

# Lecture 11 - Time Series Analysis



# 1. Overview

**Time series analysis** is crucial for financial data, as stock prices, economic indicators, and sales forecasts are often dependent on time.

## What is Time Series Data?

- A time series is a sequence of data points recorded at successive and equally spaced points in time.
  - Examples in Finance: Stock prices, interest rates, GDP growth, and exchange rates.

## Components of Time Series Data

- **Trend:** Long-term increase or decrease in the data.
- **Seasonality:** Repeating patterns or cycles (e.g., sales increasing during the holiday season).
- **Noise/Residual:** Random fluctuations that are not explained by the model.



This notebook covers:

1. The **basics of time series** data and its components.
2. How to **manipulate** and **visualize** time series data with `pandas` and `matplotlib`.
3. Apply **basic time series models** such as **moving averages** and **correlations**.

Setting the environment

```
In [1]: import numpy as np
import pandas as pd
from pylab import mpl, plt
plt.style.use('seaborn-v0_8-dark')
mpl.rcParams['font.family'] = 'serif'
%matplotlib inline
```

## 2. Data inspection

The first part of the analysis is to **inspect** the data set containing the timeseries.

### **Inspection steps:**

1. **Import** data
2. Generate **summary statistics**
3. Analysis **changes over time**
4. Adjust **frequency (resampling)**

## 2.1 Data import

For this part, we work with a standard `csv` database obtained from the **Thomson Reuters Eikon Data**. The data contains **end-of-day (EOD) price data** for a selection of instruments.

The following parameters apply:

```
file_path = 'Data/11/'  
file_name = 'tr_eikon_eod_data.csv'
```

Check file

```
In [2]: # Data from the Thomson Reuters (TR) Eikon Data API  
file_path = 'Data/11/'  
file_name = 'tr_eikon_eod_data.csv'  
file = open(file_path + file_name, 'r')
```

```
In [3]: file.readlines()[:5]
```

```
Out[3]: ['Date,AAPL.O,MSFT.O,INTC.O,AMZN.O,GS.N,SPY,.SPX,.VIX,EUR=,XAU  
=,GDX,GLD\n',  
'2010-01-04,30.57282657,30.95,20.88,133.9,173.08,113.33,1132.  
99,20.04,1.4411,1120.0,47.71,109.8\n',  
'2010-01-05,30.62568366000004,30.96,20.87,134.69,176.14,113.  
63,1136.52,19.35,1.4368,1118.65,48.17,109.7\n',  
'2010-01-06,30.13854129000003,30.77,20.8,132.25,174.26,113.7  
1,1137.14,19.16,1.4412,1138.5,49.34,111.51\n',  
'2010-01-07,30.08282706000003,30.452,20.6,130.0,177.67,114.1  
9,1141.69,19.06,1.4318,1131.9,49.1,110.82\n']
```

```
In [4]: file.close()
```

Import into `dataframe`

```
In [5]: # index_col = 0: the first column shall be handled as an index.  
# parse_dates = True: the index values are of type datetime.  
data = pd.read_csv(file_path + file_name, index_col = 0, parse_dates =
```

- Use **time as label** on `index_col`
- Explicitly interpret as `datetime` object on `parse_dates`
  - from documentation: If `True` -> try parsing the index.

Inspect `dataframe`

In [6]: `data.info()`

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 1972 entries, 2010-01-04 to 2017-10-31
Data columns (total 12 columns):
 #   Column   Non-Null Count   Dtype  
--- 
 0   AAPL.O    1972 non-null    float64
 1   MSFT.O    1972 non-null    float64
 2   INTC.O    1972 non-null    float64
 3   AMZN.O    1972 non-null    float64
 4   GS.N     1972 non-null    float64
 5   SPY       1972 non-null    float64
 6   .SPX      1972 non-null    float64
 7   .VIX      1972 non-null    float64
 8   EUR=      1972 non-null    float64
 9   XAU=      1972 non-null    float64
 10  GDX       1972 non-null    float64
 11  GLD       1972 non-null    float64
dtypes: float64(12)
memory usage: 200.3 KB
```

```
In [7]: data.head()
```

```
Out[7]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	E
Date									
2010-01-04	30.572827	30.950	20.88	133.90	173.08	113.33	1132.99	20.04	1.
2010-01-05	30.625684	30.960	20.87	134.69	176.14	113.63	1136.52	19.35	1.
2010-01-06	30.138541	30.770	20.80	132.25	174.26	113.71	1137.14	19.16	1.
2010-01-07	30.082827	30.452	20.60	130.00	177.67	114.19	1141.69	19.06	1.
2010-01-08	30.282827	30.660	20.83	133.52	174.31	114.57	1144.98	18.13	1.



```
In [8]: data.tail()
```

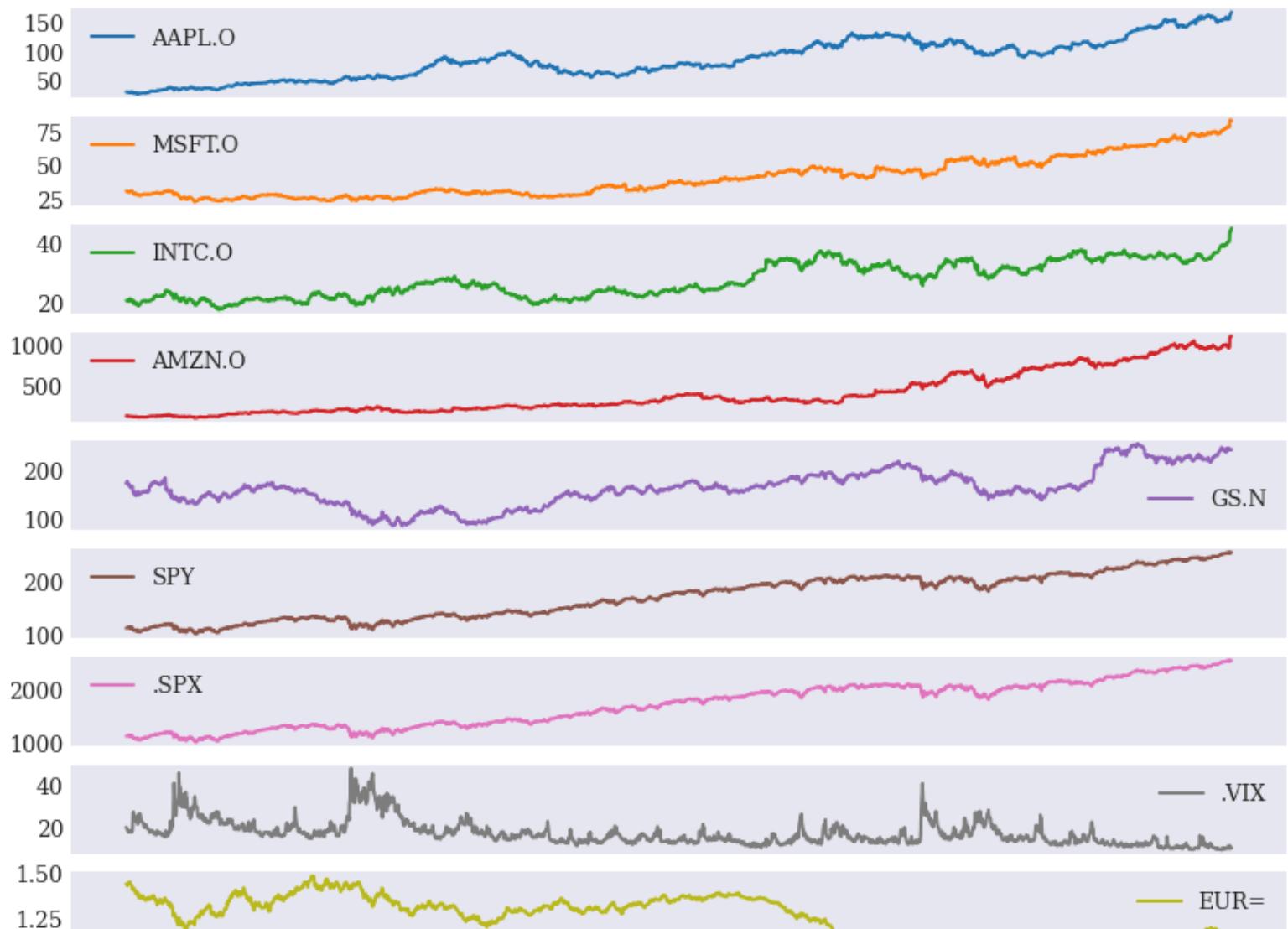
```
Out[8]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EU
Date									
2017-10-25	156.41	78.63	40.78	972.91	241.71	255.29	2557.15	11.23	1.18
2017-10-26	157.41	78.76	41.35	972.43	241.72	255.62	2560.40	11.30	1.16
2017-10-27	163.05	83.81	44.40	1100.95	241.71	257.71	2581.07	9.80	1.16
2017-10-30	166.72	83.89	44.37	1110.85	240.89	256.75	2572.83	10.50	1.16
2017-10-31	169.04	83.18	45.49	1105.28	242.48	257.15	2575.26	10.18	1.16



## Visualize timeseries

```
In [9]: data.plot(figsize = (10,12), subplots = True);
```



## Add labels

### Labeling from *Reuters Instrument Codes* (RICs)

```
In [10]: instruments = ['Apple Stock', 'Microsoft Stock',
                     'Intel Stock', 'Amazon Stock', 'Goldman Sach
                     'SPDR S&P 500 ETF Trust', 'S&P 500 Index',
                     'VIX Volatility Index', 'EUR/USD Exchange Ra
                     'Gold Price', 'VanEck Vectors Gold Miners ET
                     'SPDR Gold Trust']
```

```
In [11]: for ric, name in zip(data.columns, instruments):
            print('{:8s} | {}'.format(ric, name))
```

AAPL.O	Apple Stock
MSFT.O	Microsoft Stock
INTC.O	Intel Stock
AMZN.O	Amazon Stock
GS.N	Goldman Sachs Stock
SPY	SPDR S&P 500 ETF Trust
.SPX	S&P 500 Index
.VIX	VIX Volatility Index
EUR=	EUR/USD Exchange Rate
XAU=	Gold Price
GDX	VanEck Vectors Gold Miners ETF
GLD	SPDR Gold Trust

## 2.2 Summary statistics

## Built-in tools

```
In [12]: data.describe().round(2)
```

```
Out[12]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX
<b>count</b>	1972.00	1972.00	1972.00	1972.00	1972.00	1972.00	1972.00	1972.00
<b>mean</b>	86.53	40.59	27.70	401.15	163.61	172.84	1727.54	17.21
<b>std</b>	34.04	14.39	5.95	257.12	37.17	42.33	424.35	5.92
<b>min</b>	27.44	23.01	17.66	108.61	87.70	102.20	1022.58	9.19
<b>25%</b>	57.57	28.12	22.23	202.66	144.23	132.64	1325.53	13.25
<b>50%</b>	84.63	36.54	26.41	306.42	162.09	178.80	1783.81	15.65
<b>75%</b>	111.87	50.08	33.74	559.45	184.11	208.01	2080.15	19.20
<b>max</b>	169.04	83.89	45.49	1110.85	252.89	257.71	2581.07	48.00

```
In [13]: data.mean()
```

```
Out[13]:
```

AAPL.O	86.530152
MSFT.O	40.586752
INTC.O	27.701411
AMZN.O	401.154006
GS.N	163.614625
SPY	172.835399
.SPX	1727.538342
.VIX	17.209498
EUR=	1.252613
XAU=	1352.471593
GDX	34.499391
GLD	130.601856
	dtype: float64

Customized statistics

```
In [14]: data.aggregate(['min', 'mean', 'std', 'median', 'max']).round(2)
```

```
Out[14]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EU
min	27.44	23.01	17.66	108.61	87.70	102.20	1022.58	9.19	1
mean	86.53	40.59	27.70	401.15	163.61	172.84	1727.54	17.21	1
std	34.04	14.39	5.95	257.12	37.17	42.33	424.35	5.92	0
median	84.63	36.54	26.41	306.42	162.09	178.80	1783.81	15.65	1
max	169.04	83.89	45.49	1110.85	252.89	257.71	2581.07	48.00	1

## 2.3 Changes over time

Statistical analysis methods are often based on **changes over time** and not the absolute values themselves.

There are multiple options to calculate the changes in a time series over time:

- Absolute differences
- Percentage changes
- Logarithmic (log) returns.

## Absolute differences

`.diff()`: subtracts each row's value from the value in the previous row.

- It reveals the exact change in values from one time step to the next.
- The method returns a `dataframe`

```
In [15]: data.diff().head()
```

```
Out[15]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EUR=
Date									
2010-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-05	0.052857	0.010	-0.01	0.79	3.06	0.30	3.53	-0.69	-0.0043
2010-01-06	-0.487142	-0.190	-0.07	-2.44	-1.88	0.08	0.62	-0.19	0.0044
2010-01-07	-0.055714	-0.318	-0.20	-2.25	3.41	0.48	4.55	-0.10	-0.0094
2010-01-08	0.200000	0.208	0.23	3.52	-3.36	0.38	3.29	-0.93	0.0094



```
In [16]: data.diff(periods=2)
```

Out [16]:	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EUR
Date									
2010-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-05	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-06	-0.434285	-0.180	-0.08	-1.65	1.18	0.38	4.15	-0.88	0.000
2010-01-07	-0.542857	-0.508	-0.27	-4.69	1.53	0.56	5.17	-0.29	-0.005
2010-01-08	0.144286	-0.110	0.03	1.27	0.05	0.86	7.84	-1.03	0.000
...	...	...	...	...	...	...	...	...	...
2017-10-25	0.240000	-0.200	-0.05	6.61	-0.42	-0.82	-7.83	0.16	0.006
2017-10-26	0.310000	-0.100	0.40	-3.47	-3.12	-0.94	-8.73	0.14	-0.010
2017-10-27	6.640000	5.180	3.62	128.04	0.00	2.42	23.92	-1.43	-0.020



```
In [17]: data.diff().mean()
```

```
Out[17]: AAPL.O      0.070252  
MSFT.O      0.026499  
INTC.O      0.012486  
AMZN.O      0.492836  
GS.N       0.035211  
SPY         0.072968  
.SPX        0.731745  
.VIX        -0.005003  
EUR=        -0.000140  
XAU=        0.076712  
GDX         -0.012801  
GLD         0.005515  
dtype: float64
```

## Percentage changes

`.pct_change()` : calculates the percentage change between consecutive rows

- It reveals the relative change in values from one time step to the next.
- The method returns a `dataframe`

```
In [18]: data.pct_change().round(3).head()
```

```
Out[18]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EUR=
Date									
2010-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-05	0.002	0.000	-0.000	0.006	0.018	0.003	0.003	-0.034	-0.003
2010-01-06	-0.016	-0.006	-0.003	-0.018	-0.011	0.001	0.001	-0.010	0.003
2010-01-07	-0.002	-0.010	-0.010	-0.017	0.020	0.004	0.004	-0.005	-0.007
2010-01-08	0.007	0.007	0.011	0.027	-0.019	0.003	0.003	-0.049	0.007



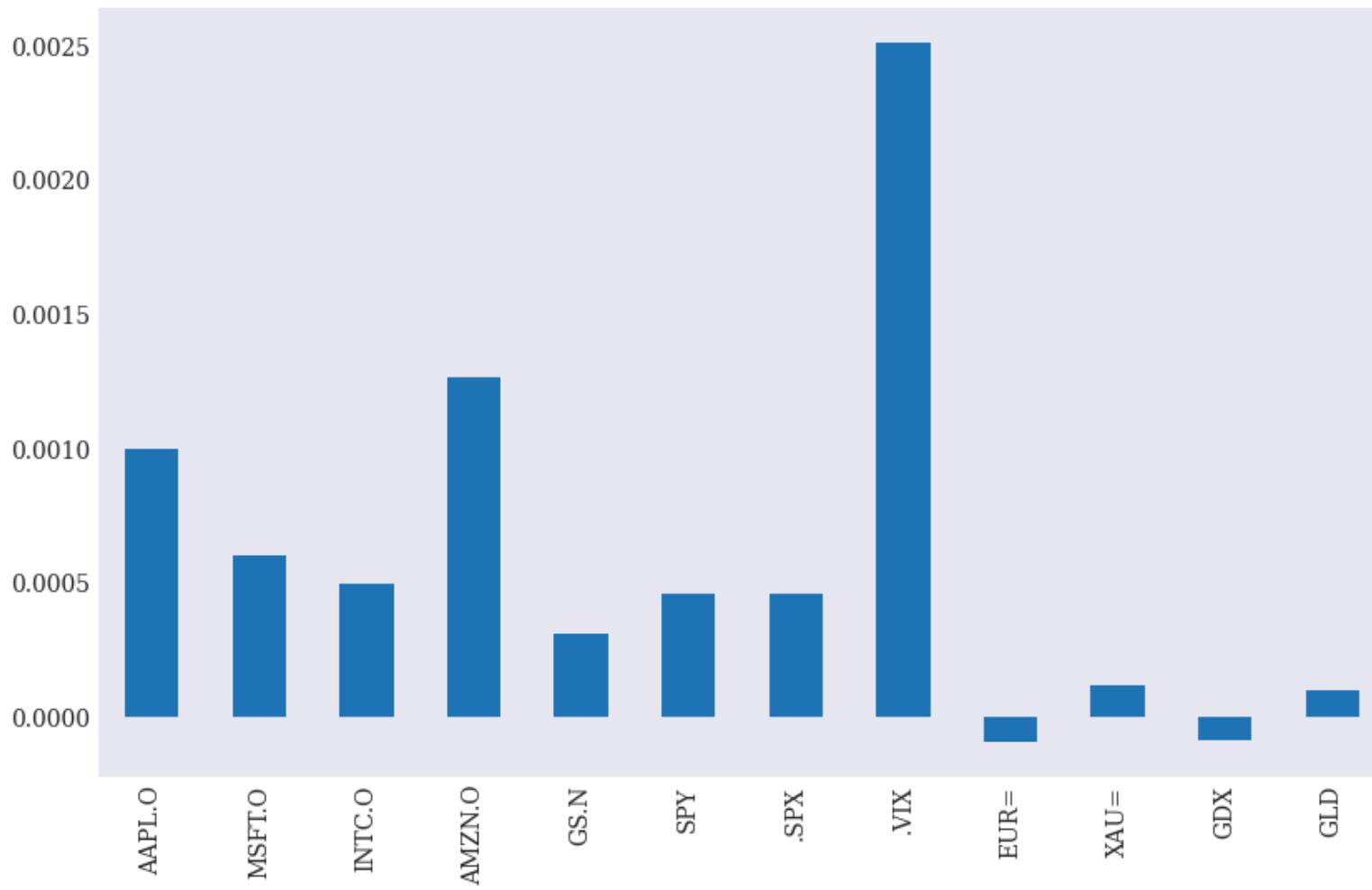
```
In [19]: data.pct_change(periods = 7).round(3).head(10)
```

```
Out[19]: AAPL.O  MSFT.O  INTC.O  AMZN.O  GS.N  SPY  .SPX  .VIX  EUR
```

Date	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EUR
2010-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-05	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-06	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-07	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-08	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-11	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-12	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2010-01-13	-0.016	-0.019	0.004	-0.036	-0.023	0.011	0.011	-0.109	0.00
2010-01-14	-0.023	0.000	0.029	-0.054	-0.043	0.011	0.011	-0.089	0.00



```
In [20]: data.pct_change().mean().plot(kind = 'bar', figsize = (10,6));
```



## Log Returns

**Logarithmic (log) returns** of time series data are the standard means to analyze returns on investments over time.

The formula is given by

$$\text{Log Return} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

In `pandas`, the denominator naturally obtains by shifting data by one row using the `.shift()` method.

```
In [21]: rrets = np.log(data / data.shift(1))
```

```
In [22]: rets.head().round(2)
```

```
Out[22]:
```

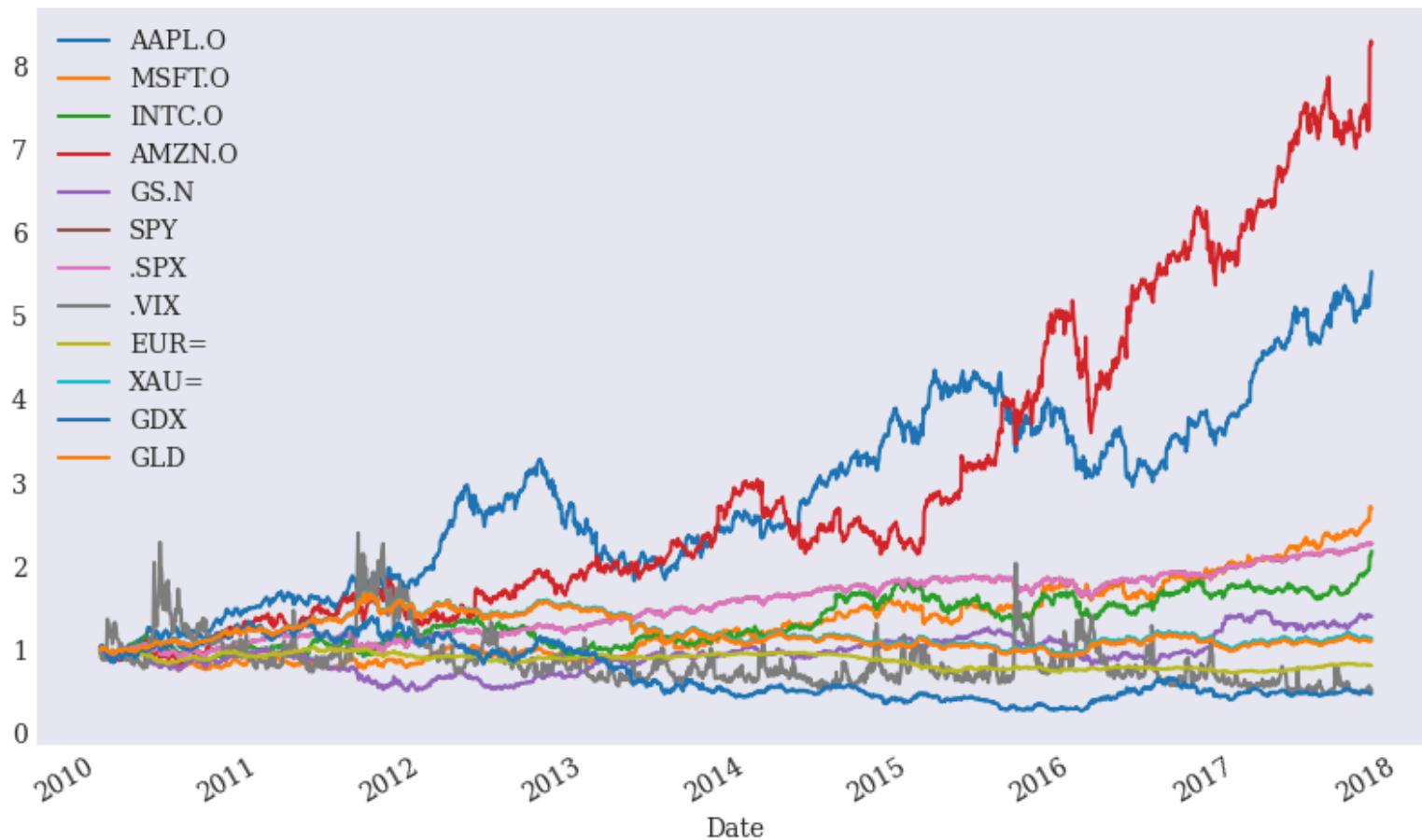
	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	EUR=	XAU
Date										
2010-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	N
2010-01-05	0.00	0.00	-0.00	0.01	0.02	0.0	0.0	-0.04	-0.00	-0.
2010-01-06	-0.02	-0.01	-0.00	-0.02	-0.01	0.0	0.0	-0.01	0.00	0.
2010-01-07	-0.00	-0.01	-0.01	-0.02	0.02	0.0	0.0	-0.01	-0.01	-0
2010-01-08	0.01	0.01	0.01	0.03	-0.02	0.0	0.0	-0.05	0.01	0.



**Cumulative returns** over a period are obtained by summing up the log returns for each interval and then exponentiate the result:

$$\text{Cumulative Return} = e^{\sum \text{Log Returns}}$$

```
In [23]: rets.cumsum().apply(np.exp).plot(figsize = (10,6));
```



## 2.4 Resampling

**Resampling** of financial time series data refers to the process of **converting the frequency of data points** in a time series.

The `resample()` method in `pandas` is used to change the frequency of time series data.

```
data.resample(rule, label='right', closed='right', kind='timestamp')
```

Parameters:

1. `rule`: This is a required parameter and specifies the new frequency for resampling.

Some common time-based frequency strings are:

- '`D`' : Day
- '`W`' : Week
- '`M`' : Month
- '`Q`' : Quarter
- '`A`' : Year

One can also specify intervals like '5min', '15T' (15 minutes), '3H' (3 hours), etc.

2. `label`: Determines how the timestamp labels in the resulting data are aligned:
  - `'right'` : Assigns the label to the end of the resampling period (e.g., a week ending on Sunday will be labeled as Sunday).
  - `'left'` : Assigns the label to the beginning of the resampling period (e.g., the first day of the week).

3. `closed` : Specifies which side of each interval is closed:

- `'right'` : The interval includes the right endpoint.
- `'left'` : The interval includes the left endpoint.

4. `kind` : Defines the type of index used:

- `'timestamp'` : Generates a DatetimeIndex.
- `'period'` : Generates a PeriodIndex.

**Aggregation functions:** After resampling, you can apply an aggregation method directly, like `mean()`, `sum()`, `last()`, `first()`, `count()`, etc. These specify how to aggregate data within each new time interval.

```
In [24]: data.resample('1W', label='right').last().head()
```

Out [24]:

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX	E
Date									
2010-01-10	30.282827	30.66	20.83	133.52	174.31	114.57	1144.98	18.13	1.
2010-01-17	29.418542	30.86	20.80	127.14	165.21	113.64	1136.03	17.91	1.
2010-01-24	28.249972	28.96	19.91	121.43	154.12	109.21	1091.76	27.31	1.
2010-01-31	27.437544	28.18	19.40	125.41	148.72	107.39	1073.87	24.62	1.
2010-02-07	27.922829	28.02	19.47	117.39	154.16	106.66	1066.19	26.11	1.

```
In [25]: # Resample to quarterly data, labeling periods at the start of the quarter
data.resample('QE', label='left').mean().head()
```

```
Out[25]:
```

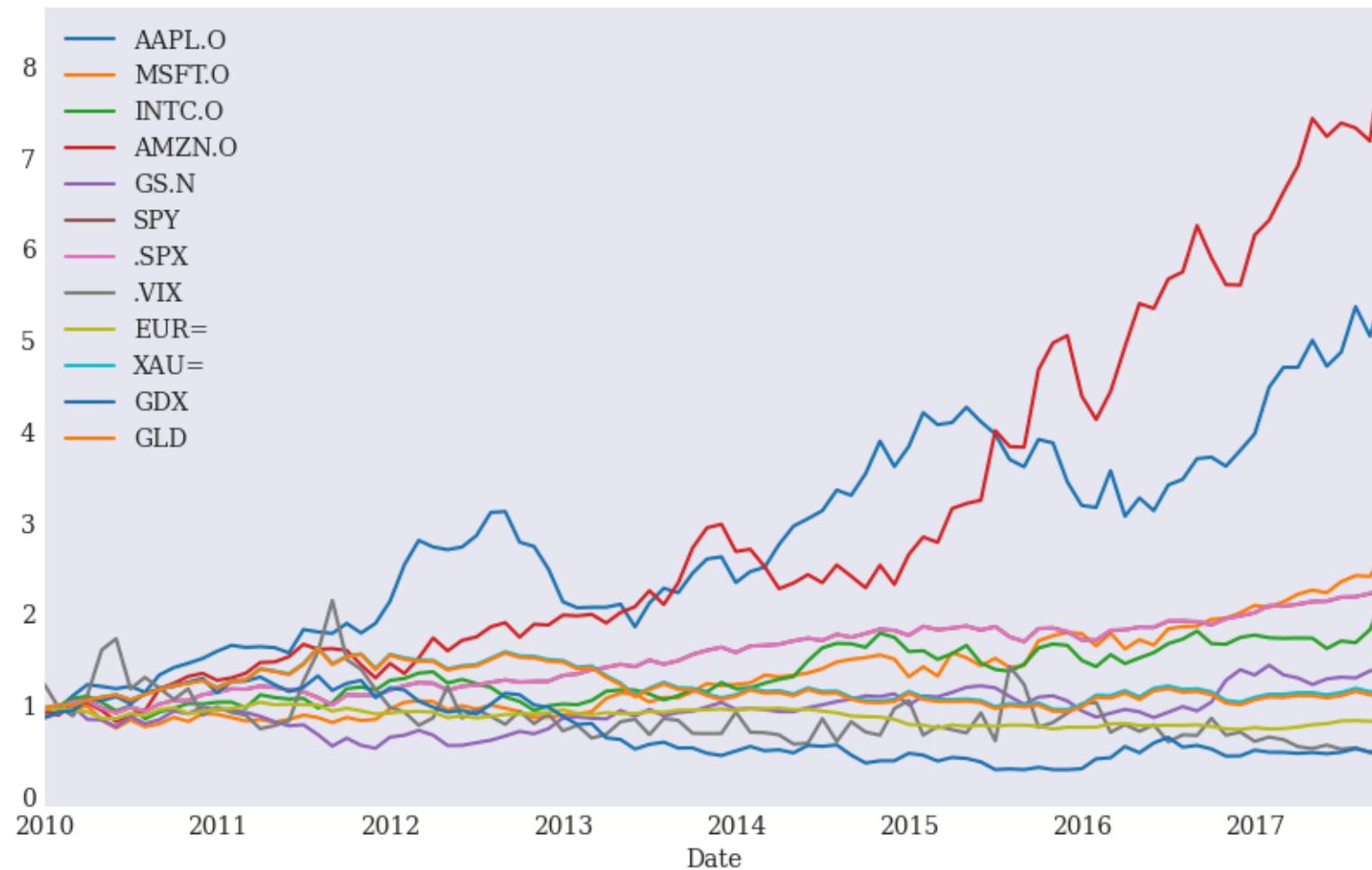
	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY
Date						
2009-12-31	30.122098	29.254115	20.850836	125.804541	164.081639	112.546607
2010-03-31	36.391239	27.977135	21.917363	130.020095	148.985873	113.694198
2010-06-30	37.129258	24.790383	19.658259	130.454059	146.242969	109.856767
2010-09-30	44.500887	26.330313	20.678320	169.520000	161.686406	120.645516
2010-12-31	49.383274	26.989895	21.107540	176.948145	164.398387	130.385081

```
In [26]: data.resample('1ME', label = 'right').last().head()
```

```
Out[26]:
```

	AAPL.O	MSFT.O	INTC.O	AMZN.O	GS.N	SPY	.SPX	.VIX
Date								
2010-01-31	27.437544	28.1800	19.40	125.41	148.72	107.3900	1073.87	24.62
2010-02-28	29.231399	28.6700	20.53	118.40	156.35	110.7400	1104.49	19.50
2010-03-31	33.571395	29.2875	22.29	135.77	170.63	117.0000	1169.43	17.59
2010-04-30	37.298534	30.5350	22.84	137.10	145.20	118.8125	1186.69	22.05
2010-05-31	36.697106	25.8000	21.42	125.46	144.26	109.3690	1089.41	32.07

```
In [27]: rets.cumsum().apply(np.exp).resample('1ME', label='right').last().plot()
```



### 3. Rolling statistics

A **rolling window** is a technique used to **apply a calculation to a specific, fixed-size subset** of data, which “rolls” or **moves across a dataset** as a window.

The purpose of a rolling window is to compute statistics, like the mean or standard deviation, for consecutive subsets of data points, creating a dynamic, time-dependent **view of trends, averages, or variability**.

This technique is commonly used in time series analysis, especially in finance, to **understand patterns over time while smoothing out short-term fluctuations**.



In Python, the `.rolling()` method in `pandas` is used to apply a rolling window to a `DataFrame` or `Series`.

This method returns a “**rolling**” object that can apply various aggregation functions, like `.mean()`, `.std()`, `.min()`, etc., over the rolling window.

```
data.rolling(window=window_size).function()
```

```
In [28]: # Let's focus on a single financial time series  
sym = 'AAPL.O'  
data = pd.DataFrame(data[sym]).dropna()  
data.tail()
```

```
Out[28]:
```

AAPL.O	
Date	
2017-10-25	156.41
2017-10-26	157.41
2017-10-27	163.05
2017-10-30	166.72
2017-10-31	169.04

```
In [29]: window = 20
```

- Calculate rolling minimum (`min`) and maximum (`max`): identify the range of prices over the past 20 days.

```
In [30]: data['min'] = data[sym].rolling(window=window).min()
```

```
In [31]: data['max'] = data[sym].rolling(window=window).max()
```

- Calculate rolling mean ( `mean` ) and standard deviation ( `std` ): The rolling mean provides a smoothed version of the price series. It smooths out short-term fluctuations, highlighting the medium-term trend. The standard deviation statistic shows the volatility of the stock price over each 20-day period.

```
In [32]: data['mean'] = data[sym].rolling(window=window).mean()
```

```
In [33]: data['std'] = data[sym].rolling(window=window).std()
```

```
In [34]: data['median'] = data[sym].rolling(window=window).median()
```

- **Calculate Exponentially Weighted Moving Average ( `ewma` )**: Unlike a simple moving average, which weights all points equally, the **EWMA** gives more importance to recent observations, allowing it to react faster to recent price changes. The `halflife` parameter controls how quickly the weights decay, with a shorter halflife emphasizing more recent data.

```
In [35]: data['ewma'] = data[sym].ewm(halflife=0.5, min_periods=window).mean()
```

```
In [36]: data.head(25)
```

	AAPL.O	min	max	mean	std	median
Date						
2010-01-04	30.572827	NaN	NaN	NaN	NaN	NaN
2010-01-05	30.625684	NaN	NaN	NaN	NaN	NaN
2010-01-06	30.138541	NaN	NaN	NaN	NaN	NaN
2010-01-07	30.082827	NaN	NaN	NaN	NaN	NaN
2010-01-08	30.282827	NaN	NaN	NaN	NaN	NaN
2010-01-11	30.015684	NaN	NaN	NaN	NaN	NaN
2010-01-12	29.674256	NaN	NaN	NaN	NaN	NaN
2010-01-13	30.092827	NaN	NaN	NaN	NaN	NaN
2010-01-14	29.918542	NaN	NaN	NaN	NaN	NaN



```
In [37]: data.dropna().head()
```

```
Out[37]:
```

	AAPL.O	min	max	mean	std	median
Date						
2010-02-01	27.818544	27.437544	30.719969	29.580892	0.933650	29.821542
2010-02-02	27.979972	27.437544	30.719969	29.451249	0.968048	29.711113
2010-02-03	28.461400	27.437544	30.719969	29.343035	0.950665	29.685970
2010-02-04	27.435687	27.435687	30.719969	29.207892	1.021129	29.547113
2010-02-05	27.922829	27.435687	30.719969	29.099892	1.037811	29.419256

## - Plotting the Rolling Statistics:

```
In [40]: ax = data[['min', 'mean', 'max']].iloc[-200:].plot(  
    figsize = (10,6), style = ['g--', 'r--', 'g--'], lw = 0.8)  
data[sym].iloc[-200:].plot(ax = ax, lw = 2.0) ;
```



## Technical Analysis Example: SMAs

A decades-old trading strategy based on technical analysis is using **two simple moving averages** (SMAs):

### Trading strategy

- Go long on a stock (or financial instrument in general) when the shorter-term SMA is above the longer-term SMA
- Go short when the opposite holds true.

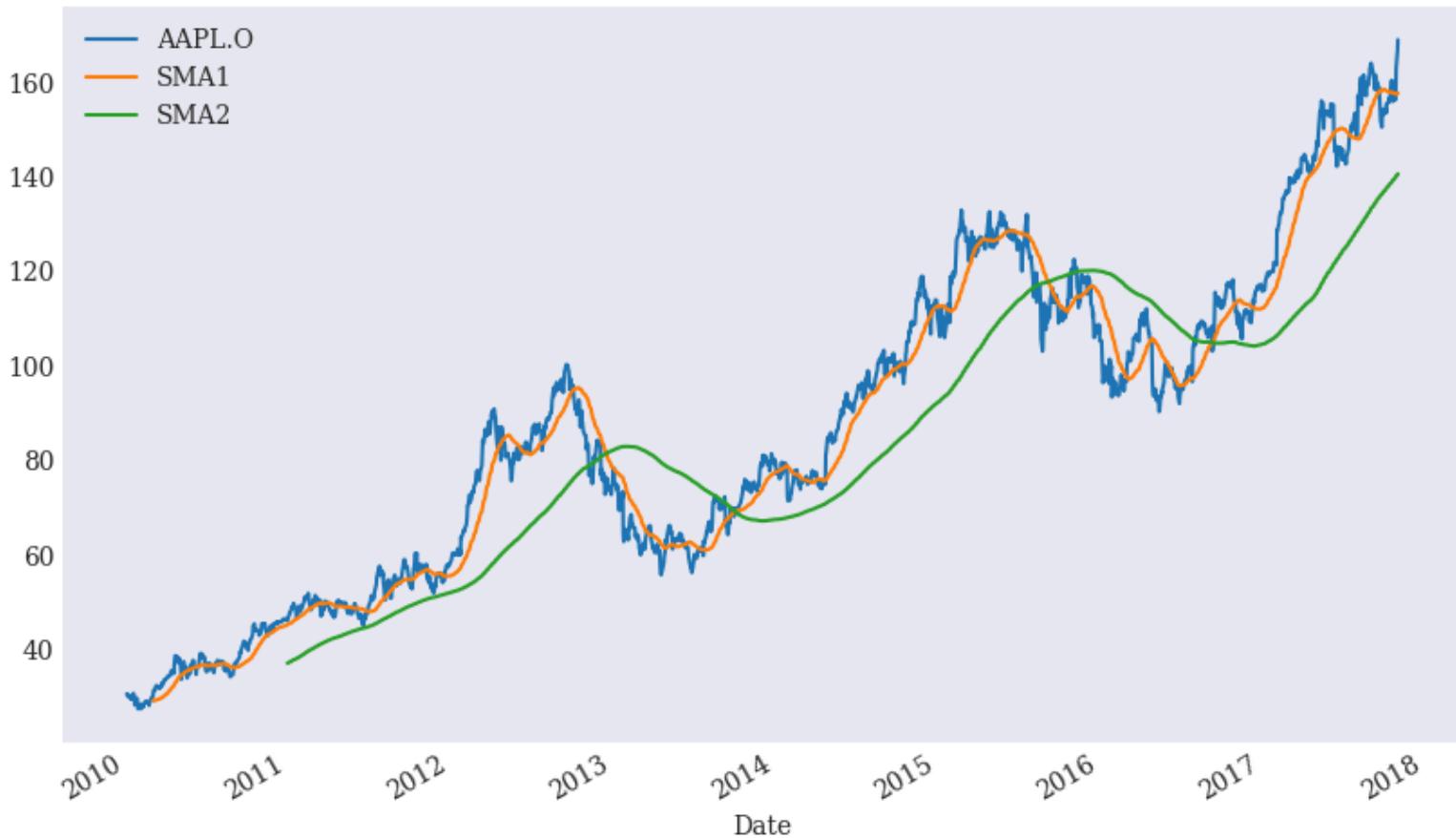
```
In [41]: data['SMA1'] = data[sym].rolling(window=42).mean()  
data['SMA2'] = data[sym].rolling(window=252).mean()
```

```
In [42]: data[[sym, 'SMA1', 'SMA2']].tail()
```

Out [42]:

	AAPL.O	SMA1	SMA2
Date			
2017-10-25	156.41	157.610952	139.862520
2017-10-26	157.41	157.514286	140.028472
2017-10-27	163.05	157.517619	140.221210
2017-10-30	166.72	157.597857	140.431528
2017-10-31	169.04	157.717857	140.651766

```
In [43]: data[[sym, 'SMA1', 'SMA2']].plot(figsize=(10, 6));
```



SMA<sub>s</sub> are then used to derive positions to implement a trading strategy.

Denote

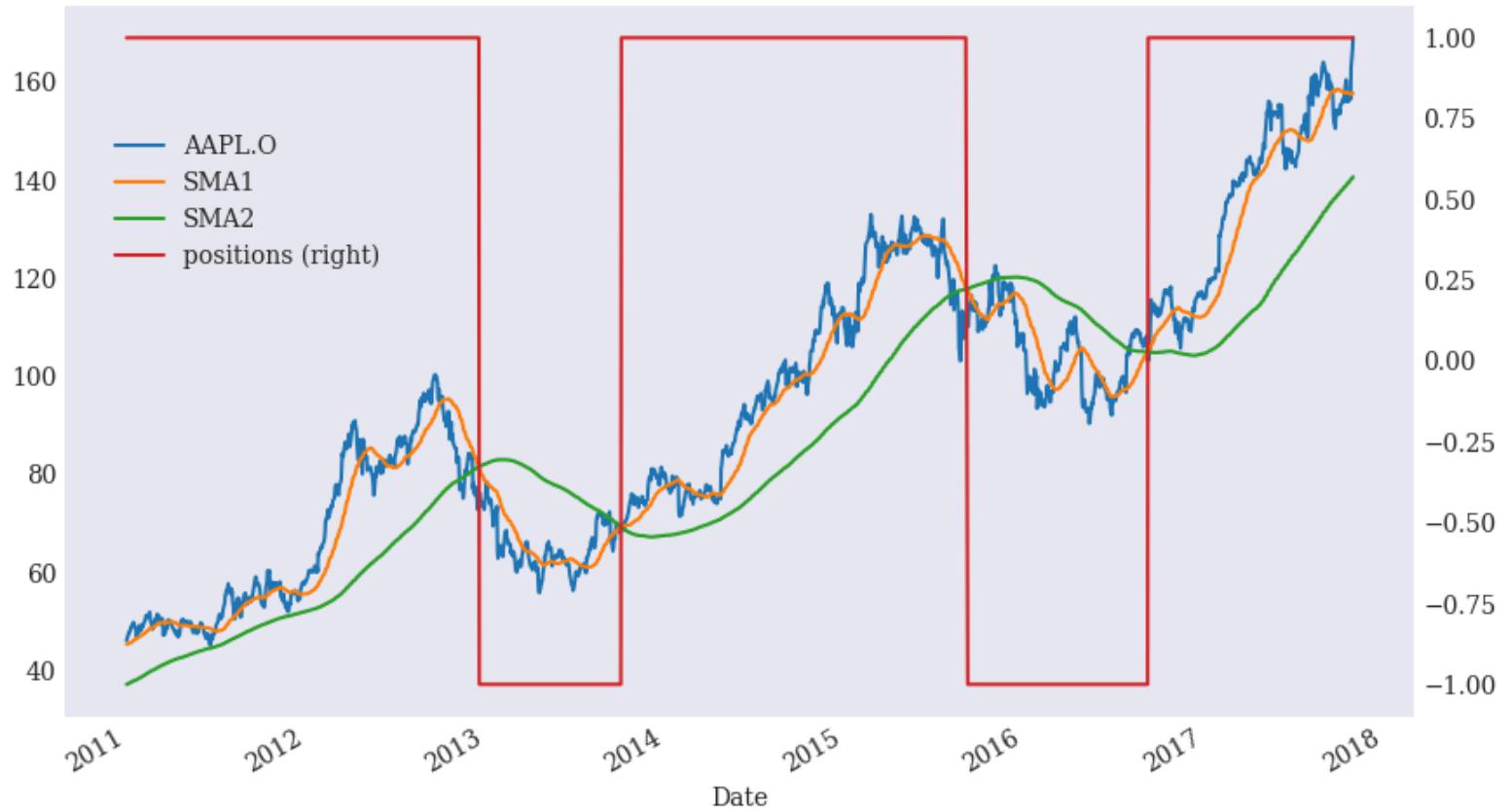
- a long position by a value of 1
- a short position by a value of -1.

The change in the position is triggered by a crossover of the two lines representing the SMA time series:

```
In [44]: data.dropna(inplace = True)
```

```
In [45]: data['positions'] = np.where(data['SMA1'] > data['SMA2'], 1, -1)
```

```
In [46]: ax = data[[sym, 'SMA1', 'SMA2', 'positions']].plot(  
    figsize = (10,6), secondary_y = 'positions')  
ax.get_legend().set_bbox_to_anchor((0.25,0.85));
```



## 4. Correlation analysis

## 4.1 Inspection of 2 timeseries

Let us consider the correlation analysis between two financial time series: the **S&P 500 Index (.SPX)** and the **VIX volatility index (.VIX)**.

- The S&P 500 is a benchmark index for U.S. stocks
- the VIX measures market volatility expectations.

Typically, these indices have an inverse relationship: when the S&P 500 falls, the VIX tends to rise, indicating higher market fear or uncertainty.

```
In [47]: raw = pd.read_csv(file_path + file_name, index_col=0, parse_dates=True)
```

```
In [48]: data = raw[['SPX', '.VIX']].dropna()
```

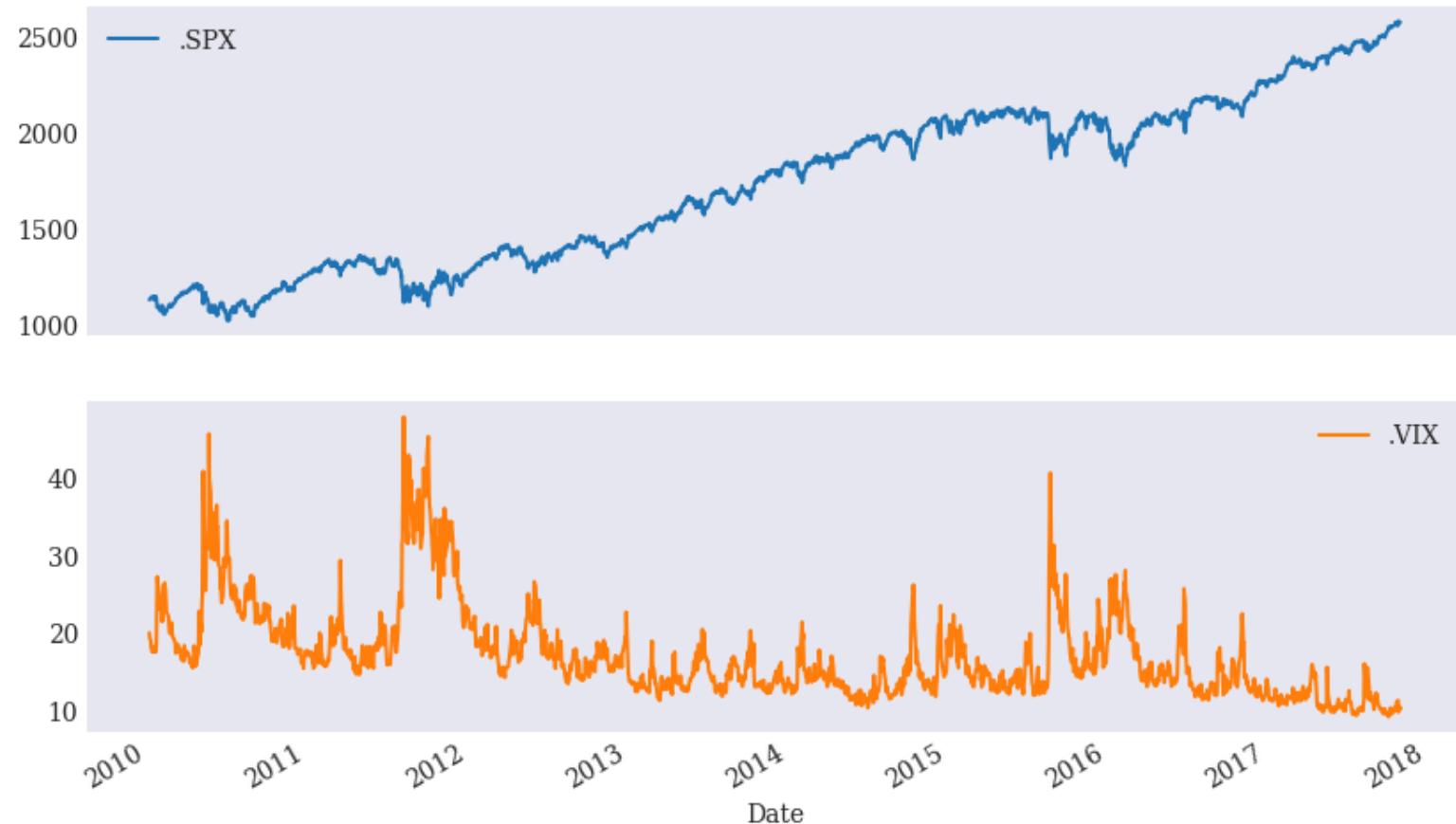
```
In [49]: data.tail()
```

```
Out[49]:
```

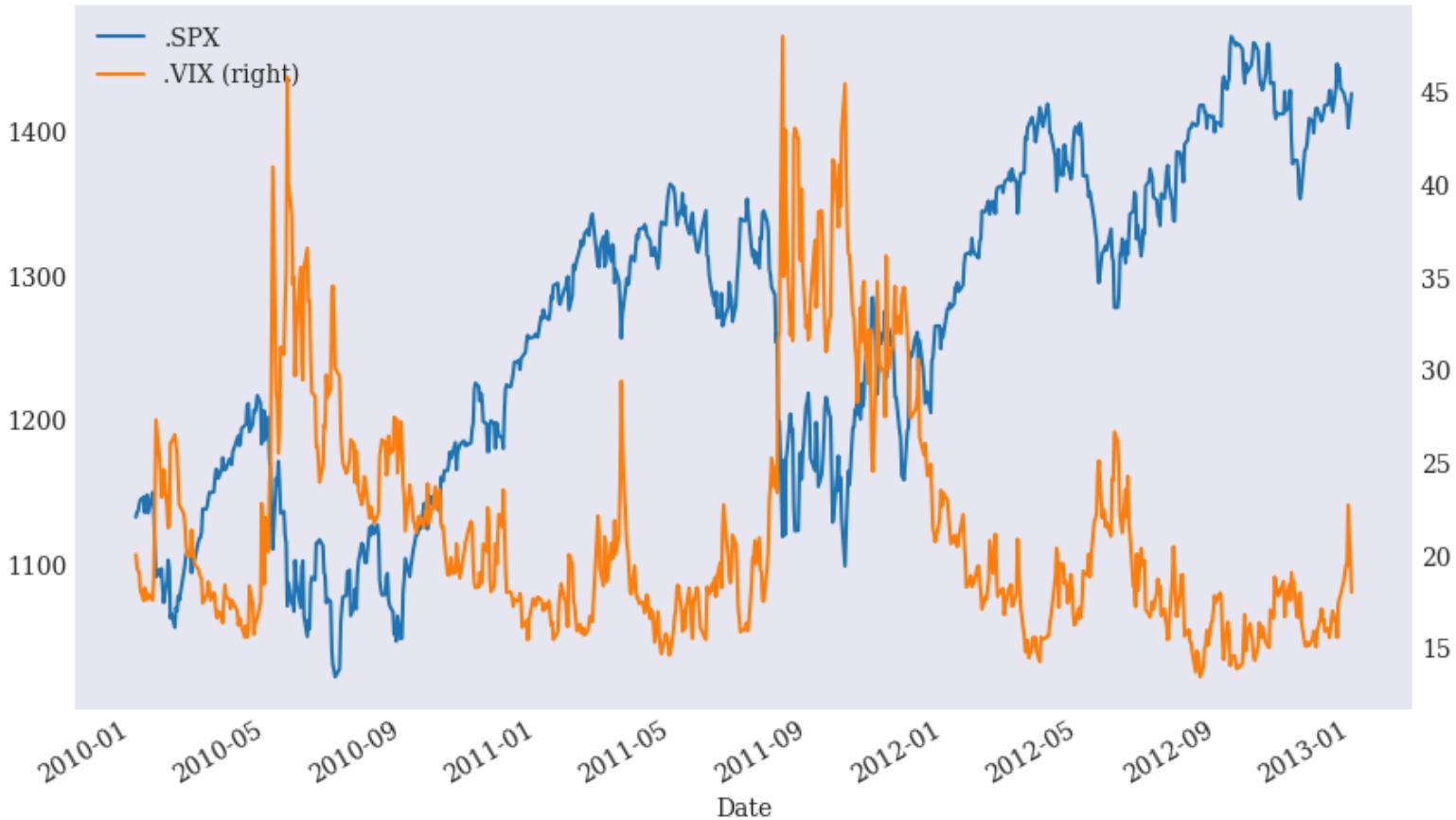
	.SPX	.VIX
Date		
2017-10-25	2557.15	11.23
2017-10-26	2560.40	11.30
2017-10-27	2581.07	9.80
2017-10-30	2572.83	10.50
2017-10-31	2575.26	10.18

## Visual inspection

```
In [50]: data.plot(subplots=True, figsize=(10, 6));
```



```
In [51]: data.loc[:'2012-12-31'].plot(secondary_y='.VIX', figsize=(10, 6));
```



## 4.2 Logarithmic Returns

Producing and processing output

```
In [52]: rets = np.log(data / data.shift(1))
```

```
In [53]: rets.head()
```

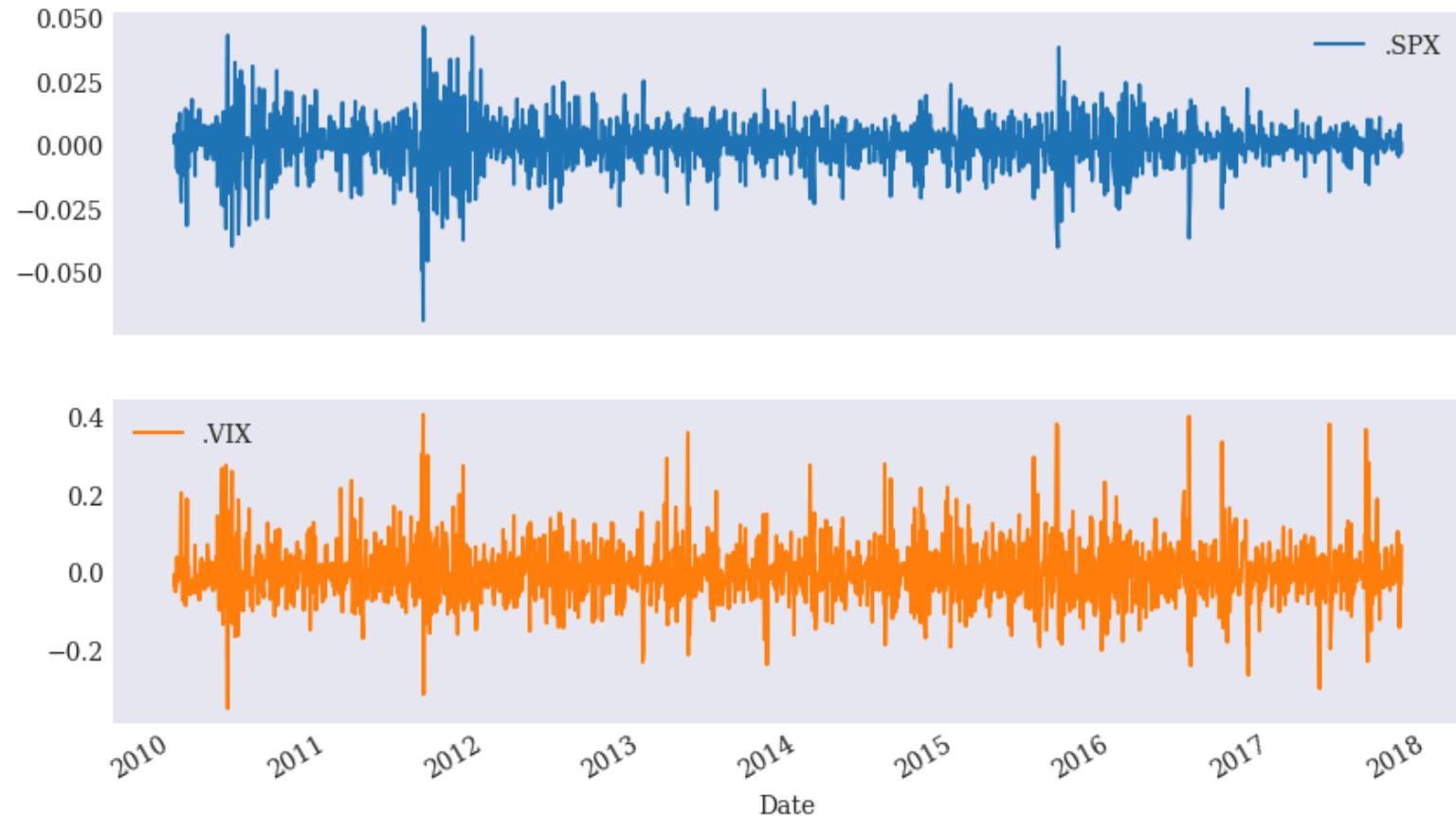
```
Out[53]:
```

	.SPX	.VIX
Date		
2010-01-04	NaN	NaN
2010-01-05	0.003111	-0.035038
2010-01-06	0.000545	-0.009868
2010-01-07	0.003993	-0.005233
2010-01-08	0.002878	-0.050024

```
In [54]: rets.dropna(inplace=True)
```

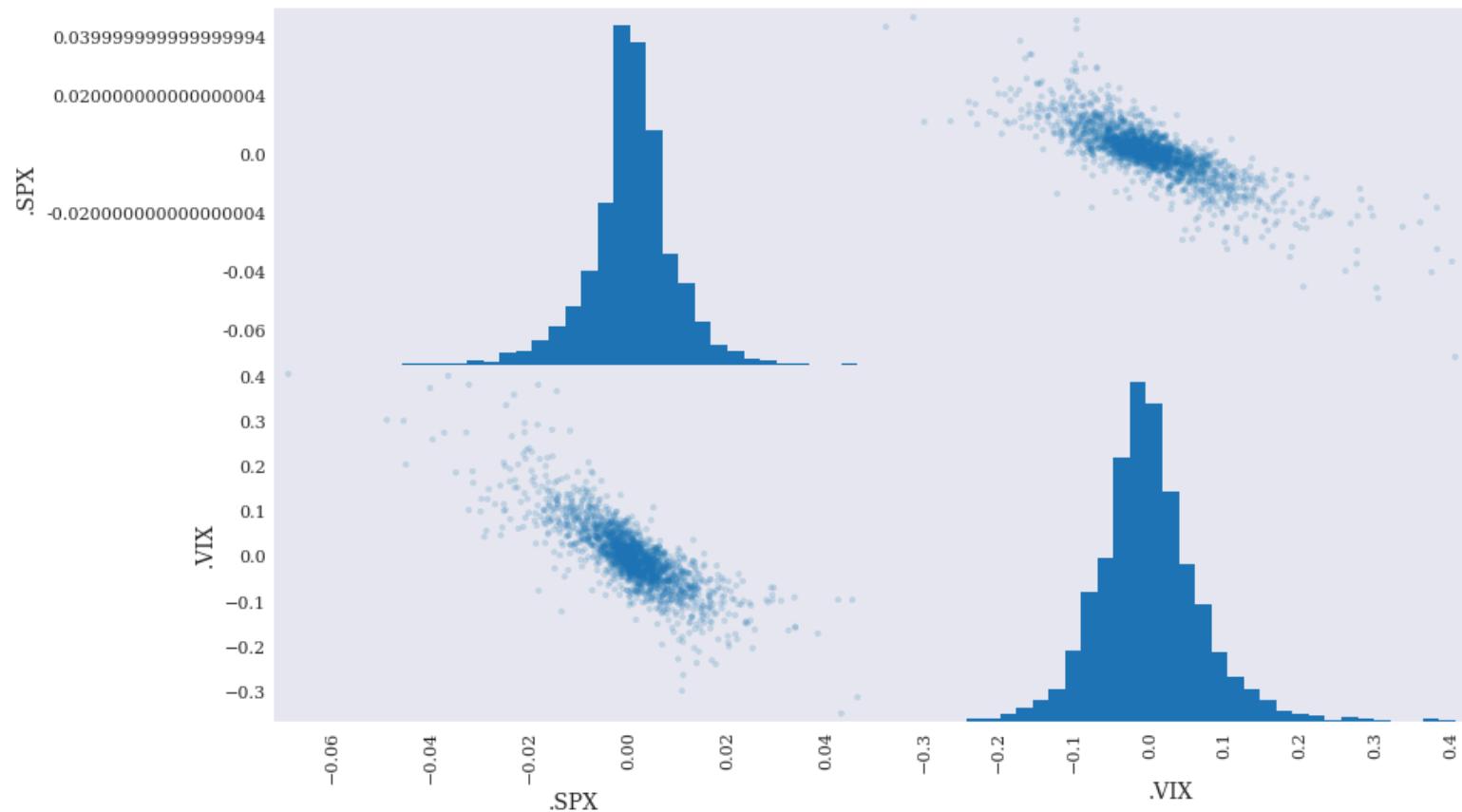
## Visual inspection

```
In [55]: rets.plot(subplots=True, figsize=(10, 6));
```



The `.plotting.scatter_matrix()` produces correlation analysis plots within and across timeseries.

```
In [56]: pd.plotting.scatter_matrix(rets,
                                alpha=0.2,
                                diagonal='hist',
                                hist_kwds={'bins': 35},
                                figsize=(10, 6));
```



## 4.3 OLS Regression

**Ordinary Least Square** regression provide a formal way to inspect the correlation between two variables.

`np.polyfit()` is a function in NumPy that fits a polynomial to a set of data points using least squares regression and returns the coefficient.

In other words, it finds the polynomial function of a specified degree that best fits the data in terms of minimizing the sum of squared errors between the fitted polynomial values and the actual data points

```
np.polyfit(x, y, deg, rcond=None, full=False, w=None, cov=False)
```

Parameters:

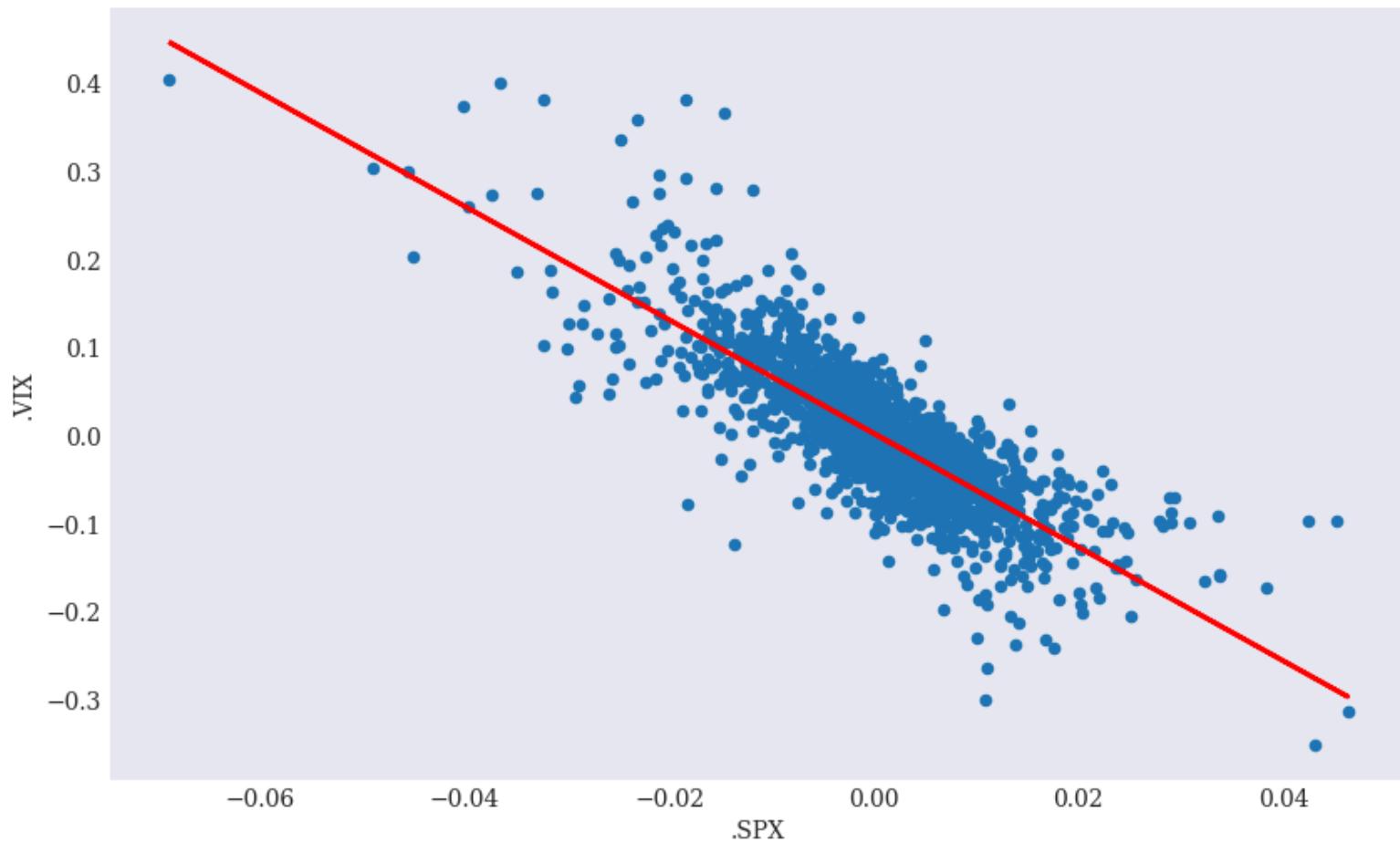
- `x` : The x-coordinates (independent variable) of the data points.
- `y` : The y-coordinates (dependent variable) of the data points.
- `deg` : Degree of the polynomial to be fit to the data. For example:
  - `deg=1` fits a line (linear regression),
  - `deg=2` fits a quadratic curve, and so on.
- `cov` (optional): If `True`, the function also returns the covariance matrix of the polynomial coefficients.

```
In [57]: reg, cov_matrix = np.polyfit(rets['.SPX'], rets['.VIX'], deg=1, cov=True)
print (f"The regression results in: VIX = {reg[0].round(2)} SPX + {reg[1].round(4)}")
# print (cov_matrix)
```

The regression results in: VIX = -6.45 SPX + 0.0023

## Visual inspection

```
In [58]: ax = rets.plot(kind='scatter', x='.SPX', y='.VIX', figsize=(10, 6))
ax.plot(rets['.SPX'], np.polyval(reg, rets['.SPX'])), 'r', lw=2);
```



where `np.polyval()` is a function in NumPy used to evaluate (calculate) the value of a polynomial for a given set of values. Essentially, given a polynomial's coefficients, `np.polyval` compute the y-values for corresponding x-values on that polynomial.

```
np.polyval(p, x)
```

- `p` : Array of polynomial coefficients in decreasing order of power.
  - For example, for a polynomial equation of the form  $y = ax^2 + bx + c$ , the coefficients array should be  $[a, b, c]$ .
- `x` : Value(s) at which to evaluate the polynomial. This can be a single number or an array of x-values.

## 4.4 Correlation

`.corr()` computes the **Pearson correlation** coefficient between pairs of columns in a `DataFrame`, a measure of the strength and direction of their linear relationship.

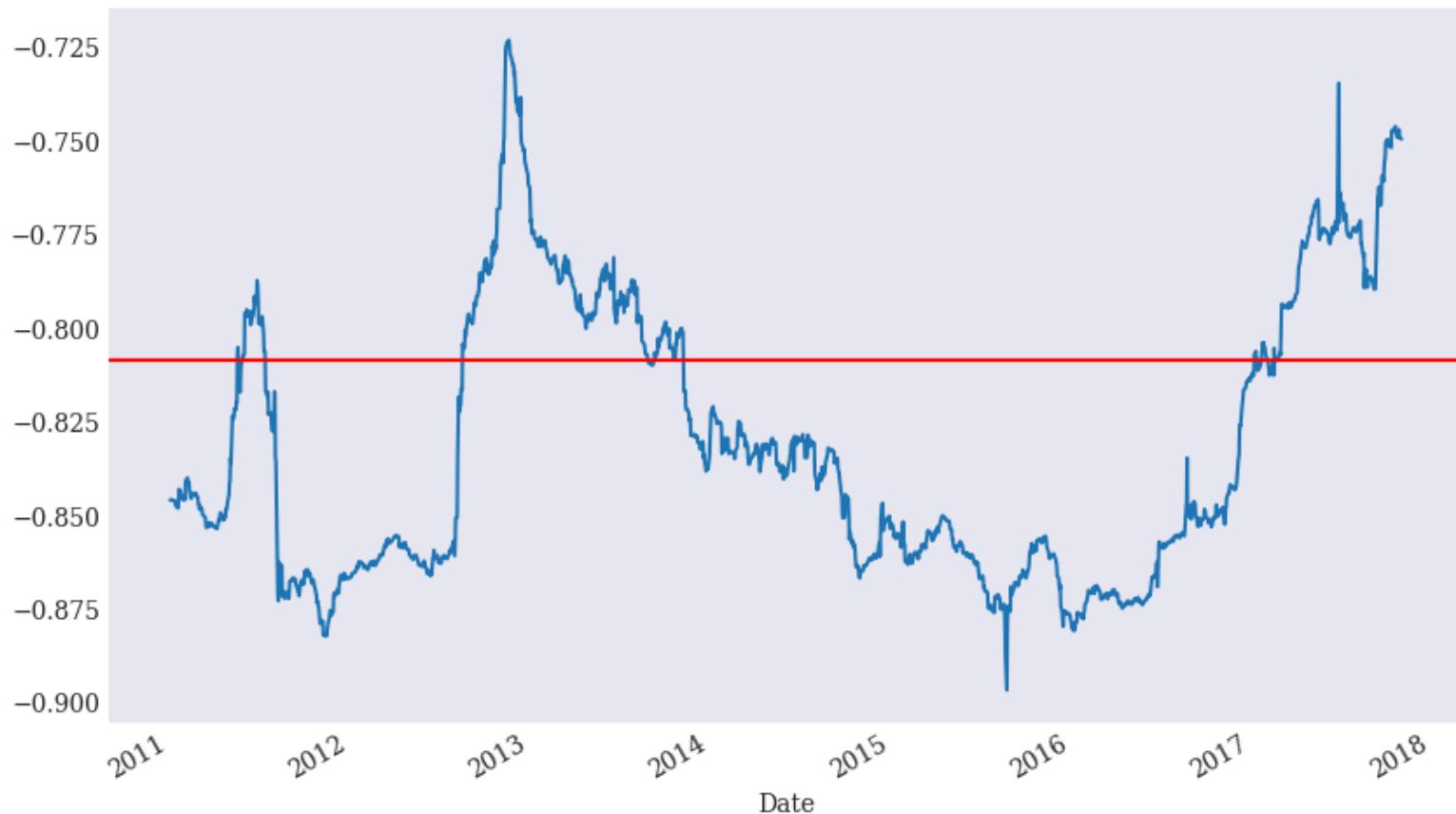
- Values range from -1 (perfect negative correlation) to +1 (perfect positive correlation).
  - For `.SPX` and `.VIX`, a strong negative correlation is expected.
- Calling the method can be
  - applied directly to a `DataFrame` to calculate correlations between each column pair.
  - used with one column to calculate correlation with another column, e.g.,  
`df['col1'].corr(df['col2'])`.

```
In [59]: rets.corr()
```

```
Out[59]:
```

	.SPX	.VIX
.SPX	1.000000	-0.808372
.VIX	-0.808372	1.000000

```
In [60]: ax = rets['.SPX'].rolling(window=252).corr(  
    rets['.VIX']).plot(figsize=(10, 6))  
ax.axhline(rets.corr().iloc[0, 1], c='r');
```



## 5. A glimpse into high-frequency data

**High-frequency data** in finance refers to data captured at very short time intervals, often seconds or milliseconds, typically related to trades, bids, and asks.

It provides detailed insights into market activity but requires careful handling due to its high volume and potential noise. Such data is commonly used in trading, market analysis, and to identify short-term price movements or anomalies.



For this part, we're loading **tick** data from a `csv` file for **EUR/USD**, which contains high-frequency information, like **bid** and **ask prices**, captured by the **FXCM broker**.

The following parameters apply:

```
file_path = 'Data/11/'  
file_name = 'fxcm_eur_usd_tick_data.csv'
```

Check file

```
In [61]: file_path = 'Data/11/'  
file_name = 'fxcm_eur_usd_tick_data.csv'  
file = open(file_path + file_name, 'r')
```

```
In [62]: file.readlines()[:10]
```

```
Out[62]: ['Bid,Ask\n',  
          '2017-11-10 12:00:00.007,1.16395,1.16394\n',  
          '2017-11-10 12:00:00.053,1.16394,1.16394\n',  
          '2017-11-10 12:00:00.740,1.16394,1.16393\n',  
          '2017-11-10 12:00:00.746,1.16394,1.16391\n',  
          '2017-11-10 12:00:00.756,1.16394,1.16392\n',  
          '2017-11-10 12:00:00.761,1.16393,1.16392\n',  
          '2017-11-10 12:00:00.772,1.16393,1.16391\n',  
          '2017-11-10 12:00:00.783,1.16391,1.16391\n',  
          '2017-11-10 12:00:00.819,1.16391,1.16392\n']
```

```
In [63]: file.close()
```



Import and inspect data

```
In [64]: tick = pd.read_csv(file_path + file_name,  
                         index_col=0, parse_dates=True)
```

In [65]: `tick.head()`

Out [65]:

		Bid	Ask
<b>2017-11-10 12:00:00.007</b>	1.16395	1.16394	
<b>2017-11-10 12:00:00.053</b>	1.16394	1.16394	
<b>2017-11-10 12:00:00.740</b>	1.16394	1.16393	
<b>2017-11-10 12:00:00.746</b>	1.16394	1.16391	
<b>2017-11-10 12:00:00.756</b>	1.16394	1.16392	

```
In [66]: tick.info()
```

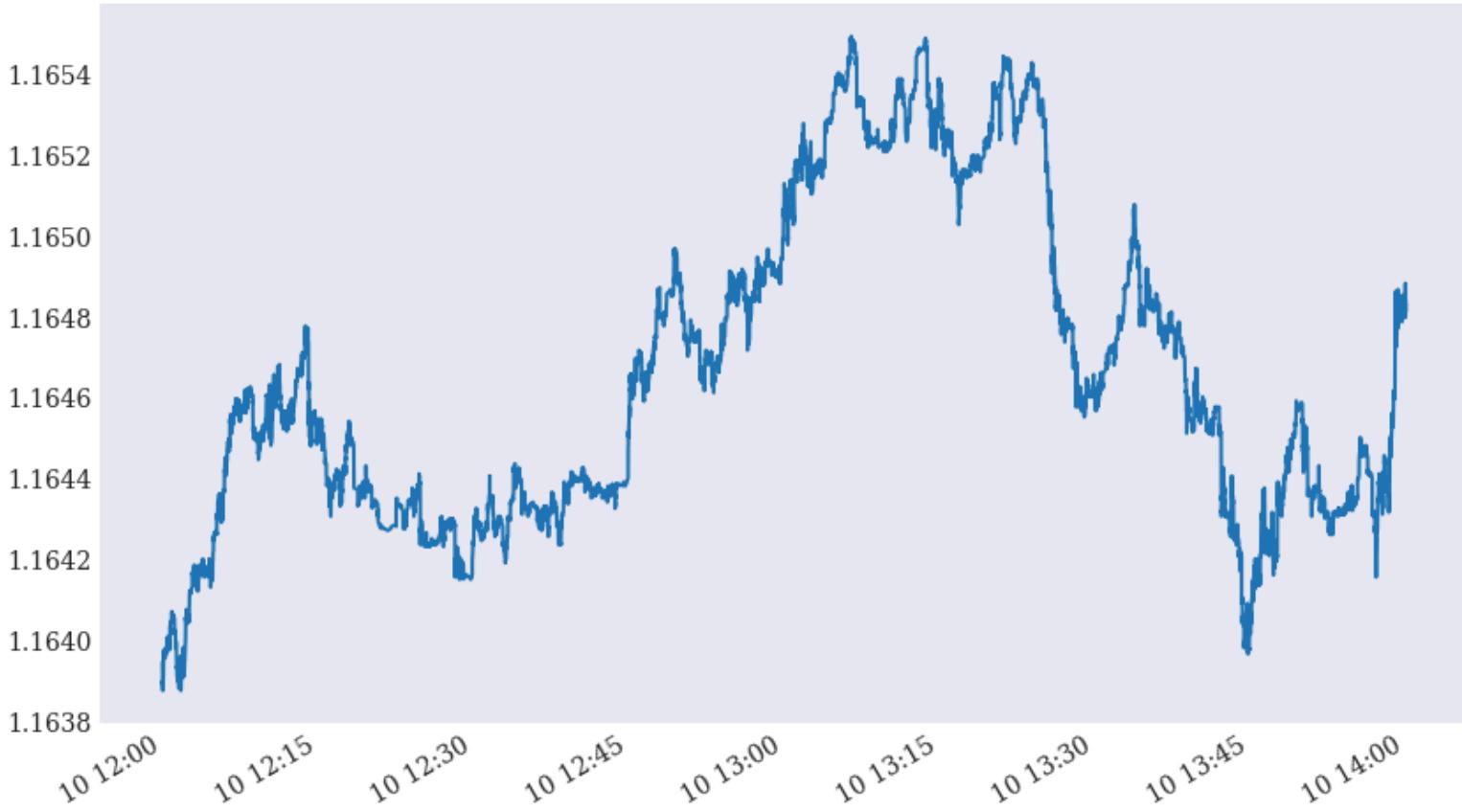
```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 17352 entries, 2017-11-10 12:00:00.007000 to 201
7-11-10 14:00:00.131000
Data columns (total 2 columns):
 #   Column  Non-Null Count  Dtype  
---  --     --          --    
 0   Bid      17352 non-null   float64 
 1   Ask      17352 non-null   float64 
dtypes: float64(2)
memory usage: 406.7 KB
```

Compute mid-prices

$$\text{Mid Price} = \frac{\text{Bid} + \text{Ask}}{2}$$

```
In [67]: tick['Mid'] = tick.mean(axis = 1)
```

```
In [68]: tick['Mid'].plot(figsize = (10,6));
```



Resampling to 5-minute intervals

```
In [69]: tick_resam = tick.resample(rule='5min', label='right').last()
```

```
In [70]: tick_resam.head()
```

```
Out[70]:
```

	Bid	Ask	Mid
<b>2017-11-10 12:05:00</b>	1.16425	1.16427	1.164260
<b>2017-11-10 12:10:00</b>	1.16454	1.16455	1.164545
<b>2017-11-10 12:15:00</b>	1.16449	1.16449	1.164490
<b>2017-11-10 12:20:00</b>	1.16437	1.16437	1.164370
<b>2017-11-10 12:25:00</b>	1.16429	1.16430	1.164295

```
In [71]: tick_resam['Mid'].plot(figsize=(10, 6));
```

