

EXPLORING PASSWORD-AUTHENTICATED KEY-EXCHANGE ALGORITHMS

A PROJECT REPORT SUBMITTED TO THE UNIVERSITY OF MANCHESTER
FOR THE DEGREE OF BACHELORS OF SCIENCE
IN THE FACULTY OF SCIENCE AND ENGINEERING

2023

Sam Leonard
f41751sl
Supervisor: Professor Bernardo Magri

Department of Computer Science

Declaration

No portion of the work referred to in this project report has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright

- i. The author of this project report (including any appendices and/or schedules to this thesis) owns certain copyright or related rights in it (the “Copyright”) and they have given The University of Manchester certain rights to use such Copyright, including for administrative purposes.
- ii. Copies of this thesis, either in full or in extracts and whether in hard or electronic copy, may be made **only** in accordance with the Copyright, Designs and Patents Act 1988 (as amended) and regulations issued under it or, where appropriate, in accordance with licensing agreements which the University has from time to time. This page must form part of any such copies made.
- iii. The ownership of certain Copyright, patents, designs, trade marks and other intellectual property (the “Intellectual Property”) and any reproductions of copyright works in the thesis, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property and Reproductions cannot and must not be made available for use without the prior written permission of the owner(s) of the relevant Intellectual Property and/or Reproductions.
- iv. Further information on the conditions under which disclosure, publication and commercialisation of this thesis, the Copyright and any Intellectual Property and/or Reproductions described in it may take place is available in the University IP Policy (see <http://documents.manchester.ac.uk/DocuInfo.aspx?DocID=24420>), in any relevant Thesis restriction declarations deposited in the University Library, The University Library’s regulations (see <http://www.library.manchester.ac.uk/about/regulations/>) and in The University’s policy on presentation of Theses

Abstract

Exploring Password-Authenticated Key-Exchange Algorithms

Sam Leonard, Supervisor: Professor Bernardo Magri

Password-Authenticated Key-Exchange (PAKE) algorithms are a niche kind of cryptography where parties seek to establish a strong shared key, from a low entropy secret such as a password. This makes the particularly attractive to some domains, such as Industrial Internet of Things (IIOT). However many PAKE algorithms are unsuitable for Internet of Things (IOT) applications, due to their heavy computational requirements. Augmented Composable Password Authenticated Connection Establishment (AuCPace) is a new PAKE protocol which aims to make PAKEs accessible to IIOT by utilising Elliptic Curve Cryptography (ECC), Verifier based PAKEs (V-PAKEs) and a novel augmented approach. This project aims to provide an approachable and developer-focused implementation of AuCPace in Rust and to contribute this implementation back to RustCrypto to promote wider adoption of PAKE algorithms.

Acknowledgements

I would like to thank everyone at absolutely wonderful supervisor, RustCrypto, Crypto Hack and my CTF Team (Organizers). Without your help none of this would have been possible.

Contents

1	Context	7
1.1	Background on PAKEs	7
1.1.1	What is a PAKE?	7
1.1.2	A brief history of PAKE algorithms	8
1.2	Elliptic Curve Cryptography	11
1.2.1	But what actually is an elliptic curve?	11
1.2.2	How do we do Cryptography with curves?	11
1.2.3	Where can Elliptic Curve Cryptography go wrong?	13
1.3	Modern PAKEs	13
1.4	AuCPace	13
1.5	Who are RustCrypto?	13
2	Design	14
2.1	Why Rust?	14
2.2	Developer Focussed Design	14
3	Implementation	15
3.1	Overview of RustCrypto and Dalek Cryptography	15
4	Testing	16
4.1	Creating Test Vectors	16
5	Reflection and Conclusion	17
5.1	Achievements	17
5.2	Reflection	17
5.3	Future Work	17
	Glossary	18
A	Python implementation of EKE	22
B	Python implementation of SRP	25

Chapter 1

Context

1.1 Background on PAKEs

1.1.1 What is a PAKE?

PAKEs are interactive, two party cryptographic protocols where each party shares knowledge of a password (a low entropy secret) and seeks to obtain a strong shared key e.g. for use later with a symmetric cipher. Critically an eavesdropper who can listen in two all messages of the key negotiation cannot learn enough information to bruteforce the password. Another way of phrasing this is that brute force attacks on the key must be "online".

There are two main types of PAKE algorithm - Augmented PAKEs and Balanced PAKEs.

- **Balanced PAKEs** are PAKEs where both parties share knowledge of the same secret password.
- **Augmented PAKEs** are PAKEs where one party has the password and the other has a "verifier" which is computed via a one-way function from the secret password.

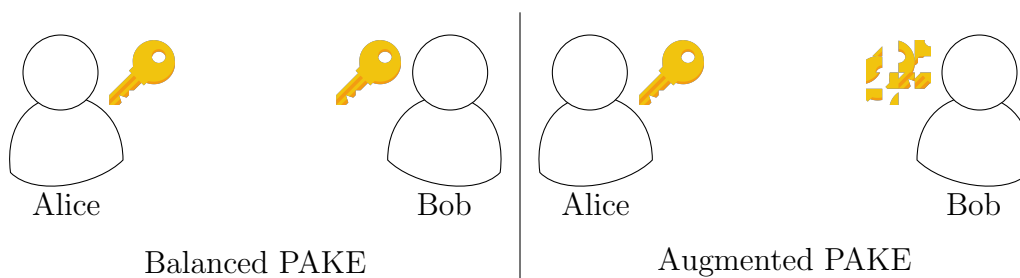


Figure 1.1: An illustration of the difference between Augmented PAKEs and Balanced PAKEs

1.1.2 A brief history of PAKE algorithms

The first **PAKE** algorithm was Bellovin and Merritt's **Encrypted Key Exchange (EKE)** scheme[BM92]. It works using a mix of **Symmetric Cryptography** and **Asymmetric Cryptography** to perform a key exchange. This comes with many challenges and subtle mistakes that are easy to make; primarily for the security of the system whatever is encrypted by the shared secret key(P) must be indistinguishable from random data. Otherwise an attacker can determine whether their guess at a trial decryption is valid. The **Rivest-Shamir-Adleman (RSA)** variant of **EKE** has this issue - the **RSA** parameter e is what is encrypted and sent in the first message. For **RSA** all valid values of e are odd, so this would prevent it being used. This is solved by adding 1 to e with a 50% chance. Figure 1.3 shows this protocol in full. While many of the initial variants on **EKE** have been shown to flawed/vulnerable, later variants have made it into real world use, such as in **Extensible Authentication Protocol (EAP)**[Vol+04] where it is available as **EAP-EKE**[She+11]. In appendix A you can find a Python implementation of this scheme

An Aside on Notation

- \leftarrow : Assignment - $x \leftarrow 5$ means x is assigned a value of 5.
- $\leftarrow_{\$}$: Sampling from a given set - $x \leftarrow_{\$} \mathbb{R}$ means to choose x at random from the set of real numbers.

Shared Parameter	Secret	Explanation
P	yes	the shared password

Figure 1.2: **EKE** shared parameters

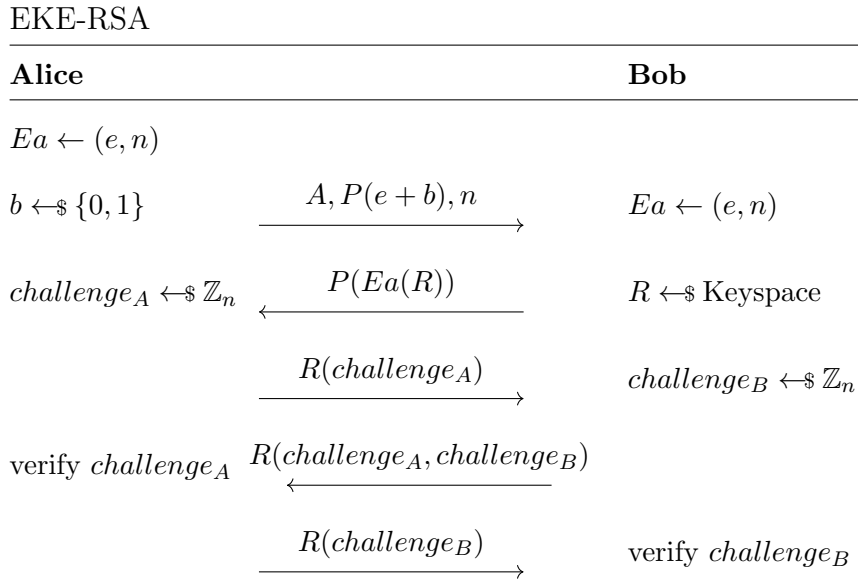


Figure 1.3: Implementing **EKE** using **RSA**

SPAKE

SPAKE1 and SPAKE2 are [Balanced PAKEs](#)’ which were introduced slightly later on by Michel Abdalla and David Pointcheval[AP05] as variations on [EKE](#). SPAKE2 and SPAKE2 are very similar so we will just explore SPAKE2 as we are more interested in [online](#) algorithms. SPAKE2 is also [Simple Password-Authenticated Key-Exchange \(SPAKE\)](#) differs from EKE in the following ways:

1. The encryption function is replaced by a simple one-time pad.
2. The [Asymmetric Cryptography](#) is provided by [Diffie-Hellman \(DH\)](#)
3. There is no explicit mutual authentication phase where challenges are exchanged. This has the advantage of reducing the number of messages that need to be sent.

Shared Parameter	Secret	Explanation
pw	yes	the shared password encoded as an element of \mathbb{Z}_p
\mathbb{G}	no	the mathematical group in which we will perform all operations
g	no	the generator of \mathbb{G}
p	no	the safe prime which defines the finite field for all operations in \mathbb{G}
M	no	an element in \mathbb{G} associated with user A
N	no	an element in \mathbb{G} associated with user B
H	no	a secure hash function

Figure 1.4: SPAKE shared parameters

SPAKE2

Alice

$$x \leftarrow \$\mathbb{Z}_p$$

$$X \leftarrow g^x$$

$$X^* \leftarrow X \cdot M^{pw}$$

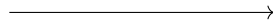
Bob

$$y \leftarrow \$\mathbb{Z}_p$$

$$Y \leftarrow g^y$$

$$Y^* \leftarrow Y \cdot N^{pw}$$

$$X^*$$



$$Y^*$$



$$K_A \leftarrow (Y^*/N^{pw})^x$$

$$SK_A \leftarrow H(A, B, X^*, Y^*, K_A)$$

$$K_B \leftarrow (X^*/M^{pw})^y$$

$$SK_B \leftarrow H(A, B, X^*, Y^*, K_B)$$

Figure 1.5: SPAKE2 Protocol

SRP

Finally we will look at [Secure Remote Password \(SRP\)](#) an [Augmented PAKE](#) first published in 1998, unlike [SPAKE2](#) it is not a modification of [EKE](#). [SRP](#) has gone through many revisions, at time of writing [SRP6a](#) is the latest version. [SRP](#) is likely the most used [PAKE](#) protocol in the world due to it's use in Apple's iCloud Keychain[\[Sec21\]](#) and it's availability as a [Transport Layer Security \(TLS\)](#) ciphersuite[\[Wu+07\]](#). However it is quite weird for what it does and there is no security proof for it[\[Gre18\]](#). An implementation of the protocol in Python can be found in [appendix B](#).

Parameter	Secret	Explanation
v	yes	the verifier stored by the server: $v = g^{H(s,I,P)}$
P	yes	the user's password
I	no	the user's name
g	no	the generator of \mathbb{G}
p	no	the safe prime which defines the finite field for all operations in \mathbb{G}
H	no	a secure hash function

Figure 1.6: [SRP](#) parameters

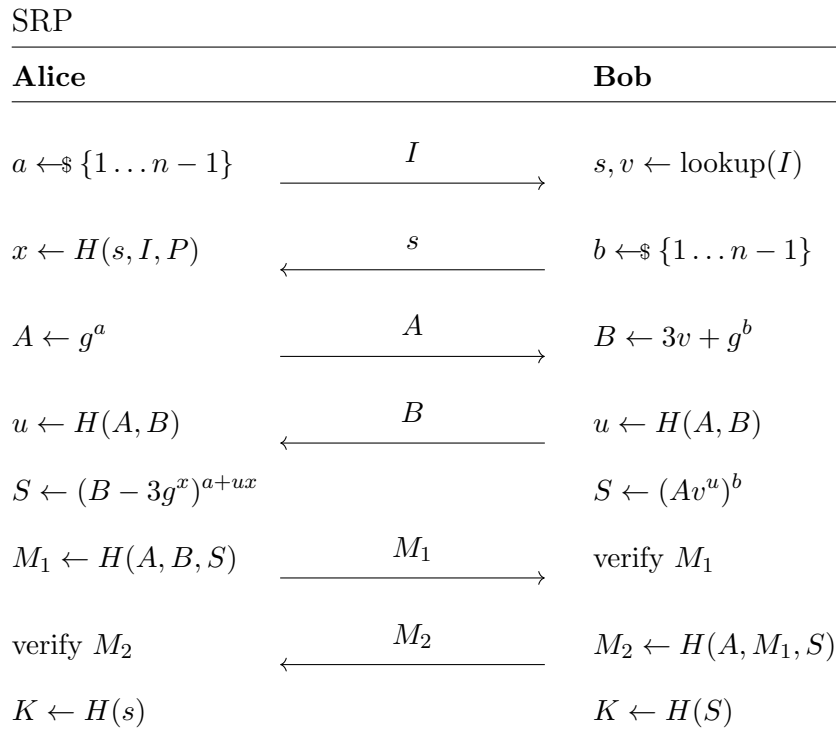


Figure 1.7: SRP-6 Protocol

1.2 Elliptic Curve Cryptography

Many modern Cryptographic protocols make use of a mathematical object known as an elliptic curve. First proposed in 1985 independently by Neal Koblitz[Kob87] and Victor S. Miller[Mil86]. Elliptic curves are attractive to cryptographers as they maintain a very high level of strength at smaller key sizes, this allows for protocols to consume less bandwidth, less memory and execute faster[KMV00]. To illustrate just how great the size savings are - National Institute of Standards and Technology (NIST) suggests that an elliptic curve key of just 256 bits provides the same level of security as an RSA key of 3072 bits[ST20].

1.2.1 But what actually is an elliptic curve?

With regards to Cryptography elliptic curves tend to come in one of two forms:

- Short Weierstraß Form: $y^2 = x^3 + ax + b$
- Montgomery Form: $by^2 = x^3 + ax^2 + x$

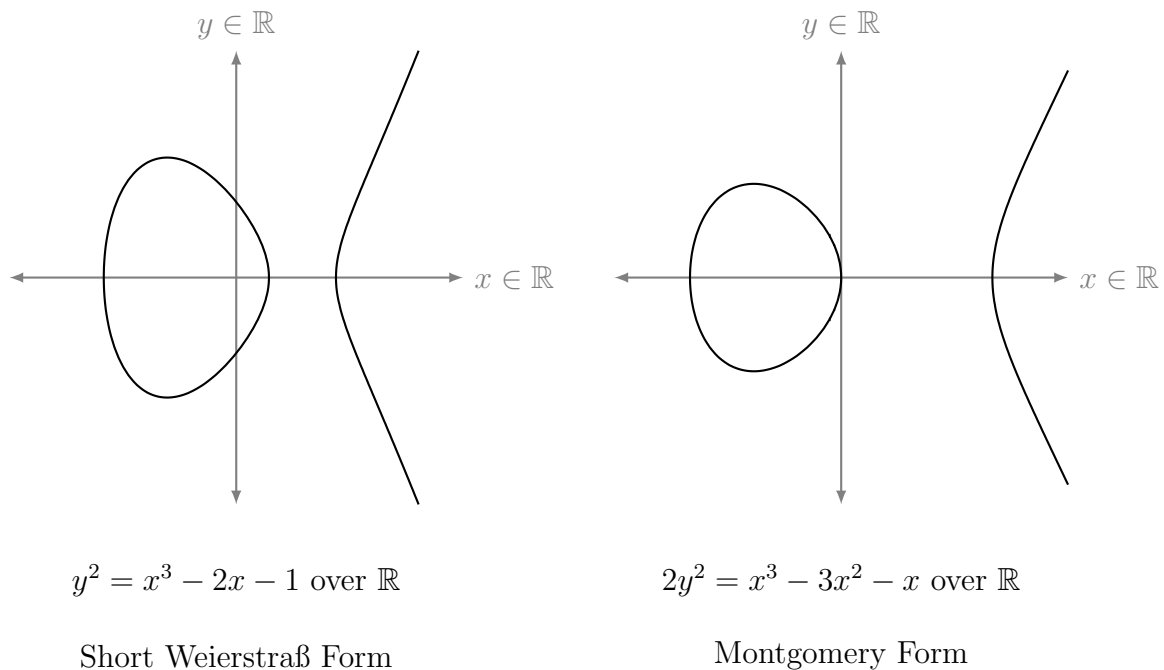


Figure 1.8: Elliptic curves over \mathbb{R} , Adapted from TikZ for Cryptographers[Jea16]

Weierstraß form is special as it is the general case for all elliptic curves, meaning all elliptic curves can be expressed as a Weierstraß curve. This property means that it is commonly used for expressing various curves. Montgomery form isn't quite as flexible, however it is favourable because it leads to significantly faster multiplication and addition operations via Montgomery's ladder[BL17].

1.2.2 How do we do Cryptography with curves?

To perform Cryptography with elliptic curves we need to define an "Abelian Group" to work in. An Abelian Group is a group whose group operation is also commutative, for

example the addition operator over the integers: $(+, \mathbb{Z})$ is an [Abelian Group](#). [Abelian Groups](#) form the basis of many modern Cryptographic algorithms, a [DH](#) key exchange can be performed in any [Abelian Group](#) for instance.

Our [Abelian Group](#) is built on the idea of "adding" points on the curve. To add two points, we find the line which passes through our two points and we continue along that line until we hit our curve again. We then reflect this point in the x -axis to get our result. What if we want to add our point to itself? Now there isn't a unique line through one point, however we are making the rules so in this case we will take the tangent to the curve at that point and then we can treat it the same as before. What if our line doesn't intersect with the curve? In this case we define a new point called the "neutral element" - \mathcal{O} . It is also called the point at infinity as it can be considered to be the single point at the end of every vertical line at infinity. Figure 1.9 illustrates all of these rules and edge cases.

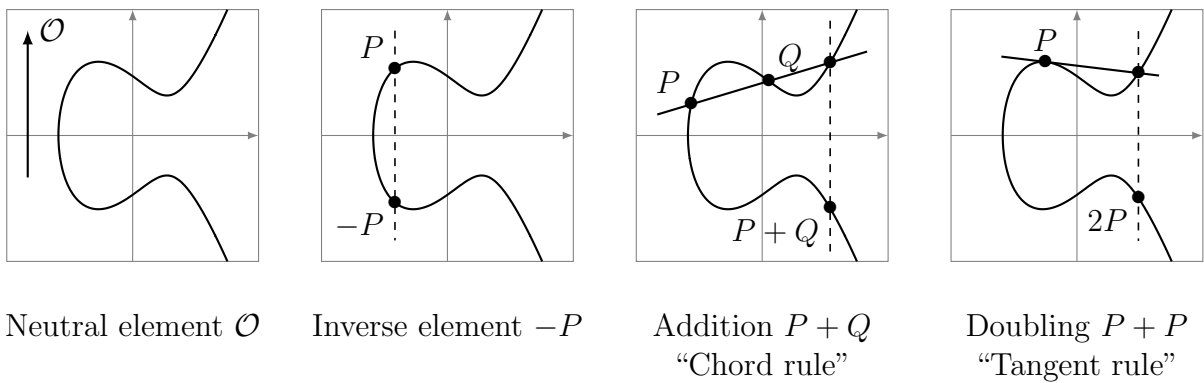


Figure 1.9: Elliptic Curve Group Operations, reproduced from TikZ for Cryptographers[Jea16]

However it's not quite that simple for us. We cannot use \mathbb{R} as computers only have finite resources we need a finite set to work in. Instead we define our operations over a [Finite Field](#), we will use the [Finite Field](#) of the integers mod a prime, denoted \mathbb{Z}_p for some prime p . Lets take a look at what our finite fields look like in a small finite field - \mathbb{Z}_{89} .

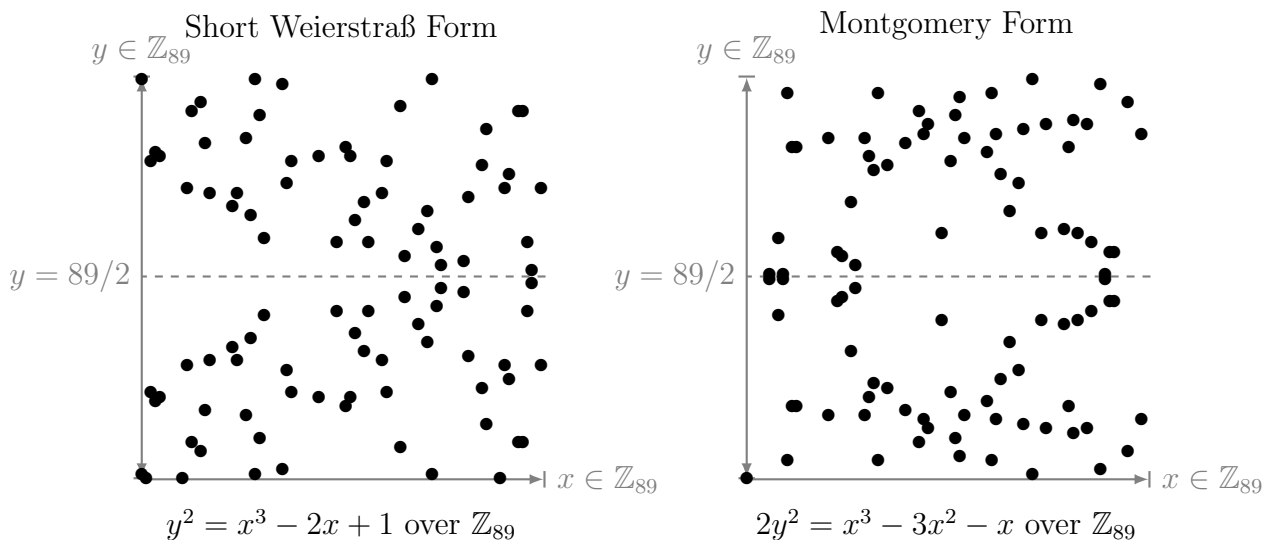


Figure 1.10: Elliptic curves over \mathbb{Z}_{89} , Adapted from TikZ for Cryptographers[Jea16]

This now looks very different to when we were looking at them in \mathbb{R} , however it shows very clearly what the elements of our set look like. They are points in the 2d coordinate plane with a symmetry around $p/2$. This might not feel intuitive but it is actually exactly what we should expect to happen, in our finite field when we negate our point's y coordinate, instead of flipping it around the y-axis, our points get wrapped around $y = p$. Hence our new point is the same distance from $y = p$ as our first point was with $y = 0$, this is where our symmetry arises.

1.2.3 Where can Elliptic Curve Cryptography go wrong?

There are many attacks against various aspects of Elliptic Curves, in general they fall into the following categories:

- Attacks against the [Elliptic Curve Discrete Logarithm Problem \(ECDLP\)](#) security of the curve:
 - The rho method[[Pol78](#)]
 - Transfer Security[[MVO91](#); [Sem98](#)]
 - CM Field Discriminants[[BL](#)]
 - Curve Rigidity[[BL13](#)]
- Attacks against the concrete implementation of [ECC](#):
 - Ladders required for safe and fast point-scalar multiplication[[BL](#)]
 - Twist Security[[LL97](#); [BMM00](#)]
 - Completeness[[IT02](#)]
 - Indistinguishability[[Ber+13](#)]

All of these attacks individually can weaken or even break the security of a given cryptosystem if not accounted for. However choosing the right curve is a good step in the right direction and can mitigate

1.3 Modern PAKEs

1.4 AuCPace

1.5 Who are RustCrypto?

Glossary

Abelian Group A group whose operator is also commutative. e.g. Addition over \mathbb{Z} . . [11](#), [12](#)

AES Advanced Encryption Scheme. [19](#)

Asymmetric Cryptography Asymmetric Cryptography is where the the sender and receiver each have two keys - a public key which can be freely shared, and a private key which must be kept secret. Common examples of this are the RSA scheme and the various DH flavours. [8](#), [9](#)

AuCPace Augmented Composable Password Authenticated Connection Establishment. [4](#)

Augmented PAKE A Balanced PAKE is one in which both parties share knowledge the same secret. This is in contrast to other schemes such as Verifier-based/Augmented PAKEs. . [7](#), [10](#)

Balanced PAKE A Balanced PAKE is one in which both parties share knowledge the same secret. This is in contrast to other schemes such as Verifier-based/Augmented PAKEs. . [7](#), [9](#)

DH Diffie-Hellman. [9](#), [12](#), [25](#)

EAP Extensible Authentication Protocol. [8](#)

ECC Elliptic Curve Cryptography. [4](#), [13](#)

ECDLP Elliptic Curve Discrete Logarithm Problem. [13](#)

EKE Encrypted Key Exchange. [8](#), [9](#), [10](#)

Finite Field A Finite Field is a finite set with an associated addition and multiplication operator, where the operators satisfy the field axioms. Namely they are: Associative, Commutative, Distributive, they have inverses and identity elements. . [12](#)

IIOT Industrial Internet of Things. [4](#)

IOT Internet of Things. [4](#)

NIST National Institute of Standards and Technology. [11](#)

Online Cryptography Online cryptography is where interactions with the cryptosystem are only possible via real-time interactions with the server. Primarily this is to prevent offline computation. [7](#), [9](#)

PAKE Password-Authenticated Key-Exchange. [4](#), [7](#), [8](#), [10](#)

RSA Rivest-Shamir-Adleman. [8](#), [11](#)

Safe Prime A number $2n + 1$ is a Safe Prime if n is prime, it is the effectively the other part of a Sophie Germain prime. . [9](#), [10](#)

SPAKE Simple Password-Authenticated Key-Exchange. [9](#), [10](#)

SRP Secure Remote Password. [10](#), [25](#)

Symmetric Cryptography Symmetric Cryptography is where the both the sender and receiver share the same secret key. It is normally computationally more efficient, the most common such scheme is [Advanced Encryption Scheme \(AES\)](#). [8](#)

TLS Transport Layer Security. [10](#)

Verifier A representation of the user's password put through some one-way function. This could be as simple as just storing a hash of the password, though for most PAKEs the verifier is an element of whatever group we are working in. An example can be seen on page [10](#). [7](#), [10](#)

V-PAKE Verifier based PAKE. [4](#)

Bibliography

- [AP05] Michel Abdalla and David Pointcheval. “Simple Password-Based Encrypted Key Exchange Protocols”. In: *Topics in Cryptology – CT-RSA 2005*. Ed. by Alfred Menezes. Berlin, Heidelberg: Springer Berlin Heidelberg, 2005, pp. 191–208. ISBN: 978-3-540-30574-3.
- [Ber+13] Daniel J Bernstein et al. “Elligator: elliptic-curve points indistinguishable from uniform random strings”. In: *Proceedings of the 2013 ACM SIGSAC conference on Computer & communications security*. 2013, pp. 967–980.
- [BL] Daniel J. Bernstein and Tanja Lange. *Daniel J. Bernstein and Tanja Lange. SafeCurves: choosing safe curves for elliptic-curve cryptography*. <https://safecurves.cr.yp.to>, accessed 3rd April 2023.
- [BL13] Daniel J Bernstein and Tanja Lange. “Security dangers of the NIST curves”. In: *Invited talk, International State of the Art Cryptography Workshop, Athens, Greece*. 2013.
- [BL17] Daniel J. Bernstein and Tanja Lange. *Montgomery curves and the Montgomery ladder*. Cryptology ePrint Archive, Paper 2017/293. <https://eprint.iacr.org/2017/293>. 2017. URL: <https://eprint.iacr.org/2017/293>.
- [BM92] Steven Michael Bellovin and Michael Merritt. *Encrypted key exchange: Password-based protocols secure against dictionary attacks*. May 1992.
- [BMM00] Ingrid Biehl, Bernd Meyer, and Volker Müller. “Differential fault attacks on elliptic curve cryptosystems”. In: *Advances in Cryptology—CRYPTO 2000: 20th Annual International Cryptology Conference Santa Barbara, California, USA, August 20–24, 2000 Proceedings 20*. Springer. 2000, pp. 131–146.
- [Gre18] Matthew Green. *Should you use SRP?* <https://blog.cryptographyengineering.com/should-you-use-srp/>, accessed 3rd April, 2023. 2018. URL: <https://blog.cryptographyengineering.com/should-you-use-srp/>.
- [IT02] Tetsuya Izu and Tsuyoshi Takagi. “Exceptional procedure attack on elliptic curve cryptosystems”. In: *Public Key Cryptography—PKC 2003: 6th International Workshop on Practice and Theory in Public Key Cryptography Miami, FL, USA, January 6–8, 2003 Proceedings 6*. Springer. 2002, pp. 224–239.
- [Jea16] Jérémy Jean. *TikZ for Cryptographers*. <https://www.iacr.org/authors/tikz/>. 2016.
- [KMV00] Neal Koblitz, Alfred Menezes, and Scott Vanstone. “The state of elliptic curve cryptography”. In: *Designs, codes and cryptography 19* (2000), pp. 173–193.
- [Kob87] Neal Koblitz. “Elliptic curve cryptosystems”. In: *Mathematics of computation* 48.177 (1987), pp. 203–209.

- [LL97] Chae Hoon Lim and Pil Joong Lee. “A key recovery attack on discrete log-based schemes using a prime order subgroup”. In: *Advances in Cryptology—CRYPTO’97: 17th Annual International Cryptology Conference Santa Barbara, California, USA August 17–21, 1997 Proceedings* 17. Springer. 1997, pp. 249–263.
- [Mil86] Victor S. Miller. “Use of Elliptic Curves in Cryptography”. In: *Advances in Cryptology — CRYPTO ’85 Proceedings*. Ed. by Hugh C. Williams. Berlin, Heidelberg: Springer Berlin Heidelberg, 1986, pp. 417–426. ISBN: 978-3-540-39799-1.
- [MVO91] Alfred Menezes, Scott Vanstone, and Tatsuaki Okamoto. “Reducing elliptic curve logarithms to logarithms in a finite field”. In: *Proceedings of the twenty-third annual ACM symposium on Theory of computing*. 1991, pp. 80–89.
- [Pol78] John M Pollard. “Monte Carlo methods for index computation (mod p)”. In: *Mathematics of computation* 32.143 (1978), pp. 918–924.
- [Sec21] Apple Platform Security. *Escrow security for iCloud Keychain*. <https://support.apple.com/en-gb/guide/security/sec3e341e75d/web>, accessed 3rd April, 2023. May 2021. URL: <https://support.apple.com/en-gb/guide/security/sec3e341e75d/web>.
- [Sem98] Igor Semaev. “Evaluation of discrete logarithms in a group of ρ -torsion points of an elliptic curve in characteristic ρ ”. In: *Mathematics of computation* 67.221 (1998), pp. 353–356.
- [She+11] Y. Sheffer et al. *An EAP Authentication Method Based on the Encrypted Key Exchange (EKE) Protocol*. rfc 6124. <https://www.rfc-editor.org/rfc/rfc6124.txt>. Feb. 2011. URL: <https://www.rfc-editor.org/rfc/rfc6124.txt>.
- [ST20] National Institute of Standards and Technology. *Recommendation for Key Management: Part 1 – General*. Tech. rep. Federal Information Processing Standards Publications (FIPS PUBS) 140-2, Change Notice 2 December 03, 2002. Washington, D.C.: U.S. Department of Commerce, 2020. DOI: [10.6028/10.6028/NIST.SP.800-57pt1r5](https://doi.org/10.6028/10.6028/NIST.SP.800-57pt1r5).
- [Vol+04] John Vollbrecht et al. *Extensible Authentication Protocol (EAP)*. rfc 3748. <https://www.rfc-editor.org/rfc/rfc3748.txt>. RFC Editor, June 2004. URL: <https://www.rfc-editor.org/rfc/rfc3748.txt>.
- [Wu+07] T. Wu et al. *Using the Secure Remote Password (SRP) Protocol for TLS Authentication*. RFC 5054. <https://www.rfc-editor.org/rfc/rfc5054.txt>. RFC Editor, Nov. 2007. URL: <https://www.rfc-editor.org/rfc/rfc5054.txt>.
- [Wu00] Tom Wu. *The SRP Authentication and Key Exchange System*. RFC 2945. <https://www.rfc-editor.org/rfc/rfc2945.txt>. RFC Editor, Sept. 2000. URL: <https://www.rfc-editor.org/rfc/rfc2945.txt>.

Appendix A

Python implementation of EKE

While researching Bellovin and Merritt's EKE scheme[BM92], I created a full implementation of the scheme in Python. The full code can be found at https://github.com/tritoke/eke_python. The core negotiation functions for the client and server have been included below:

Listing 1: Client Negotiate

```
negotiate(self):
    # generate random public key Ea
    Ea = RSA.gen()

    # instantiate AES with the password
    P = AES.new(self.password.ljust(16).encode(), AES.MODE_ECB)

    # send a negotiate command
    self.send_json(
        action="negotiate",
        username=self.username,
        enc_pub_key=b64e(P.encrypt(Ea.encode_public_key())),
        modulus=Ea.n
    )

    # receive and decrypt R
    self.recv_json()
    key =
        ↪ 12b(Ea.decrypt(b2l(P.decrypt(b64d(self.data["enc_secret_key"]))))))
    R = AES.new(key, AES.MODE_ECB)

    # send first challenge
    challengeA = randbytes(16)
    self.send_json(challenge_a=b64e(R.encrypt(challengeA)))

    # receive challenge response
    self.recv_json()
    challenge_response = R.decrypt(b64d(self.data["challenge_response"]))
```

```

assert challenge_response[:16] == challengeA, "Challenge A failed."

challengeB = challenge_response[16:]

# response with challengeB
self.send_json(challenge_b=b64e(R.encrypt(challengeB)))

# receive success message
self.recv_json()
assert self.data["success"], self.data.get("message",
    ↪ "ChallengeB failed.")

# store the shared key
self.R = R

```

Listing 2: Server Negotiate

```

handle_eke_negotiate_key(self):
# decrypt Ea using P
P = AES.new(self.database[self.data["username"]].ljust(16).encode(),
    ↪ AES.MODE_ECB)
e = b2l(P.decrypt(b64d(self.data["enc_pub_key"])))

# e is always odd, but we add 1 with 50% probability
if e % 2 == 0:
    e -= 1

# generate secret key R
R = randbytes(16)
Ea = RSA.from_pub_key(e, self.data["modulus"])
self.send_json(enc_secret_key=b64e(P.encrypt(l2b(Ea.encrypt(b2l(R))))))
x = b64e(P.encrypt(l2b(Ea.encrypt(b2l(R)))))

# transform R into a cipher instance
R = AES.new(R, AES.MODE_ECB)

# receive encrypted challengeA and generate challengeB
self.recv_json()
challengeA = R.decrypt(b64d(self.data["challenge_a"]))
challengeB = randbytes(16)

# send challengeA + challengeB
self.send_json(challenge_response=b64e(R.encrypt(challengeA+challengeB))
    ↪ ))

# receive challengeB back again
self.recv_json()
success = R.decrypt(b64d(self.data["challenge_b"])) == challengeB

```

```
self.send_json(success=success)
self.R = R
```

Appendix B

Python implementation of SRP

While conducting my initial research on PAKEs I came across [SRP\[Wu00\]](#). SRP is the first protocol I looked at which took the approach of encoding values as DH group elements. To understand this approach better I chose to create a toy implementation. The full code can be found at https://github.com/tritoke/srp_python. The core negotiation functions for the client and server have been included below:

Listing 3: Client Negotiate

```
negotiate(self):
# send a negotiate command
self.send_json(action="negotiate", username=self.username)

# receive the salt back from the server
self.recv_json()
s = int(self.data["salt"])
x = H(s, H(f"{self.username}:{self.password}"))

# generate an ephemeral key pair and send the public key to the server
a = strong_rand(KEYSIZE_BITS)
A = pow(g, a, N)
self.send_json(user_public_ephemeral_key=A)

# receive the servers public ephemeral key back
self.recv_json()
B = self.data["server_public_ephemeral_key"]

# calculate u and S
u = H(A, B)
S = pow((B - 3 * pow(g, x, N)), a + u * x, N)

# calculate M1
M1 = H(A, B, S)
self.send_json(verification_message=M1)

# receive M2
self.recv_json()
```

```

M2 = self.data["verification_message"]

if M2 != H(A, M1, S):
    print("Failed to agree on shared key.")

K = H(S)

return K

```

Listing 4: Server Negotiate

```

handle_srp_negotiate_key(self):
# receive the username I from the client
# lookup data in database
user = self.data["username"]
I = b2l(user.encode())

if (db_record := self.database.get(user)) is None:
    self.send_json(success=False,
        ↳ message=f"Failed to find user in DB.")
    return

s = db_record["salt"]
v = db_record["verifier"]

# send s to the client
self.send_json(salt=s)

# receive A from the user
self.recv_json()
A = self.data["user_public_ephemeral_key"]

# calculate B
b = strong_rand(KEYSIZE_BITS)
B = 3 * v + pow(g, b, N)

# send B to the client
self.send_json(server_public_ephemeral_key=B)

# calculate u and S
u = H(A, B)
S = pow(A * pow(v, u, N), b, N)

# receive M1 from the client
self.recv_json()
M1 = self.data["verification_message"]

# verify M1

```

```

if M1 != H(A, B, S):
    self.send_json(success=False,
        ↪ message=f"Failed to agree shared key.")
    return

# calculate M2
M2 = H(A, M1, S)
self.send_json(verification_message=M2)

# calculate key
K = H(S)

# log the derived key - not part of the protocol
print(f"Derived K={K:X}")

# encrypt our final message to the client using our shared key
key = l2b(K)
nonce = get_random_bytes(16)

cipher = AES.new(key, AES.MODE_GCM, nonce=nonce)
ct, mac = cipher.encrypt_and_digest(f_
    ↪ "Successfully agreed shared key for {user}.".encode())

# notify the client of the success
self.send_json(success=True, nonce=b64e(nonce), enc_message=b64e(ct),
    ↪ tag=b64e(mac))

```