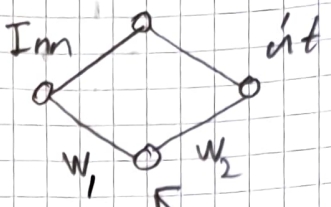
NTK - Kernel Pelingar

$f: \mathbb{R} \to \mathbb{R}$

$$y = \underset{1\times 2}{W^{[2]}}\left(\underset{2\times 1}{W^{[1]}}\underset{1\times 1}{x} + \underset{2\times 1}{b^{[1]}}\right) + \underset{1\times 1}{b^{[2]}}$$



Tauganet

$$W^{[1]} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad b^{[1]} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad x_1 = 1 \quad x_2 = 2$$

Random datapoints

$$W^{[2]} = [1, 0] \quad b^{[2]} = 0$$

$$f_\theta = W^{[2]}\left(W^{[1]}x + b^{[1]}\right) + b^{[2]} = \boxed{W^{[2]}_{1,1} W^{[1]}_{1,1} x + W^{[2]}_{1,2} W^{[1]}_{2,1} x + W^{[2]}_{1,1} b^{[1]}_{1,1} + W^{[2]}_{1,2} b^{[1]}_{2,1} + b^{[2]}_{1,1}}$$

Tauganetið okkar skilgreint sem fall eftir fylkjamargföldun

$$\frac{\partial f}{\partial \theta_1} = \frac{\partial f}{\partial W^{[1]}_{1,1}}$$

$$\theta = \begin{bmatrix} W^{[1]}_{1,1} \\ W^{[1]}_{2,1} \\ b^{[1]}_{1,1} \\ b^{[1]}_{2,1} \\ W^{[2]}_{1,1} \\ W^{[2]}_{1,2} \\ b_{1,1} \end{bmatrix} \Rightarrow \boxed{\theta \text{ er } 7 \text{ stakka vigur}}$$

$$2\times 1 + 2\times 1 + 1\times 2 + 1\times 1 = 7$$

$$\frac{\partial f}{\partial \theta_1} = W^{[2]}_{1,1} x \qquad \frac{\partial f}{\partial \theta_2} = W^{[2]}_{1,2} x \qquad \frac{\partial f}{\partial \theta_3} = W^{[2]}_{1,1}$$

$$\frac{\partial f}{\partial \theta_4} = W^{[2]}_{1,2} \qquad \frac{\partial f}{\partial \theta_5} = b^{[1]}_{1,1} \qquad \frac{\partial f}{\partial \theta_6} = b^{[1]}_{2,1} \qquad \frac{\partial f}{\partial \theta_7} = 0$$

Reikna kernel

$$K_{1,2} \Rightarrow \sum_k \frac{\partial f_{x_i}}{\partial \theta_k} \cdot \frac{\partial f_{x_j}}{\partial \theta_k} = \underset{\theta_1}{[1\cdot 2]} + \underset{\theta_2}{[0\cdot 0]} + \underset{\theta_3}{[1\cdot 1]} + \underset{\theta_4}{[0\cdot 0]} + \underset{\theta_5}{[0\cdot 0]} + \underset{\theta_6}{[1\cdot 1]} + \underset{\theta_7}{[0]} = 4$$

$$K_{2,1} \overset{!}{=} K_{1,2} = 4$$

$$K_{1,1} = \underset{\theta_1}{[1\cdot 1]} + \underset{\theta_2}{[0]} + \underset{\theta_3}{[1\cdot 1]} + \underset{\theta_4}{[0\cdot 0]} + \underset{\theta_5}{[0\cdot 0]} + \underset{\theta_6}{[1\cdot 1]} + \underset{\theta_7}{[0]} = 3$$

$$K_{2,2} = \underset{\theta_1}{[2\cdot 2]} + \underset{\theta_2}{[0\cdot 0]} + \underset{\theta_3}{[1\cdot 1]} + \underset{\theta_4}{[0\cdot 0]} + \underset{\theta_5}{[0\cdot 0]} + \underset{\theta_6}{[1\cdot 1]} + \underset{\theta_7}{[0]} = 6$$

$$K = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \qquad \text{eigen}(K).\text{values} = \begin{bmatrix} 0.22799 \\ 8.77200 \end{bmatrix}$$