EECS 343: Theoretical Computer Science **Final Exam**
Harold Connamacher, Instructor **May 1, 2020**

Name: ______________________________

You have 3 hours to take this test. The test is six questions plus two extra credit questions and closed book and closed notes.

At the end of the test, please write "I have neither given nor received aid on this examination, and I did not exceed the allowed time" and sign your name.

Problem 1: (20 points)
Let A and B be regular languages. Define the language AB^* as

$$AB^* = \{ab_1 \ldots b_k \mid a \in A, b_i \in B, k \geq 0\}$$

That is, a string from A followed by 0 or more strings from B. Prove that AB^* is regular.

Problem 2: (20 points)
Let L be the subset of the language generaged by the regular expression $(x + y + z)^*$ where every string in L has the same number of x's as z's.

Prove that L is not regular.

Problem 3: (20 points)
Consider the same language from problem 2. Is L context-free? Prove or disprove.

Problem 4: (20 points)
Suppose we have a system of m $(+1,-1)$-linear functions over n variables $x_1, \ldots, x_n$ and that we wish to find a point at which each function is non-negative. We can write this as:

$$\begin{aligned}
a_{1,1}x_1 + a_{1,2}x_2 + ... + a_{1,n}x_n &\geq 0 \\
a_{2,1}x_1 + a_{2,2}x_2 + ... + a_{2,n}x_n &\geq 0 \\
&\cdots \\
a_{m,1}x_1 + a_{m,2}x_2 + ... + a_{m,n}x_n &\geq 0
\end{aligned}$$

where each $a_{i,j} \in \{-1, 0, 1\}$. Is there an assignment of -1 or $+1$ to the variables $x_1, \ldots, x_n$ such that each linear function above is non-negative? Prove that finding such an assignment is an NP-complete problem.

Problem 5: (20 points)
Define B_{TM} as follows

$$B_{TM} = \{\langle M, a, b\rangle \mid \ M \text{ is a Turing machine and } \ a \notin \mathcal{L}(M) \vee b \in \mathcal{L}(M)\rangle\}.$$

That is, if M accepts a then it must also accept b.

Prove that B_{TM} is undecidable.

Problem 6: (20 points)
Consider the language

$REACH = \{\langle G, s\rangle \mid \ G$ is a directed graph and there is a path from s to all other vertices of $G\}$

Prove that $REACH$ is NL-complete.

Extra Credit 1: (10 points, must be mostly correct for partial credit)
Let $A \in \text{SPACE}(f(n))$ and $B \in \text{SPACE}(g(n))$ with $A \subseteq B$ and $f(n)$ is $\Omega(\log n)$. Prove that $B^A \subseteq B$.

Extra Credit 2: (10 points, must be mostly correct for partial credit)
Prove that there is no infinite subset of the set of inccompressible strings that is Turing recognizable.