Summary: Universal Statistical Simulator

1 Introduction

The Quantum Fourier Transform (QFT) is a well-known example of a quantum algorithm that shows exponential speed-up over classical computation. However, such speed-up is often explained with assumptions about classical computational complexity. This paper introduces a *Quantum Galton Board* (QGB) — an intuitive quantum circuit model that mimics the physical Galton Board, using only three types of quantum gates. The QGB can calculate 2^n trajectories with $O(n^2)$ resources and low circuit depth. By removing pegs and altering left/right probabilities, the QGB can be extended to a *Universal Statistical Simulator*.

2 Background

2.1 Classical Galton Board

A classical n-level Galton Board produces a binomial distribution:

$$P(k) = \binom{n}{k} p^k q^{n-k}, \quad p = q = 0.5$$

which approximates a normal distribution via the De Moivre–Laplace theorem.

2.2 Quantum Galton Board Concept

The QGB models each *peg* as a quantum subcircuit:

- All qubits start in $|0\rangle$.
- A control qubit is placed in superposition (Hadamard).
- A "ball" qubit is flipped with an X-gate.
- Controlled-SWAP operations simulate left/right paths.

This *quantum peg* module is repeated for successive board levels.

3 Complexity and Features

- n output bits require n ancilla qubits.
- Maximum gate count: $2n^2 + 5n + 2$.
- Circuit depth is less than half of some prior methods, reducing error rates.
- Post-processing is required to match the classical Galton Board output format.

4 Experimental Results

4.1 Remote Simulation

A 4-level QGB was run on IBM-QX simulator, producing the expected normal distribution on 5 output qubits.

4.2 Hardware Runs

The quantum peg module was tested on an actual IBM-QX device. While the target states had the largest peaks, noise was significant due to the decomposition of *controlled-SWAP* gates into many basic gates.

4.3 Local Simulation and Rescaling

Local simulation allowed output scaling to a larger range while preserving the normal distribution shape.

5 Biased Quantum Galton Board (B-QGB)

5.1 Biased Peg Design

Replacing the Hadamard with a rotation $R_x(\theta)$ controls the probability of each path. For example, $\theta = 2\pi/3$ yields a 75% vs. 25% output probability split.

5.2 B-QGB Gate Count

A biased peg adds minimal overhead: RESET, $R_x(\theta)$, two controlled-SWAPs, and up to two CNOTs. Overall complexity: $3(n^2 + n) + n + 2$.

6 Fine-Grained Bias Control

Bias can be adjusted per peg by assigning different θ values, with corrective CNOT and RESET gates to maintain consistency. Complexity: $\approx 3n^2 + 3n + 1$.

7 Conclusions

The QGB approach offers:

- An intuitive quantum simulation of statistical processes.
- Low circuit depth and flexible bias control.
- Applicability to generating various statistical distributions.

Current NISQ hardware noise limits practical large-scale implementation, especially due to non-native controlled-SWAP gates. Nevertheless, simulations confirm the method's potential for high-quality quantum-based random number generation and statistical modeling.