Towards Aerodynamic Simulations in Julia with Trixi.jl

JuliaCon 2024

Daniel Doehring[1]

in collaboration with

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Ahmad Peyvan[5], and Andrew Winters[6]

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[5]Applied Mathematics, Brown University [6]TIMA, Linköping University

First Things First

Acknowledgements:

- **Arpit Babbar:**
  Drag, Lift, Pressure and Friction Coefficients
- **Simon Candelaresi:**
  Restarting simulations
- **Jesse Chan & Ahmad Peyvan:**
  Parabolic adaptive mesh refinement (AMR), particularly mortars
- **Andrew Winters:**
  Boundary identification for `Gmsh` generated meshes

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Reproducibility:

- Public repository on GitHub:
  - → `https://github.com/trixi-framework/talk-2024-juliacon_aerodynamics`

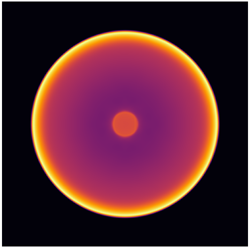
Outline

1. Scope of the Talk
2. Case Study: NACA 4412 Airfoil
3. Extensions
4. Conclusion & Outlook

About Trixi.jl

Trixi.jl → `https://github.com/trixi-framework/Trixi.jl`

- is a Julia package for simulation of *compressible* flows (PDEs)



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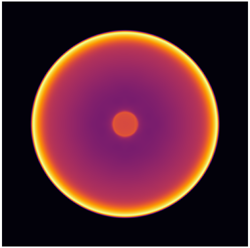

- is a Julia package for simulation of *compressible* flows (PDEs)
- implements the Discontinuous Galerkin (DG) method

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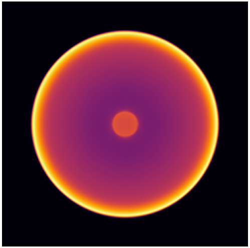
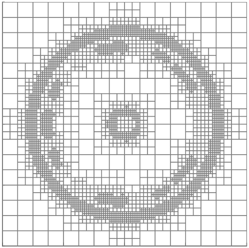
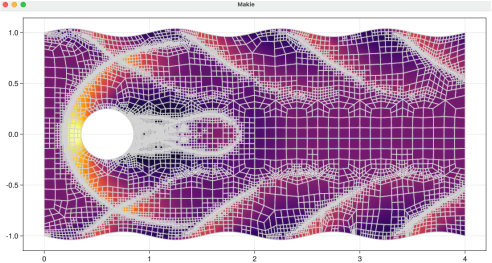




- is a Julia package for simulation of *compressible* flows (PDEs)
- implements the Discontinuous Galerkin (DG) method
- is a **grid-based** solver with adaptive mesh refinement (AMR)

Picking up where we left off

Andrew's talk at JuliaCon '22:



- `HOHQMesh`
- Inviscid (Hyperbolic) problems

Picking up where we left off

Andrew's talk at JuliaCon '22:

- `HOHQMesh`
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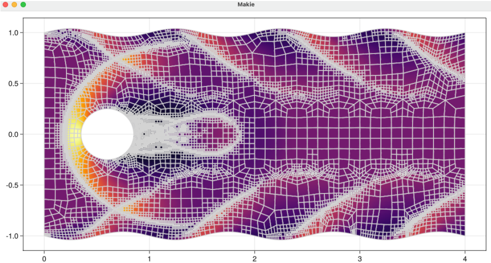
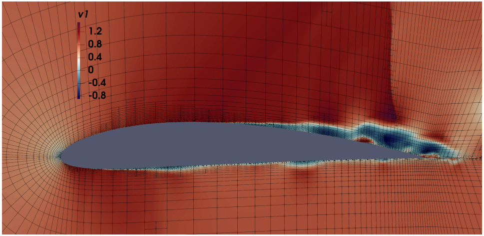
Today:

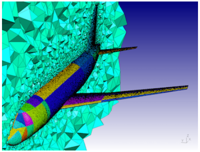
- Usage of existing meshes
- Viscous (Hyperbolic-Parabolic) problems





Motivation and Objectives

Mesh generation is a non-trivial task, especially for "true", i.e., non-extruded 3D meshes!



Picture taken from
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[a] `https://github.com/trixi-framework/P4est.jl`

[b] https://github.com/cburstedde/p4est

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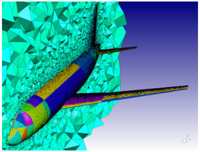
Goals: Be able to use existing

- Meshes in standardized format (`.inp`)
- Tools for mesh generation ⇒ `Gmsh`



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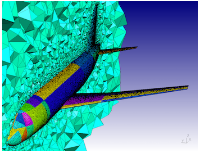
While **maintaining features** like

- Adaptive mesh refinement (AMR)
- Parallelization
- Visualization
- Restarting



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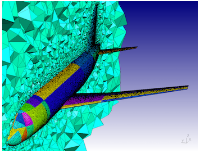
While **maintaining features** like

- Adaptive mesh refinement (AMR) ✓
- Parallelization ✓
- Visualization ✓
- Restarting ✓

⇒ Use `P4est.jl`[a], wrapper around `p4est`[b] library



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NASA Turbulence Modeling Resource (TMR)

Plenty high-quality meshes for

- Airfoils
- Jets
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- ...

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Langley Research Center

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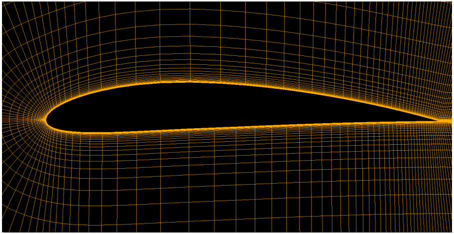
Langley Research Center
Turbulence Modeling Resource

Here:
Focus on **2D NACA 4412 airfoil**

→ 14336 Quadrilateral elements



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Case Study: NACA 4412 Airfoil

Mesh import workflow

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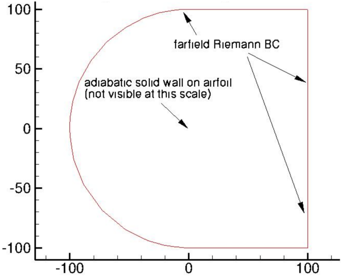

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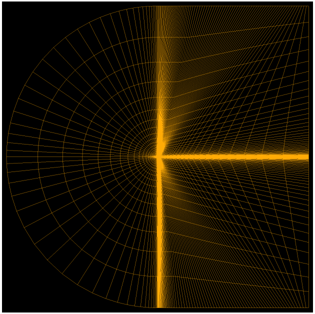

Case Study: NACA 4412 Airfoil

Mesh import workflow

1. Download 3D mesh & boundary information file
2. Convert `.p3d` to `.msh` via `p3d2gmsh.py`
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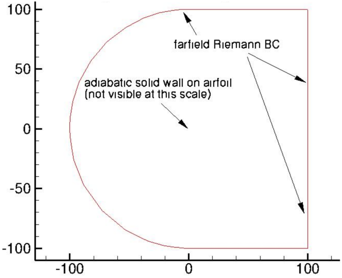

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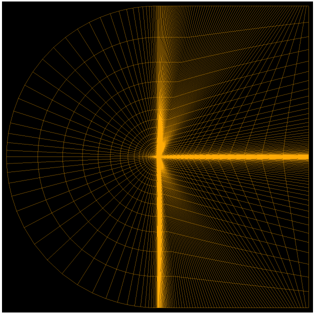

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3. Geometric ID assignment
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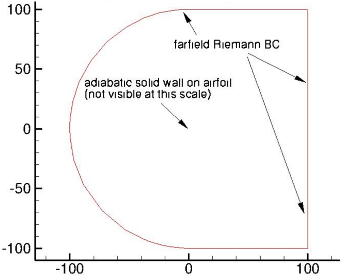

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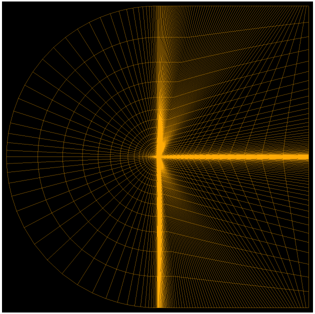

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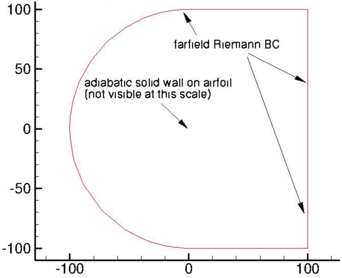
4. Export `.msh` via `Gmsh` to `.inp`



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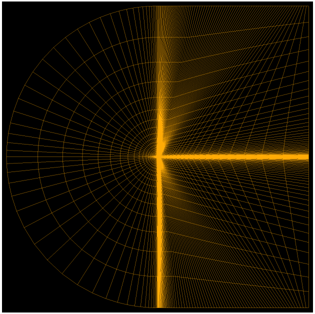

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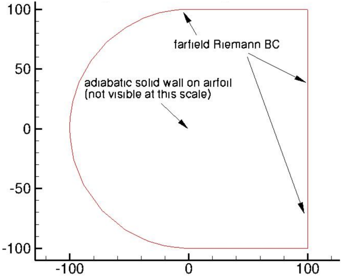
4. Export `.msh` via `Gmsh` to `.inp`
5. Truncate 3D to 2D in `.inp` format
   → `Abaqus3Dto2D.jl`



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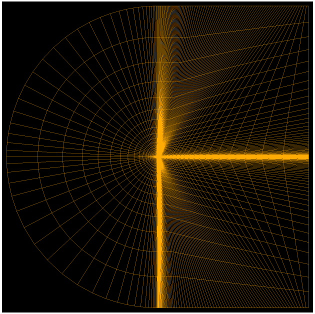

Case Study: NACA 4412 Airfoil

→ Ready for Trixi.jl:

- Mesh

```julia
using Trixi

# 225x65 C-grid
mesh_file = "NACA4412_2_2D_unique.inp"
boundary_symbols = [# Farfield (Riemann-invariant) boundaries
                    :b2_symmetry_y_strong, :b4_farfield_riem,
                    :b5_farfield_riem, :b7_farfield_riem,
                    :b8_to_stitch_a,
                    # Airfoil
                    :b6_viscous_solid]

k = 3 # Local polynomial degree
mesh = P4estMesh{2}(mesh_file, polydeg = k,
                    boundary_symbols = boundary_symbols)
```

Case Study: NACA 4412 Airfoil

Inviscid transonic flow

- Equations & Solver

```
gamma = 1.4 # Ratio of specific heats
equations = CompressibleEulerEquations2D(gamma)
```

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gamma = 1.4 # Ratio of specific heats
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basis = LobattoLegendreBasis(k)
shock_indicator = IndicatorHennemannGassner(equations,
                                            basis,
                                            variable = density_pressure)
volume_integral = VolumeIntegralShockCapturingHG(shock_indicator;
                                                 volume_flux_dg = flux_chandrashekar,
                                                 volume_flux_fv = flux_hllc)
# About 1.15 · 10^6 degrees of freedom
solver = DGSEM(polydeg = k,
               surface_flux = flux_hllc,
               # Use split-form DG
               volume_integral = volume_integral)
```

Case Study: NACA 4412 Airfoil

Inviscid transonic flow

- Initial Condition
  $\mathrm{Ma}_\infty = 0.8, \alpha = 1.25^\circ$

```julia
@inline function initial_condition_mach08_flow(x, t, equations)
    # Non-dimensionalized (for gas with γ = 1.4)
    rho_freestream = 1.4
    v1 = 0.7998096216639273   # 0.8 * cos(1.25°)
    v2 = 0.017451908027648896 # 0.8 * sin(1.25°)
    p_freestream = 1.0

    prim = SVector(rho_freestream, v1, v2, p_freestream)
    return prim2cons(prim, equations)
end

initial_condition = initial_condition_mach08_flow
```

Case Study: NACA 4412 Airfoil

Inviscid transonic flow

- Boundary Conditions & Semidiscretization

```julia
bc_free_stream = BoundaryConditionDirichlet(initial_condition)

bc = Dict(# Farfield boundaries
          :b2_symmetry_y_strong => bc_free_stream,
          :b4_farfield_riem => bc_free_stream,
          :b5_farfield_riem => bc_free_stream,
          :b7_farfield_riem => bc_free_stream,
          :b8_to_stitch_a => bc_free_stream,
          # Airfoil
          :b6_viscous_solid => boundary_condition_slip_wall)
```

Case Study: NACA 4412 Airfoil

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bc_free_stream = BoundaryConditionDirichlet(initial_condition)

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semi = SemidiscretizationHyperbolic(mesh, equations,
                                    initial_condition, solver;
                                    boundary_conditions = bc)
```

Case Study: NACA 4412 Airfoil

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semi = SemidiscretizationHyperbolic(mesh, equations,
                                    initial_condition, solver;
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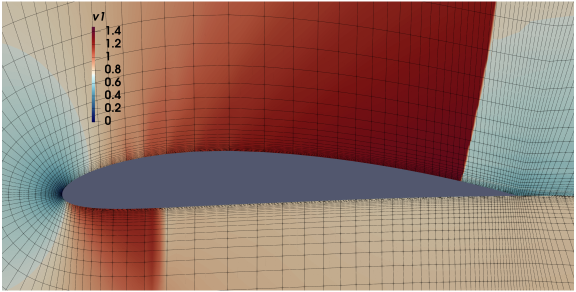
tspan = (0.0, 10.0)
ode = semidiscretize(semi, tspan) # Compatible with OrdinaryDiffEq.jl
```

Case Study: NACA 4412 Airfoil

Inviscid transonic flow

$t_0 = 10.0 \rightarrow t_f = 10.5$

- NASA TMR mesh



Case Study: NACA 4412 Airfoil

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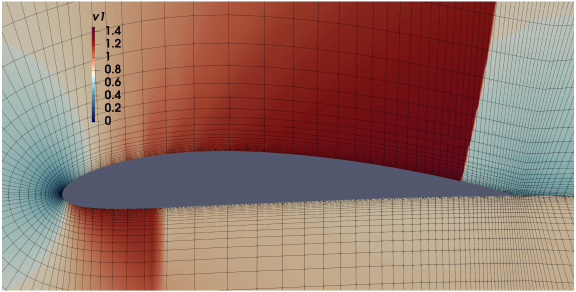
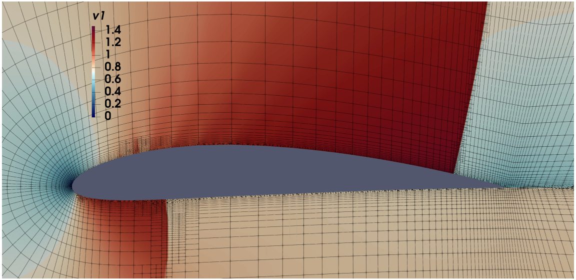


- NASA TMR mesh
  + 3-Level AMR



Case Study: NACA 4412 Airfoil

Viscous transonic flow

- Viscous Equations & Boundary Conditions

```
prandtl_number() = 0.72 # Pr = μc_p/λ
mu() = 1.12e-7 # Constant dynamic viscosity μ ⇒ Re = 10^7
equations_parabolic = CompressibleNavierStokesDiffusion2D(equations, # Euler equations
                                                          mu = mu(),
                                                          Prandtl = prandtl_number())
```

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velocity_airfoil = NoSlip((x, t, equations) -> SVector(0.0, 0.0))
heat_airfoil = Adiabatic((x, t, equations) -> 0.0)

bc_airfoil = BoundaryConditionNavierStokesWall(velocity_airfoil,
                                               heat_airfoil)
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bc_parabolic = Dict(:b2_symmetry_y_strong => bc_free_stream,
                    #= Other free-stream BCs =#
                    :b6_viscous_solid => bc_airfoil)
```

Case Study: NACA 4412 Airfoil

Viscous transonic flow

- Semidiscretization

```
#                                         ⇓                                 ⇓
semi = SemidiscretizationHyperbolicParabolic(mesh, (equations, equations_parabolic),
                                              initial_condition, solver;
                                              boundary_conditions = (bc, # ⇓
                                                                     bc_parabolic))
```

Case Study: NACA 4412 Airfoil

Viscous transonic flow

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#                                         ⇓                            ⇓
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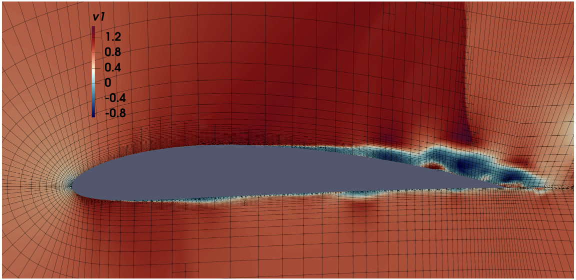
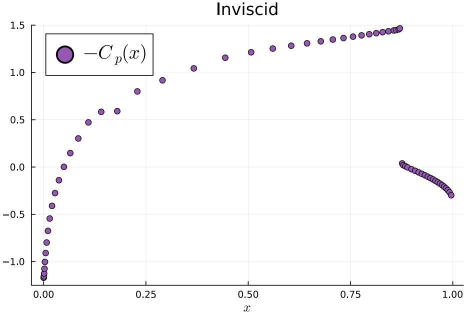


Case Study: NACA 4412 Airfoil

Pressure Coefficients

$$C_p(x) = \frac{p(x) - p_\infty}{2\rho_\infty L_\infty u_\infty^2}$$



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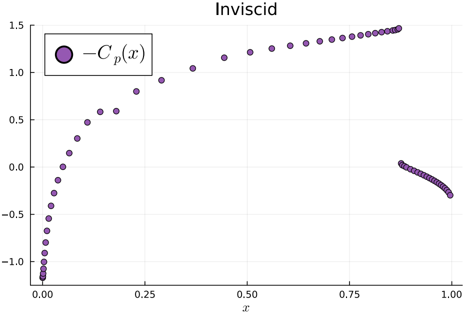

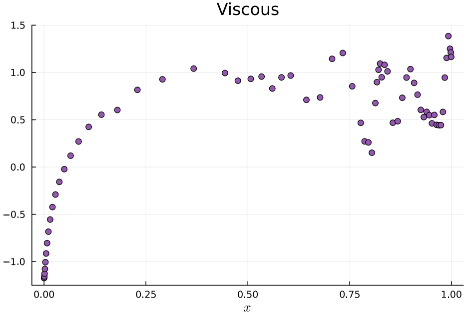

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Case Study: SD7003 Airfoil

Viscous laminar (incompressible) flow

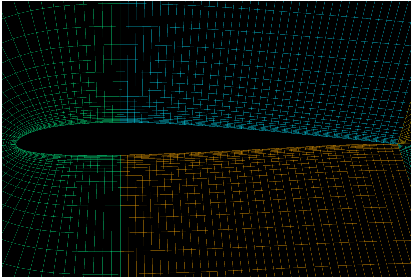
- $\mathrm{Re} = 10^4, \mathrm{Ma} = 0.2$
- $\mathrm{Pr} = 0.72, \gamma = 1.4$
- $\alpha = 4°$

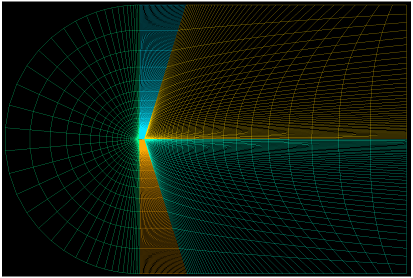
Mesh from ESM in[a]

- $x_{\mathrm{min}} = -20, x_{\mathrm{max}} = 60,$
  $y_{\mathrm{min}} = -20, y_{\mathrm{max}} = 20$
- 7605 elements

Discretization:

- Space: 4th-order DGSEM
- Entropy-stable flux-differencing
- Time: 4th-order Multirate P-ERK

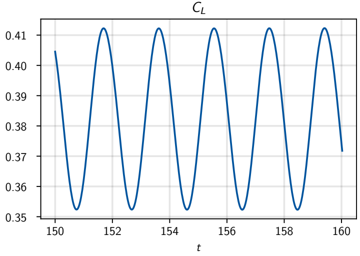


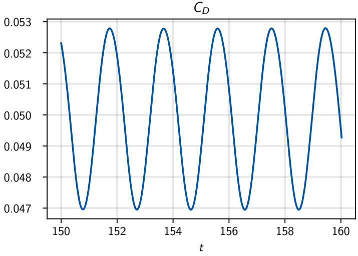


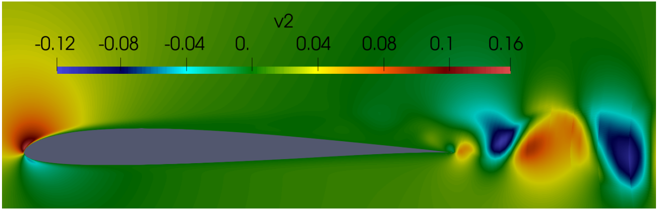
[a] B. C. Vermeire, P-ERK for Stiff Systems of Equations, JCP, 2019.

Case Study: SD7003 Airfoil

Viscous laminar (incompressible) flow







Case Study: SD7003 Airfoil

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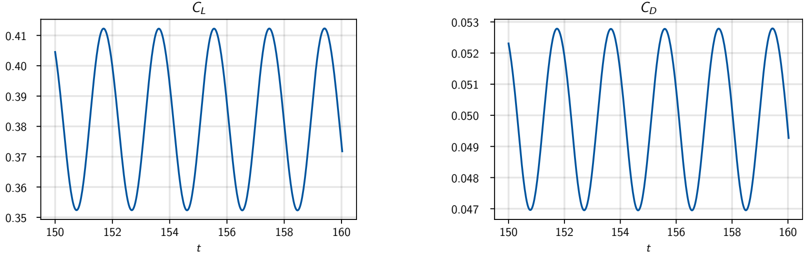

Source	$\overline{C}_L = \overline{C}_{L,p}$	$\overline{C}_D = \overline{C}_{D,p} + \overline{C}_{D,\mu}$
Trixi.jl	0.3827	0.04995
Vermeire[a]	0.3841	0.04990
Uranga et al.[b]	0.3755	0.04978
López-Morales et al.[c]	0.3719	0.04940

[a] B. C. Vermeire, JCP, 2019.

[b] A. Uranga et al., Int. J. Num Methods Engineering, 2011

[c] M. López-Morales et al., 32nd AIAA AAC, 2014

Viscous flows: Extensions

Sutherland's law

$$\mu(T) = \mu_{\text{ref}} \left(\frac{T_{\text{ref}} + C}{T + C} \right) \left(\frac{T}{T_{\text{ref}}} \right)^{1.5}$$

```julia
#mu() = 1.12e-7
@inline function mu(u, equations)
    T_ref = 291.15

    R_specific_air = 287.052874
    T = R_specific_air * Trixi.temperature(u, equations) # Re-dim. temperature

    C_air = 120.0
    mu_ref_air = 1.827e-5

    return mu_ref_air * (T_ref + C_air) / (T + C_air) * (T / T_ref)^1.5
end
```

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end

equations_parabolic = CompressibleNavierStokesDiffusion2D(equations,
                                                          mu = mu, # mu = mu(),
                                                          Prandtl = prandtl_number())
```

Outline

1. Scope of the Talk
2. Case Study: NACA 4412 Airfoil
3. Extensions
4. Conclusion & Outlook

Conclusion & Outlook

Summary:

- Import of existing quadrilateral meshes in `.inp` format

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ToDo:

- Higher order (curved) quad/hex elements from `Gmsh`/ in `.inp` format

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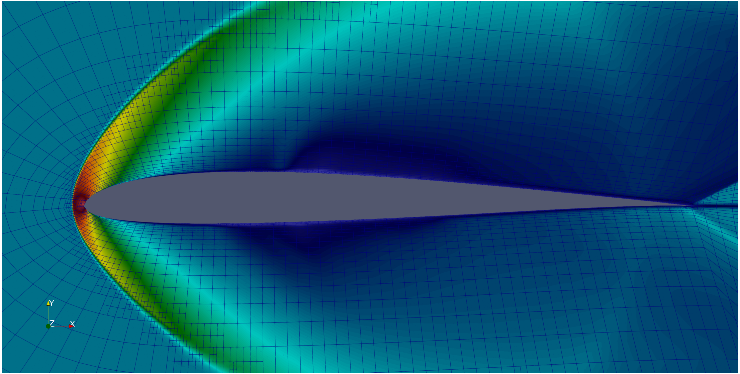
ToDo:

- Higher order (curved) quad/hex elements from `Gmsh`/ in `.inp` format
- Beyond quads/hexes with `T8Code.jl`
  → `https://github.com/DLR-AMR/T8code.jl`

Thank you for your attention!

Questions?



Density profile for invsicid Mach 2 flow around SD7003 airfoil with AMR.