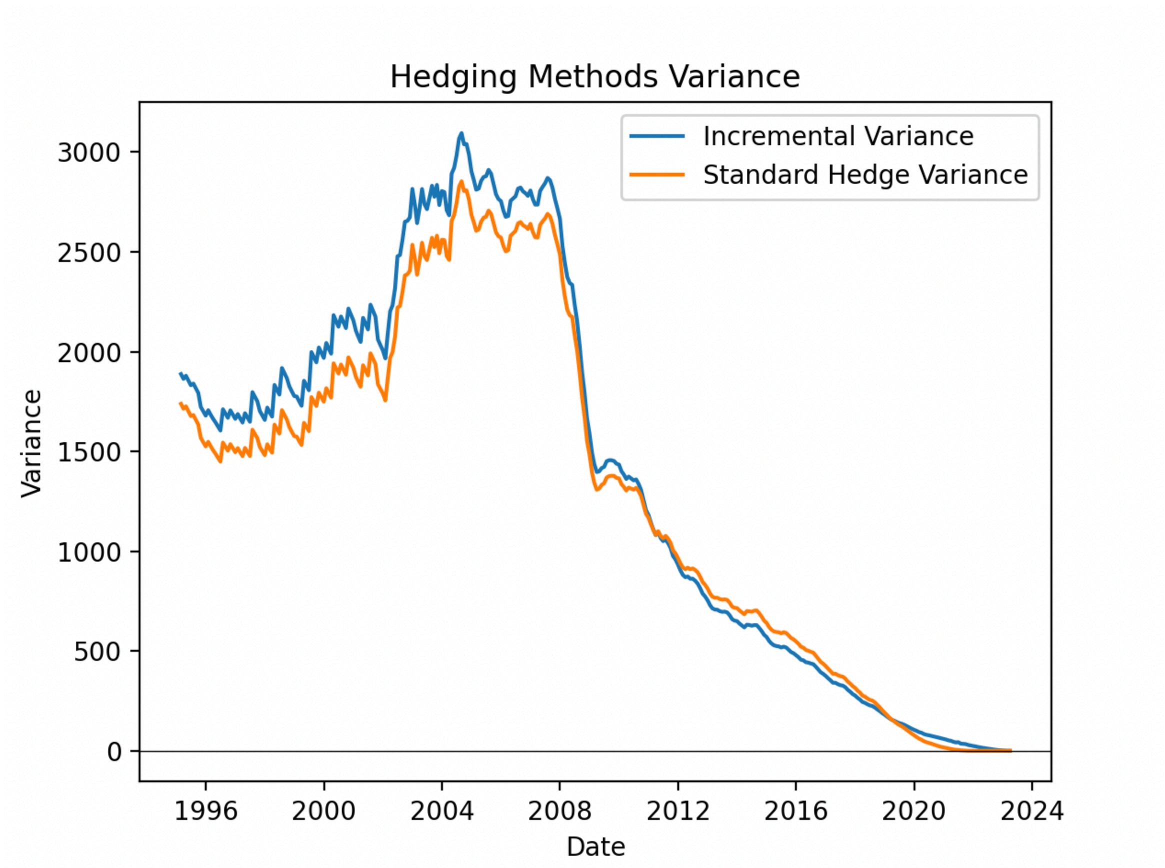
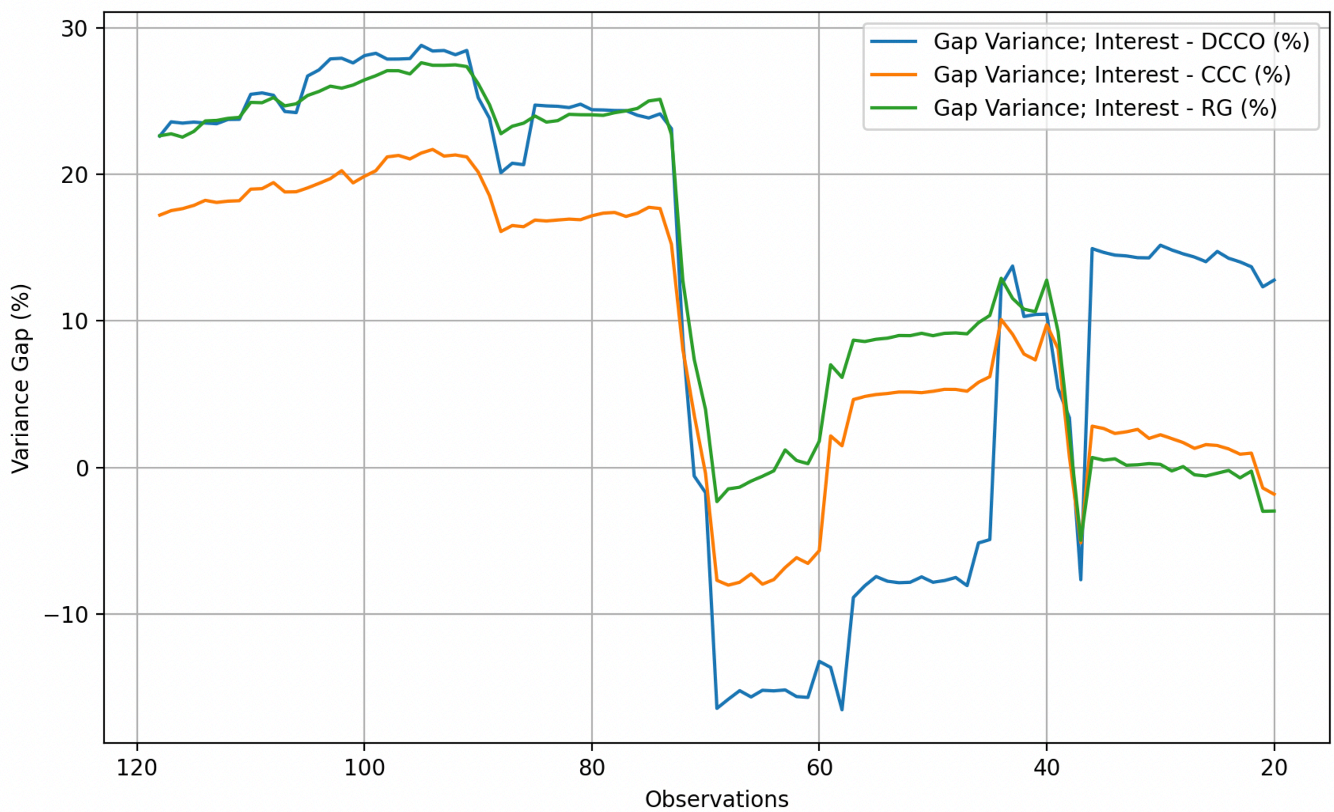


Figure 5: Comparison of Hedging Methods' Variances



correlation is negative, the hedge ratio is calculated based on the variance-covariance matrix established by the DCC GARCH model. However, if the forecasted correlation turns out to be positive, the DCCO GARCH hedging strategy refrains from taking any hedging position in the sterling LIBOR future. This decision stems from the observation that a positive correlation between the sterling LIBOR future price and the variable rate implies a negative correlation between the sterling LIBOR rate and the variable rate. Yet, as outlined in the "Aims and Objectives" section, there exists no logical or even moderate-term economic basis for these rates to be negatively correlated. Consequently, under such circumstances, we assert in this study that the market behavior is influenced by random fluctuations rather than indicative of an effective hedge. We leverage the forecasting capability of the

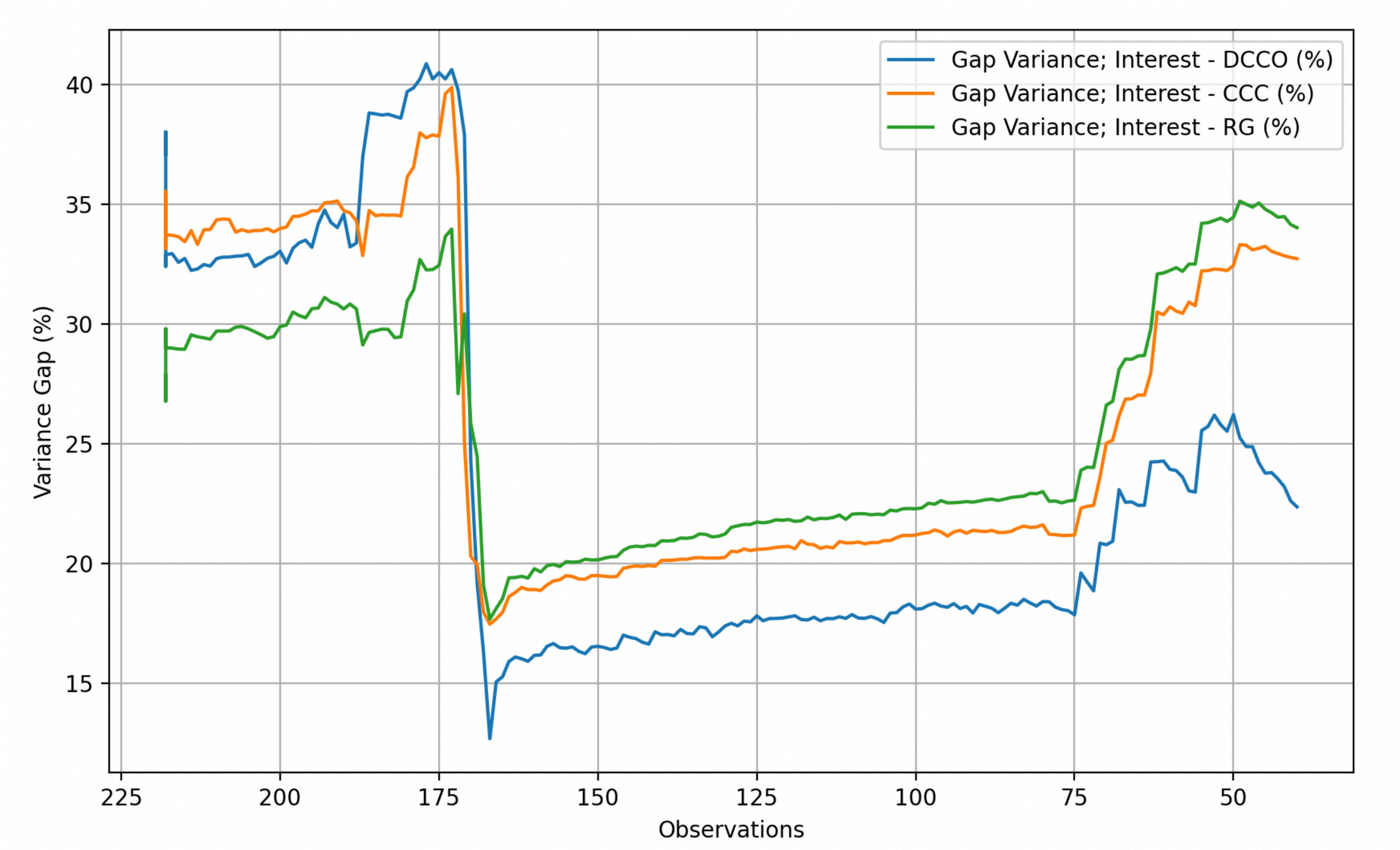
Chart 5: In-Sample Variance Improvement



DCC GARCH model for correlation to integrate this economic rationale into a novel dynamic hedging model: the DCCO GARCH model.

The in-sample and out-sample simulations were conducted without any additional information beyond December 31, 2004, for the in-sample, and without using any ex-post information prior to January 31, 2005, for the out-sample. Charts 5 and 6 encapsulate the variance improvement across

Chart 6: Out-Sample Variance Improvement

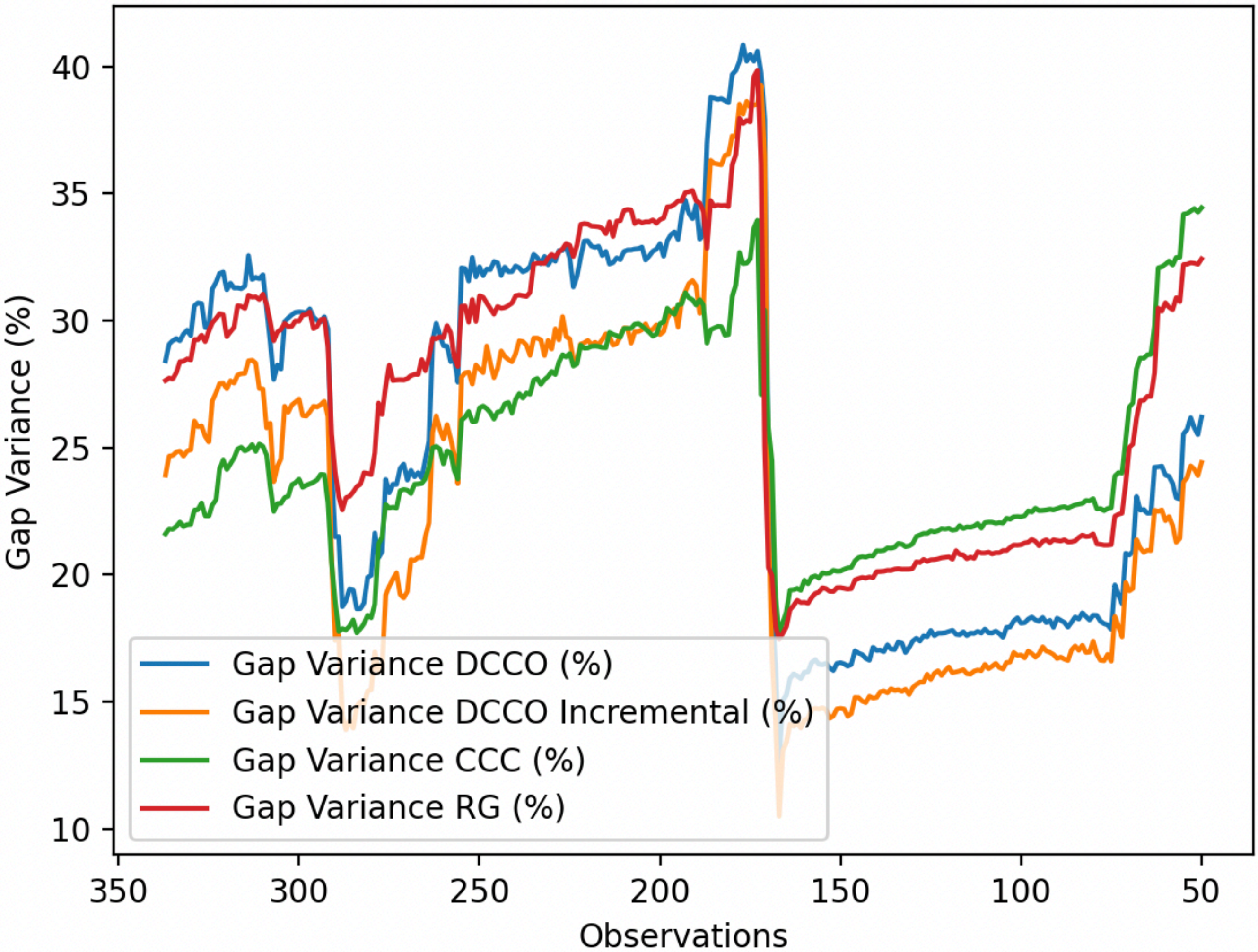


different hedging models compare to the initial interest payments variance. Each plotted point represents a simulated loan with its associated interest payments. The variance is calculated based on the interest payment series for each loan and depicted on the graph. Over time, newer loans with fewer associated payments are illustrated along the x-axis, representing the declining number of observations for each loan. As we progress along the x-axis, the decreasing number of observations diminishes the relevance of the variance in assessing its informational quality. The OLS model will be referred to as 'RG' on most of the informative graphs, serving as an acronym for 'regression'.

The method that initially reduces the variance most is unclear in the beginning of the in-sample graph (chart 5). On average, there's an approximate 10% improvement in variance overall. The top performers are competing against the OLS (also referred to as 'RG' in this study) model and the DCCO model (an acronym for the synthetic DCCO GARCH model). A similar scenario is observed in the out-sample chart (chart 6). However, the average overall variance improvement exceeds 20%. More importantly, the OLS model shows greater consistency in reducing variance the most. Chart 7 depicts the complete simulation, showcasing the variance improvement of the hedging methods in comparison to the variance of interest payments. The OLS method consistently emerges as one of the top performers, while the DCCO model also demonstrates competitive performance.

The OLS model operates incrementally, addressing solely the volatility linked to variable rate movements without incorporating our specific volatility model. Its limitation lies in the inability to account for the natural reduction in interest payments as the loan is amortised. To assess if the OLS model's superiority is inherent in its incremental approach, I included the DCCO incremental model, deriving the minimum-variance hedge ratio from the DCC GARCH model based on revert-to-rate volatility. Surprisingly, the DCCO incremental model demonstrated one of the poorest performances, notably underperforming even the standard DCCO model. This discrepancy suggests that the regression model's superiority is not solely due to its

Chart 7: Variance Improvement Across Hedging Models



incremental nature. This observation supports the notion highlighted in the section 'Insight into the Best Volatility Model' that incremental methodologies might inadvertently introduce unnecessary variance.

Considering Chart 7, one might be tempted to conclude that the OLS model is more reliable and effective compared to complex multivariate stochastic models. However, this paper specifically aims to provide a protective solution when financial burdens are at their peak, offering a flexible mechanism to mitigate risks and uncertainties. Hence, it is necessary to test these models in high-volatility environments.

Evaluating Hedging Methods' Robustness Against Volatility

This simulation is three-dimensional. Firstly, it involves a similar process illustrated in chart 7. For each loan, we simulate its repayment over time. As we progress through time, new mortgages are initiated each month based on the current house price reference. But instead we compute the variance of the monthly interest payments within the loan and plot this variance against the number of monthly payments it includes, representing the number of observations or, in other words, its reliability. Once this entire simulation is completed, the results are illustrated on chart 7. However, in this scenario, we repeat the process, but each time only considering interest payments that exhibit a volatility higher than the threshold volatility. We iterate this process, gradually elevating the volatility threshold, recalculating the variance for these payments and respective hedging methods, and plotting the results against the number of observations. This process yields multiple charts similar to Chart 7, each corresponding to different considered volatility

Chart 8: Variance Evolution Relative to Observations

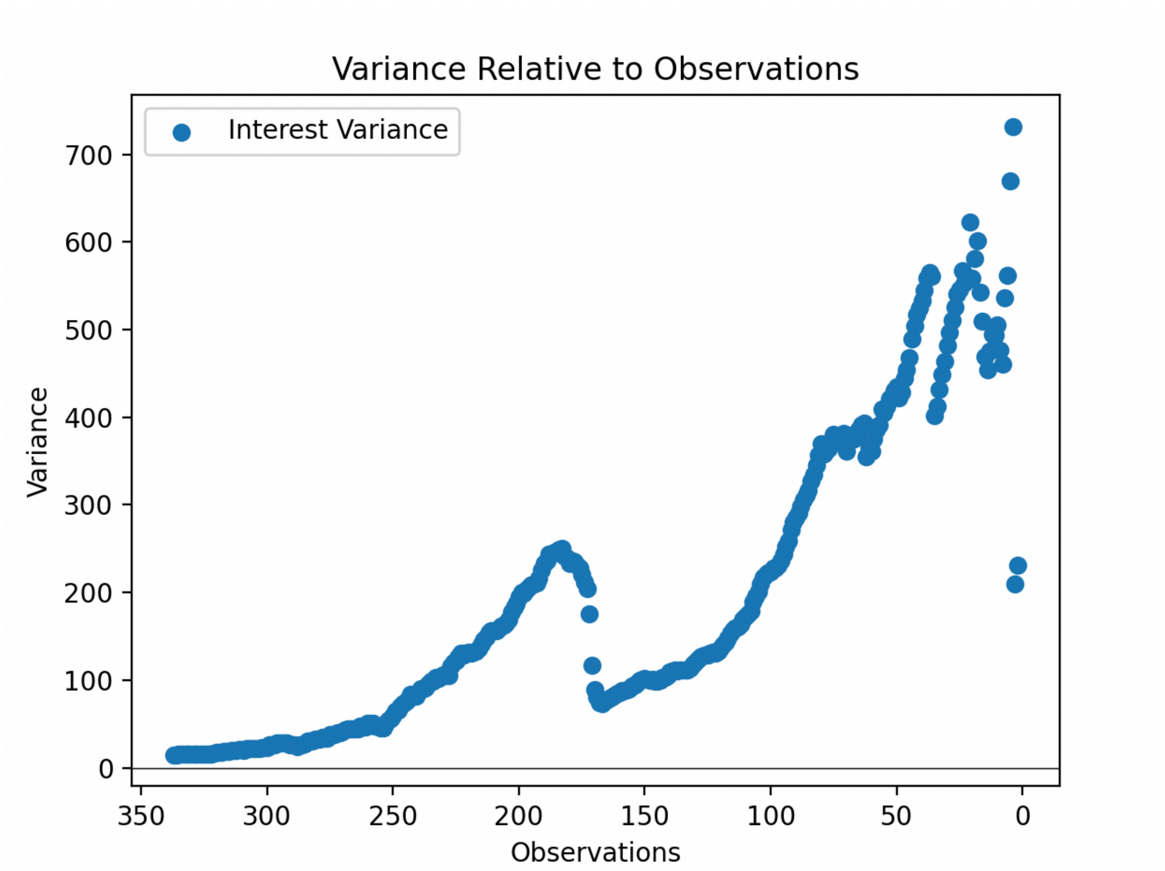


Chart 9: Fitted Function

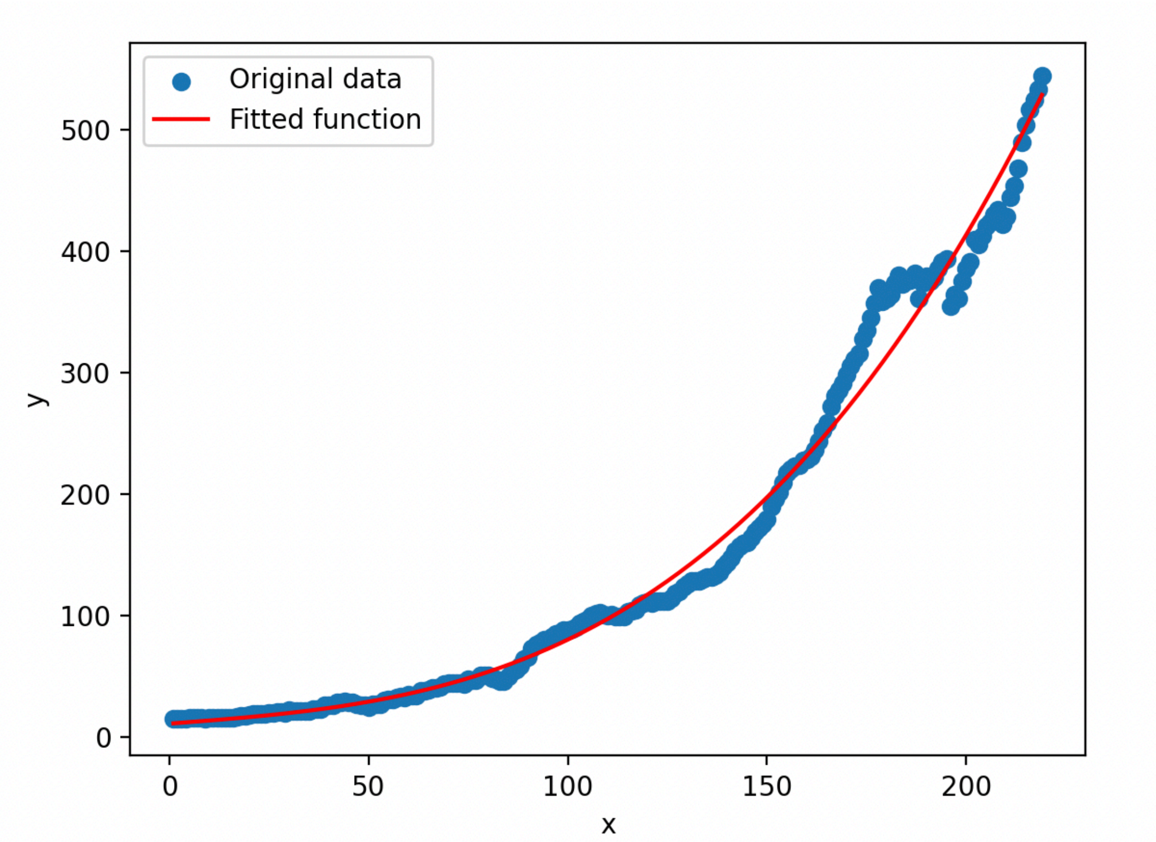
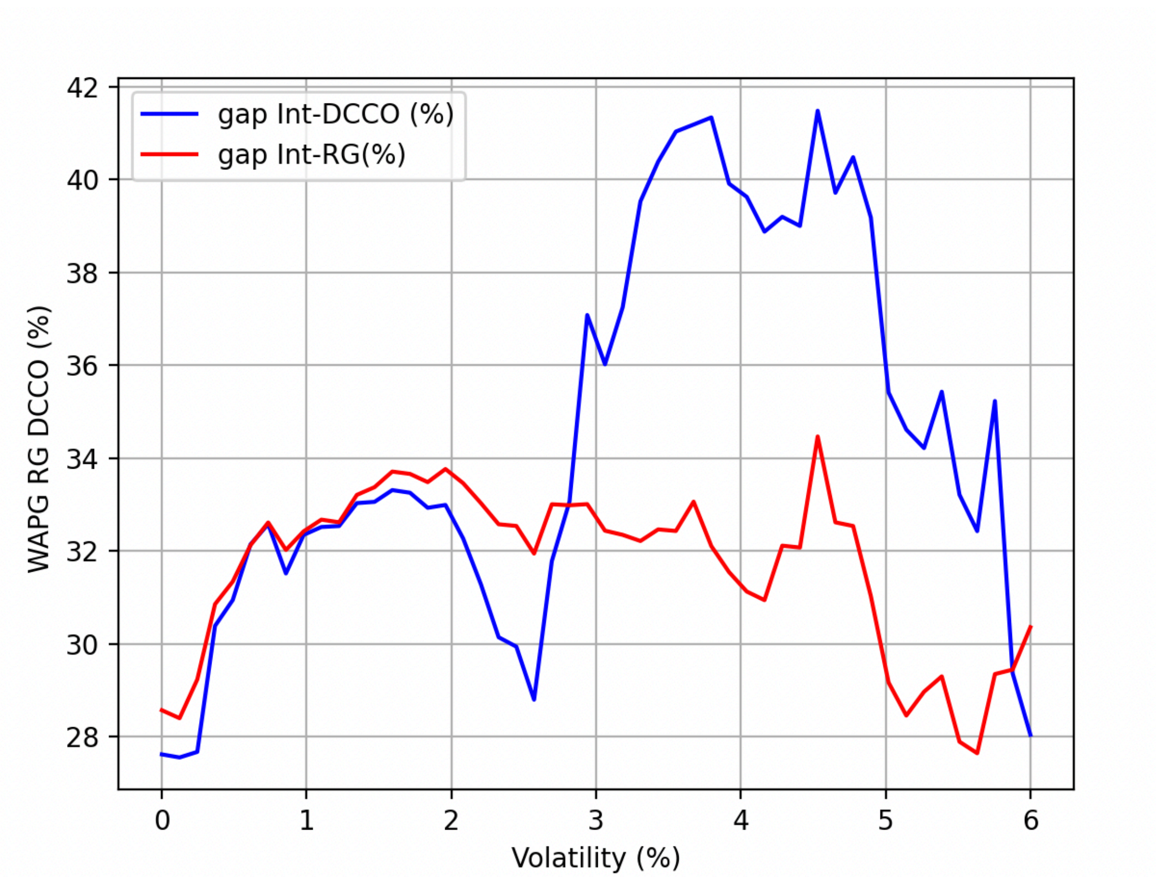


Chart 10: Weighted Average Variance Gap in Relation to Volatility



thresholds (where Chart 7 represents a threshold of zero). We will refer to these generated charts, each corresponding to a particular threshold volatility, as 'sub-worlds' to streamline our analysis. Each sub-world contains a significant amount of variance data, which becomes less informative with decreasing observations. Plotting a three-dimensional chart with observations on the x-axis, variance on the y-axis, and sub-worlds on the z-axis would result in unreadable visualisations. Instead, we will consolidate the variance information from each sub-world into a single data point.

To accomplish this, we aim to compute the average percentage variance gaps between the hedging methods variances and the interest variances in each sub-world. However, for this average to hold significance, it should consider the varying information content within the sub-worlds. Not all variances in these sub-worlds are equivalent; some contain more observations, rendering them more reliable. Hence, constructing a weighted average is necessary, with weights proportional to the information quantity. As we traverse the x-axis within the sub-worlds, the decline in observations doesn't align linearly with the reduction in information relevance; instead, it diminishes at a significantly accelerated pace. Understanding the correlation between the diminishing observations and the variance information quality is crucial. This correlation will enable us to appropriately weight each variance within the sub-worlds, facilitating the derivation of a precise weighted average.

Chart 8 highlights the variance behavior as the number of observations decreases. The observed relationship appears exponential. To emphasise this relationship, we eliminated variance outliers. Subsequently, we applied a third-degree polynomial function to ensure the relationship is strictly convex and increasing:

$a * x ** 3 + b * x + c$

We obtained the coefficients for this equation by fitting the curve to the original data, as shown in Chart 9:

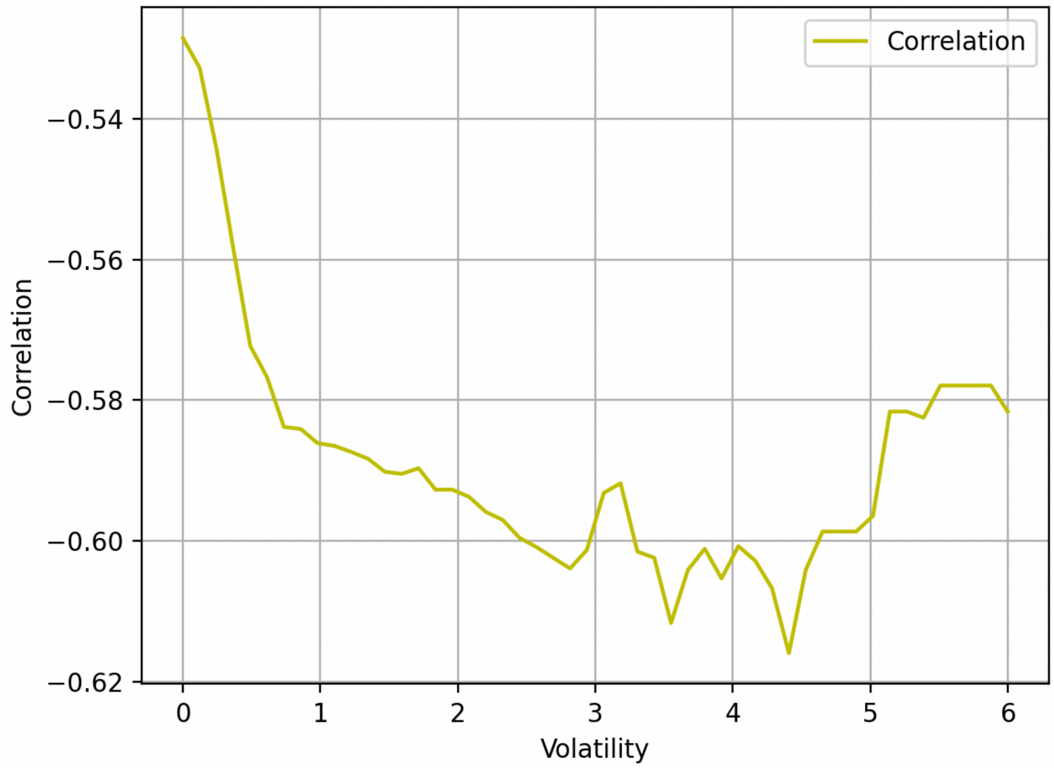
a = 0.000044

b = 0.25

c = 10.93

We posit that the rate of increase in this function signifies the mathematical relationship between the loss of information and the quantity of observations. Consequently, the weighting of variance in the sub-world will decrease proportionately. By incorporating this diminishing rate into our series of observation numbers, we assign weights to each variance, calculating the weighted average variance gap for each sub-world. Chart 10 illustrates these computed values. The x-axis represents the volatility threshold required for interest payments to be considered in the weighted average. On the y-axis, we observe the weighted average variance improvement of the hedging method concerning the initial interest variances. Both methods exhibit similar performance up to the 2% threshold. However, beyond this point, the OLS model notably outperforms the DCCO model. As volatility rises above approximately 2.81%, the DCCO's performance shows a significant surge. The rationale behind this trend is depicted in chart 11. The DCCO’s performance is intricately linked to the correlation between the two interest rates. As volatility escalates, the correlation also increases (refer to chart 11), aligning with the economic logic governing these rates. During substantial market shifts, these rates mirror economic dynamics rather than random fluctuations. Notably, owing to its

Chart 11: Correlation Dynamics: Future vs. Variable Rate with Changing Volatility



parameters (a = 0.9999 and b = 0), the multivariate model displays heightened sensitivity to correlation breakdowns.

Discussions

The parameters that modelled the covariance matrix in DCC GARCH model (a = 0.9999, b = 0) are unusual parameters and could be a sign that we reach a local optimum instead of a global optimum. This is because as the number of parameters to estimate increase the likelihood function become flat (Regnesentral, Orskaug, 2009). Despite removing outliers and adjusting starting values, the maximum likelihood estimation consistently converges toward these parameters. However, it could also signal significant correlation breakdowns as major shocks impact the relationship between the two interest rates. The parameters heavily weigh instantaneous past conditional correlation, making the hedging method valuable for navigating unpredictable market fluctuations. We capitalised on this by synthesising the DCCO GARCH model, which refrains from taking a position when a correlation breakdown is forecasted. Its significant performance and its ability to compete with the OLS model highlight the substantial impact of leveraging correlation breakdown and the utility of the estimated parameters. Although it's challenging to determine the most suitable model for mortgage owners, our analysis reveals mixed signals. However, it presents crucial insights, particularly demonstrating that the DCCO model outperforms others as volatility rises. This finding is significant as we seek to mitigate variable rate risks for mortgage owners facing financial burdens. Given the high economic uncertainty, mortgage holders prefer flexibility over fixed-rate contracts. The DCCO model offers such adaptability while effectively managing risks, especially during uncertain times, as its performance improves with increasing volatility. The standard DCC GARCH model was not retained as its inferior performance was assessed with certainty. The performance of the CCC GARCH model fell short of meeting the reliability criteria for consideration as a viable hedging approach in this study. The OLS model, however, displayed unexpected resilience, showcasing consistent performance relative to the other models throughout the analysis. However, our analysis lacks important details that could make the OLS model more attractive, such as transactional factors inherent in hedging decisions. The statistical summaries highlighted in table 5 raise concerns regarding the GARCH models' capability to precisely explain asset variance. These findings prompt a consideration that a t-student distribution might potentially offer a better fit than the Gaussian distribution used in our model. Another important point made in our analysis, is how we assessed the relationship between the quality of information that offer the variance and the number of observations. We utilised an analytical approach to explore this relationship, although the outcome was more of an interpretative process than a strictly scientific derivation. Defining and understanding this relationship lacks a precise framework. Our process involved numerous assumptions, particularly in assuming a cubic correlation between the loss of information and the removal of data. It's essential to acknowledge that this relationship might adhere to an exponential or alternate function. This uncertainty significantly impacts the weighted average variance gap, which, in turn, influences the conclusions drawn in this paper.

Conclusion

This research delves into a specific area that lacks comprehensive literature, as outlined in the 'Literature Review' section. Our study demonstrates the superior performance of our volatility model, purposely biased downward only when the variable rate declines, over the incremental approach when implemented within the synthesised DCCO GARCH model. Furthermore, we highlight how this model's efficacy escalates with an increase in correlation between the two interest rates, particularly in highly volatile scenarios, outperforming the OLS model. This discovery substantiates the viability of dynamic hedge ratios, especially in high-risk and uncertain environments where users are most susceptible. Overall, the study successfully achieves its objective by proposing a flexible hedging mechanism for mortgage owners to manage interest rate risk while minimising missed opportunities during rate declines. Yet, as delineated in the discussion section, this paper encounters several limitations. Among these are challenges in appropriately specifying error distributions and an insufficient explanation of variances derived from our GARCH model. Additionally, further analysis is necessary to determine whether our volatility model for interest payments genuinely offers a superior specification compared to the incremental approach for instance.

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