EECS 101: HOMEWORK #2

Due: January 23, 2019

1. Consider an amplified measurement C in electrons for a collection site (potential well) in a cooled CCD camera described by

$$C = (S + N_A + N_P)A$$

where S is the signal in electrons, N_A is a zero-mean amplifier noise source with a variance of 1 electron, N_P is the photon noise source, A is the constant amplifier gain, and N_A and N_P are independent. (Hint: read problem 3 before working on this problem)

a) Write an expression for the variance of the measurement C.

b) Define the signal-to-noise of the quantity C as its mean divided by its standard deviation. What is the signal-to-noise for the measurement C?

c) For what minimum value of S will the signal-to-noise exceed 100?

2. Consider an imaging system using a lens with focal length 4cm having an image plane 6cm behind the lens. Assume the lens diameter is 2cm.

a) How far in front of the lens on the optical axis of the system must we place a point to get an image without blur?

b) Suppose the image plane has an active area of 2cm $\times$ 2cm which is partitioned into 500 $\times$ 500 square potential wells (collection sites). Supose the point in part a) images without blur to the center of a potential well. How far can we move the point in focus towards the lens before the image extends to more than one potential well.

3. Assume that we have a CCD camera system that is cooled so that noise due to dark current is negligible. Then digitized pixel values will be given by

$$D = (S + N_A + N_P)A + N_Q \tag{1}$$

where S is the signal in electrons, N_A is the zero-mean amplifier noise source with variance σ_A^2, N_P is the zero-mean photon noise with variance S, A is the gain of the amplifier, and N_Q is the zero-mean quantization noise with variance σ_Q^2. Assume that the noise sources are independent.

a) Show that for a constant signal level S the expected value of D is

$$\mu = SA \tag{2}$$

and the variance of D is given by

$$\sigma_D^2 = A\mu + \sigma_C^2 \tag{3}$$

where

$$\sigma_C^2 = A^2\sigma_A^2 + \sigma_Q^2 \tag{4}$$

b) Write a program that reads a digital image $I(x, y)$ of size $N \times N$ where $N = 100$ and estimates μ by

$$\widehat{\mu} = \frac{1}{N^2} \sum_{1\leq x\leq N} \sum_{1\leq y\leq N} I(x, y) \tag{5}$$

and σ_D^2 by

$$\widehat{\sigma}_D^2 = \frac{1}{N^2 - 1} \sum_{1\leq x\leq N} \sum_{1\leq y\leq N} (I(x, y) - \widehat{\mu})^2 \tag{6}$$

Run your program to find $\widehat{\mu}$ and $\widehat{\sigma}_D^2$ for each of four images provided by your TA.

c) Plot $\widehat{\sigma}_D^2$ versus $\widehat{\mu}$ for these four points. Estimate A and σ_C^2 using a least squares fit of the line given by (3) to your data. A program that shows how to read an image will be provided in lab. You are required to demonstrate your program to your TA during lab.