# Homework lmeca2323 - Airfoils: XFLR5 results compared to potential flow results

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Consider a boundary layer on a flat plate aligned with the $x$-axis, and with irrotational flow at velocity $u_e(x) \geq 0$ at the edge of the boundary layer. We then have that the vorticity within the boundary layer is $\omega = -\frac{\partial u}{\partial y}$. The "circulation per unit length" of the boundary layer at some location $x$ is $\gamma(x) = -u_e(x)$.

The total vorticity flux at some location $x$ is $Q_\omega(x) = \int_0^{\delta(x)} \omega(x,y)\, u(x,y)\, dy$. We thus obtain that $Q_\omega(x) = -\frac{u_e(x)^2}{2}$ and thus also that $Q_\omega(x) = -\frac{\gamma(x)^2}{2}$.

In the limit of very high Reynolds numbers, the boundary layer becomes an "infinitely thin vortex sheet" that still has $\gamma(x) = -u_e(x)$ as its circulation per unit length. This constitutes the basis of the method used in this homework: the "panel method".

We will first study the flow past an airfoil (i.e., an aerodynamic profile), and under the assumption that the boundary layers are infinitely thin and remain attached to the profile: thus a potential flow, and using the panel method. Then, we will investigate what happens for a real flow at high Reynolds number, with boundary layers and separation. For that, we will use the simplified tool XFLR5. The results obtained with this simplified model will be compared to a more accurate RANS simulation.

**The vortex panel method.** For the potential flow, we use the distance coordinate $s$ measured along the airfoil, starting from the trailing edge and going clockwise. The vortex sheet $\gamma(s)$ on the airfoil surface must be determined. For that, the sheet will be discretised using $N$ linear vortex panels, (i.e. each having a linear distribution of $\gamma(s)$). The condition that the normal velocity at the centre of each panel must be zero provides $N$ linear equations. The total vorticity flux at the trailing edge, from both the extrados and intrados boundary layers that meet at the trailing edge, is zero: we thus have that $\gamma_{N+1}^2 = \gamma_1^2$, and hence that $\gamma_{N+1} = -\gamma_1$. This is the equivalent of the Kutta-Joukowski condition.

The formulas for the fields $\psi$, $u$ and $v$ induced by one vortex panel are provided in Appendix.

**The airfoil geometry.** We are here interested in airfoils designed for airborne wind energy systems. Therefore, we have chosen to study the "MRevE" airfoil, introduced by [2]. This airfoil has been reverse-engineered based on the pictures of the M600 prototype of the Makani project. The coordinates of the extrados and intrados, noted $y_e$ and $y_i$, can be obtained from the mean camber line coordinates, $y_c$, and the thickness distribution, $y_t$, as follows (using the notation $x^* = x/c$, $y_e^* = y_e/c$, $y_i^* = y_i/c$, $y_c^* = y_c/c$, and $y_t^* = y_t/c$):

$$
\begin{aligned}
y_e^* &= \quad y_c^* + y_t^* \ , \\
y_i^* &= \quad y_c^* - y_t^* \ .
\end{aligned}
$$

$y_c^*$ and $y_t^*$ are given by the following expressions:

$$
\begin{aligned}
y_c^* &= 0.50511\,(x^*) - 0.97016\,(x^*)^2 + 0.79535\,(x^*)^3 - 0.3303\,(x^*)^4 \ , \\
y_t^* &= 0.39981\sqrt{x^*} - 0.09876\,(x^*) - 0.6376\,(x^*)^2 + 0.34907\,(x^*)^3 - 0.01252\,(x^*)^4 \ .
\end{aligned}
$$

To generate the panels, you need to discretise the airfoil. This is done by applying the equations above to adequate $x^*$ coordinates. To choose these abscissa, we know that we will need more panels at the leading and trailing edges. Their value $x_i^*$ can thus be computed using:

$$x_i^* = \frac{1}{2}\,(1 - \cos\theta_i) = \sin^2(\theta_i/2) \tag{1}$$

where the $\theta_i$ are spread uniformly between 0 and $\pi$. However, this results in extremely small panels at the trailing edges when we use a large number $N$ of panels. The problem is not present at the leading edge since the slope of the profile function is infinite there. To compensate for this problem, the discretisation is modified using instead:

$$x_i^* = \frac{(1 - \cos\theta_i)}{(1 - \cos\theta_{end})} \tag{2}$$

where the $\theta_i$ are spread uniformly between 0 and $\theta_{end}$. A good choice for $\theta_{end}$ can be $\theta_{end} = 0.95\,\pi$.

**XFLR5**[1]

XFLR5 is an API designed to analyze airfoils, wings, and planes. Built on top of Xfoil [1], it combines an inviscid vortex panel method and a model representing the viscous layers, both solved simultaneously using a Newton method. The software also encloses a compressibility correction (the Karman-Tsien correction) that also allows to investigate airfoil where compressibility effects are significant; yet of course still well in the subsonic regime. It is, therefore, a convenient tool for quickly analysing the real flow past an airfoil.

[1] Software and documentation can be found here: http://www.xflr5.tech/xflr5.htm

**Part 1: Potential flow**

You are expected to:

1. Provide a visualisation of the discretisation of the airfoil by the panels (e.g., using small *bullets* to show the points chosen along the profile)

2. Investigate the effect of the angle of attack $\alpha$, by considering three cases. Provide:

   (a) the plot of the vortex sheet, $\frac{\gamma(s)}{U_\infty}$, as a function of $\frac{s}{c}$. For simple comparisons, the graphs corresponding to the three $\alpha$ cases will be put on the same plot.

   (b) the plot of the pressure coefficient, $C_p(s)$, as a function of $\frac{s}{c}$; also all on the same plot.

   (c) the plots of the streamlines, so as to illustrate the obtained flow. Also indicate clearly the dividing streamline (i.e., the streamline that hits the profile at the stagnation point).

   (d) the lift coefficient, $C_l = \frac{l}{\frac{1}{2}\rho U_\infty^2 c}$, obtained for each $\alpha$ considered. We recall that the lift per unit span, $l$, is obtained as $l = -\rho U_\infty \Gamma$ where $\Gamma$ is the total circulation of the flow past the airfoil. In the potential flow, $\Gamma = \oint \gamma(s)\, ds$, and this integral evaluated numerically:

$$\Gamma = \sum_{i=1}^{N} \frac{(\gamma_i + \gamma_{i+1})}{2} (2\, b_i) \ , \tag{3}$$

   where $(2\, b_i)$ is the length of the panel $i$, and also recalling that $\gamma_{N+1} = -\gamma_1$

   (e) comments on the results.

**Part 2: XFLR5**

You are expected to:

1. Investigate the effect of the Reynolds number $Re$, by considering three cases. Provide:

   (a) the plot of the lift coefficient, $C_l$, as a function of the angle of attack $\alpha$. For simple comparisons, the graphs corresponding to the three $Re$ cases will be put on the same plot.

   (b) the plot of the drag coefficient, $C_d$, as a function of the angle of attack $\alpha$. Again, the graphs corresponding to the three $Re$ cases will be put on the same plot.

   (c) the plot of the airfoil polar (i.e., the plot of $C_l - C_d$).

   (d) comments on the results.

2. Compare the results obtained with XFLR5 to those of a RANS simulation of the "MRevE" airfoil at $Re = 6\,10^6$ provided on Moodle. Comments the results.

3. Compare the results obtained for the potential flow to those obtained using XFLR5. Comments the results.

**Important informations:**

- The project shall be realised alone or in a group of two students.
- An analysis of the *physics of the problem* is expected, as well as a *critical view* of the obtained results.
- It is also expected to put some effort in providing a good quality presentation of the obtained results: quality plots, quality writing.
- No tolerance will be accepted concerning plagiarism. The code for the potential flow and the report must both be an original production. Any use, even partial, of the work of another person must be mentioned explicitly.
- The source code for the potential flow and the report shall both be provided on Moodle, at last, **the 5th of April**. They will be systematically compared to those of others, and to those of previous years. The code and the report must be compressed into a single archive file before uploading it. The name of the archive should be HW1-2323-LASTNAME1-LASTNAME2, and contain "REPORT-LASTNAME1-LASTNAME2.pdf" and one or several "CODE- LASTNAME1-LASTNAME2-whatever you want.py". If done alone, then only put LASTNAME1.

# References


[1] Mark Drela. Xfoil: An analysis and design system for low reynolds number airfoils. In *Low Reynolds number aerodynamics*, pages 1–12. Springer, 1989.

[2] Dylan Eijkelhof. Design and optimisation framework of a multi-mw airborne wind energy reference system. 2019.

# A Formulas for linear vortex panels

We consider a panel of span $2b$ and aligned with the x-axis ($-b \le x' \le b$ and $y' = 0$). The distribution of $\gamma(x')$ over the panel is linear:

$$
\begin{aligned}
\gamma(x') &= \alpha x' + \beta \,, \\
\alpha &= \frac{(\gamma_R - \gamma_L)}{2b} \,, \\
\beta &= \frac{(\gamma_R + \gamma_L)}{2} \,.
\end{aligned}
$$

We obtain, by integration and using $s = x - x'$:

$$
\begin{aligned}
\psi(x,y) &= \frac{1}{4\pi} \left[ (\alpha x + \beta) \left( s \, \log(s^2 + y^2) - 2s + 2y \, \arctan\left(\frac{s}{y}\right) \right) - \right. \\
&\quad \left. \frac{\alpha}{2} \left( (s^2 + y^2) \log(s^2 + y^2) - s^2 \right) \right]_{x+b}^{x-b} \,, \\
u(x,y) &= -\frac{1}{2\pi} \left[ \frac{\alpha y}{2} \log\left(s^2 + y^2\right) - (\alpha x + \beta) \left( \arctan\left(\frac{s}{y}\right) \right) \right]_{x+b}^{x-b} \,, \\
v(x,y) &= -\frac{1}{2\pi} \left[ \alpha \left( -s + y \arctan\left(\frac{s}{y}\right) \right) + \frac{1}{2} (\alpha x + \beta) \log\left(s^2 + y^2\right) \right]_{x+b}^{x-b} \,.
\end{aligned}
$$

For $-b < x < b$, those functions are:

$$
\begin{aligned}
\lim_{y \to 0^-} u(x,y) &= \frac{1}{2}(\alpha x + \beta) \quad = \frac{\gamma(x)}{2} \,, \\
\lim_{y \to 0^+} u(x,y) &= -\frac{1}{2}(\alpha x + \beta) \; = -\frac{\gamma(x)}{2} \,, \\
v(x,0) &= -\frac{1}{2\pi} \left( 2\alpha b + (\alpha x + \beta) \log\left(\frac{b - x}{b + x}\right) \right) \,.
\end{aligned}
$$