EECS 343: Theoretical Computer Science **Test 2**
Harold Connamacher, Instructor **March 20, 2020**

Name: _____

You have 2 hours to take this test. The test is close book and closed notes.

At the end of the test, please write "I have neither given nor received aid on this examination, and I did not exceed the allowed time" and sign your name.

Problem 1:
Let A and B be regular languages over $\{0, 1\}$. Define the language

$$I_{A,B} = \{a_1b_1a_2b_2 \dots a_nb_n \mid a_1a_2 \dots a_n \in A, \text{ and } b_1b_2 \dots b_n \in B\}.$$

That is, every string in $I_{A,B}$ is created by taking a string from A and a string from B of the same length and alternating the characters of the two strings. For example, if $a = 1001$ and $b = 0101$, then the string 10010011 is in $I_{A,B}$.

Prove that $I_{A,B}$ is regular.

Problem 2:
Consider the language $N = \{x\overline{x} \mid x \in \{0, 1\}^*\}$ where $\overline{x}$ is the bit-wise complement of x. The bit-wise complement is formed by taking x and flipping each 0 to a 1 and each 1 to a 0. For example, if x is 0110, then $\overline{x}$ is 1001, and the string 01101001 is in the language.

a) Prove that N is not regular.

b) Prove that N is context-free.

Problem 3:
Consider the language $L = \{0^{2^k} \mid k \geq 0\}$.
That is, L contains all strings of 0's such that the length of the string is a power of 2. Prove that L is not context-free.