EXPLORING PASSWORD-AUTHENTICATED KEY-EXCHANGE ALGORITHMS

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Declaration

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Abstract

Exploring Password-Authenticated Key-Exchange Algorithms

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Password-Authenticated Key-Exchange (PAKE) algorithms are a niche kind of cryptography where parties seek to establish a strong shared key, from a low entropy secret such as a password. This makes the particularly attractive to some domains, such as Industrial Internet of Things (IIOT). However many PAKE algorithms are unsuitable for Internet of Things (IOT) applications, due to their heavy computational requirements. Augmented Composable Password Authenticated Connection Establishment (AuCPace) is a new PAKE protocol which aims to make PAKEs accessible to IIOT by utilising Elliptic Curve Cryptography (ECC), Verifier based PAKEs (V-PAKEs) and a novel augmented approach. This project aims to provide an approachable and developer-focused implementation of AuCPace in Rust and to contribute this implementation back to RustCrypto to promote wider adoption of PAKE algorithms.

Acknowledgements

I would like to thank everyone at absolutely onderful supervisor, RustCrypto, Crypto Hack and my CTF Team (0rganizers). Without your help none of this would have been possible.

Contents

Chapter 1

Context

1.1 Background on PAKEs

1.1.1 What is a PAKE?

PAKEs are interactive, two party cryptographic protocols where each party shares knowledge of a password (a low entropy secret) and seeks to obtain a strong shared key e.g. for use later with a symmetric cipher. Critically an eavesdropper who can listen in two all messages of the key negotiation cannot learn enough information to bruteforce the password. Another way of phrasing this is that brute force attacks on the key must be "online".

There are two main types of PAKE algorithm - Augmented PAKEs and Balanced PAKEs.

- Balanced PAKEs are PAKEs where both parties share knowledge of the same secret password.
- Augmented PAKEs are PAKEs where one party has the password and the other has a "verifier" which is computed via a one-way function from the secret password.

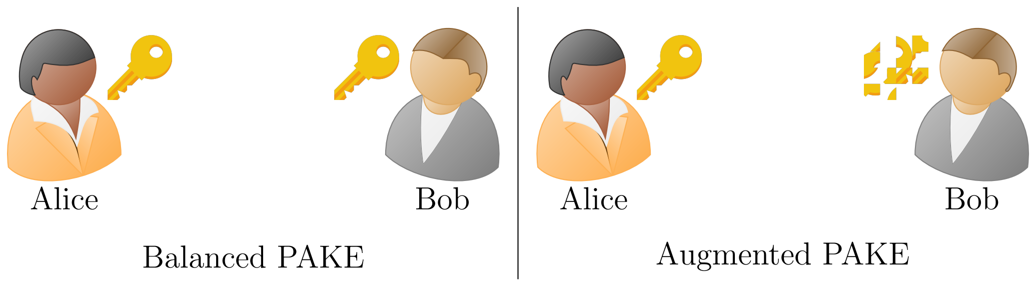


Figure 1.1: An illustration of the difference between Augmented PAKEs and Balanced PAKEs

1.1.2 A brief history of PAKE algorithms

The first PAKE algorithm was Bellovin and Merritt's Encrypted Key Exchange (EKE) scheme.? It works using a mix of Symmetric Cryptography and Asymmetric Cryptography to perform a key exchange. This comes with many challenges and subtle mistakes that are easy to make; primarily for the security of the system whatever is encrypted by the shared secret key(P) must be indistinguishable from random data. Otherwise an attacker can determine whether their guess at a trial decryption is valid. The Rivest-Shamir-Adleman (RSA) variant of EKE has this issue - the RSA parameter e is what is encrypted and sent in the first message. For RSA all valid values of e are odd, so this would prevent it being used. This is solved by adding 1 to e with a 50% chance. Figure 1.3 shows this protocol in full. While many of the initial variants on EKE have been shown to flawed/vulnerable, later variants have made it into real world use, such as in Extensible Authentication Protocol (EAP)? where it is available as EAP-EKE.? In appendix A you can find a Python implementation of this scheme

An Aside on Notation

- $\leftarrow$: Assignment - $x \leftarrow 5$ means x is assigned a value of 5.
- $\leftarrow\!\$$: Sampling from a given set - $x \leftarrow\!\$ \mathbb{R}$ means to choose x at random from the set of real numbers.

Shared Parameter	Secret	Explanation
P	yes	the shared password

Figure 1.2: EKE shared parameters

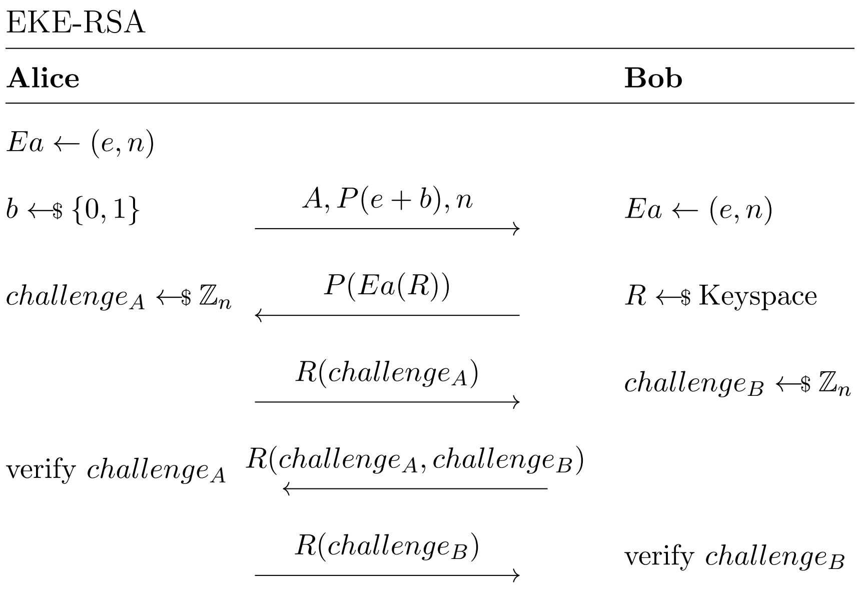

EKE-RSA
Alice
Bob
$Ea \leftarrow (e, n)$
$b \leftarrow\!\$ \{0,1\}$
$A, P(e+b), n$
$Ea \leftarrow (e, n)$
$challenge_A \leftarrow\!\$ \mathbb{Z}_n$
$P(Ea(R))$
$R \leftarrow\!\$$ Keyspace
$R(challenge_A)$
$challenge_B \leftarrow\!\$ \mathbb{Z}_n$
verify $challenge_A$
$R(challenge_A, challenge_B)$
$R(challenge_B)$
verify $challenge_B$

Figure 1.3: Implementing EKE using RSA

SPAKE

SPAKE1 and SPAKE2 are Balanced PAKEs' which were introduced slightly later on by Michel Abdalla and David Pointcheval[?] as variations on EKE. SPAKE2 and SPAKE2 are very similar so we will just explore SPAKE2 as we are more interested in online algorithms. SPAKE2 is also Simple Password-Authenticated Key-Exchange (SPAKE) differs from EKE in the following ways:

1. The encryption function is replaced by a simple one-time pad.
2. The Asymmetric Cryptography is provided by Diffie-Hellman (DH)
3. There is no explicit mutual authentication phase where challenges are exchanged. This has the advantage of reducing the number of messages that need to be sent.

Shared Parameter	Secret	Explanation
pw	yes	the shared password encoded as an element of $\mathbb{Z}_p$
$\mathbb{G}$	no	the mathematical group in which we will perform all opertions
g	no	the generator of $\mathbb{G}$
p	no	the safe prime which defines the finite field for all operations in $\mathbb{G}$
M	no	an element in $\mathbb{G}$ associated with user A
N	no	an element in $\mathbb{G}$ associated with user B
H	no	a secure hash function

Figure 1.4: SPAKE shared parameters

SPAKE2

Alice		**Bob**
$x \leftarrow\$ \mathbb{Z}_p$		$y \leftarrow\$ \mathbb{Z}_p$
$X \leftarrow g^x$		$Y \leftarrow g^y$
$X^* \leftarrow X \cdot M^{pw}$		$Y^* \leftarrow X \cdot N^{pw}$
	$\xrightarrow{X^*}$	
	$\xleftarrow{Y^*}$	
$K_A \leftarrow (Y^*/N^{pw})^x$		$K_B \leftarrow (X^*/M^{pw})^y$
$SK_A \leftarrow H(A, B, X^*, Y^*, Ka)$		$SK_B \leftarrow H(A, B, X^*, Y^*, Kb)$

Figure 1.5: SPAKE2 Protocol

SRP

Finally we will look at Secure Remote Password (SRP) an Augmented PAKE first published in 1998, unlike SPAKE2 it is not a modification of EKE. SRP has gone through many revisions, at time of writing SRP6a is the latest version. SRP is likely the most used PAKE protocol in the world due to it's use in Apple's iCloud Keychain[?] and it's availability as a Transport Layer Security (TLS) ciphersuite.[?] However it is quite weird for what it does and there is no security proof for it.[?] An implementation of the protocol in Python can be found in appendix B.

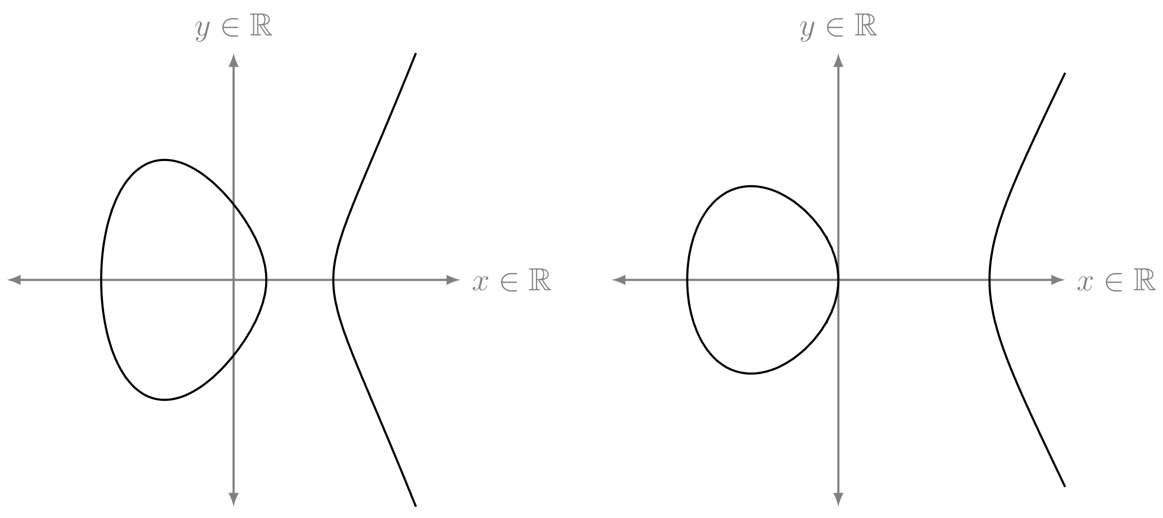
Parameter	Secret	Explanation
v	yes	the verifier stored by the server: $v = g^{H(s,I,P)}$
P	yes	the user's password
I	no	the user's name
g	no	the generator of $\mathbb{G}$
p	no	the safe prime which defines the finite field for all operations in $\mathbb{G}$
H	no	a secure hash function

Figure 1.6: SRP parameters

SRP

Alice		**Bob**
$a \leftarrow\!\$ \{1 \ldots n-1\}$	$\xrightarrow{I}$	$s, v \leftarrow \text{lookup}(I)$
$x \leftarrow H(s, I, P)$	$\xleftarrow{s}$	$b \leftarrow\!\$ \{1 \ldots n-1\}$
$A \leftarrow g^a$	$\xrightarrow{A}$	$B \leftarrow 3v + g^b$
$u \leftarrow H(A, B)$	$\xleftarrow{B}$	$u \leftarrow H(A, B)$
$S \leftarrow (B - 3g^x)^{a+ux}$		$S \leftarrow (Av^u)^b$
$M_1 \leftarrow H(A, B, S)$	$\xrightarrow{M_1}$	verify M_1
verify M_2	$\xleftarrow{M_2}$	$M_2 \leftarrow H(A, M_1, S)$
$K \leftarrow H(s)$		$K \leftarrow H(S)$

Figure 1.7: SRP-6 Protocol



$y^2 = x^3 - 2x - 1$ over $\mathbb{R}$ $\qquad$ $2y^2 = x^3 - 3x^2 - x$ over $\mathbb{R}$

Short Weierstraß Form $\qquad$ Montgomery Form

Figure 1.8: Elliptic curves over $\mathbb{Z}_{89}$, Inspired by TikZ for Cryptographers[?]

1.2 Elliptic Curve Cryptography

Many modern Cryptograhpic protocols make use of a mathematical object known as an elliptic curve. First proposed in 1985 independently by Neal Koblitz[?] and Victor S. Miller.[?] Elliptic curves are attractive to cryptographers as they maintain a very high level of strength at smaller key sizes, this allows for protocols to consume less bandwidth, less memory and execute faster.[?] To illustrate just how great the size savings are - National Institute of Standards and Technology (NIST) suggests that an elliptic curve key of just 256 bits provides the same level of security as an RSA key of 3072 bits.[?]

1.2.1 But what actually is an elliptic curve?

With regards to Cryptography elliptic curves tend to come in one of two forms:

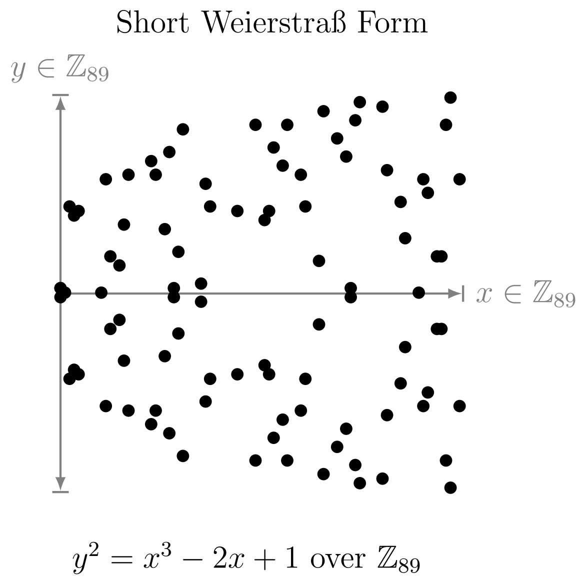
- Short Weierstraß Form: $y^2 = x^3 + ax + b$
- Montgomery Form: $by^2 = x^3 + ax^2 + x$

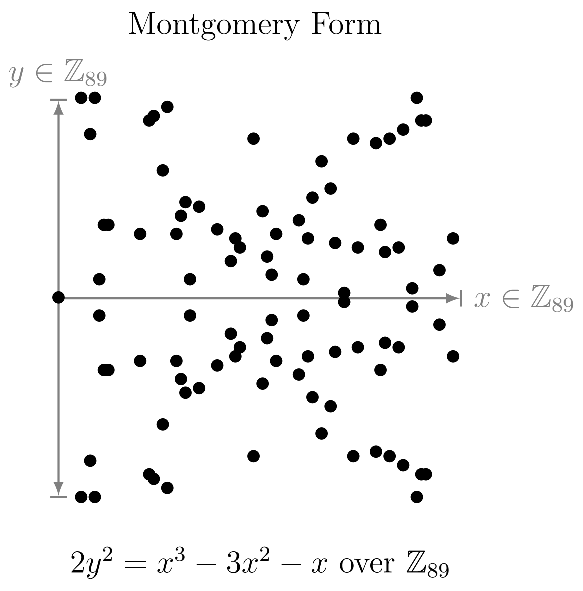
Weierstraß form is special as it is the general case for all elliptic curves, meaning all elliptic curves can be expressed as a Weierstraß curve. This property means that it is commonly used for expressing various curves. Montgomery form isn't quite as flexible, however it is favourable because it leads to significantly faster multiplication and addition operations via Montgomery's ladder.[?]

1.2.2 How do we do Cryptography with curves?

To perform Cryptography with elliptic curves we need to define an "Abelian Group" to work in. An Abelian Group is a group whose group operation is also commutative, for example the addition operator over the integers: $(+, \mathbb{Z})$ is an Abelian Group. Abelian

Groups form the basis of many modern Cryptographic algorithms, a DH key exchange can be performed in any Abelian Group for instance.
To define an abelian group over our elliptic curves we need a set G, our set will be the set of integer points on the curve mod a prime p - $\{x|x \in \mathbb{Z}_p \text{ and } \exists(y \in \mathbb{Z}_p)[By^2 = x^3+Ax^2+x]\}$. It is helpful to visualalise this so lets take our curves from before and see how they look in a small group - $\mathbb{Z}_{89}$.





Inspired by TikZ for Cryptographers?

This now looks very different to when we were looking at them in $\mathbb{R}$, however it shows very clearly what the elements of our set look like. They are points in the 2d coordinate plane with a symmetry around the Y-axis, this is worth noting as it is the basis for forming our group's binary operator.
Lets build our binary operator, for this to work we need to define the notion of adding two points together, and it needs to be associative, have an inverse, have some notion of an "identity" and it needs to be commutative. To start with we will define point addition using the "Chord rule" and then handle the edge cases. The Chord rule

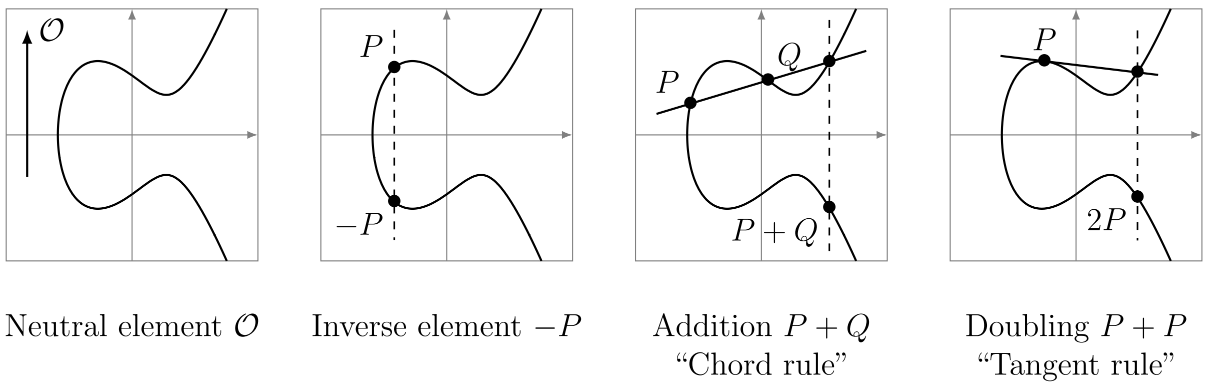


Figure 1.9: Elliptic Curve Group Operations, reproduced from TikZ for Cryptographers?

1.3 Modern PAKEs

1.4 AuCPace

1.5 Who are RustCrypto?

Chapter 2

Design

2.1 Why Rust?

2.2 Developer Focussed Design

Chapter 3

Implementation

3.1 Overview of RustCrypto and Dalek Cryptography

Chapter 4

Testing

4.1 Creating Test Vectors

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Reflection and Conclusion

5.1 Achievements

5.2 Reflection

5.3 Future Work

Glossary

Abelian Group A group whose operator is also commutative. e.g. Addition over $\mathbb{Z}$. . 11, 12

AES Advanced Encryption Scheme. 19

Asymmetric Cryptography Asymmetric Cryptography is where the the sender and receiver each have two keys - a public key which can be freely shared, and a private key which must be kept secret. Common examples of this are the RSA scheme and the various DH flavours. 8, 9

AuCPace Augmented Composable Password Authenticated Connection Establishment. 4

Augmented PAKE A Balanced PAKE is one in which both parties share knowledge the same secret. This is in contrast to other schemes such as Verifier-based/Augmented PAKEs. . 7, 10

Balanced PAKE A Balanced PAKE is one in which both parties share knowledge the same secret. This is in contrast to other schemes such as Verifier-based/Augmented PAKEs. . 7, 9

DH Diffie-Hellman. 9, 12, 23

EAP Extensible Authentication Protocol. 8

ECC Elliptic Curve Cryptography. 4

EKE Encrypted Key Exchange. 8, 9, 10

IIOT Industrial Internet of Things. 4

IOT Internet of Things. 4

NIST National Institute of Standards and Technology. 11

Online Cryptography Online cryptography is where interactions with the cryptosystem are only possible via real-time interactions with the server. Primarily this is to prevent offline computation. 7, 9

PAKE Password-Authenticated Key-Exchange. 4, 7, 8, 10

RSA Rivest-Shamir-Adleman. 8, 11

Safe Prime A number $2n + 1$ is a Safe Prime if n is prime, it is the effectively the other part of a Sophie Germain prime. . 9, 10

SPAKE Simple Password-Authenticated Key-Exchange. 9, 10

SRP Secure Remote Password. 10, 23

Symmetric Cryptography Symmetric Cryptography is where the both the sender and receiver share the same secret key. It is normally computationally more efficient, the most common such scheme is Advanced Encryption Scheme (AES). 8

TLS Transport Layer Security. 10

Verifier A representation of the user's password put through some one-way function. This could be as simple as just storing a hash of the password, though for most PAKEs the verifier is an element of whatever group we are working in. An example can be seen on page 10. 7, 10

V-PAKE Verifier based PAKE. 4

Appendix A

Python implementation of EKE

While researching Bellovin and Merritt's EKE scheme,? I created a full implementation of the scheme in Python. The full code can be found at https://github.com/tritoke/eke_python. The core negotiation functions for the client and server have been included below:

Listing 1: Client Negotiate

```python
def negotiate(self):
    # generate random public key Ea
    Ea = RSA.gen()

    # instantiate AES with the password
    P = AES.new(self.password.ljust(16).encode(), AES.MODE_ECB)

    # send a negotiate command
    self.send_json(
        action="negotiate",
        username=self.username,
        enc_pub_key=b64e(P.encrypt(Ea.encode_public_key())),
        modulus=Ea.n
    )

    # receive and decrypt R
    self.recv_json()
    key = l2b(Ea.decrypt(b2l(P.decrypt(b64d(self.data["enc_secret_key"]
    ↪  ])))))
    R = AES.new(key, AES.MODE_ECB)

    # send first challenge
    challengeA = randbytes(16)
    self.send_json(challenge_a=b64e(R.encrypt(challengeA)))

    # receive challenge response
    self.recv_json()
    challenge_response =
    ↪  R.decrypt(b64d(self.data["challenge_response"]))
    assert challenge_response[:16] == challengeA, "Challenge A failed."

    challengeB = challenge_response[16:]

    # response with challengeB
    self.send_json(challenge_b=b64e(R.encrypt(challengeB)))

    # receive success message
    self.recv_json()
    assert self.data["success"], self.data.get("message",
     ↪  "ChallengeB failed.")

    # store the shared key
    self.R = R
```

Listing 2: Server Negotiate

```
def handle_eke_negotiate_key(self):
    # decrypt Ea using P
    P = AES.new(self.database[self.data["username"]].ljust(16).encode(⌋
     ↪  ),
     ↪  AES.MODE_ECB)
    e = b2l(P.decrypt(b64d(self.data["enc_pub_key"])))

    # e is always odd, but we add 1 with 50% probability
    if e % 2 == 0:
        e -= 1

    # generate secret key R
    R = randbytes(16)
    Ea = RSA.from_pub_key(e, self.data["modulus"])
    self.send_json(enc_secret_key=b64e(P.encrypt(l2b(Ea.encrypt(b2l(R)⌋
     ↪  )))))
    x = b64e(P.encrypt(l2b(Ea.encrypt(b2l(R)))))

    # transform R into a cipher instance
    R = AES.new(R, AES.MODE_ECB)

    # receive encrypted challengeA and generate challengeB
    self.recv_json()
    challengeA = R.decrypt(b64d(self.data["challenge_a"]))
    challengeB = randbytes(16)

    # send challengeA + challengeB
    self.send_json(challenge_response=b64e(R.encrypt(challengeA+challe⌋
     ↪  ngeB)))
    # receive challengeB back again
    self.recv_json()
    success = R.decrypt(b64d(self.data["challenge_b"])) == challengeB

    self.send_json(success=success)
    self.R = R
```

Appendix B

Python implementation of SRP

While conducting my initial research on PAKEs I came across SRP.[?] SRP is the first protocol I looked at which took the approach of encoding values as DH group elements. To understand this approach better I chose to create a toy implementation. The full code can be found at `https://github.com/tritoke/srp_python`. The core negotiation functions for the client and server have been included below:

Listing 3: Client Negotiate

```python
def negotiate(self):
    # send a negotiate command
    self.send_json(action="negotiate", username=self.username)

    # receive the salt back from the server
    self.recv_json()
    s = int(self.data["salt"])
    x = H(s, H(f"{self.username}:{self.password}"))

    # generate an ephemeral key pair and send the public key to the
    ↪  server
    a = strong_rand(KEYSIZE_BITS)
    A = pow(g, a, N)
    self.send_json(user_public_ephemeral_key=A)

    # receive the servers public ephemeral key back
    self.recv_json()
    B = self.data["server_public_ephemeral_key"]

    # calculate u and S
    u = H(A, B)
    S = pow((B - 3 * pow(g, x, N)), a + u * x, N)

    # calculate M1
    M1 = H(A, B, S)
    self.send_json(verification_message=M1)

    # receive M2
    self.recv_json()
    M2 = self.data["verification_message"]

    if M2 != H(A, M1, S):
        print("Failed to agree on shared key.")

    K = H(S)

    return K
```

Listing 4: Server Negotiate

```
def handle_srp_negotiate_key(self):
    # receive the username I from the client
    # lookup data in database
    user = self.data["username"]
    I = b2l(user.encode())

    if (db_record := self.database.get(user)) is None:
        self.send_json(success=False,
         ↪  message=f"Failed to find user in DB.")
        return

    s = db_record["salt"]
    v = db_record["verifier"]

    # send s to the client
    self.send_json(salt=s)

    # receive A from the user
    self.recv_json()
    A = self.data["user_public_ephemeral_key"]

    # calculate B
    b = strong_rand(KEYSIZE_BITS)
    B = 3 * v + pow(g, b, N)

    # send B to the client
    self.send_json(server_public_ephemeral_key=B)

    # calculate u and S
    u = H(A, B)
    S = pow(A * pow(v, u, N), b, N)

    # receive M1 from the client
    self.recv_json()
    M1 = self.data["verification_message"]
```

```
# verify M1
if M1 != H(A, B, S):
    self.send_json(success=False,
     ↪  message=f"Failed to agree shared key.")
    return

# calculate M2
M2 = H(A, M1, S)
self.send_json(verification_message=M2)

# calculate key
K = H(S)

# log the derived key - not part of the protocol
print(f"Derived K={K:X}")

# encrypt our final message to the client using our shared key
key = l2b(K)
nonce = get_random_bytes(16)

cipher = AES.new(key, AES.MODE_GCM, nonce=nonce)
ct, mac = cipher.encrypt_and_digest(f⌋
 ↪  "Successfully agreed shared key for {user}.".encode())

# notify the client of the success
self.send_json(success=True, nonce=b64e(nonce),
 ↪  enc_message=b64e(ct), tag=b64e(mac))
```