EECS 496: Sequential Decision Making

Soumya Ray

sray@case.edu

Office: Olin 516

Office hours (today): T 4:30-5:30 or by appointment

Recap

- What is the Markov Blanket?
- How do we represent probabilities in a BN?
- There are two kinds of inference algorithms for BNS: ____ and _____. Why do we need two kinds?
- What is the intuition behind variable elimination?
- In VE we first ______. In each iteration we _______ the _____ involving the variable, then _____ it _____.
- How does the first step work? How does the second step work?
- What happens if there is evidence?
- The key idea in approximate inference is to generate _____, then _____ how many meet a specified condition.
- How does Monte Carlo inference work from a joint pdf?
- In order to construct a sample from a BN, we first _______ . Then we _____ each variable given its ______.

Today

- Inference in Bayesian Networks (Ch 14, Russell and Norvig)
- Reasoning over time (Ch 15)

Approximate Inference 1 (Monte Carlo)

- How to generate a sample from a BN?
- Idea: Topologically sort the variables according to the graph structure
- Sample each according to the conditional distribution (well-defined due to the sorting)
- Count the samples with desired values
- Easy!
  – Right?

Approximate Inference 2: Incorporating Evidence

- What if we have evidence?
- Well, let's just throw away the samples that have the evidence variables wrong
  - "Rejection Sampling"

Approximate Inference 3: Incorporating Evidence

- What if we have evidence?
- Rejection Sampling is obviously wasteful

- Alternatively, sample only the non-evidence variables, and weight the samples according to the likelihood of the evidence given the rest
  – "Likelihood Weighting"

Problems

- If there's:
  - Lots of evidence,
  - Highly improbable evidence (according to the BN),
  - Evidence occurring late in the topological sort
- We will end up with
  - Many samples thrown away (Rejection sampling)
  - A large set of low-likelihood samples (Likelihood weighting)
- What to do?

Approximate Inference 4: MCMC to the Rescue

- Key idea: Stop drawing independent samples
  - Let the $i+1^{st}$ sample depend on the i^{th} sample
- This creates a "Markov chain" of samples
- Under appropriate conditions, this chain will converge to a "stationary distribution", where the frequency of encountering a sample is proportional to its (true) probability
- This idea is called "Markov Chain Monte Carlo", or MCMC

MCMC Algorithm Loop

- Initialize the chain (choose first sample, arbitrarily)
- Run the chain (choose the next sample, MCMC algorithms differ here)
- After a sufficiently long time (burn-in time), start counting the samples with the desired property
- Count however long you want to get an accurate estimate

Gibbs Sampling

- Simplest MCMC algorithm for BN inference
- The "state" of the chain corresponds to all the variables in the BN being assigned values (evidence variables are fixed)
- Sample a random non-evidence variable V proportional to $Pr(V|MB(V))$, where MB is the Markov Blanket

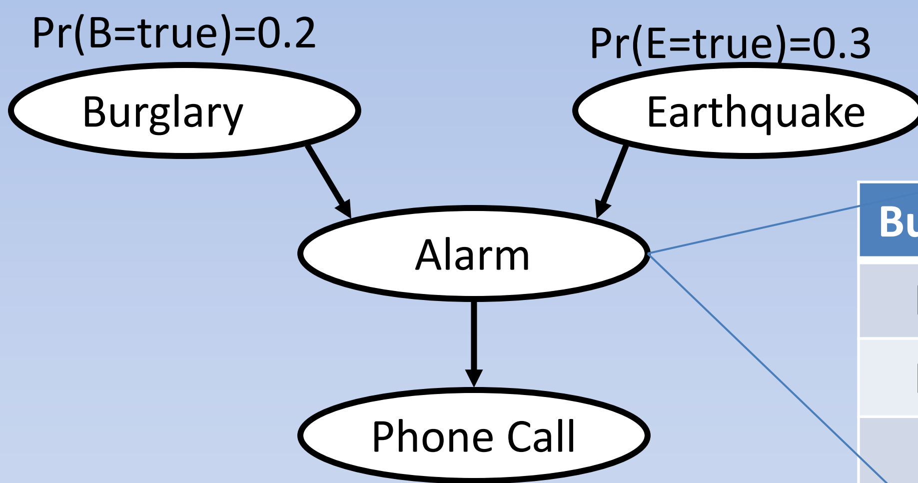
Distribution given the MB

- We can show that:

$$\Pr(V = v \mid MB(V)) \propto$$

$$\Pr(V = v \mid Pa(V)) \times \prod_{C_j \in Children(V)} \Pr(C_j = c_j \mid Pa(C_j))$$

MCMC



Burglary	Earthquake	A=true	A=false
False	False	0.2	0.8
False	True	0.5	0.5
True	False	0.75	0.25
True	True	0.9	0.1

Alarm	P=true	P=false
False	0.25	0.75
True	0.75	0.25

Find $Pr(P)$ using MCMC

Why does this work?

- Let $\pi_t(x)$ be the probability of the chain being in state x at time t
- Let $T(x, x')$ be the probability of the chain transitioning to x' after x
  - We choose this and it is independent of time
- We'll assume the chain is "regular," i.e. the chain can't have "pieces" so you can't get from one piece to another (also called "irreducible")

Stationary Distribution

- Note:

$$\pi_{t+1}(x') = \sum_{x} \pi_t(x) T(x, x')$$

- A "stationary distribution" satisfies:

$$\pi_{t+1}(x) = \pi_t(x)$$

Detailed Balance

- A Markov chain satisfies "detailed balance" if there exists some distribution π so that

$$\pi(x)T(x,x') = \pi(x')T(x',x)$$

- If a regular chain satisfies detailed balance, then there exists some t so that for all $T > t$, $\pi_t = \pi$ (detailed balance implies convergence to a stationary distribution)

Convergence of Gibbs Sampling for BNs

- We can show that the MC produced by Gibbs sampling satisfies detailed balance with π =the probability distribution represented by the Bayes net
- Further, this distribution is unique–the chain has no other stationary distribution
- So Gibbs sampling will produce the correct answer–eventually