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CSCI 3656-001

CSCI 3656 - Final Project: 737 MAX and the Cholesky Equations

Introduction:

New engine locations on the *Boeing 737 MAX* caused it to pitch up slightly during climbing attitudes, creating the risk of an unintended stall. The *MCAS* (Maneuvering Characteristics Augmentation System) was added to mitigate this risk by automatically applying nose down trim to the horizontal stabilizer if a dangerous angle of attack was detected. The system was deemed the likely cause of two major accidents in 2018 and 2019 that ultimately caused the *MAX* to be temporarily recalled.

Much speculation has ensued as to the exact cause of the accidents: sensor failure, *MCAS* error, poor pilot training/pilot error… I will not attempt to validate or invalidate any causal theories in this project. The *MCAS* system's algorithms, from a very simple and uneducated perspective, likely sought to create a linear relationship between a multitude of values (AoA, airspeed, force on the yolk) in order to assist the pilot in countering instability in climbing attitudes. "To achieve the desired handling qualities, MCAS was introduced to generate a near linear relation between the required stick force and G, which makes the handling of the aircraft predictable for the flight crew" (*Bechai*).

In the case that some form of linear regression was used in the MCAS algorithms, it would have needed to carry out calculations as quickly and accurately as possible. One of our lectures in *Numerical Computation* mentioned the 737 MAX in the context of Cholesky/Normal equations as solutions to the least squares problem. In this project I intend to explore how such quick solutions can accrue error quickly. I have no doubt the development of MCAS underwent extensive testing to reduce error in their chosen methods, I hope if nothing else to emphasize the importance of such practices.

In 2020, stale data being displayed on avionics systems caused the FAA to mandate a power-down of 787s every 51 days (*Cornfield*). At first, this seemed to me to be analogous to the common pilot error that has taken many lives in general aviation. A pilot who logs fuel added and fuel burned continuously over many flights but never corroborates these numbers by actually checking the fuel in the tank, eventually may embark on a flight where that error has accrued beyond a few decimal points and left them stranded in the air with plenty of fuel on their notepads but nothing left for the engine. However, it is more likely that the 787's stale data issue is in the realm of computing error. The choice of 51 days is somewhat peculiar, but some interesting speculation exists: rollback error, integer storage (*Administrator*). This is off-topic, but further emphasizes the importance of sophisticated error control.

Experimentation:

A matrix (A) in the context of aerodynamics would likely parameterize some equations of motion, which it would be advantageous to linearize, hence the application of a least squares solution. This is far outside the scope of my knowledge, I only aim to compare cost/benefit/error of certain methods.

A least squares solution is an *x* which minimizes the equation

$$||Ax - b||$$

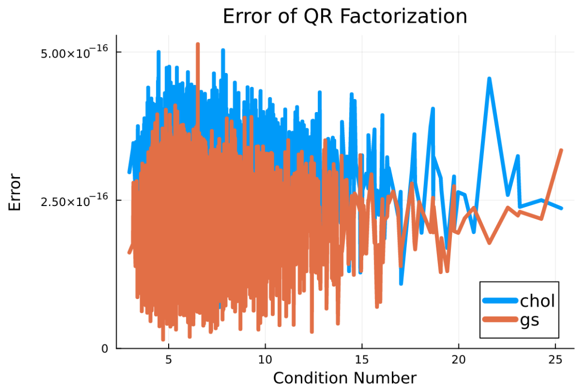
X needs to minimize the sum of the squared residuals to find a best fit. If b is not in the range of A, it needs to be orthogonally projected into the range of A.

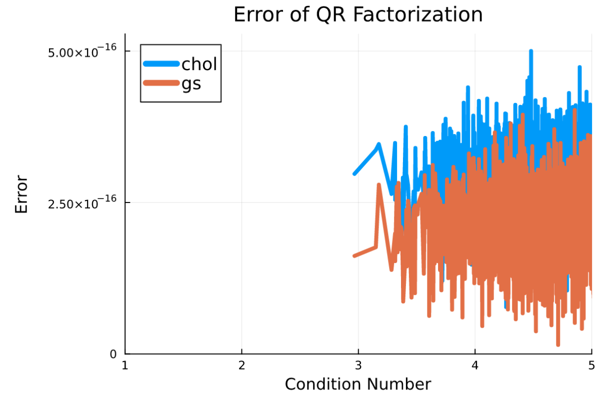
We can obtain the Cholesky Factor of A^TA directly from its QR factorization, if A is non-singular:

$$A = QR, \qquad A^TA = R^TQ^TQR = LL^T$$

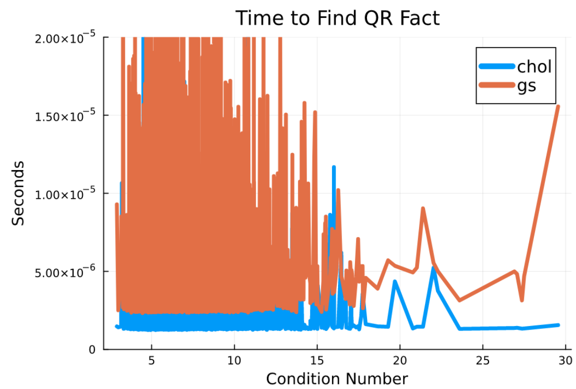
The Cholesky normal equation A^TA allows us to utilize the efficiency of a Cholesky decomposition without A being strictly Hermetian. However, A^TA is very poorly conditioned.

With the help of some lecture code (*Brown*), I plotted the error of Cholesky solutions vs. error of the pure Grahm-Schmidt QR decomposition by order of ascending condition number. I suspected that a direct relationship between condition number and error would exist, i.e. the higher the condition number the greater the magnitude of the least squares solution. This was not the case… Note that a b-value is not factored into the "magnitude of the least square solution"

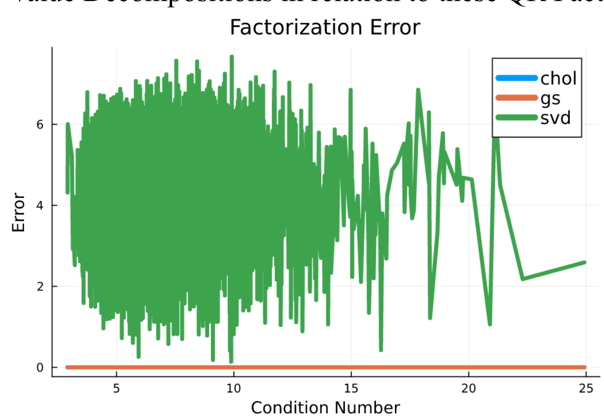




Despite the lack of a defining relation between error and condition number, the Gram-Schmidt QR Factorization is more accurate than the partial Cholesky Factor, but takes more time. Also consider that condition numbers generated by the random 10,000 matrices are all greater than 3. Perhaps a better correlation could be observed for more well-conditioned matrices.



Next, observe the error of Singular Value Decompositions in relation to these QR Factorizations…

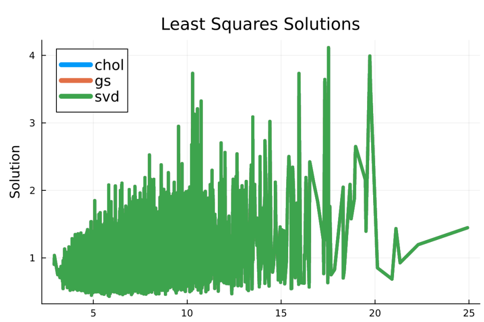


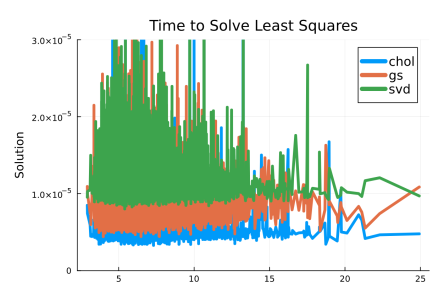
It is much higher, at least for small matrices. However, their respective least squares solutions are almost identical. For solutions via QR (from Cholesky factor and Gram-Schmidt):

$$\hat{x} = R^{-1}Q^{T}b$$

For solutions via SVD:

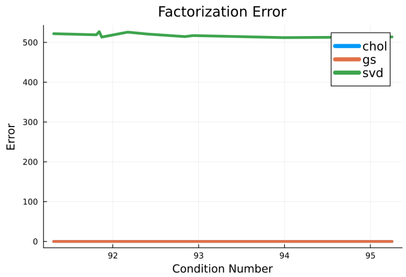
$$\hat{x} = V\Sigma^{+}U^{T}b$$

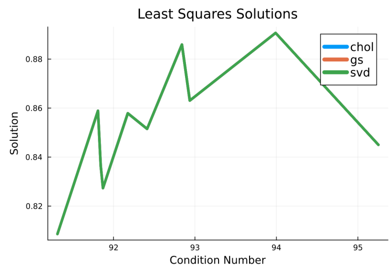


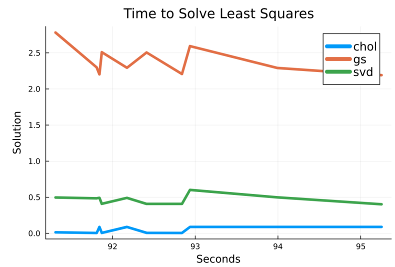


Note that R is rarely non-singular, and a Moore-Penrose pseudo-inverse is used in its place. Just as expected, the time to obtain these solutions is largest for the SVD. But will these results look the same for significantly larger matrices?

Significantly increasing the size of the matrices being operated on and reducing from 10,000 calculations to 10 renders the same results…







Conclusion:

When first beginning my experimentation, I fully expected to discover a direct correlation between condition number and the error of factorizations and/or least square solutions. This lack of correlation observed in my graphs could be a result of the poorly conditioned randomly generated matrices, as acknowledged previously. The time/accuracy trade-off of partial Cholesky factorizations argued quite strongly in favor of the method, much to my surprise.

I also sought out to make specific connections to some of Boeing's mathematical errors in recent years, but failed to find any such specific information–the explanation of events with Boeing in the introduction is more of an abstract about the importance of analyzing error carefully. The consequences of my many mistakes in this project will not lead to the loss of many hundreds of lives, perhaps only a bit of confused drooling and hair pulling by yours truly. I hope at the very least that this project was thought-provoking, thank you for reading.

Sources:

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- Brown, J. (n.d.). *Numerical computation*. Welcome - Numerical Computation. Retrieved May 6, 2023, from https://cu-numcomp.github.io/spring23/

Links:

- This google document (for commenting): https://docs.google.com/document/d/1CEy3co3wabkQLSrDC4FvzQlhk_TsN4mIDoCMVSJ78z4/edit?usp=sharing
- Github Repository: https://github.com/triszlers/NumComp_FinalProject.git
- Video explanation: https://drive.google.com/file/d/13-rW5h1sUbT7aOeYH450eEyS6eLFiw9C/view?usp=sharing