EECS 496: Sequential Decision Making

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Recap

- In the Baum-Welch algorithm's E step, we estimate the ___ ___ of each parameter given our dataset. In the M step we find the ___ ___ ___ with these.
- To do the E step for emission distributions, we check each position in each sequence where _____. Then we find the probability that ____. Then we ___ the probabilities.
- To do the E step for transition distributions, we check ___ in each sequence. Then we find the probability that _____. Then we ___ the probabilities.
- One procedure that is used as a subroutine in this process is the "backward" algorithm. This computes Pr({o__,…,o___}|s__=__).
- What is the difference between a dynamic Bayesian network and an HMM? Why is this important/useful?
- A DBN is represented by a pair of ____. Each ___ is a ___ ___. There are symmetric edges that ______and asymmetric edges that ______.
- CPTs in a DBN represent Pr(__ __ | __ __).
- It is generally impossible to use exact inference in a DBN because _____. To avoid ____, we must maintain a representation of the ____ ____ at ____ ___ ___. However, this could be very complex.
- An approximate inference technique we can use is ___ ___. This extends the ___ ___ approach from BNs. It represents the ___ ___ at some time step with a set of ____ _____.
- For each time step, (i) the set of ___ are _____ _____ in time by _____ from the _____.
- Then (ii) the new _____ are _____ by the _____ of the ____ (if any), at that time step.
- Finally a set of samples are _______ from _______. This set is ______.

Today

- Sequential Probabilistic Models (Ch 15)
- Review of part 1

Discrete Time, Continuous State Sequential Probabilistic Models

Continuous State Processes

- Many natural processes are best modeled as continuous processes
- Examples:
  - A robot is trying to localize itself as it moves
  - You are looking at blips on a radar screen and trying to track an object
- The underlying processes are continuous but we sample the states at discrete intervals

Kalman Filtering

- Specify the process state using continuous variables (e.g. position, velocity, acceleration)
- To model the evolution of the process state, we need a distribution $Pr(s_{t+1}|s_t)$
- One possibility is to use a *linear Gaussian model*: s_{t+1} is a linear function of s_t, with some Gaussian noise
  – This is called a Kalman Filter

Example

- Motion tracking: state has position and velocity (assume no acceleration)

$$x(t+\Delta t) = x(t) + v_x(t)\Delta t$$

$$y(t+\Delta t) = y(t) + v_y(t)\Delta t$$

Linear dynamics

- Then:

$$\Pr(X(t+\Delta t) = x(t+\Delta t) \mid x(t), v_x(t)) =$$
$$N(x(t+\Delta t); \mu = x(t) + v_x(t)\Delta t, \sigma)$$

Gaussian noise

Example contd.

- The previous model has "transition noise" but no "sensor noise"
- In general:

Transition Distribution:
how likely are we to move to $\mathbf{s}(t+1)$ given we are in $\mathbf{s}(t)$

$$\Pr(\mathbf{s}(t+1) \mid \mathbf{s}(t)) = N(\mathbf{s}(t+1); A\mathbf{s}(t), \Sigma_s)$$

$$\Pr(\mathbf{o}(t) \mid \mathbf{s}(t)) = N(\mathbf{o}(t); B\mathbf{s}(t), \Sigma_o)$$

Sensor Distribution:
how likely are we to see $\mathbf{o}(t)$ given we are in $\mathbf{s}(t)$

Prediction Problems

- In continuous state processes, we are often interested in *online* prediction problems
  - We see a sequence of observations and need to make predictions about the state *as it evolves*

Prediction Problems

- Filtering: what is the current state given the observations so far? $Pr(\mathbf{s}(t)|\mathbf{o}(1),\ldots,\mathbf{o}(t))$
- Prediction: what is the next (k) state I am likely to be in? $Pr(\mathbf{s}(t+k)|\mathbf{o}(1),\ldots,\mathbf{o}(t))$
- Smoothing: How likely was I to have been in some state in the past, given the observations so far? $Pr(\mathbf{s}(t-k)|\mathbf{o}(1),\ldots,\mathbf{o}(t))$
- Most likely path: $argmax_{\mathbf{s}}\, Pr(\mathbf{s}|\mathbf{o}(1),\ldots,\mathbf{o}(t))$

Filtering

- Estimate the state at time t as:

$$\Pr(\mathbf{s}(t)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t))$$

$$\propto \Pr(\mathbf{o}(t)\,|\,\mathbf{s}(t),\mathbf{o}(1),\ldots,\mathbf{o}(t-1))\Pr(\mathbf{s}(t)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t-1))$$

Bayes rule

$$\propto \Pr(\mathbf{o}(t)\,|\,\mathbf{s}(t))\Pr(\mathbf{s}(t)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t-1))$$

Markov property

$$\propto \Pr(\mathbf{o}(t)\,|\,\mathbf{s}(t))\int_{\mathbf{s}(t-1)}\Pr(\mathbf{s}(t),\mathbf{s}(t-1)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t-1))d\mathbf{s}(t-1)$$

Total prob.

$$\propto \Pr(\mathbf{o}(t)\,|\,\mathbf{s}(t))\int_{\mathbf{s}(t-1)}\Pr(\mathbf{s}(t)\,|\,\mathbf{s}(t-1),\mathbf{o}(1),\ldots,\mathbf{o}(t-1))\Pr(\mathbf{s}(t-1)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t-1))d\mathbf{s}(t-1)$$

Cond. prob.

$$\propto \Pr(\mathbf{o}(t)\,|\,\mathbf{s}(t))\int_{\mathbf{s}(t-1)}\Pr(\mathbf{s}(t)\,|\,\mathbf{s}(t-1))\Pr(\mathbf{s}(t-1)\,|\,\mathbf{o}(1),\ldots,\mathbf{o}(t-1))d\mathbf{s}(t-1)$$

Markov property

Sensor Distrib

Transition Distrib

Result of filtering at previous step

Filtering with the Kalman Filter

- Key observation: If every distribution is linear Gaussian, the filtering computation has a closed form solution that is also linear Gaussian
- We'll need to assume a Gaussian prior for $\Pr(\mathbf{s}(0))$

Example

- Consider a process with a single variable x
- Let $Pr(x(0))=N(0,1)$, $Pr(x(t+1)|x(t))=N(x(t),1)$, $Pr(o(t)|x(t))=N(x(t),1)$
- Suppose the first observation is $o(1)$ and we want to find $Pr(x(1))$

Example

$$\Pr(x(1) \mid o(1))$$

$$\propto \Pr(o(1) \mid x(1)) \int_{x(0)} \Pr(x(1) \mid x(0)) \Pr(x(0)) dx(0)$$

$$\int_{-\infty}^{\infty} N(x(1); x(0), 1) N(x(0); 0, 1) dx(0)$$

$$= c \int_{-\infty}^{\infty} e^{-\frac{1}{2}((x(1)-x(0))^2 + x(0)^2)} dx(0)$$

Integrate by completing the square

$$= ce^{-\frac{1}{2}\left(\frac{x(1)^2}{2}\right)}$$

Example contd.

$$\Pr(x(1) \mid o(1))$$

$$\propto \Pr(o(1) \mid x(1)) \int_{x(0)} \Pr(x(1) \mid x(0)) \Pr(x(0)) dx(0)$$

$$\Pr(x(1) \mid o(1))$$

$$\propto N(o(1); x(1), 1) e^{-\frac{1}{2}\left(\frac{x(1)^2}{2}\right)}$$

$$\propto N\left(\frac{2o(1)}{3}, \frac{2}{3}\right)$$

Weighted mean of previous mean and current obs (weighted by variance)

Updated variance (does not depend on observation)

Learning: Parameter Estimation

- Given a sequence of observations, estimate the linear sensor and transition models and the Gaussian covariances
- Closed form solution (least squares)

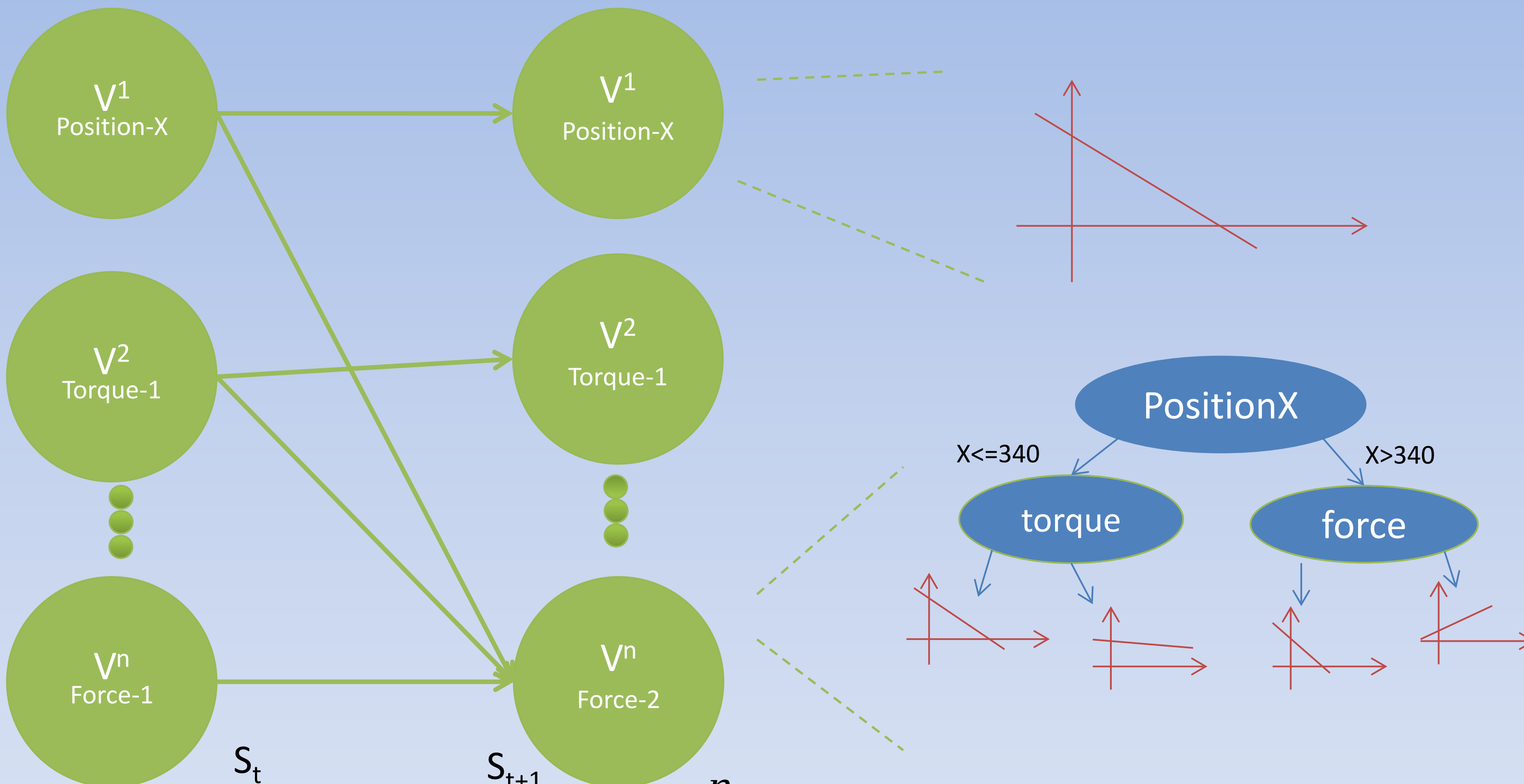
Variations

- Kalman Filters are easy to work with because the math is nice
- But they make some very strong assumptions about the process being modeled
- People have looked at various ways to relax these assumptions
  - Extended KF uses nonlinear models
  - Switching KF models other distributions as mixtures of Gaussians
- Continuous time variants of the KF also exist

DBNs with KFs

- DBNs can be used with Kalman filters to provide a powerful, flexible, general purpose representation
- In this case, each CPT is represented as a linear Gaussian distribution
- For more flexibility, can use regression trees with linear Gaussian leaves
  - Learning needs lots of data but provides exceptionally good fits

DBNs with KFs



$$\Pr(\mathbf{S}_{t+1} \mid \mathbf{S}_t) = \prod_{i=1}^{n} \Pr(V_{t+1}^i \mid Pa(V_{t+1}^i))$$

Review of part 1

- In this part, we have learned about various languages to represent the world and various inference algorithms to reason with these languages

Propositional Logic

- How is the world represented?
- How do we do inference?
- What are some pros and cons for this representation?

Basic Probability Theory

- How is the world represented?
- How do we do inference?
- What are some pros and cons for this representation?

Bayesian Networks

- How is the world represented?
- How do we do inference?
- What are some pros and cons for this representation?

Sequential Probabilistic Models

- How is the world represented?
- How do we do inference?
- What are some pros and cons for this representation?