

HIGH PERFORMANCE COMPUTING PROFILING

# Implementation of Durand Kerner Method to solve Polynomial Equations

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Durand  
Kerner

$$\begin{aligned}r_{n+1} &= r_n - \frac{p(r_n)}{(r_n - s_n)(r_n - t_n)} \\s_{n+1} &= s_n - \frac{p(s_n)}{(s_n - r_n)(s_n - t_n)} \\t_{n+1} &= t_n - \frac{p(t_n)}{(t_n - r_n)(t_n - s_n)}\end{aligned}$$

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## Serial Code:

```
#include <stdio.h>
#include <math.h>
#include <complex.h>
#include <omp.h>

#define M_PI 3.14159265358979323846
#define coff_size 500

double R=0;
double complex z[coff_size];
double complex deltaZ[coff_size];
double deltaZMax;
double epsilon = 1e-6;
double complex QsubJ,fz;
int max_iter = 1000;

//-----Function Prototypes-----
void durand_kerner(); //Prototypes
void calc_theta();
double max_cof();
void printz();
void update_z();
void update_fz();

int main() {

    double complex cList[coff_size]; //List of coefficients
    double complex z;
    double x,y; //x for real and y for imaginary parts of the coefficient
    int n=0; //n is number degree of polynomial

    //-----Read Coefficients-----
```

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```

    printf("Enter coefficients and enter any char other than number when
done:\n");

    while(scanf("%lf %lf",&x,&y) == 2) { //Read coefficients from stdin
        cList[n] = (x + y*I);
        n++;
    }
    x = 1; //Cn = 1, because the equation has to be normalized
    y = 0;
    z = (x + y*I);
    cList[n] = z; //Store in cList[]

    durand_kerner(cList,n);

}

//-----Function
Definition-----

void durand_kerner(double complex cList[],int n) {
    float st;
    st=omp_get_wtime();
    R = 1 + max_cof(cList,n); //End Equation 5

    calc_theta(n);
    int k;
    for(k=1;k <= max_iter;k++) {

        //printz(cList,n,k);

        deltaZMax = 0;

        update_fz(cList,n);
        update_z(n);

        if(deltaZMax <= epsilon) {

```

---

```

        break;
    }

}

st=omp_get_wtime()-st;
printf("%d\n",k);
printz(cList,n);
printf("Time Taken=%f\n",st);
}

void calc_theta(int n) {
    for(int j=0;j < n;j++) {
        z[j] = ( cos( j*((2*M_PI)/n) ) + (I*sin( j*((2*M_PI)/n) )) ) * R;
    }
}

double max_cof(double complex cList[],int n)
{
    double r;
    for(int j=0;j < n;j++) {
        if(cabs(cList[j]) > R) {
            r = cabs(cList[j]);
        }
    }

    return r;
}

void printz(double complex cList[],int n)
{
    printf("Final Output:(Note: if the roots repeat then there exist less than n-1 roots for the equation)\n");
    for(int i=0;i < n;i++) {
        printf("z[%d] = %0.10f + %0.10f*I\n",i,creal(z[i]),cimag(z[i]));
    }
}

```

```
        fflush(stdout);
    }
}

void update_z(int n)
{
    for(int j=0;j < n;j++) {
        z[j] = z[j] + deltaZ[j];
    }
}

void update_fz(double complex cList[],int n)
{
    for(int j=0;j < n;j++) {

        QsubJ = 1;
        for(int i=0;i < n;i++) {
            if(i != j) {
                QsubJ = (z[j]-z[i])*QsubJ;
            }
        }
        fz = 1;
        for(int k = n-1;k >= 0;k--) {
            fz = fz*z[j] + cList[k];
        }

        deltaZ[j] = (-fz/QsubJ);

        if(cabs(deltaZ[j]) > deltaZMax) {
            deltaZMax = cabs(deltaZ[j]);
        }
    }
}
```

---

## Profiling:

Three types of profiling are followed in this report. Namely

- Function Profiling
- Line Profiling
- Hardware Profiling

### Function Profiling:

For function profiling we will be using GPROF. GPROF helps to identify the number of times each function is called. Using the following commands, we can get a GPROF report.

### Commands Used:

- Use -pg command during gcc compilation to enable profiling
- Execute the binary file to generate the profiling data in the form of gmon.out file

Following is the gprof report:

### Flat profile:

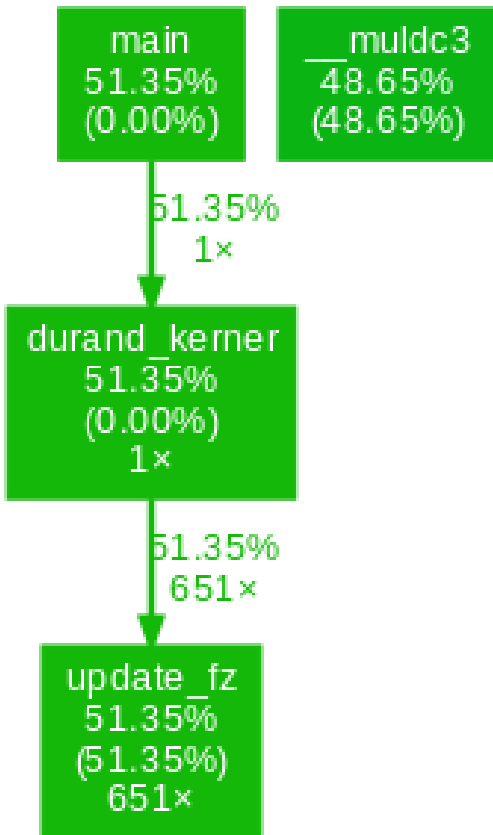
```
Flat profile:
```

Each sample counts as 0.01 seconds.

% time	cumulative seconds	self seconds	calls	self ms/call	total ms/call	name
51.49	0.19	0.19	651	0.29	0.29	update_fz
48.78	0.37	0.18				__muldc3
0.00	0.37	0.00	651	0.00	0.00	update_z
0.00	0.37	0.00	1	0.00	0.00	calc_theta
0.00	0.37	0.00	1	0.00	190.53	durand_kerner
0.00	0.37	0.00	1	0.00	0.00	max_cof
0.00	0.37	0.00	1	0.00	0.00	printz

## Call Graph :

Call graph (explanation follows)						
granularity: each sample hit covers 2 byte(s) for 2.70% of 0.37 seconds						
index	% time	self	children	called	name	
[1]	51.4	0.19	0.00	651/651	durand_kerner [2]	
		0.19	0.00	651	update_fz [1]	
-----						
[2]	51.4	0.00	0.19	1/1	main [3]	
		0.00	0.19	1	durand_kerner [2]	
		0.19	0.00	651/651	update_fz [1]	
		0.00	0.00	651/651	update_z [5]	
		0.00	0.00	1/1	max_cof [7]	
		0.00	0.00	1/1	calc_theta [6]	
		0.00	0.00	1/1	printz [8]	
-----						
[3]	51.4	0.00	0.19		<spontaneous>	
		0.00	0.19		main [3]	
		0.00	0.19	1/1	durand_kerner [2]	
-----						
[4]	48.6	0.18	0.00		<spontaneous>	
					__muldc3 [4]	
-----						
[5]	0.0	0.00	0.00	651/651	durand_kerner [2]	
		0.00	0.00	651	update_z [5]	
-----						
[6]	0.0	0.00	0.00	1/1	durand_kerner [2]	
		0.00	0.00	1	calc_theta [6]	
-----						
[7]	0.0	0.00	0.00	1/1	durand_kerner [2]	
		0.00	0.00	1	max_cof [7]	
-----						
[8]	0.0	0.00	0.00	1/1	durand_kerner [2]	
		0.00	0.00	1	printz [8]	
-----						



### Observation:

- It is to be noted that the predominant operations are `update_fz` and `__muldc3` which is called 51% and 48% of the time respectively .
- `__muldc3` is not a user-built function but rather of the c programming inbuilt function which is invoked when two complex numbers are being multiplied together. The Runtime Function is of the form : complex double `__muldc3 (double a, double b, double c, double d)` which returns the product of  $a + ib$  and  $c + id$ , following the rules of C99 Annex G.



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## Line Based Profiling:

GCOV helps to find the no of times each line is executed ,number of times a branch has been taken and other informations related to each line of the code.

Following commands can be used to generate GCOV report:

- Enable profiling using -fprofile-arcs -ftest-coverage during compilation
- Execute binary file to generate the data
- Execute gcov <filename> with -b -c parameters to generate coverage data
- Open <filename>.gcov to see the report

```
(base) sharan@Omen:~/Desktop/Sem7/HPC/Project$ cat durand-kerner.c.gcov
-: 0:Source:durand-kerner.c
-: 0:Graph:durand-kerner.gcno
-: 0:Data:durand-kerner.gcda
-: 0:Runs:1
-: 1:#include <stdio.h>
-: 2:#include <math.h>
-: 3:#include <complex.h>
-: 4:#include <omp.h>
-: 5:
-: 6:#define M_PI 3.14159265358979323846
-: 7:#define coff size 500
-: 8:
-: 9:double R=0;
-: 10:double complex z[coff size];
-: 11:double complex deltaZ[coff size];
-: 12:double deltaZMax;
-: 13:double epsilon = 1e-6;
-: 14:double complex QsubJ,fz;
-: 15:int max iter = 1000;
-: 16:
-: 17:
-: 18://-----Function Prototypes-----
-: 19:void durand_kerner(); //Prototypes
-: 20:void calc_theta();
-: 21:double max_cof();
-: 22:void printz();
-: 23:void update_z();
-: 24:void update_fz();
-: 25:
-: 26:
1: 27:int main() {
-: 28:
-: 29:     double complex cList[coff size]; //List of coefficients
-: 30:     double complex z;
-: 31:     double x,y; //x for real and y for imaginary parts of the coefficient
1: 32:     int n=0; //n is number degree of polynomial
-: 33:
-: 34:
-: 35://-----Read Coefficients-----
1: 36:     printf("Enter coefficients and enter any char other than number when done:\n");
201: 37:     while(scanf("%lf %lf",&x,&y) == 2) { //Read coefficients from stdin
200: 38:         cList[n] = (x + y*I);
200: 39:         n++;
-: 40:     }
1: 41:     x = 1; //Cn = 1, because the equation has to be normalized
1: 42:     y = 0;
1: 43:     z = (x + y*I);
1: 44:     cList[n] = z; //Store in cList[]
-: 45:
-: 46:
1: 47:     durand_kerner(cList,n);
```

```

-: 46:
1: 47:     durand kerner(cList,n);
-: 48:
-: 49:
-: 50:}
-: 51://-----Function Definition-----
-: 52:
-: 53:
1: 54:void durand kerner(double complex cList[],int n) {
-: 55:     float st;
1: 56:     st=omp get wtime();
1: 57:     R = 1 + max cof(cList,n); //End Equation 5
-: 58:
1: 59:     calc theta(n);
-: 60:     int k;
651: 61:     for(k=1;k <= max iter;k++) {
-: 62:
-: 63:         //printz(cList,n,k);
-: 64:
651: 65:         deltaZMax = 0;
-: 66:
651: 67:         update fz(cList,n);
651: 68:         update z(n);
-: 69:
651: 70:         if(deltaZMax <= epsilon) {
1: 71:             break;
-: 72:         }
-: 73:
-: 74:     }
1: 75:     st=omp get wtime()-st;
1: 76:     printf("%d\n",k);
1: 77:     printz(cList,n);
1: 78:     printf("Time Taken=%f\n",st);
-: 79:
1: 80:}
-: 81:
1: 82:void calc theta(int n) {
201: 83:     for(int j=0;j < n;j++) {
200: 84:         z[j] = ( cos( j*((2*M PI)/n) ) + (I*sin( j*((2*M PI)/n) )) )*R;
-: 85:     }
-: 86:
1: 87:}
-: 88:
1: 89:double max cof(double complex cList[],int n)
-: 90:{
-: 91:     double r;
201: 92:     for(int j=0;j < n;j++) {
200: 93:         if(cabs(cList[j]) > R) {
200: 94:             r = cabs(cList[j]);
-: 95:         }
-: 96:     }
-: 97:

```

```

1: 89:double max cof(double complex cList[],int n)
-: 90:{
-: 91:    double r;
201: 92:    for(int j=0;j < n;j++) {
200: 93:        if(cabs(cList[j]) > R) {
200: 94:            r = cabs(cList[j]);
-: 95:        }
-: 96:    }
-: 97:
1: 98:    return r;
-: 99:}
-: 100:
1: 101:void printz(double complex cList[],int n)
-: 102:{
1: 103:    printf("Final Output:(Note: if the roots repeat then there exist less than n-1 roots for the equation)\n");
201: 104:    for(int i=0;i < n;i++) {
200: 105:        printf("z[%d] = %0.10f + %0.10f*I\n",i,creal(z[i]),cimag(z[i]));
200: 106:        fflush(stdout);
-: 107:    }
1: 108:}
-: 109:
651: 110:void update z(int n)
-: 111:{
130851: 112:    for(int j=0;j < n;j++) {
130200: 113:        z[j] = z[j] + deltaZ[j];
-: 114:    }
651: 115:}
-: 116:
651: 117:void update fz(double complex cList[],int n)
-: 118:{
130851: 119:    for(int j=0;j < n;j++) {
-: 120:
130200: 121:        QsubJ = 1;
26170200: 122:        for(int i=0;i < n;i++) {
26040000: 123:            if(i != j) {
25909800: 124:                QsubJ = (z[j]-z[i])*QsubJ;
-: 125:            }
-: 126:        }
130200: 127:        fz = 1;
26170200: 128:        for(int k = n-1;k >= 0;k--) {
26040000: 129:            fz = fz*z[j] + cList[k];
-: 130:        }
-: 131:    }
-: 132:
130200: 133:    deltaZ[j] = (-fz/QsubJ);
-: 134:
130200: 135:    if(cabs(deltaZ[j]) > deltaZMax) {
44584: 136:        deltaZMax = cabs(deltaZ[j]);
-: 137:    }
-: 138:
651: 139:}

```

## Observation:

- From the above gcov report,we can see that update\_fz has lines of code that are being run for the highest no of time.For example:updating fz value executed more no of times.
- Second most executed lines are updating z value and QsubJ value which belongs to update\_fz and update\_z function.

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## Hardware Profiling :

LIKWID tool is used for hardware profiling. Following Commands are used to generate hardware related data:

- `likwid-topology`: to generate hardware data
- `likwid-perfctr -c 0-3 -g <Hardware/Option> <binary file>`: To generate execution of the program related to Hardware. Example Giving option as L2 gives data related to execution of program and how L2 cache is affected during execution.

## Hardware Specification:

```

(base) sharan@Omen:~$ likwid-topology
-----
CPU name:      Intel(R) Core(TM) i5-7300HQ CPU @ 2.50GHz
CPU type:      Intel Coffeelake processor
CPU stepping:  9
*****
Hardware Thread Topology
*****
Sockets:      1
Cores per socket: 4
Threads per core: 1
-----
HWThread      Thread      Core      Socket      Available
0              0              0          0            *
1              0              1          0            *
2              0              2          0            *
3              0              3          0            *
-----
Socket 0:      ( 0 1 2 3 )
-----
*****
Cache Topology
*****
Level:         1
Size:          32 kB
Cache groups:  ( 0 ) ( 1 ) ( 2 ) ( 3 )
-----
Level:         2
Size:          256 kB
Cache groups:  ( 0 ) ( 1 ) ( 2 ) ( 3 )
-----
Level:         3
Size:          6 MB
Cache groups:  ( 0 1 2 3 )
-----

```

```

*****
NUMA Topology
*****
NUMA domains:  1
-----
Domain:        0
Processors:    ( 0 1 2 3 )
Distances:     10
Free memory:   4769.25 MB
Total memory:  15965.7 MB
-----

```

## Likwid FLOPS\_DP Report:

```
(base) sharan@Omen:~/Desktop/Sem7/HPC/Project$ sudo likwid-perfctr -c 0-7 -g FLOPS DP ./dk < input.in
[sudo] password for sharan:
```

```
-----
CPU name:      Intel(R) Core(TM) i5-7300HQ CPU @ 2.50GHz
CPU type:      Intel Coffeelake processor
CPU clock:     2.50 GHz
-----
```

```
Sleeping longer as likwid sleep() called without prior initialization
Sleeping longer as likwid sleep() called without prior initialization
Enter coefficients and enter any char other than number when done:
495
Time Taken=0.295464
-----
```

Group 1: FLOPS DP

Event	Counter	Core 0	Core 1	Core 2	Core 3
INSTR RETIRED ANY	FIXC0	135775957	119238819	2468335368	204992325
CPU CLK UNHALTED CORE	FIXC1	163749715	133500282	909249738	187059720
CPU CLK UNHALTED REF	FIXC2	132371720	108792320	706702360	150110792
FP ARITH INST RETIRED 128B PACKED DOUBLE	PMC0	1060	8343	2112	4820
FP ARITH INST RETIRED SCALAR DOUBLE	PMC1	198342	20038	315201691	39002
FP ARITH INST RETIRED 256B PACKED DOUBLE	PMC2	2	0	0	0

Event	Counter	Sum	Min	Max	Avg
INSTR RETIRED ANY STAT	FIXC0	2928342469	119238819	2468335368	7.320856e+08
CPU CLK UNHALTED CORE STAT	FIXC1	1393559455	133500282	909249738	3.483899e+08
CPU CLK UNHALTED REF STAT	FIXC2	1097977192	108792320	706702360	274494298
FP ARITH INST RETIRED 128B PACKED DOUBLE STAT	PMC0	16335	1060	8343	4083.7500
FP ARITH INST RETIRED SCALAR DOUBLE STAT	PMC1	315459073	20038	315201691	7.886477e+07
FP ARITH INST RETIRED 256B PACKED DOUBLE STAT	PMC2	2	0	2	0.5000

Metric	Core 0	Core 1	Core 2	Core 3
Runtime (RDTSC) [s]	0.3056	0.3056	0.3056	0.3056
Runtime unhaltd [s]	0.0656	0.0535	0.3643	0.0749
Clock [MHz]	3087.6345	3062.8413	3211.3467	3110.3475
CPI	1.2060	1.1196	0.3684	0.9125
DP [MFLOP/s]	0.6560	0.1202	1031.5049	0.1592
AVX DP [MFLOP/s]	2.617983e-05	0	0	0
Packed [MUOPS/s]	0.0035	0.0273	0.0069	0.0158
Scalar [MUOPS/s]	0.6491	0.0656	1031.4910	0.1276
Vectorization ratio	0.5326	29.3964	0.0007	10.9990

Metric	Sum	Min	Max	Avg
Runtime (RDTSC) [s] STAT	1.2224	0.3056	0.3056	0.3056
Runtime unhaltd [s] STAT	0.5583	0.0535	0.3643	0.1396
Clock [MHz] STAT	12472.1700	3062.8413	3211.3467	3118.0425
CPI STAT	3.6065	0.3684	1.2060	0.9016
DP [MFLOP/s] STAT	1032.4403	0.1202	1031.5049	258.1101
AVX DP [MFLOP/s] STAT	2.617983e-05	0	2.617983e-05	6.544957e-06
Packed [MUOPS/s] STAT	0.0535	0.0035	0.0273	0.0134
Scalar [MUOPS/s] STAT	1032.3333	0.0656	1031.4910	258.0833
Vectorization ratio STAT	40.9287	0.0007	29.3964	10.2322

## Likwid L3 Report:

```
(base) sharan@Omen:~/Desktop/Sem7/HPC/Project$ sudo likwid-perfctr -c 0-7 -g L3 ./dk < input.in
-----
CPU name:      Intel(R) Core(TM) i5-7300HQ CPU @ 2.50GHz
CPU type:      Intel Coffeelake processor
CPU clock:     2.50 GHz
-----
Sleeping longer as likwid sleep() called without prior initialization
Sleeping longer as likwid sleep() called without prior initialization
Enter coefficients and enter any char other than number when done:
495
Time Taken=0.292505
-----
Group 1: L3
-----+-----+-----+-----+-----+-----+
| Event          | Counter | Core 0 | Core 1 | Core 2 | Core 3 |
+-----+-----+-----+-----+-----+-----+
| INSTR RETIRED ANY | FIXC0   | 151683170 | 80226953 | 2454470654 | 131935662 |
| CPU CLK UNHALTED CORE | FIXC1   | 139069053 | 91748366 | 893507653 | 103578560 |
| CPU CLK UNHALTED REF | FIXC2   | 109833568 | 72960888 | 685560512 | 81989856 |
| L2 LINES IN ALL | PMC0    | 5043529 | 4731644 | 333723 | 4019020 |
| L2 TRANS L2 WB | PMC1    | 858862 | 867494 | 68571 | 789864 |
+-----+-----+-----+-----+-----+-----+
```

Event	Counter	Sum	Min	Max	Avg
INSTR RETIRED ANY STAT	FIXC0	2818316439	80226953	2454470654	7.045791e+08
CPU CLK UNHALTED CORE STAT	FIXC1	1227903632	91748366	893507653	306975908
CPU CLK UNHALTED REF STAT	FIXC2	950344824	72960888	685560512	237586206
L2 LINES IN ALL STAT	PMC0	14127916	333723	5043529	3531979
L2 TRANS L2 WB STAT	PMC1	2584791	68571	867494	646197.7500

Metric	Core 0	Core 1	Core 2	Core 3
Runtime (RDTSC) [s]	0.2947	0.2947	0.2947	0.2947
Runtime unhaltd [s]	0.0557	0.0368	0.3580	0.0415
Clock [MHz]	3160.1109	3138.4495	3252.8153	3152.9470
CPI	0.9168	1.1436	0.3640	0.7851
L3 load bandwidth [MBytes/s]	1095.1393	1027.4173	72.4638	872.6799
L3 load data volume [GBytes]	0.3228	0.3028	0.0214	0.2572
L3 evict bandwidth [MBytes/s]	186.4911	188.3655	14.8893	171.5091
L3 evict data volume [GBytes]	0.0550	0.0555	0.0044	0.0506
L3 bandwidth [MBytes/s]	1281.6304	1215.7828	87.3531	1044.1890
L3 data volume [GBytes]	0.3778	0.3583	0.0257	0.3078

Metric	Sum	Min	Max	Avg
Runtime (RDTSC) [s] STAT	1.1788	0.2947	0.2947	0.2947
Runtime unhaltd [s] STAT	0.4920	0.0368	0.3580	0.1230
Clock [MHz] STAT	12704.3227	3138.4495	3252.8153	3176.0807
CPI STAT	3.2095	0.3640	1.1436	0.8024
L3 load bandwidth [MBytes/s] STAT	3067.7003	72.4638	1095.1393	766.9251
L3 load data volume [GBytes] STAT	0.9042	0.0214	0.3228	0.2260
L3 evict bandwidth [MBytes/s] STAT	561.2550	14.8893	188.3655	140.3137
L3 evict data volume [GBytes] STAT	0.1655	0.0044	0.0555	0.0414
L3 bandwidth [MBytes/s] STAT	3628.9553	87.3531	1281.6304	907.2388
L3 data volume [GBytes] STAT	1.0696	0.0257	0.3778	0.2674

## Observation:

- From the above report it can be seen that the CPI from both L3 and FLOPS\_DP report that are not equal across each core ,which confirms that the program is yet to be optimized which is expected from the serial code.
- The L3 data volume and bandwidth also indicates that the load is not balanced across each core
- The Clock across each core seems to be stable