EECS 343: Theoretical Computer Science **Test 3**
Harold Connamacher, Instructor **April 17, 2020**

Name: _____

You have 2 hours to take this test. The test is close book and closed notes.

At the end of the test, please write "I have neither given nor received aid on this examination, and I did not exceed the allowed time" and sign your name.

Problem 1:
Consider the following language:

$$L = \{\langle a, b, c\rangle \mid a, b, c \in \mathbb{Z}^+ \text{ and there exists } 0 < x < c \text{ such that } x^2 = a \bmod b\}.$$

Prove that L is in NP.

(Given three positive integers, is there a positive integer x less than c such that dividing x^2 by b gives remainder a?)

Problem 2:
Let a *three-time solvable* cnf-formula be a Boolean formula in conjunctive normal form with n variables that has at least 3 different satisfying assignments. Prove that determining if a specific cnf-formula is a *three-time solvable* formula is NP-complete.

Problem 3:
Consider a 4-way graph game. The game is for 4 players and is played on a directed graph with one vertex specified as the starting vertex. A token is placed on the start vertex, and each player, in order, takes a turn moving the token along an edge to an adjacent vertex. The token cannot be moved to a previously visited vertex nor can it move "backward" on a directed edge. The last player to take a legal move wins the game.

$$\textit{4-Way Graph Game} = \{\langle G, s\rangle \mid \text{Player 1 has a winning strategy in the 4-player game played on graph } G \text{ starting at vertex } s.\}$$

Prove that *4-Way Graph Game* is PSPACE-complete.