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* P&S 2024 solution

Q1 (A):- Define and give example of Random variable (3m)

=> Random variable is mathematical function that associates numerical value with each outcome of random experiment.
=> Value of random variable varies with each trial of experiment.
Ex:- Tossing coin
=> Types:- i) Discrete ii) continuous

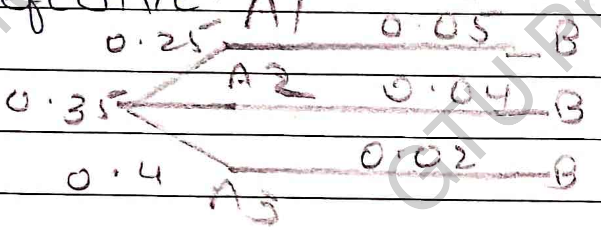
Q1(B) What is Probability that leap year selected at random will have 53 sunday? (4m)

=> leap year has 366 days or 52 week & 2 days
=> Two days can be = {(mon, tue), (Tue, wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, mon)}.
=> So there are 7 possibility of which 2 have sunday.
=> So Prob. of 53 sunday = $\boxed{P(A) = \frac{2}{7}}$.

Q1(C) :- In bolt Factory 3 machine A, B, C produce 25%, 35%, 40% of total output It was Found that 5%, 4% & 2% are defective bolt in production by machine A, B, C. A Bolt is choosen at random from total output & is found to be defective. Find probability that it is manufactured from. i) machine A, ii) machine B, iii) machine C ? (7m)

=> Let A_1, A_2, A_3 be events that bolt manufacture from machine A, B, C

=> Let B be event that bolt drawn is defective.



=> $P(A_1) = 25/100 = 0.25$

$P(A_2) = 35/100 = 0.35$

$P(A_3) = 40/100 = 0.4$

=> Prob. that bolt drawn is defective given that is manufacture from

- Machine A = $5/100 = 0.05$
- Machine B = $4/100 = 0.04$
- Machine C = $2/100 = 0.02$

=> using bias theorem :-

$$P(A_1/B) = \frac{P(A_1)\,P(B/A)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$\text{machine A} = \frac{0.25 \times 0.05}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 * 0.02)}$$

$$= \boxed{0.3623}$$

(ii) machine B = $\dfrac{P(A_2)P(B/A_2)}{P(A1)P(B/A1) + P(A2)P(B/A2) + P(A3)P(B/A3)}$

$$P(A_2/B) = \frac{0.35 \times 0.04}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)}$$

$$= \boxed{\cancel{0.2319}} \quad \boxed{0.4058}$$

(iii) machine C :- $P(A3/B) = \dfrac{P(A3)P(B/A3)}{P(A1)P(B/A1) + P(A2)P(B/A2) + P(A3)P(B/A3)}$

$$= \frac{0.4 \times 0.02}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)}$$

$$= \boxed{0.2319}$$

Q2 (A) Bag contain 3 Red & 4 white balls. Two draws are made without Replacement. What is probability that both ball are red (3m)

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⇒ Let A be event that ball drawn is red in first draw = $P(A) = \frac{3}{7}$

⇒ Let B be event that ball drawn is red in second draw given that 1st ball drawn is red: $P(B) = \frac{2}{6}$

⇒ Prob. that both ball red = $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{3}{7} \times \cancel{\frac{2}{6}}\ \frac{2}{6}$$

$$= \boxed{\frac{1}{7}}$$

Q2(B) Prob. that student A solve maths problem is $\frac{2}{5}$ & student B solve is $\frac{2}{3}$ what is prob. that :- i) Problem not solve
ii) both ~~A~~ A & B solve problem, working independently of each other (4m)

⇒ Let A & B be events that student A & B solve problem : $P(A) = \frac{2}{5}$ $P(B) = \frac{2}{3}$

⇒ ~~(i)~~ Prob. A does not solve problem :

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{2}{5}$$

$$= \boxed{\frac{3}{5}}$$

⇒ Prob. that B does not solve problem :

$$P(\bar{B}) = 1 - P(B)$$
$$= 1 - \frac{2}{3}$$
$$= \boxed{\frac{1}{3}}$$

(i) Prob. that Problem not solved:

$$P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$$
$$= \frac{3}{5} \times \frac{1}{3}$$
$$= \boxed{\frac{1}{5}}$$

(ii) Prob. that both A & B solve problem:

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{2}{5} \times \frac{2}{3}$$
$$= \boxed{\frac{4}{15}}$$

Q2(C) Verify following function $F(x)$ is ~~density~~ distribution function:

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/4} & x \geq 0 \end{cases}$$

Also find prob $P(X \leq 4)$, $P(X \leq 8)$, $P(X \leq 4 \leq 8)$ (7m).

=> for function $F(x)$,

(i) $F(-\infty) = 0$

(ii) $F(\infty) = 1 - e^{-\infty} = 1 - 0 = 1$

(iii) $0 \leq F(x) \leq 1 \qquad -\infty < x < \infty$

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$\Rightarrow$ If $F(x)$ is corresponding prob. density function : $F(x) = F'(x) = 0 \quad x < 0$

$$= \frac{1}{4} e^{-x/4} \quad x \geq 0$$ given

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} \frac{1}{4} e^{-x/4}\,dx$$

$$= \frac{1}{4} \left| \frac{e^{-x/4}}{-1/4} \right|_{0}^{\infty} \cancel{dx}$$

$$= \left| e^{-x/4} \right|_{0}^{\infty}$$

$$= -(0-1)$$

$$= \boxed{1}$$

Hence $f(x)$ is ~~den~~ distribution function

(i) $P(X \leq 4) = F(4) = 1 - e^{-1}$

$$= 1 - 1/e$$

$$= \cancel{e-1} \qquad = \boxed{\frac{e-1}{e}}$$

(iii) $P(X \geq 8) = 1 - P(X \leq 8)$

$$= 1 - F(8)$$

$$= 1 - (1 - e^{-2})$$

$$= e^{-2}$$

$$= \boxed{\frac{1}{e^2}}$$

(iii) $P(4 \leq x \leq 8) = F(8) - F(4)$

$$= (1-e^{2}) - (1-e^{-1})$$

$$= e^{-1} - e^{-2}$$

$$= \boxed{e-1/e^2}$$

OR

Q2(c) The prob. mass Function of random variable X is 0 except at points X = 0, 1, 2. At these point it has values $P(X=0): 3c^3$, $P(X=1): 4c-10c^2$, $P(X=2): 5c-1$ find:-

(i) c?

$\Rightarrow$ Since $P(X=x)$ is prob. mass Function:

$\therefore \Sigma P(X=x)=1$

$\therefore P(X=0)+P(X=1)+P(X=2)=1$

$\therefore 3c^3+4c-10c^2+5c-1=1$

$\therefore (3c-1)(c-2)(c-1)=0$

$\therefore 3c^3-10c^2+9c-2=0$

$\therefore (3c-1)(c-2)(c-1)=0$

$c=\frac{1}{3}, 2, 1$ But $c<1$, Otherwise given prob will greater than 1 or less than zero so $\boxed{c=\frac{1}{3}}$

(ii) $P(X<1) = P(X=0)$

$P(X<1 = \frac{1}{9}$

(iii) $P(1<X\leq 2) = P(X=2)$

$P(1<X\leq 2) = \frac{2}{9}$

$\rightarrow P(0\leq X\leq 2)$

$= P(X=1)+P(X=2)$

$= \frac{2}{9}+\frac{2}{3}$

$= \boxed{\frac{8}{9}}$

Prob. distribution:

X	0	1	2
$P(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{3}$

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Q3 (A) find constant K such that function

$$f(x) = \begin{cases} Kx^2; & 0 < X < 3 \\ 0; & \text{Otherwise} \end{cases}$$

is prob. density function (3m)

$\Rightarrow$ since $f(x)$ is prob. density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$= \int_{\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$$

$$= 0 + \int_{0}^{3} f(x)\,dx + 0 = 0$$

$$= k \left| \frac{x^3}{3} \right|_{0}^{3} = 1$$

$$= k/3\,(27-0) = 1$$

$$= \boxed{k = \frac{1}{9}}$$

Q3 (B) Random variable X has following distribut

X	1	2	3	4	5	6
P(X=x)	1/36	3/36	5/36	7/36	9/36	11/36

(4m)

find

(i) $\text{mean} = \Sigma x\, P(x)$

$$= 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(11/36\right)$$

$$= \boxed{\frac{161}{36}} = \boxed{4.47}$$

(ii) Variance $= \sigma^2 = \Sigma x^2 p(x) - \mu^2$

$= 1\left(\frac{1}{36}\right) + 4\left(\frac{3}{36}\right) + 9\left(\frac{5}{36}\right) + 16\left(\frac{7}{36}\right) + 25\left(\frac{9}{36}\right)$

$+ 36\left(\frac{11}{36}\right) - (4.47)^2$

$= \frac{791}{36} - 19.98$

$= \boxed{1.99}$

Q3(c) Compute Karl Pearsons coefficient of correlation between x & y for following data (7m)

x	10	14	18	22	26	30
y	18	12	24	6	30	36

$\Rightarrow$ let $a = 22$ $b = 24$ assumed mean of x & y

$\Rightarrow$ Let $h = 4$, $k = 6$, $n = 6$

$dx = \frac{x-a}{h} = \frac{x-22}{4}$

$dy = \frac{y-b}{h} = \frac{y-24}{4}$

x	y	dx	dy	dx^2	dy^2	dx dy
10	18	-3	-1	9	1	3
14	12	-2	-2	4	4	4
18	24	-1	0	1	0	0
22	6	0	-3	0	9	0
26	30	1	1	1	1	1
30	36	2	2	4	4	4

$$r = \frac{\Sigma dx dy - \Sigma dx\, dy / n}{\sqrt{\Sigma dx^2 - \frac{(\Sigma dx)^2}{n}} \cdot \sqrt{\Sigma dy^2 - \frac{(\Sigma dy)^2}{n}}}$$

$$= \frac{12 - \frac{(-3)(-3)}{6}}{\sqrt{19 - \frac{(-3)^2}{6}} \cdot \sqrt{19 - \frac{(-3)^2}{6}}}$$

$$= \boxed{0.6}$$

OR

Q3(A) Find coefficient of correlation between x & y if regression line are: $x + 6y = 6$ & $3x + 2y = 10$ (3m)

=> Regression line pass through line $(\bar{x}, \bar{y})$

$$\bar{x} + 6\bar{y} = 6 \quad --(1)$$

$$3\bar{x} + 2\bar{y} = 10 \quad --(2)$$

Solving eqs (1) & (2)

$$\bar{x} = 3, \quad \bar{y} = \frac{1}{2}$$

(i) Let line $x + 6y = 6$ be line of regression of y on x :-

$$6y = -x + 6$$

$$y = -\frac{1}{6}x + 1$$

~~$b_{yx} =$~~

$$\boxed{b_{yx} = -\frac{1}{6}}$$

=> Let line $3x + 2y = 10$ be line of regression of x on y

$$3x = -2y + 10$$

$$x = -\frac{2}{3}y + \frac{10}{3}$$

$$b_{xy} = -\frac{2}{3}$$

$$\Rightarrow \quad r = \sqrt{b_{yx}\, b_{xy}}$$

$$= \sqrt{\left(-\frac{1}{6}\right) \times \left(-\frac{2}{3}\right)}$$

$$= \frac{1}{3}$$

since b_{xy} & b_{yx} are $-ve$, r is $-ve$

so $\boxed{r = -\frac{1}{3}}$

Q3(B) Calculate 1st four moments from following data (4m)

x	0	1	2	3	4	5	6	7	8
y	5	10	15	20	25	20	15	10	5

$\Rightarrow$ $N = \Sigma f = 125$

$$\mu = \frac{\Sigma f(x)}{N} = \frac{500}{125} = \boxed{4}$$

x	f	fx	$x-\mu$	$f(x-\mu)$	$f(x-\mu)^2$	$f(x-\mu)^3$	$f(x-\mu)^4$
0	5	0	-4	-20	80	-320	1280
1	10	10	-3	-30	90	-270	810
2	15	30	-2	-30	60	-120	240
3	20	60	-1	-20	20	-20	20
4	25	100	0	0	0	0	0
5	20	90	1	20	20	20	20

6	15	90	2	30	60	120	240
7	10	70	3	30	90	270	810
8	5	40	4	20	80	320	1280
	125	500		0	500	0	4700

$\Rightarrow$ moments about actual mean:-

$$\mu_1 = \frac{\Sigma f(x-\mu)}{N} = \frac{0}{125} = \boxed{0}$$

$$\mu_2 = \frac{\Sigma f(x-\mu)^2}{N} = \frac{500}{125} = \boxed{4}$$

$$\mu_3 = \frac{\Sigma f(x-\mu)^3}{N} = \frac{0}{125} = \boxed{0}$$

$$\mu_4 = \frac{\Sigma f(x-\mu)^4}{N} = \frac{4700}{125} = \boxed{37.6}$$

Q3(c) The following data gives experience of machine operator & their rating as given by number of good parts turned out per 100 piece. Calculate regression line of performance rating on experience also estimate probable performance if operator has 11 years experience (7m)

Operator	1	2	3	4	5	6
Performance rating (x)	23	43	53	63	73	83
Experience (Y)	5	6	7	8	9	10

$\Rightarrow$ $n = 6$

x	y	y^2	xy
23	5	25	115
43	6	36	258
53	7	49	371

63	8	64	504
73	9	81	657
83	10	100	830
338	45	335	2735

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

$$= \frac{2735 - \frac{338 \times 45}{6}}{335 - \frac{(45)^2}{6}}$$

$$= \boxed{11.429}$$

$\Rightarrow \bar{x} = \Sigma x / n = 338/6 = \boxed{56.33}$

$\Rightarrow \bar{y} = \Sigma y / n = 45/6 = \boxed{7.5}$

$\Rightarrow$ Equation of regression line of x on y :-

$= x - \bar{x} = b_{xy}(y - \bar{y})$

$= x - 56.33 = 11.429(29.3875)$

$= x = 11.429y - 29.3875$

$\Rightarrow$ Estimated performance if $y = 11$ is :-

$\boxed{x = 96.3315}$

Q4(A) Define terms of hypothesis:

(i) NULL hypothesis:- It is statement in hypothesis testing that assume there is no effect or no difference between group or variable. It serve as default or straight line assumption, which reasemble aim to test again alternative hypothesis

(ii) Alternate hypothesis:- is statement that directly contradict null hypothesis. It represent hypothesis that researcher aim to support, suggesting that there is significant effect or difference between group or variable

(iii) level of significance:- is threashold set by reasearcher to determine whether null hypothesis should be rejected. It represent probability of rejecting null hypothesis when it is actually true.

Q4(B) A dice tossed 960 times & it falls with 5 upward 184 times. Is dice bias Unbiased at level of significance 0.01?
$|Z_{0.01}| = 2.58$ (4m)

$\Rightarrow$ $n = 960$

P = prob. of throwing 5 with one dice = $\boxed{1/6}$

$q = 1 - P = 1 - \frac{1}{6} = \boxed{\frac{5}{6}}$

$\mu = nPq = \frac{1}{6} \times \frac{5}{6}$

$\rightarrow$ $\mu = np = 960 \times (1/6) = \boxed{160}$

$\rightarrow$ $\sigma = \sqrt{npq} = \sqrt{960 \times \frac{1}{6} \times \frac{5}{6}} = \boxed{11.55}$

$\rightarrow$ x = no. of success = 184

(i) Null hypothesis H_0 : Dice unbaised
(ii) Alternative hypothesis H_1 : Dice biase

(iii) level of significance $\alpha = 0.01$
(iv) test statistics =

$$Z = \frac{x-\mu}{\sigma} = \frac{184-160}{11.55} = \boxed{2.08}$$

(v) critical value = $|Z_{0.01}| = 2.58$

(vi) Decision :- Since $|Z| < |Z_{0.01}|$, null hypothesis is accepted at 1% level of significance ie: dice is unbiased

Q4(c) A set of 5 similar coin tossed 320 times & result obtain as follow:

No. of head	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test hypothesis that data follow binomial distribution value (critical value $\chi^2_{0.05} = 11.07$)
(7m)

$\Rightarrow$ null hypothesis H_0 : Data follow binomical distribution.

(ii) Alternative hypothesis: H_2: Data do not follow binomial distribution.

(iii) Level of significance: $\alpha = 0.05$

(iv) Statistics = χ^2

Prob. of getting head = $p = 1/2$

Prob. of getting tail = $q = 1/2$

→ By binomial distribution:

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x} \quad ; x = 0,1,2,3,4,5$$

$N = 320$

$$P(x) = {}^{5}C_{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{5-x}$$

→ Expected ~~or~~ Frequency fe,

$$NP(x) = 320\left[{}^{5}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{5-x}\right]$$

→ Expected Frequency,

X	0	1	2	3	4	5
fe	10	50	100	100	50	10

No. of head	Observed Frequency fo	Expected Frequency fe	$\frac{(fo-fe)^2}{fe}$
0	6	10	1.6
1	27	50	10.58
2	72	100	7.84
3	112	100	1.44
4	71	50	8.82
5	32	10	48.4
			$\chi^2 = \Sigma\frac{(fo-fe)^2}{fe} = 78.68$

(v) critical value = $V = n-1 = 6-1 = 5$

$$\chi^2_{0.05} = 11.07$$

(VII) Decision :- Since $X^2 > X^2_{0.05}$ at 5% level of significance, null hypothesis rejected

OR

Q4(A) The height of 10 male of given locality are found to be 175, 168, 155, 170, 175, 160, 160 & 165 cm. Based on this sample, find 95% confidence limit for height of male in that locality ($t_{0.05}$ ($v=9$) $= 2.262$)

=> $n = 10$
$\bar{x} = 165$ $S = 7.6$ } From calculator
$v = n-1 = 10-1 = 9$

=> From t table: $t_{0.05}$ ($v=9$) $= 2.262$ (two tail)
The 95% confidence limits for μ are:

$$\left[\bar{x} - t_{0.05}\left(\frac{S}{\sqrt{n-1}}\right), \bar{x} + t_{0.05}\left(\frac{S}{\sqrt{n-1}}\right)\right]$$

$$\left[165 - \frac{2.262(7.6)}{\sqrt{10-1}}, 165 + \frac{2.26(7.6)}{\sqrt{10-1}}\right]$$

=> ie: height of male in locality are likely to be in limit 159.27 cm & 170.73 cm

Q4(B) Random sample drawn from two countries gave foll data ~~ret~~ relating to height of adult males:

SD (in inches)	Country A	Country B
No. in sample	2.58	2.50
	1000	1200

Is difference between S.D significant? $|Z_{0.05}| = 1.96$

Ans :- $n_1 = 1000$ $n_2 = 1200$ $S1 = 2.58$ $S2 = 2.50$

(i) Null hypothesis H_0 : $\sigma 1 = \sigma 2$ i.e. their is no significant difference betn 2 S.D

(ii) Alternative hypothesis H_1 : $\sigma 1 \neq \sigma 2$ (two tailed)

(iii) Level of significance : $\alpha = 0.05$

(iv) Statistics : ~~$\frac{x-\mu}{\sigma}$~~ ~~$\sqrt{S1}$~~

$$Z = \frac{S1 - S2}{\sqrt{\frac{S1^2}{2n_1} + \frac{S2^2}{2n_2}}}$$

$$= \frac{2.58 - 2.50}{\sqrt{\frac{(2.58)^2}{2 \times 1000} + \frac{(2.50)^2}{2 \times 1200}}}$$

$$= \boxed{0.977}$$

(v) critical value = $|Z_{0.05}| = 1.96$

(vi) Decision $|Z| < |Z_{0.05}|$ so hypothesis accepted at 5% level of significance

Q4(c) If Random Variable has poison dist. such that $P(X=1) = P(X=2)$, find
(i) mean (ii) $P(X=4)$ (iii) $P(X \geq 1)$ (iv) $P(1 < X < 4)$ (7m)

$\Rightarrow$ for Poison Distribution:

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} ; \quad x = 0, 1, 2 \ldots$$

$$= P(X=1) = P(X=2)$$

$$\Rightarrow \frac{e^{-\lambda}\lambda^{1}}{1!} = \frac{e^{-\lambda}\lambda^{2}}{2!}$$

$$= \lambda^2 = 2\lambda$$

$$= \lambda^2 - 2\lambda = 0$$

$$= \lambda = 0, \lambda = 2 \qquad \lambda \neq 0$$

$$\boxed{\lambda = \text{mean} = 2}$$

Hence $P(X=x) = \frac{e^{-2}2^{x}}{x!} ; \quad x = 0, 1, 2 \ldots$

(ii) $P(X=4)$

$$= \frac{e^{-2}2^{4}}{4!}$$

$$= \boxed{0.9022}$$

(iii) $P(X \geq 1)$

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{e^{-2}2^{0}}{0!}$$

$$= \boxed{0.8647}$$

(iv) $P(1 < X < 4)$

$$= P(X=3) + P(X=4)$$

$$= \sum_{x=2}^{3} P(X=x)$$

$$= \sum_{x=2}^{3} \frac{e^{-2}2^{x}}{x!}$$

$$= \boxed{0.4511}$$

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Q5(A) Fit straight line $y = ax + b$ to foll data (3m):

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

=> Let straight line to be fitted to data be:
$y = ax + b$

=> Normal equations are:

$\Sigma y = na\, b\, \Sigma x$ -- ①

$\Sigma xy = a\Sigma x + b\Sigma x^2$ -- ②

Hence $\boxed{n = 6}$

x	y	x^2	xy
1	2.4	1	2.4
2	3	4	6
3	3.6	9	10.8
4	4	16	16
6	5	36	30
8	6	64	48
24	24	130	113.2

Substitute this value in -① & -②

$24 = 6a + 2bb$ -- ③

$113.2 = 24a + 130b$ -- ④

Solving Eq (3) & Eq (4)

$a = 1.9764$; $b = 0.5059$

Hence required equation of straight line is:

$\boxed{y = 1.9764 + 0.5059x}$

Q5(B) fit curve $y = ab^x$ to following data:

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

$\Rightarrow$ $y = ab^x$, taking log on both side

$= \log_e y = \log_e a + \log_e b$

= Putting $\log_e y = Y$, $\log_e a = A$, $x = X$ & $\log_e B = B$

$= \boxed{Y = A + BX}$

$\rightarrow$ normal Eq are:

$\Sigma Y = nA + B\Sigma X$ — — ①

$\Sigma xy = a\Sigma x + b\Sigma x^2$ — — ②

$\boxed{n = 8}$

x	y	X	Y	X^2	XY
1	1	1	0.000	1	0.0000
2	1.2	2	0.1823	4	0.3646
3	1.8	3	0.5878	9	1.7634
4	2.5	4	0.9163	16	3.6652
5	3.6	5	1.2809	25	6.4045
6	4.7	6	1.5476	36	9.2856
7	6.6	7	1.8871	49	13.2097
8	9.1	8	2.2083	64	17.6664
36	8.6103			204	52.3594

Substitute these value in eq -① & -②

$8.6103 = 8A + 36B$ — ③

$52.3594 = 36A + 204B$ — ④

solving (3) & (4)

$A = -0.3823 \qquad B = 0.3241$

$\log_e a = A \qquad \log_e b = B$

$\log_e a = -0.38 \qquad \log_e b = 0.3241$

$a = 0.6823 \qquad b = 1.3828$

=) Hence required law is

$y = 0.6823 + (1.3828)^x$

Q5(c) The lifetime of certain kind of battery has mean life of 400 hour & SD = 45 years. Assume distribution of lifetime to be normal, find (i) percentage of battery life time atleast = 470 hours

(ii) Proportion of battery with lifetime between 385 & 415 hours

(iii) min life of best 5% of battery

$P(0 < z < 0.33) = 0.1293$ (7m)

=) Let x be random variable that denotes lifetime of certain kind of battery

$\mu = 400 \qquad \sigma = 45$

$z = x - \mu / \sigma$

(i) when $x = 470$

$z = \dfrac{470 - 400}{45} = \boxed{1.56}$

$P(x \geq 470) = P(z \geq 1.56)$

$= 0.5 - 0.4406$

$\boxed{P(x \geq 470 = 0.0594}$

Hence Percentage of battery with life time of atleast 470 hour = 5.94 %.

(ii) When $x = 385$

$$Z = \frac{385-400}{45} = \boxed{-0.33}$$

$X = 415$; $Z = 415-400/45 = \boxed{0.33}$

$\rightarrow P(385 < X < 485) = P(-0.33 < Z < 0.33)$

$= P(-0.33 < Z < 0) + P(0 < Z < 0.33)$

$= P(0 < Z < 0.33) + P(0 < Z < 0.33)$

$= 2P(0 < Z < 0.33)$

$= 2(0.1293)$

$= \boxed{0.2586}$

(iii) $P(X > X_1) = 0.05$

$P(X > X_1) = P(Z > Z_1)$

$\Rightarrow 0.5 = 0.5 - P(0 \le Z \le Z_1)$

$= P(0 \le Z \le Z_1) = 0.5 - 0.05$

$= P(0 \le Z \le Z_1) = 0.45$

$$= Z_1 = \frac{x_1 - 400}{45}$$

$= x_1 = 1.65(45 + 400)$

$= \boxed{x_1 = 474.25 \text{ hours}}$

OR

Q5 (A) mean & variance of binomial variates are 8 & 6 find $P(X \geq 2)$

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$\Rightarrow \mu = np = 8$, $\sigma^2 = npq = 6$

$\rightarrow \frac{npq}{np} = \frac{6}{8} = \frac{3}{4}$

$\rightarrow \boxed{q = 3/4}$

$\rightarrow p = 1 - q = 1 - 3/4 = \boxed{1/4}$

$\rightarrow np = 8$

$\rightarrow n \times 1/4 = 8$

$\Rightarrow \boxed{n = 32}$

$\Rightarrow P(X=x) = {}^nC_x p^x q^{n-x}$

$= {}^{32}C_x \left(\frac{1}{4}\right)^x q^{32-x}$; $x = 0, 1, 2 \cdots 32$

$\Rightarrow P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \sum_{x=0}^{1} P(X=x)$

$= 1 - \sum_{x=0}^{1} P\left(32C_x \left(\frac{1}{4}\right)^x \times \left(\frac{3}{4}\right)^{32-x}\right.$

$= \boxed{0.9988}$

Q5(B) Out of 800 family with 5 children each, how many would you except to have (i) 3 boys (ii) 5 girls

$\Rightarrow$ Let p be probability of having boy $= \boxed{1/2}$

$\Rightarrow q = 1 - p = 1 - 1/2 = \boxed{1/2}$

$\Rightarrow n = 5$

$\Rightarrow N = 800$

(i) Prob. of having 3 boys out of 5 children

$P(X=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$= \boxed{5/16}$

⇒ Expected no. of family having 3 boy out of 5 children

∴ $NP(x=3) = 800 \times 5/16 = \boxed{250}$

(ii) Prob. of having 5 girls out of 5 children

$P(X=3) = {}^5C_0 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$

$= \boxed{1/32}$

Expected no. of family having 5 girl out of 5 children.

$\Rightarrow NP(x=0)$

$\Rightarrow 800 \times \frac{1}{32} = \boxed{25}$

Q5(c) fit second degree parabolic curve for foll data (7m):

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

⇒ Let $X = x-10$

⇒ let $Y = y-10$

⇒ let eq of parabola be: $Y = a+bx+cx^2$

⇒ normal eq are: $\Sigma Y = na + b\Sigma x + c\Sigma x^2$ — (1)

$\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$ — (2)

$\Sigma X^2Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$ — (3)

x	y	X	Y	X^2	X^3	X^4	XY	X^2Y
1	2	-4	-8	16	-64	256	32	-128
2	6	-3	-4	9	-27	81	12	-36
3	7	-2	-3	4	-8	16	6	-12
4	8	-1	-2	1	-1	1	2	-2
5	10	0	0	0	0	0	0	0
6	11	1	1	1	1	1	1	1
7	11	2	1	4	8	16	2	4
8	10	3	0	9	27	81	0	0
9	9	4	-1	16	64	256	-4	-16
		0	-16	60	0	708	51	-189

substituting these values in eq ①, ②, ③

$$-16 = 9a + 60c \quad - ④$$

$$51 = 60b \quad - ⑤$$

$$-189 = 60a + 708c \quad - ⑥$$

Solving eq ④, ⑤, ⑥

$$a = 0.0043$$

$$b = 0.85$$

$$c = 0.2673$$

Hence req eq of parabola is

$\rightarrow \quad y = 0.0043 + 0.85x - 0.2673x^2$

$\rightarrow \quad y - 10 = 0.0043 + (0.85)(x-5) - 0.2673(x-5)$

$\rightarrow \quad y = 10 + 0.0043 + 0.85(x-5) - 0.2673(x^2 - 10x + 25$

$\rightarrow \quad y = 10 + 0.0043 + 0.85x - 4.25 - 0.2673x^2 + \cancel{2.76}\ 2.673x - 6.6825$

$\rightarrow \quad \boxed{y = -0.9282 + 3.523x - 0.2673x^2}$