

EECS 343: Theoretical Computer Science, Homework Exercise 8
due Monday, March 23, 2020 before class

Problem 1: Show that the class of languages NP is closed under union and concatenation.

Problem 2: The SUBGRAPH ISOMORPHISM problem (language) is the set

$$L_{SGI} = \{\langle G, H \rangle \mid G \text{ and } H \text{ are graphs and } H \text{ is isomorphic to a subgraph of } G\}.$$

In more detail, let G have vertex set V_1 and edge set E_1 , and let H have vertex set V_2 and edge set E_2 . H is isomorphic to a subgraph of G if there are subsets $V \subseteq V_1$ and $E \subseteq E_1$ such that $|V| = |V_2|$, $|E| = |E_2|$, and there exists a one-to-one function $f : V_2 \rightarrow V$ such that $(u, v) \in E_2$ if and only if $(f(u), f(v)) \in E$. (In non-math, we can find an exact copy of H inside V .)

Prove that SUBGRAPH ISOMORPHISM is in NP.

Problem 3: The HAMILTON CYCLE problem (language) is the set

$$L_{HC} = \{\langle G \rangle \mid G \text{ is a graph that contains a cycle that uses every vertex exactly once}\}.$$

A cycle is a sequence of vertices $v_1, v_2, v_3, \dots, v_k$ such that there is an edge connecting v_i with v_{i+1} for each i from 1 to $k-1$ as well as an edge from v_k to v_1 .

Prove that HAMILTON CYCLE \leq_P GRAPH ISOMORPHISM. (Be careful, your first idea probably fails the isomorphism test, but a slight change to that idea should work.)