AST1420 Project 1: Jeans Modeling

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https://github.com/trivnguyen/AST1420_Jeans_Modeling_Tutorials

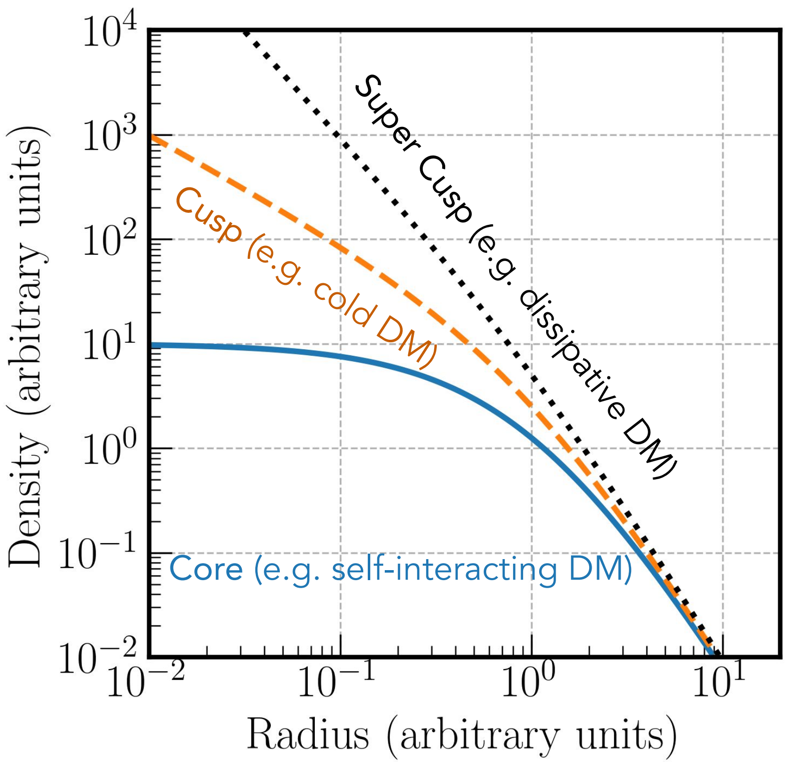
Recap: near-field cosmology and dwarf spheroids

Density profiles are sensitive to:

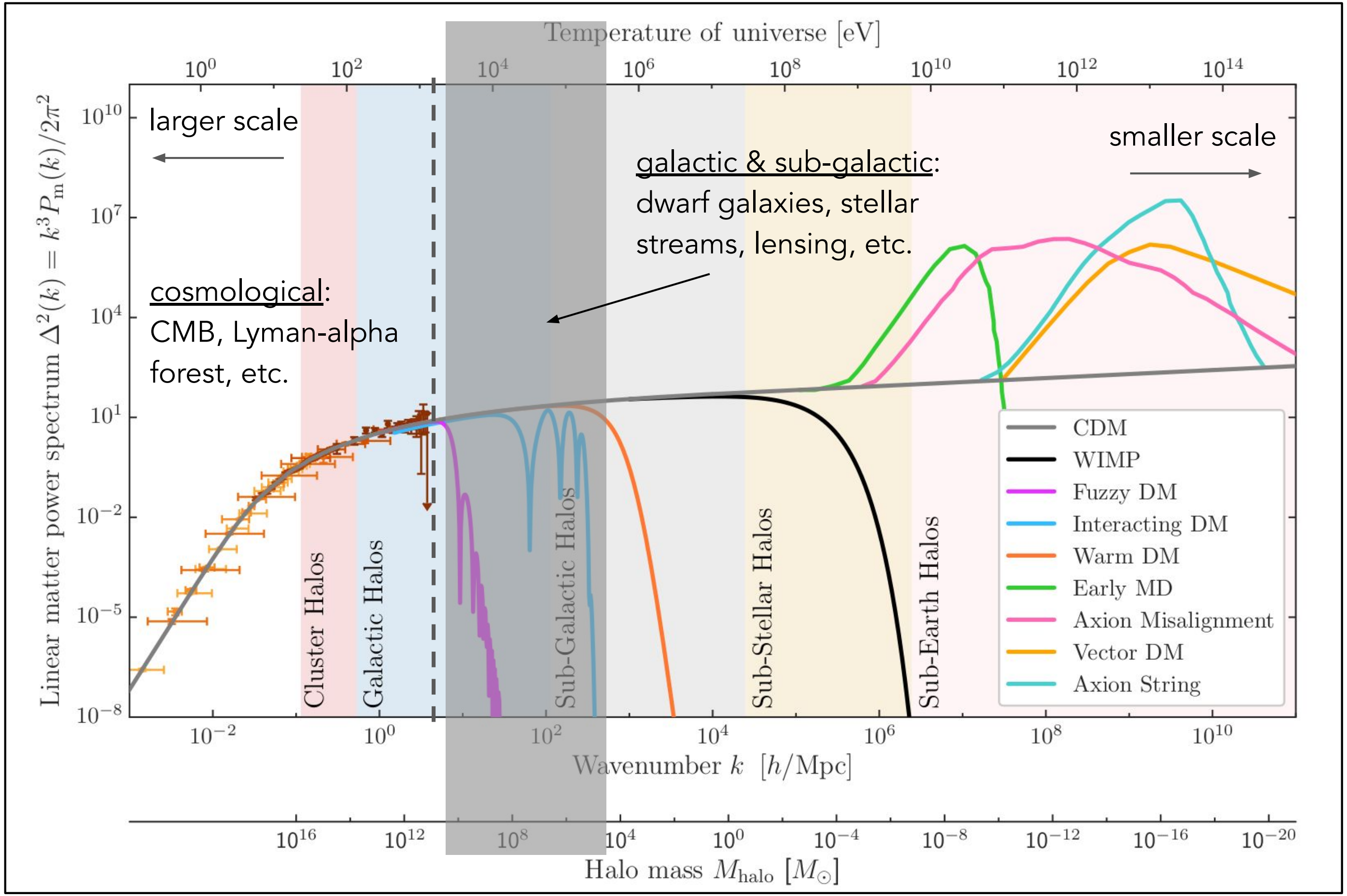
1. Dark matter physics:
   self-interaction, particle mass
2. Annihilation/decay rate:
   indirect detection experiment

Generalized NFW profile:

$$\rho_{\rm dm}^{\rm gNFW}(r) = \rho_0 \left(\frac{r}{r_{\rm dm}}\right)^{-\gamma} \left(1 + \frac{r}{r_{\rm dm}}\right)^{\gamma-3}$$



Linear Matter Power Spectrum



Dwarf galaxies:

$M_{\rm halo} = 10^9 - 10^{11}\,M_\odot$

Stellar streams:

$M_{\rm halo} = 10^6 - 10^9 M_\odot$

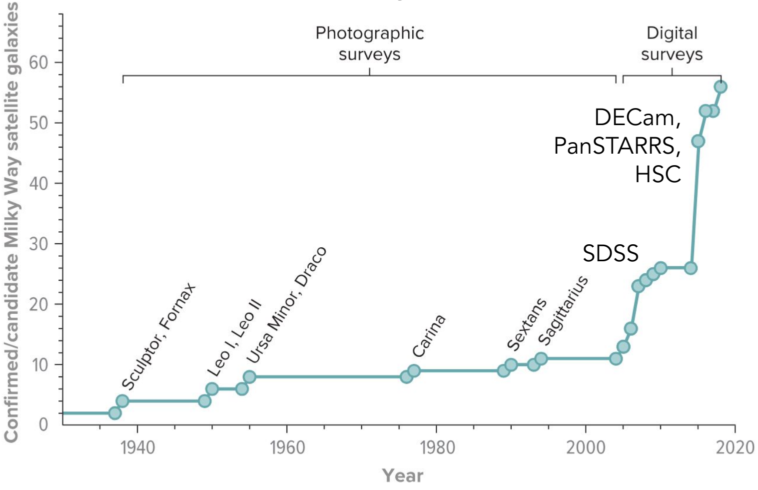
Bechtol et al.
(Snowmass2021)

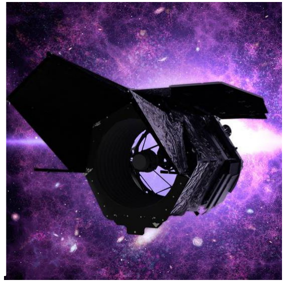
Near-field cosmology: Why Now?

Photometric surveys will discover hundreds of new ultra-faint systems and provide stellar proper motions

Spectroscopic surveys will measure stellar radial velocities

Number of dwarf galaxies over time





J.D. Simon (arXiv:1901.05465)

AST1420 Project 1: Jeans Modeling of Spherical Systems

In this project, you will implement Jeans modeling and measure the mass and mass profile of a dwarf spherical system

Tasks:

1. *(Optional) Run on mock data of constant and varied anisotropy*
2. Pick a dwarf galaxy and do a literature review
3. Run Jeans modeling on the selected galaxy and report their mass and mass profile

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Mock Catalog

4 mock galaxies: CoreIso, CuspIso, CoreOM, CuspOM

100 tracers each, 5 km/s LOS uncertainty

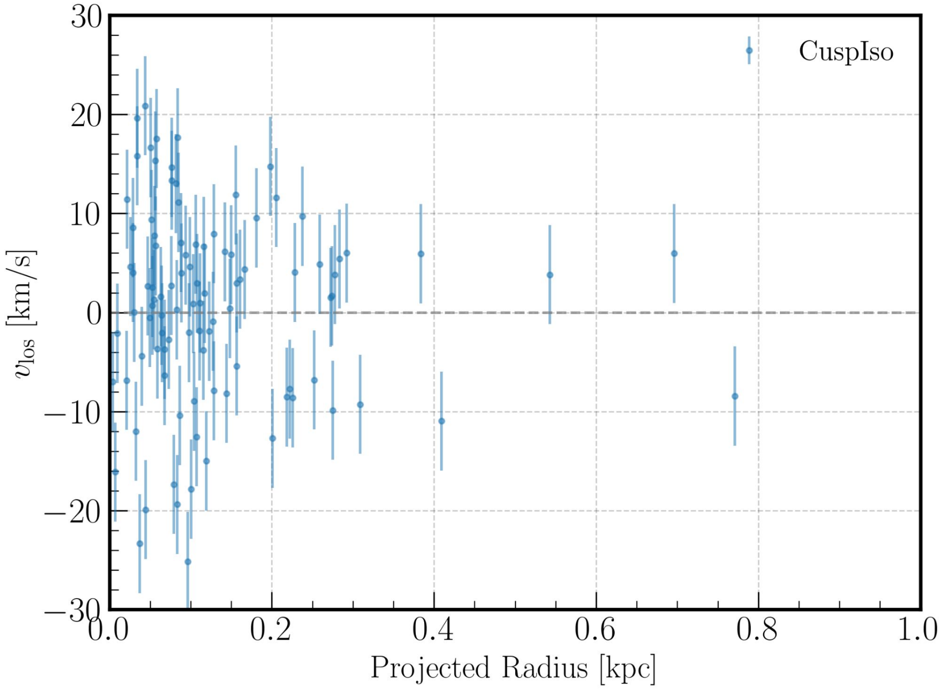
Parameter set taken from *Gaia Challenge*

Model	Anisotropy	$r_{1/2}$ [kpc]	r_a [kpc]	γ	$\rho_{\rm dm}$ [10^8 $M_\odot$ kpc^{-3}]
CoreOM	Osipkov-Merritt	0.25	0.25	0.0	4.0
CuspOM	Osipkov-Merritt	0.1	0.1	1.0	0.64
CoreIso	Isotropic	0.25	∞	0.0	4.0
CuspIso	Isotropic	0.1	∞	1.0	0.64

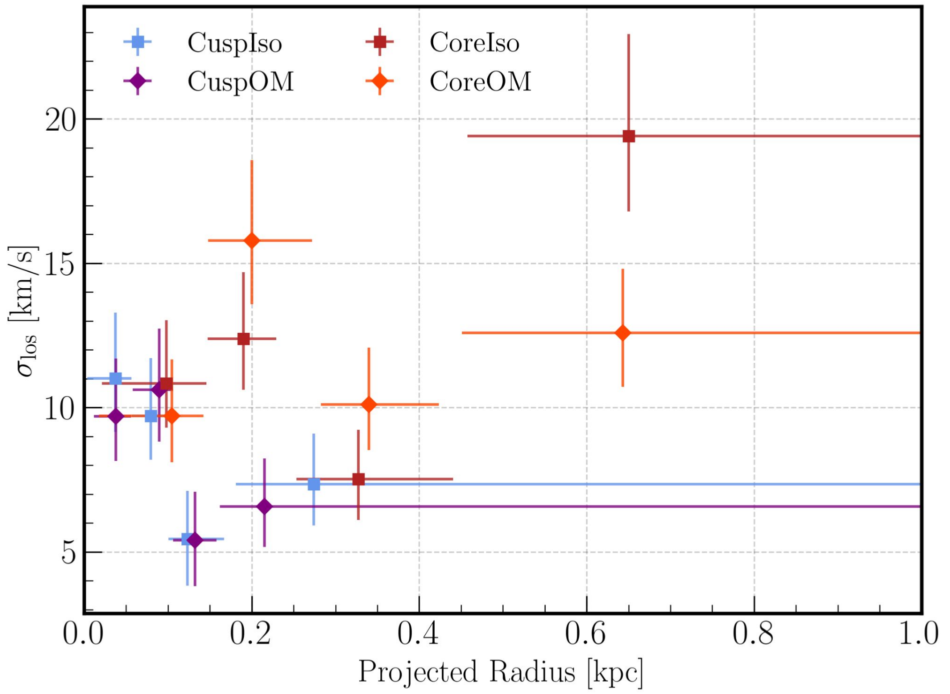
https://astrowiki.surrey.ac.uk/doku.php?id=tests:sphtri

Mock Catalog

LOS velocities + projected radius



Binned velocity dispersion



Jeans velocity dispersion

Line-of-sight dispersion:

$$\boxed{\sigma^2_{\rm los}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \beta \frac{R^2}{r^2}\right) \frac{\nu(r)\sigma_r^2(r) r\, dr}{\sqrt{r^2 - R^2}}}$$

$$\nu(r)\sigma_r^2(r) = \frac{1}{g(r)} \int_r^\infty \frac{GM(< s)\nu(s)}{s^2} g(s)\, ds$$

3D dispersion

$$g(r) = \exp\left(2 \int \frac{\beta(r)}{r} dr\right)$$

NOTE: the constant in g(r) cancels out in 3D dispersion

Jeans velocity dispersion

Proper motion velocity dispersion:

$$\sigma^2_{\mathrm{pm,R}}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \beta + \beta \frac{R^2}{r^2}\right) \frac{\nu(r)\sigma_r^2(r) r\, dr}{\sqrt{r^2 - R^2}}$$

$$\sigma^2_{\mathrm{pm,T}}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty (1 - \beta) \frac{\nu(r)\sigma_r^2(r) r\, dr}{\sqrt{r^2 - R^2}}$$

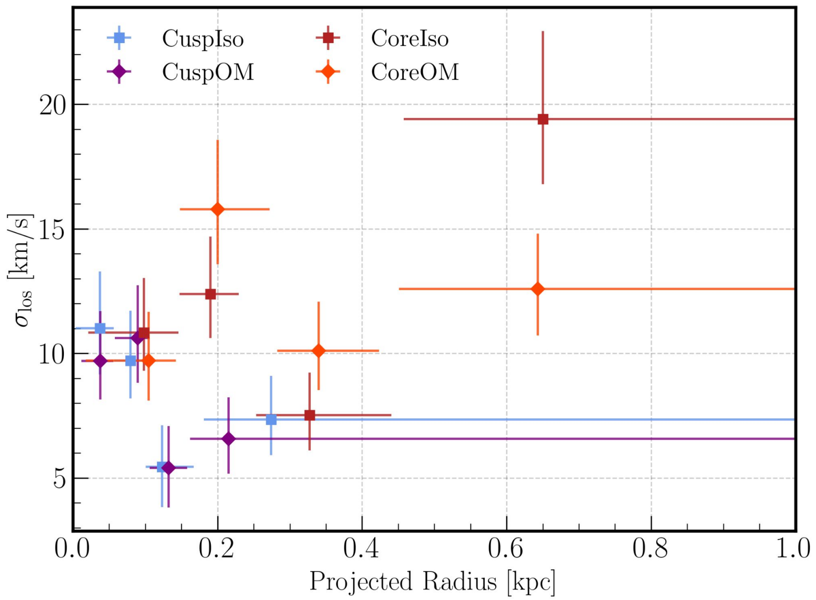
How do I connect model and observation?

Model

$$\sigma^2_{\rm los}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \beta \frac{R^2}{r^2}\right) \frac{\nu(r)\sigma_r^2(r) r \, dr}{\sqrt{r^2 - R^2}}$$

???

Observation



Bayesian inference

Bayes' Theorem allow you to construct the probability of parameters given observed data and some *a-priori* probability of the parameters

$$\underbrace{P(\theta \mid x, \mathcal{M})}_{\text{Posterior probability}} = \frac{\overbrace{P(x \mid \theta, \mathcal{M})}^{\text{Likelihood of data given parameters}}\;\overbrace{P(\theta \mid \mathcal{M})}^{\text{A priori probability of parameters (Prior)}}}{\underbrace{P(x \mid \mathcal{M})}_{\text{Evidence ("prior predictive of data)}}}$$

Sampling high-dimensional posteriors

If the posterior is high-dimensional, evaluating it on a grid is too inefficient

Markov Chain Monte Carlo (MCMC) is a general technique for sampling high-dimensional PDFs

In the tutorials, we will use pocoMC, a smarter version of MCMC that learns the shape of your posterior as it samples.

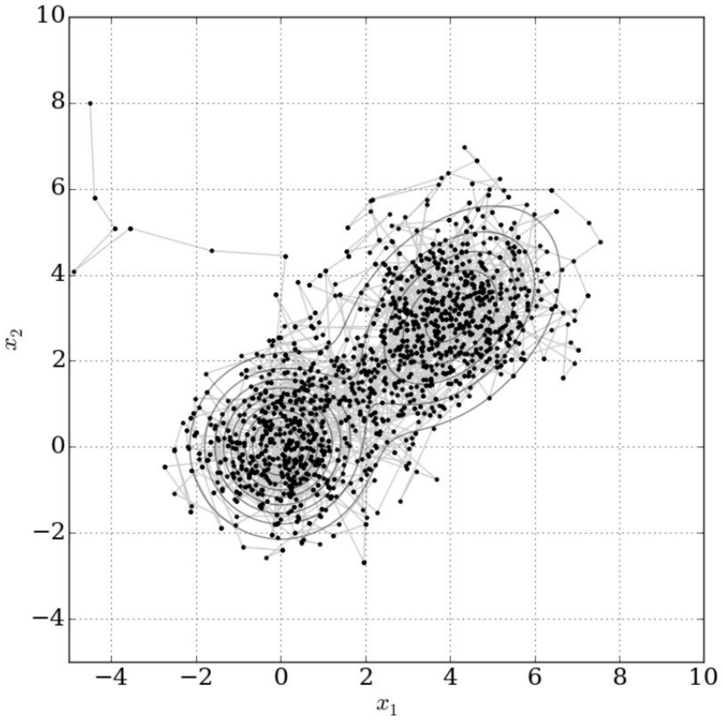
Karamanis et al. (arXiv:2207.05660)

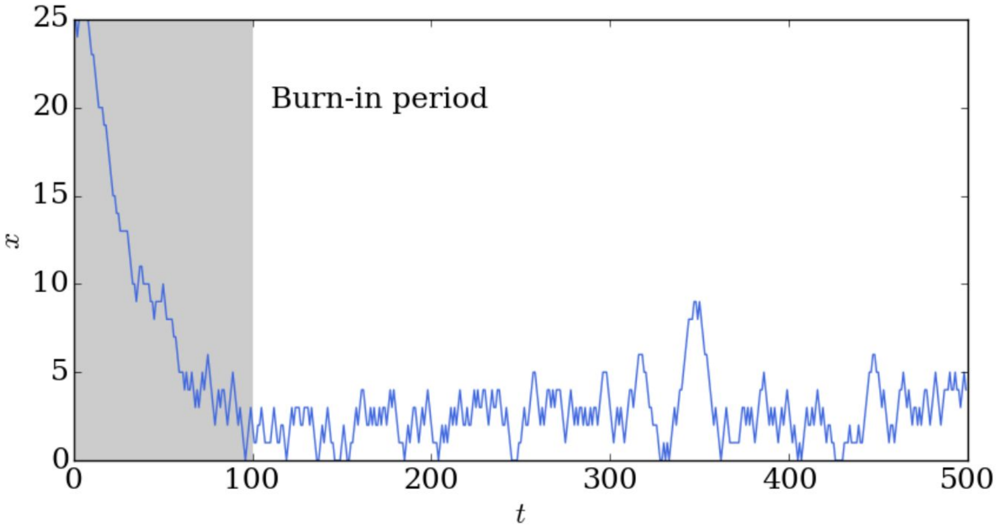
https://pocomc.readthedocs.io/en/latest/

Visualization:

https://chi-feng.github.io/mcmc-demo/app.html?

More on MCMC





Some resources:

https://www.pas.rochester.edu/~sybenzvi/courses/phy403/2016s/p403_18_mcmc.pdf

Jeans Likelihood

Strigari et al. (arXiv:0808.3772)

Under the Gaussian assumption, the likelihood is given by:

systemic velocity

$$p(x \mid \theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi(\sigma_{\rm los}^2(R_i;\theta) + \Delta_{{\rm err},i}^2)}} \exp\left(-\frac{(v_i - \bar{v})^2}{2(\sigma_{\rm los}^2(R_i;\theta) + \Delta_{{\rm err},i}^2)}\right)$$

measurement error

NOTE:

- The systemic velocity is treated as a free parameter. However, if you subtract the mean velocity beforehand, then it should be consistent with zero
- The Gaussian assumption is surprisingly robust for the *intrinsic dispersion*. This will break down if *measurement errors* are not Gaussian

Model Parameters

3 dark matter parameters:

$$\rho_{\rm dm}^{\rm gNFW}(r) = \rho_0 \left(\frac{r}{r_{\rm dm}}\right)^{-\gamma} \left(1 + \frac{r}{r_{\rm dm}}\right)^{\gamma-3}$$

1 tracer density parameter:
(M cancels out in Jeans eq)

$$\nu(r) = \frac{3M}{4\pi a^3} \left(1 + \frac{r^2}{a^2}\right)^{-5/2}$$

3 anisotropy parameters:

$$\beta(r) = \beta_0 + (\beta_\infty - \beta_0) \frac{r^2}{r^2 + r_a^2}$$

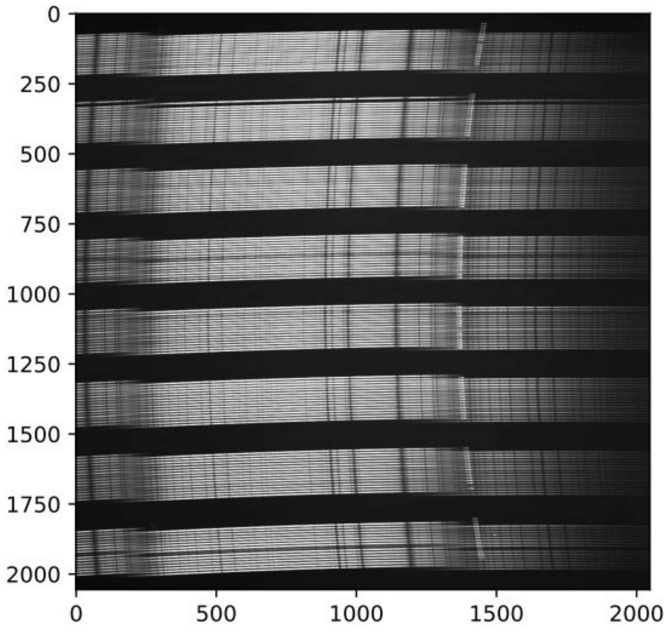
1 nuisance parameter:

$$\bar{v}$$

Systematics in Jeans modeling

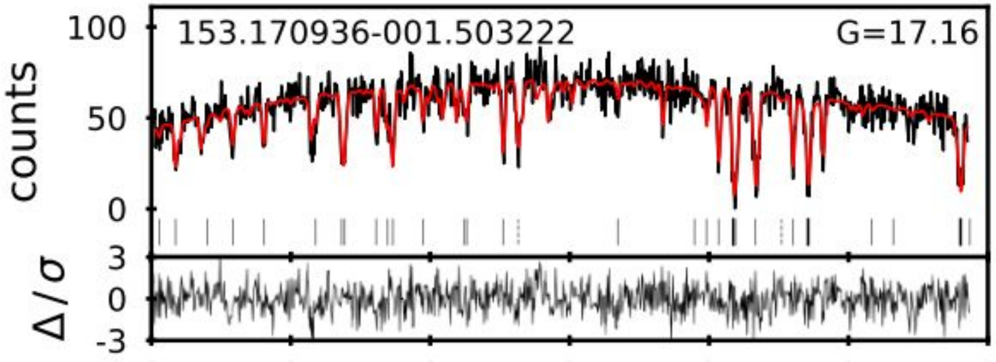
1. LOS measurement uncertainty is not Gaussian/calibrated
2. Incomplete spectroscopic selection function
3. System is not in equilibrium:
    a. baryon feedback, e.g. El-Badry et al. (arXiv:1610.04232)
    b. undergoing tidal disrupting => look for tidal features

Understanding how LOS measurements are made



raw M2FS *calibration image*

Walker et al. (arXiv:2312.1273)



spectra + best fit

Do NOT combine measurements taken from different calibration pipelines (zero-point offset) !!

Spectroscopic selection & tracer density

The tracer mass density profile is often assumed to be Plummer:

<u>3D Plummer</u>: $$\nu(r) = \frac{3M}{4\pi a^3}\left(1 + \frac{r^2}{a^2}\right)^{-5/2}$$

<u>2D Plummer</u>: $$\Sigma_\star(R) = \frac{M}{\pi a^2}\left(1 + \frac{R^2}{a^2}\right)^{-2}$$

Also see:
L instead of M

Other profiles are: King, Sersic, Exponential

Plummer, H. C. 1911, MNRAS, 71, 460 (no arxiv)

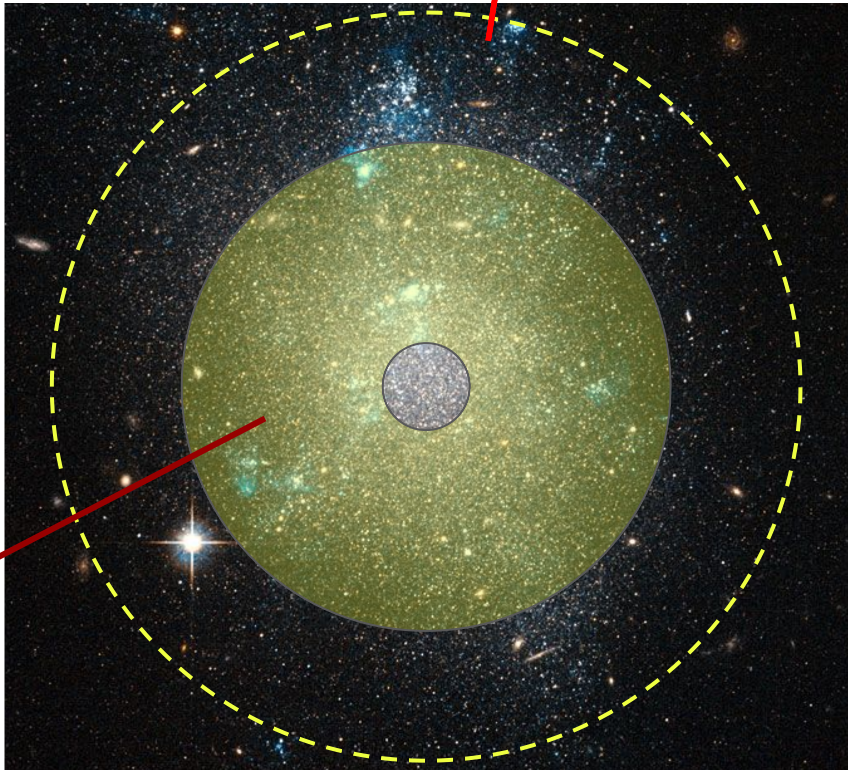
Spectroscopic selection & tracer density

Spectroscopic observations are often incomplete:

1. Large radii: fewer stars
2. Small radii: crowding & fiber collision

=> The spectropic tracer density profile is often *NOT* representative of the true profile

True radial extent



region with spectroscopic obs

Spectroscopic selection & tracer density

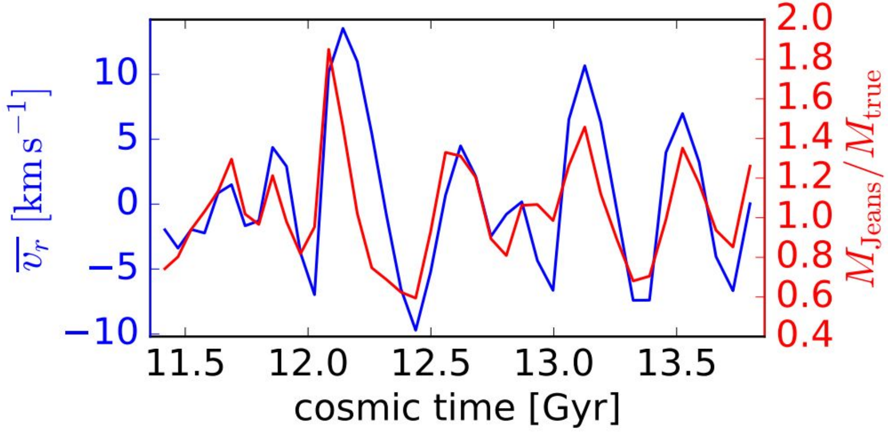
In practice, we would measure the tracer mass profile using photometric observations

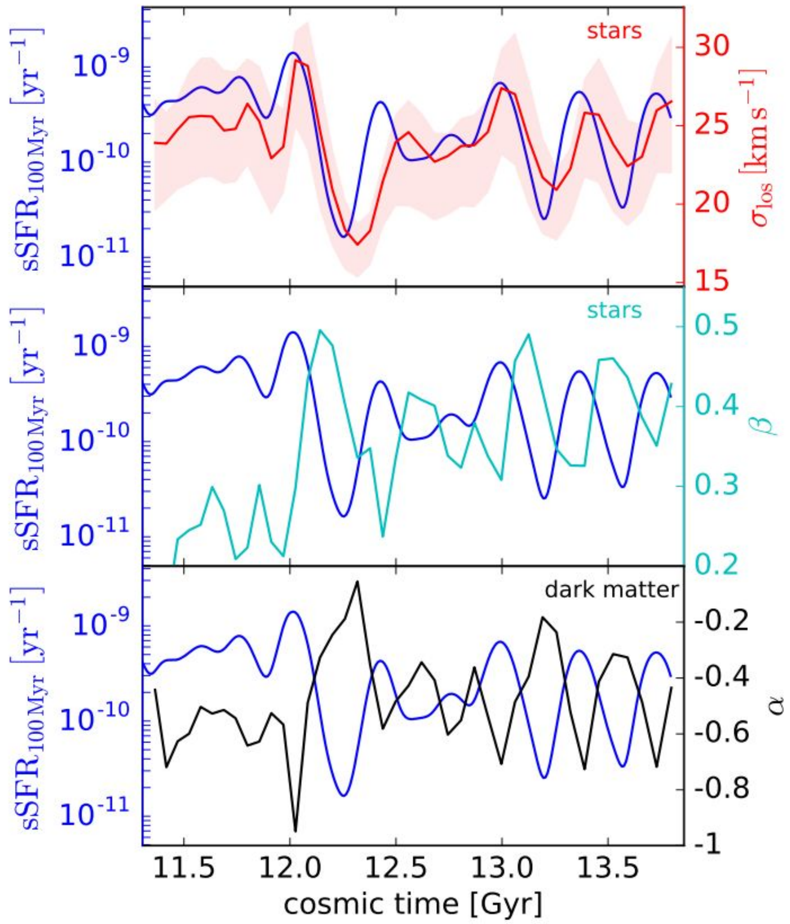
During the Jeans fit, we will still let the tracer density profile be a free parameter, but only within the uncertainty of the photometric data

NOTE: This assumes that the *tracer mass profile* matches the *tracer light profile*, which is reasonable but not always true (e.g. mass segregation)

Effect of non-equilibrium

El-Badry et al. (arXiv:1610.04232)





AST1420 Project 1 Recap

In this project, you will implement Jeans modeling and measure the mass and mass profile of a dwarf spherical system

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Part 1: Jeans modeling tutorials

The two tutorial notebooks will help you implement Jeans modeling

1. Notebook 1: Jeans modeling with constant anisotropy
2. Notebook 2: Jeans modeling with Osipkov-Merritt anisotropy

You should walk through them independently and make sure your answer is consistent with the solution

You do not need to submit these

https://github.com/trivnguyen/AST1420_Jeans_Modeling_Tutorials

Part 1: Jeans modeling tutorials

To build your intuition, try answering these questions when going over the tutorials:

1. At which radius does the posterior distribution of the mass profile the most constrained? How does that relate to the Wolf mass?
2. In the corner plot, which directions are the degeneracies between density profile parameters?
3. In case of constant anisotropy, how does increasing or decreasing the anisotropy affect the mass profile, at small and large radii?`
4. Which type of mass profile is more difficult to constrain? Cored vs cupsy profiles

Part 2: Jeans modeling of a real dwarf galaxy

Pick a dwarf galaxy from the spreadsheet below. Your task is to:

1. Conduct a literature review on the density profile of this galaxy
2. Run Jeans modeling and report the mass and mass density profile of the galaxy

You will have to find and download the spectroscopic data. A few potentially helpful resource:

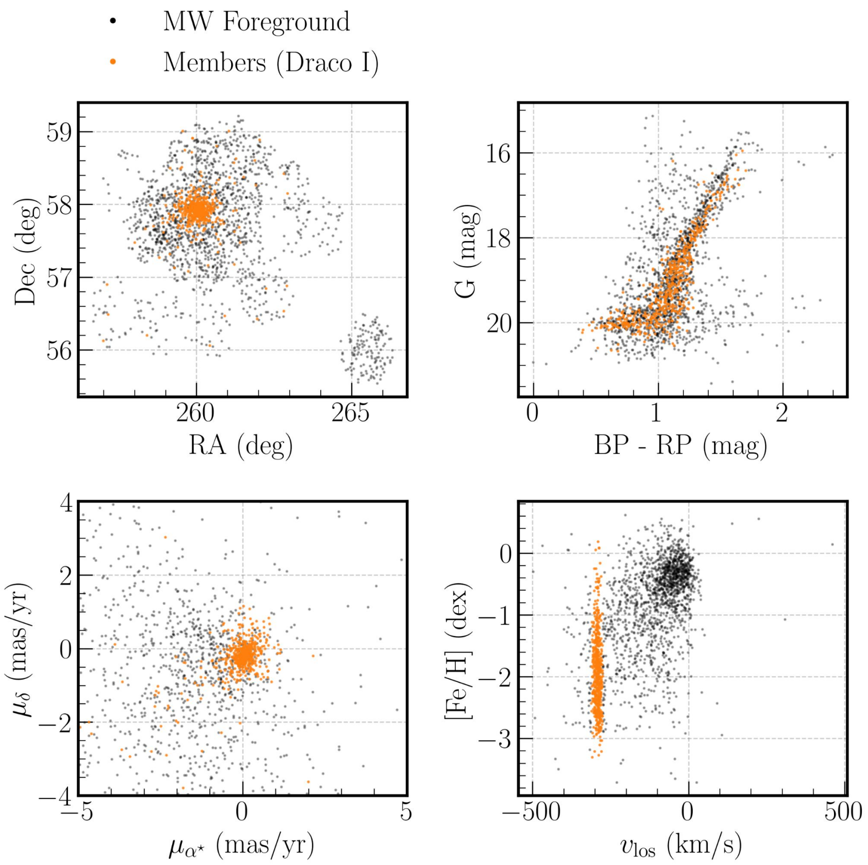
- Magellan/MMT catalog: Walker et al. (arXiv:2312.1273)
- Local Volume catalog: Pace (arXiv:2411.07424)
- Proper motion & membership: Pace, Erkal, & Li (arXiv:2205.0569)

https://docs.google.com/spreadsheets/d/1C9g81tvyyqNDzs3DahjT-RmJSYj5dCTbbuEC4drKTcQ/edit?usp=sharing

Part 2: Jeans modeling of a real dwarf galaxy

Some catalogs (e.g. Walker+23 MMT/Magellan) do not report membership probability

I suggest cross-matching with the Pace proper motion catalog and applying crude 3-sigma cut on the LOS velocities



Additional Resources

Advanced Jeans modeling techniques:

- High-order moment: Read & Steger (arXiv:1701.04833)
- Triaxial potential: Hayashi et al. (arXiv:1603.08046)
- Gaussian 3D velocity: Mamon, Biviano, & Boue (arXiv:1212.1455)
- Multi-Gaussian Expansions: Watkins et al. (arXiv:1308.4789)
- Machine learning: Nguyen et al. (arXiv:2208.12825), Lim et al. (arXiv:2505.00763)
- Comparison between techniques: Read et al. (arXiv:2011.09493)

Observation compilations:

- Geringer-Sameth et al. (arXiv:1408.0002)
- Hayashi et al. (arXiv:2206.02821)

Catalogs:

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MCMC and Jeans: under the hood

Under the hood operation (simplified):

1. Sample DM parameters from some *proposal distribution*
2. Calculate the intrinsic velocity dispersion from DM parameters
3. Evaluate the likelihood and prior
4. Evaluate the *transition probability* and accept/reject sample
5. Repeat (1) - (4) until converged

Some resources:

https://www.pas.rochester.edu/~sybenzvi/courses/phy403/2016s/p403_18_mcmc.pdf