

**EECS 343: Theoretical Computer Science, Homework Exercise 1**  
**due Monday, January 27, 2020 before class**

**Problem 1:** Let  $L_1$  and  $L_2$  be languages. Consider language  $L = L_1 \times L_2$ . ( $L$  consists of pairs  $(a, b)$  where  $a \in L_1$  and  $b \in L_2$ .) Prove (a) if  $L_1$  and  $L_2$  are decidable languages then  $L$  is also decidable, and (b) if  $L_1$  and  $L_2$  are recognizable languages then  $L$  is also recognizable.

**Problem 2:** Let  $a^n$  denote a string of length  $n$  characters, each character being an  $a$ . Write a Turing Machine that will accept strings of the form  $a^n b^{2n}$  and reject all other strings, where  $a$  and  $b$  are elements of the input alphabet  $\Sigma$  and  $n$  is some nonnegative integer. Explicitly give your machine's alphabet, set of states, and transition function. Prove that your Turing Machine is correct.

**Problem 3:** Let  $L$  be the set of all strings over the alphabet  $\Sigma = \{a, b\}$  that contain exactly twice as many  $a$ 's as  $b$ 's. For example,  $aababa$  would be in the language  $L$  and  $abab$  would not. Write a Turing machine that will accept all strings that are in  $L$  and reject all other strings. Explicitly give your machine's alphabet, set of states, and transition function. Prove that your Turing machine is correct.