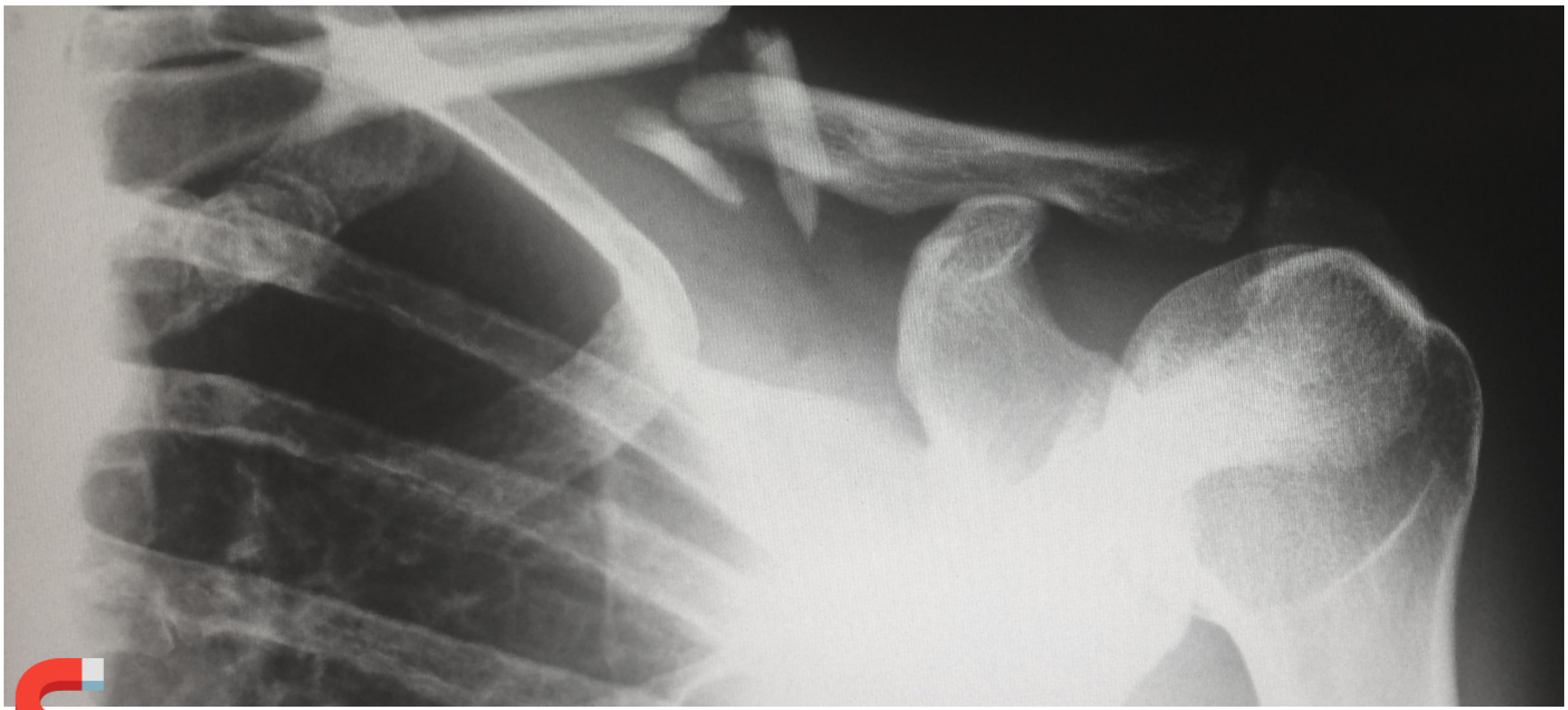

Module 6: Particles and Medical Physics

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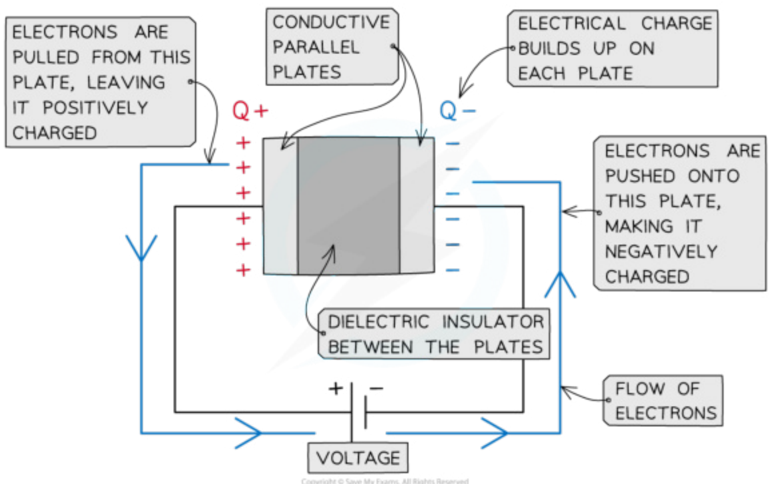
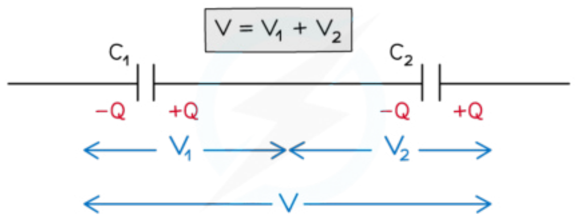
Subject	Physics A2

6.1 Capacitors



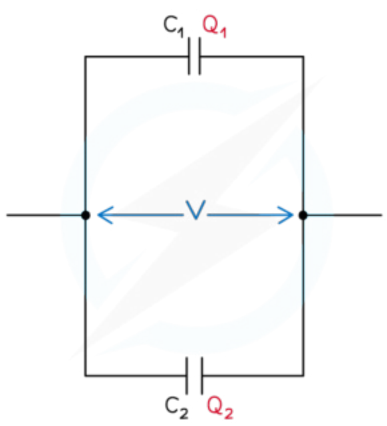
- Capacitors store electrical charge with two conducting plates separated by a dielectric
  Capacitance $C = \frac{Q}{V}$ is charge per unit pd in **farad (F)**
- In series, capacitors have the same charge across all and share the pd $\frac{1}{C} = \sum \frac{1}{C_i}$
  - ▼ Proof.

$$V = \sum V_i \implies \frac{V}{Q} = \sum \frac{V_i}{Q} \implies \frac{1}{C} = \sum \frac{1}{C_i}$$



- In parallel, capacitors have the same pd across branches $C = \sum C_i$
  - ▼ Proof.

$$Q = \sum Q_i \implies \frac{Q}{V} = \sum \frac{Q_i}{V} \implies C = \sum C_i$$

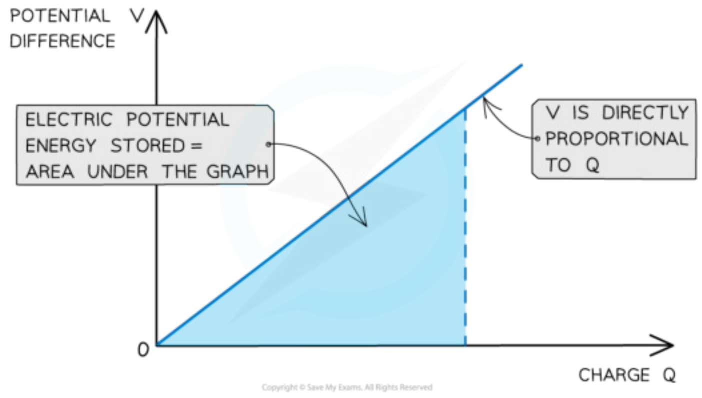


- Energy is area under V-Q graph $W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}V^2C$
  - ▼ Proof.

$$\delta W = V\delta Q \implies \int dW = \int \frac{Q}{C} dQ \implies W = \frac{1}{2}\frac{Q^2}{C}$$

- Discharging $X = X_0 e^{-\frac{t}{CR}}$ where $X \in \{I, V, Q\}$
  Time constant $\tau = CR$ is the time taken to decrease to e^{-1} of the original value

- Charging $I = I_0 e^{-\frac{t}{CR}}$ and $X = X_0\left(1 - e^{-\frac{t}{CR}}\right)$ where $X \in \{V, Q\}$
- Smoothing capacitors

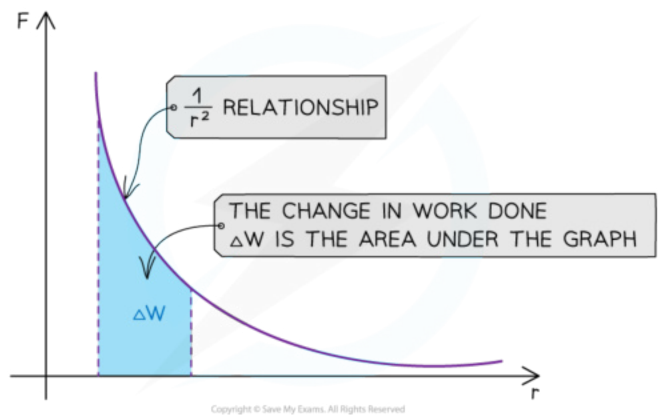


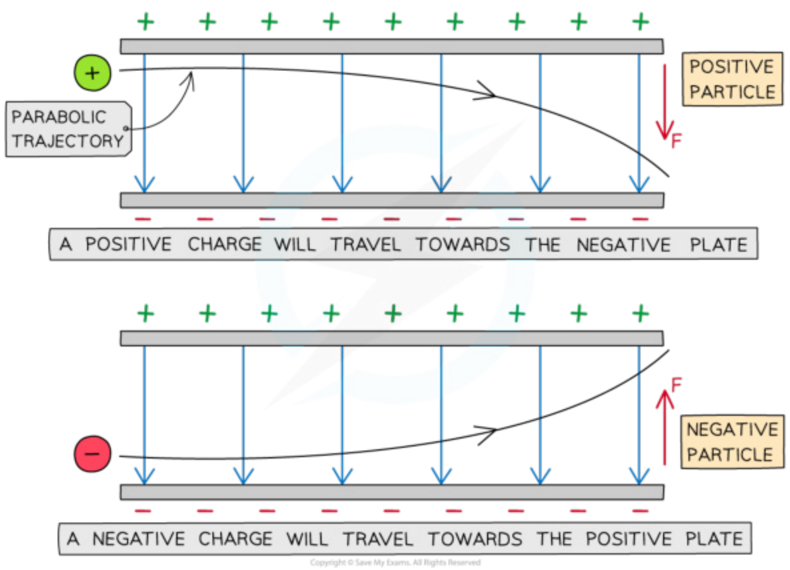
6.2 Electric Fields

- An electric field has infinite range, and affects anything with charge
  Field lines go from positive to negative

Vectors	electric field strength (E=F/Q)	electrostatic force (F=Qq/4πε0r^2)
Scalars	electric potential (V=Q/4πε0r)	electric potential energy (energy=Qq/4πε0r)

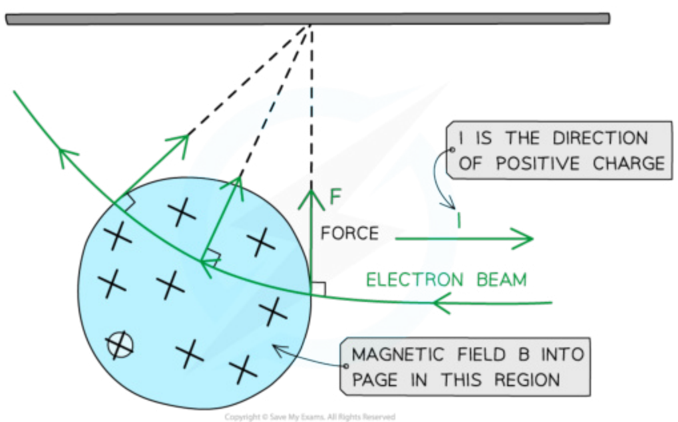
- **Electric field strength** E of an electric field at a point in space is the force experienced per unit POSITIVE charge
  In a **uniform field**, electric field strength is the same everywhere $E = \frac{V}{d}$
- Positive force indicates two charges are similar and repel
  Negative force indicates two charges are opposite and attract
- **Electric potential** V is the work done per unit POSITIVE charge to bring from infinity to a point
- Parabolic trajectory moving at right angles to an electric field with length L:
  - horizontal motion is unaffected $t = \frac{L}{v_H}$
  - vertical motion undergoes acceleration $a = \frac{F}{m} = \frac{EQ}{m}$
    hence $v_V = u + at = \frac{EQL}{mv_H}$
- Capacitance of a parallel plate capacitor $C = \varepsilon_0 \varepsilon_r A/d$ where ε_r is the relative permittivity of the dielectric
  Capacitance of an radius R isolated charge sphere $C = 4\pi\varepsilon_0 R$



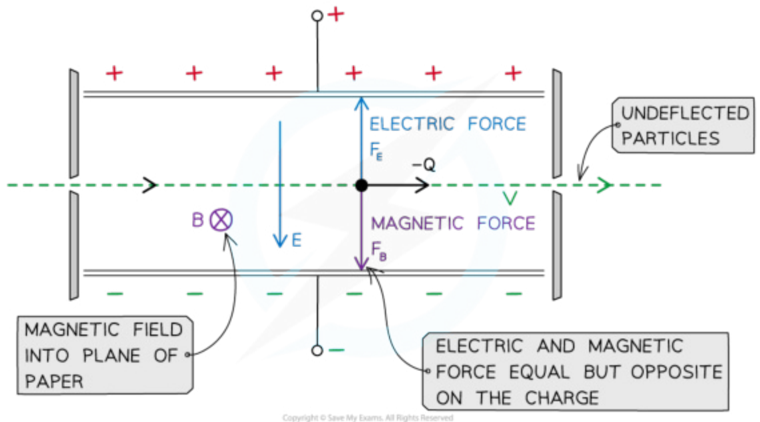


6.3 Electromagnetism

- **Fleming's left-hand rule** and **right-hand grip rule**, field lines go from north to south
- Force experienced by wire with length L with current I at angle θ between the field and the wire in a uniform magnetic field with **magnetic flux density** B, measured in **tesla (T)**, is $F = BIL\sin\theta$
- Assuming $I \perp B$, charge follows a circular path in a magnetic field $F = BQv$



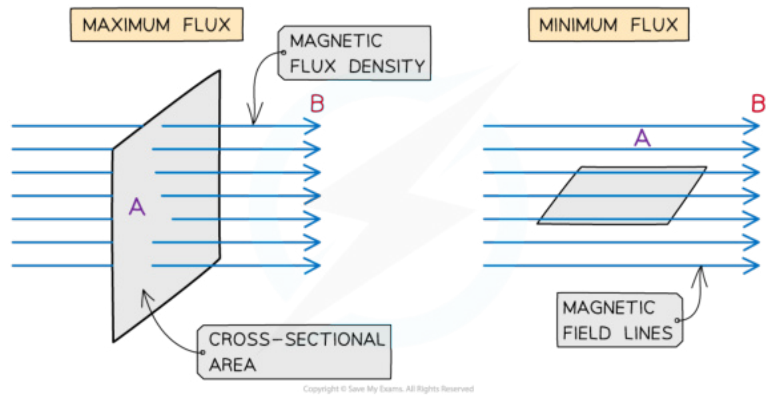
$$BQv = \frac{mv^2}{r} \implies r = \frac{mv}{BQ}$$



- **Velocity selector** works by selecting particles with the same magnetic force as electric force $EQ = BQv$
- **Magnetic flux** $\phi = BA\cos\theta$ where θ is the angle between the NORMAL to the coil and the field in **weber (Wb)**
- **Magnetic flux linkage** $N\phi$ is measured in **weber-turns**
- **Faraday's law** and **Lenz's law**:

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$$

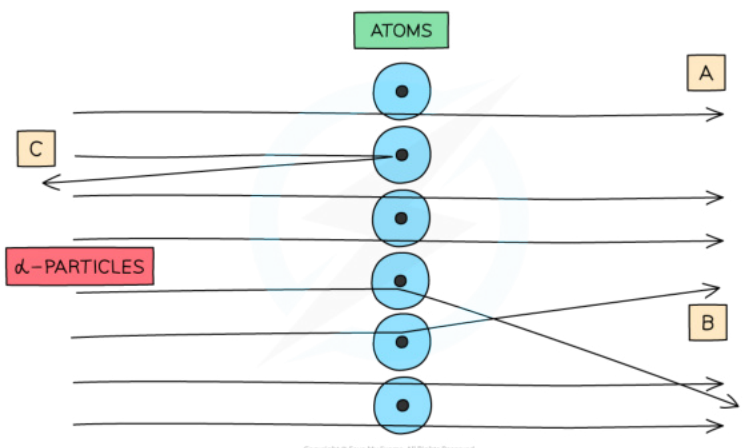
  - when there is a change in magnetic flux linkage, emf is induced
  - Lenz's law states that the direction of the induced emf or current opposes the change producing it: work must be done to induce an emf due to conservation of energy



- **AC generator** rotates a coil in a magnetic field which changes $N\phi$ hence induces an emf ε
- **Transformers** consist of a laminated iron core, input and output coils
  An alternating current in the primary coil produces varying magnetic flux in the iron core and is linked to the secondary coil $\frac{n_s}{n_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$

6.4 Nuclear and Particle Physics

- **Rutherford's scattering experiment** was performed by targeting a thin gold foil with a beam of alpha particles
  - most alpha particles passed straight through the gold foil with little scattering $\Rightarrow$ atoms are mostly empty
  - very few particles were scattered through large angles (>90°) $E_k = E_p$ $\Rightarrow$ positive nucleus radius upper bound

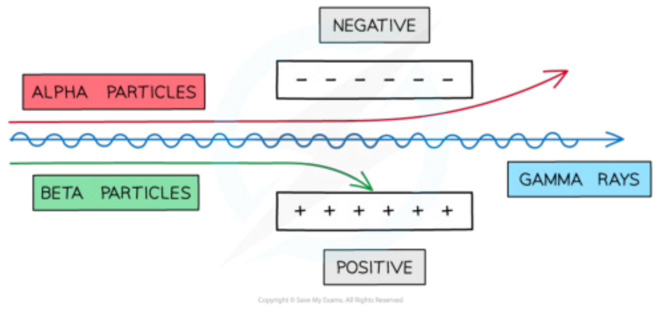


- A nucleus with **nucleon number** A has radius $R = r_0\sqrt[3]{A}$
- **Four fundamental forces**
  - **strong nuclear force** is experienced by nucleons with 10^{-15}m range
  - **electromagnetic force** is experienced by charged particles with ∞ range
  - **weak nuclear force** is responsible for beta-decay with 10^{-18}m range
  - **gravitational force** is experienced by all particles with mass with ∞ range

$^A_Z\mathrm{X}$

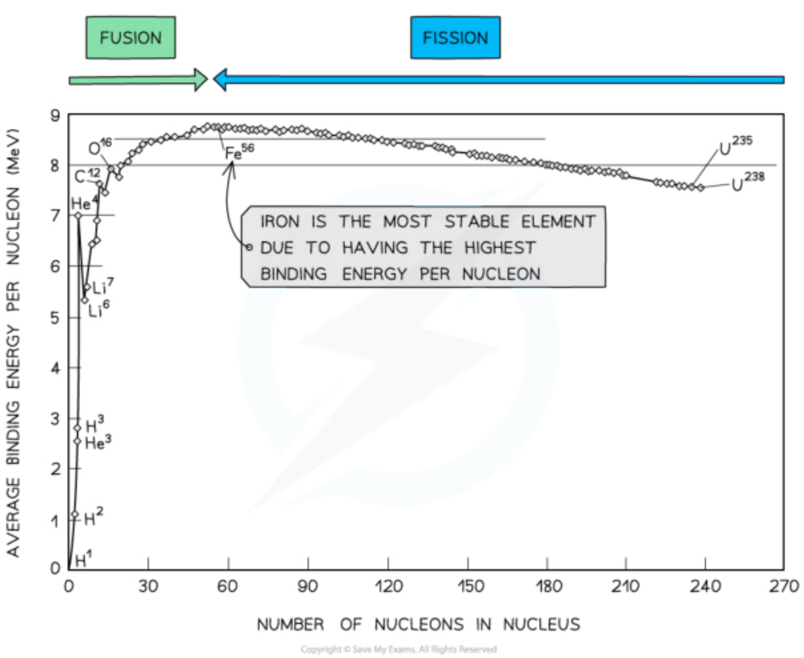
A nucleon
Z proton

- **Annihilation** happens when a particle and its antiparticle counterpart meet
- **Hadrons** are made out of **quarks**: **baryons** (3 quarks - $\mathrm{p}^+ = \mathrm{uud}$, $\mathrm{n} = \mathrm{udd}$, etc.) and **mesons** ($\mathrm{q\bar{q}}$)
  Leptons (e^-, ν_e, $\bar{\nu}_\mu$, etc.) are not affected by the strong nuclear force



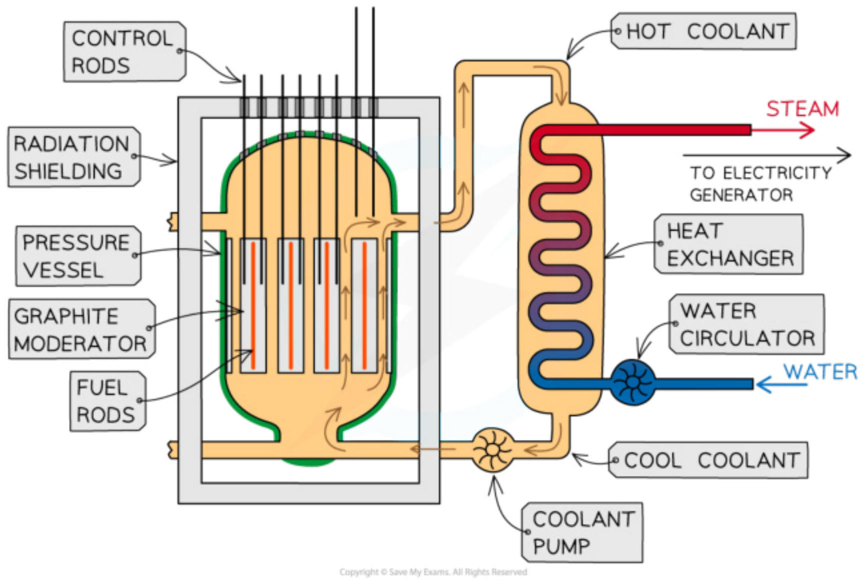
- **Radioactivity** is the random and spontaneous decay of unstable nuclei, all nuclear radiations are **ionising**
  - $^4_2\alpha$ **decay** emits a He **nuclei**, is the most ionising and has the shortest range (paper) happens in nuclei with many protons, decays into a different element $Z-2$ and $A-4$
  - β^- **decay** $^1_0\mathrm{n} \rightarrow {}^1_1\mathrm{p} + {}^{0}_{-1}\mathrm{e} + \overline{\nu_e}$, $\mathrm{d} \rightarrow \mathrm{u} + {}^{0}_{-1}\mathrm{e} + \overline{\nu_e}$ is less ionising and has longer range (few mm of Al)
    happens in "neutron-rich" nuclei, decays into a different element $Z+1$
  - β^+ **decay** $^1_1\mathrm{p} \rightarrow {}^1_0\mathrm{n} + {}^{0}_{+1}\mathrm{e} + \nu_e$, $\mathrm{u} \rightarrow \mathrm{d} + {}^{0}_{+1}\mathrm{e} + \nu_e$ and has no range (annihilated by electrons) happens in "neutron-rich" nuclei, decays into a different element $Z-1$
  - γ **decay** is a photon, the least ionising and has the longest range (few cm of Pb) happens in nuclei with surplus energy
- **Half life** $t_{\frac{1}{2}}$ of an isotope is the average time it takes for half the number of active nuclei in the sample to decay
  Activity A in **becquerel (Bq)** is the number of decays in the sample per second
  Decay constant λ in s^{-1}, min^{-1}, etc. is the probability of decay of an individual nucleus per unit time
- **Exponential decay** $X = X_0 e^{-\lambda t}$ where $X \in \{A, N\}$ and N is the number of undecayed nuclei, $\lambda t_{\frac{1}{2}} = \ln 2$
- **Radioactive dating:** uses $^{14}_{6}\mathrm{C}$ for living things $t_{\frac{1}{2}} = 5730\mathrm{y}$, $^{87}_{37}\mathrm{Rb}$ for rocks $t_{\frac{1}{2}} = 49\mathrm{By}$
- **Conservation of mass-energy:** radioactivity converts mass to energy of emitted particles and photons $\Delta E = \Delta mc^2$
- $\mathrm{p} + \bar{\mathrm{p}} \rightarrow 2\gamma$ gives **annihilation**; $\gamma \rightarrow \mathrm{p} + \bar{\mathrm{p}}$ gives **pair production**

- Energy is needed to overcome strong nuclear force in order to pull a nucleus apart $\Rightarrow$ $\text{mass of nucleus} < \text{total mass of separate nucleons}$
  - **mass defect** $\Delta m = \text{total mass of separate nucleons} - \text{mass of nucleus}$
  - **binding energy** $\Delta E = \Delta mc^2$ is the minimum energy required to completely separate a nucleus into its constituent protons and neutrons
  - **Average binding energy per nucleon** is used to compare nuclei: $^{56}_{26}\mathrm{Fe}$ has the greatest BE per nucleon
    $\Delta E = \mathrm{BE_{LHS}} - \mathrm{BE_{RHS}}$: positive $\Rightarrow$ energy converted to mass, negative $\Rightarrow$ mass converted to energy
- **Induced fission** $^1_0\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^1_0\mathrm{n}$



1. ${}^{235}_{92}\mathrm{U}$ absorbs a **thermal neutron** and becomes ${}^{236}_{92}\mathrm{U}$
2. ${}^{236}_{92}\mathrm{U}$ is very unstable and splits by fission
3. 3 fast moving neutrons emerge, which can trigger further fission reactions in a **chain reaction**

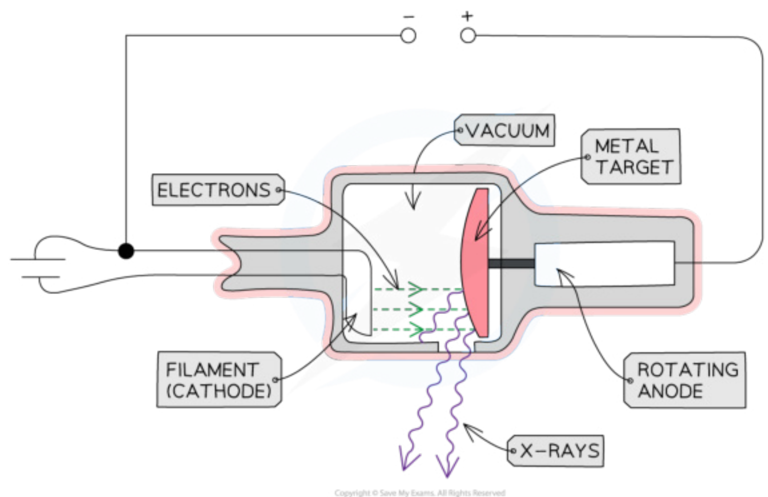
- **Fuel rods** contain enriched uranium (2-3% ${}^{235}_{92}\mathrm{U}$, ${}^{238}_{92}\mathrm{U}$ does not fission)
  Moderators (water or graphite) slow down fast neutrons without absorbing them
  Control rods (boron or cadmium) absorb neutrons to control the rate of fission by moving in and out



- Stars produce energy by **nuclear fusion**, which combines two high-speed nuclei to form a bigger nucleus
  Nuclear fusion occurs under high temperature and high pressure $\Rightarrow$ high KE to overcome the electromagnetic repulsion and get close enough for the strong nuclear force to take over, e.g. ${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{p} \rightarrow {}^{3}_{2}\mathrm{He}$

6.5 Medical Imaging

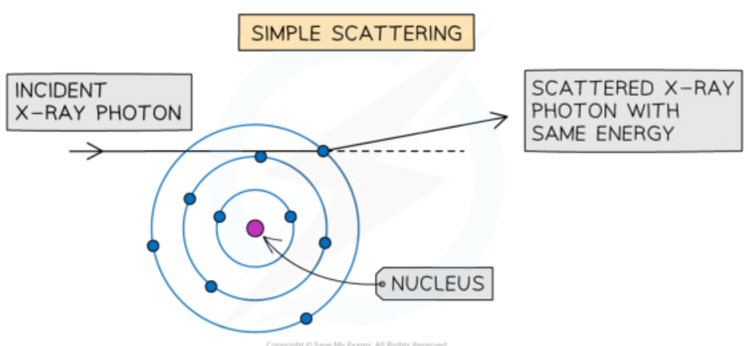
- Electrons are accelerated through a high pd, travelling from cathode to the target metal; most of the KE is transformed into heat, the rest into X-ray photons
  $\frac{hc}{\lambda} = eV \implies \lambda = \frac{hc}{eV}$



- **Attenuation** is the decrease in intensity (in the original direction) of EM radiation as it passes through matter
- $I = I_0 e^{-\mu x}$ where μ and x is the **absorption coefficient** (in m^{-1}) and the thickness respectively
- **Contrast medium** such as ${}_{53}\mathrm{I}$ and ${}_{56}\mathrm{Ba}$ could be used to improve visibility of soft tissues $\mu \propto Z^3$

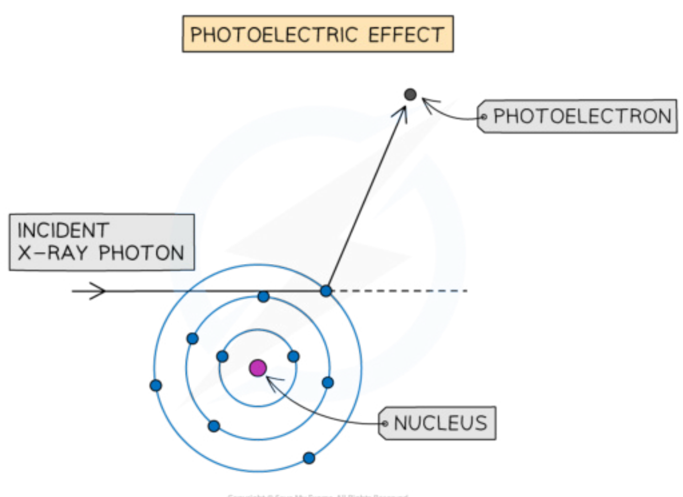
1. **Simple scattering**

   X-ray photon is scattered elastically by an electron, photon bears insufficient energy to cause ionisation (1-20keV)



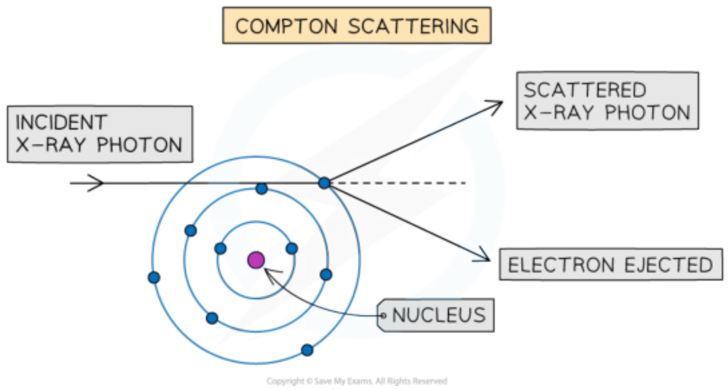
2. **Photoelectric effect** (predominant in imaging)

   X-ray photon disappears and removes an electron from the atom (<0.1MeV) $hf = \phi + KE_{\text{max}}$



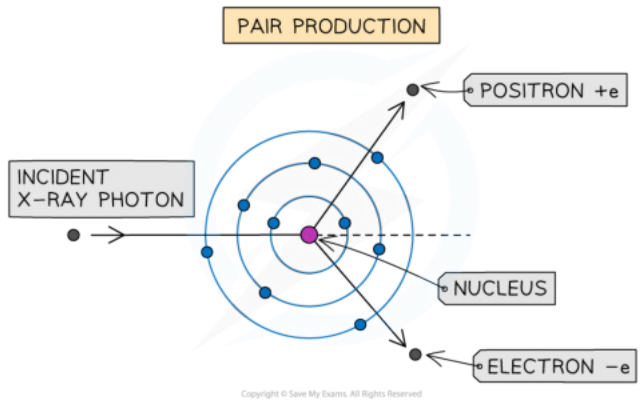
3. **Compton scattering**

Electron is ejected, and the X-ray photon is scattered with reduced energy, energy and momentum is conserved (0.5-5.0MeV)

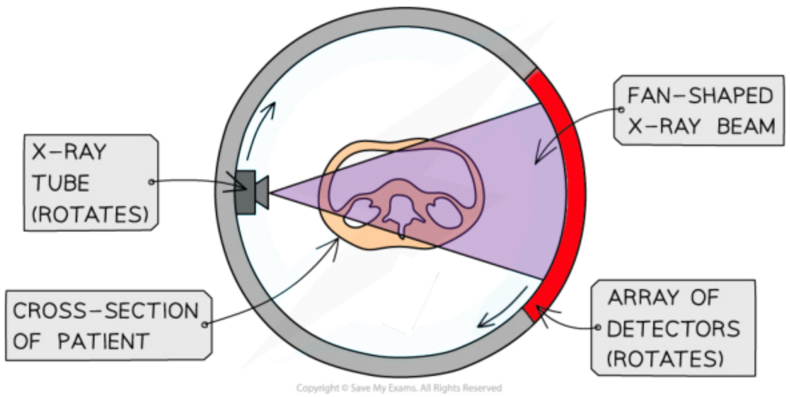


4. **Pair production**

X-ray photon interacts with the nucleus, disappears and converts into an electron-positron pair (>1.02MeV) $E_{\text{min}} = 2m_e c^2$

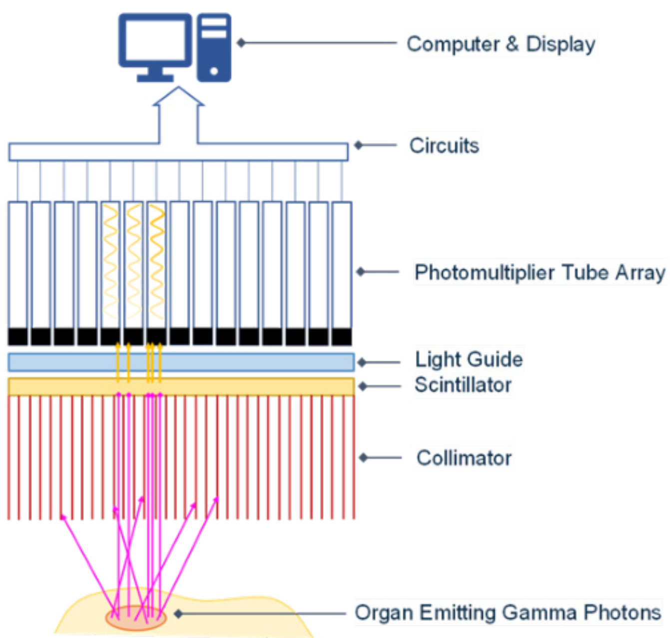


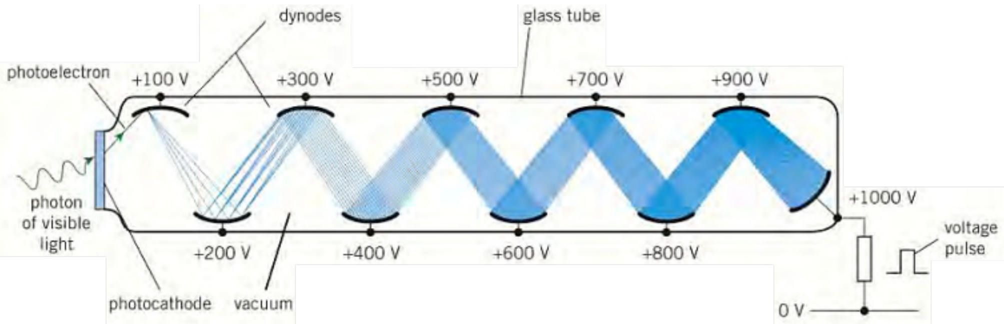
- **CAT scan** records cross-sectional scans and assemble them into a 3D image
  - patient lies in a vertical ring containing an X-ray tube and detectors on the opposite side
  - the tube and detectors rotate and move along the patient, recording each slice as a 2D image
  - software manipulation generates a 3D image



- **Medical tracers** are radioisotopes with short $t_{\frac{1}{2}}$ are chosen to ensure high activity and that the high dosage of radiation does not persist long; combines with elements that target the desired tissues
  - ${}^{99\text{m}}_{43}\text{Tc} \xrightarrow{6\text{h}} {}^{99}_{43}\text{Tc} + \gamma$ is metastable, concentration is used to identify defects
  - ${}^{18}_{9}\text{F} \xrightarrow{110\text{min}} {}^{18}_{8}\text{O} + {}^{0}_{+1}\text{e} + \nu_e + \gamma$ is used in fluorodeoxyglucose with high rate of respiration in PET scan

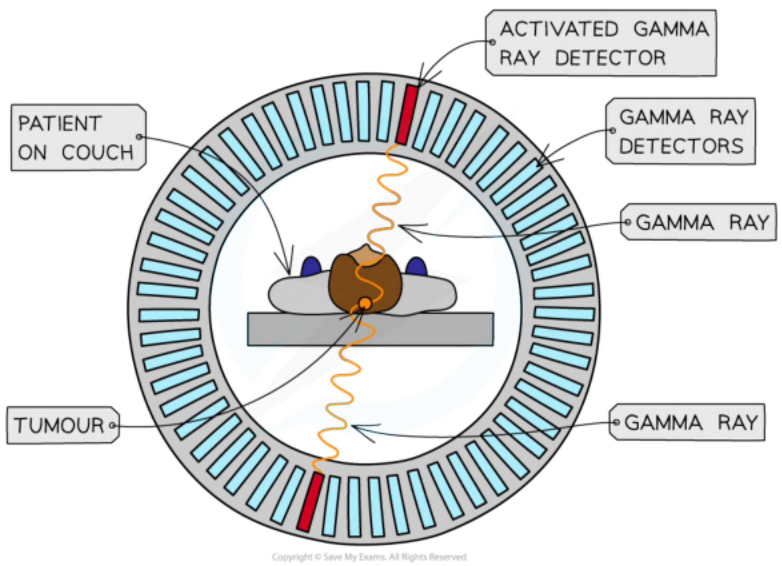
- **Gamma camera** detects gamma photons from the medical tracer:
  1. Radioisotopes in tissue emits γ photons, which travel towards the **collimator** that redirects γ to the **scintillator**
  2. A γ photon strikes scintillator and produces thousands of visible light photons, which travels through **light guide** into **photomultiplier tubes**, converting into electric pulse





Each electron creates four secondary electrons on average repeated at successive dynodes

- **PET scan** can produce slices through the body and hence a 3D image
  - detects γ photons produced when the β^+ particle from decaying ${}^{18}_{9}\mathrm{F}$ annihilates with an electron, not those directly emitted in the γ decay
  - annihilation of positron-electron pair produces two γ photons travelling in opposite directions due to conservation of momentum
  - difference in the arrival times of the photon pair and the speed of photons c is used to work out the point of annihilation and concentration
  - software manipulation generates an image on display



- **Ultrasound transducer** generates and receives ultrasound
  - **piezoelectric effect:** an emf is induced when piezoelectric materials (e.g. quartz) are compressed or extended, and vice versa
  - high-frequency alternating pd matching the natural frequency is applied between opposite ends: vibration with resonance produces intense ultrasound
  - ultrasound incident on the crystal will make it vibrate ⇒ alternating emf is generated
- **A-scan** (amplitude scan) uses a stationary transducer to record a straight line; ultrasound partially reflects at boundaries between soft tissues, $\text{thickness} = \frac{ct}{2}$
  B-scan (brightness scan) moves the transducer over the skin; brightness of dot on digital screen is proportional to intensity of reflected ultrasound
- **Acoustic impedance** of material with density ρ and speed c of ultrasound is $Z = \rho c$
  Intensity reflection coefficient $I_r : I_0 = (Z_2 - Z_1)^2 : (Z_2 + Z_1)^2$ where I_r is reflected and I_0 is incident
  Coupling gel with similar acoustic impedance to skin, i.e. acoustic matching or impedance matching, is used to fill air gaps between transducer and skin ⇒ negligible reflection
- **Doppler imaging:** speed of blood v could be measured with $\frac{\Delta f}{f} = \frac{2v\cos\theta}{c}$ where f is the frequency of ultrasound from transducer, c is the speed of ultrasound in blood, and θ is the angle between transducer and blood flow
  - frequency increases $\Delta f > 0$ when blood moves towards transducer, decreases $\Delta f < 0$ when blood moves away