Price Elasticity of Demand

Let $j = 1, \dots, M = 88$ denote each store, and $i = 1, \dots, n_j$ the individual measurements within each store. Also, let $N = \sum_{j=1}^{M} n_j$. We will write the model as

$$y_{ij} = \beta_{1j} + \beta_{2j}x_{1,ij} + \beta_{3j}x_{2,ij} + \beta_{4j}(x_{1,ij} \times x_{2,ij}) + \varepsilon_{ij},$$

where y_{ij} is the log of the sales volume (in thousands of units sold), $x_{1,ij}$ is the log of the price (in USD), and $x_{2,ij}$ is an indicator denoting whether or not advertising was present in the store; i.e., $x_{2,ij} = 1$ if the ij^{th} observation was measured when an advertisement display was present, and 0 otherwise.

In words,

$$\log \text{Quantity}_{ij} = \beta_{1j} + \beta_{2j} \log \text{Price}_{ij} + \beta_{3j}\text{Display}_{ij} + \beta_{4j}\left(\log \text{Price}_{ij} \times \text{Display}_{ij}\right) + \varepsilon_{ij},$$

or

$$\log Q_{ij} = \beta_{1j} + \beta_{2j} \log P_{ij} + \beta_{3j}D_{ij} + \beta_{4j}\left(\log P_{ij} \times D_{ij}\right) + \varepsilon_{ij},$$

where $Q_{ij} = \text{Quantity}_{ij}$, $P_{ij} = \text{Price}_{ij}$, and $D_{ij} = \text{Display}_{ij}$.

Also, we assume the ε_{ij} are iid $N(0, \sigma^2)$. We will denote $\boldsymbol{\beta}_j = \left(\beta_{1j}, \beta_{2j}, \beta_{3j}, \beta_{4j}\right)^T$ and $\boldsymbol{x}_{ij} = \left(1, x_{1,ij}, x_{2,ij}, x_{1,ij}x_{2,ij}\right)^T$, so that the model can also be written as

$$y_{ij} = \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} + \varepsilon_{ij}.$$

We further assume that $\boldsymbol{\beta}_j$ are random effects, deriving from a $\mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma})$ distribution. We then place (conditionally) conjugate prior distributions on $\boldsymbol{\theta}$, $\boldsymbol{\Sigma}$, and $\lambda = \frac{1}{\sigma^2}$. The hierarchical nature of the model can thus be specified as:

$$y_{ij} | \boldsymbol{\beta}_j, \lambda \sim N\left(\boldsymbol{\beta}_j^T \boldsymbol{x}_{ij}, \frac{1}{\lambda}\right)$$

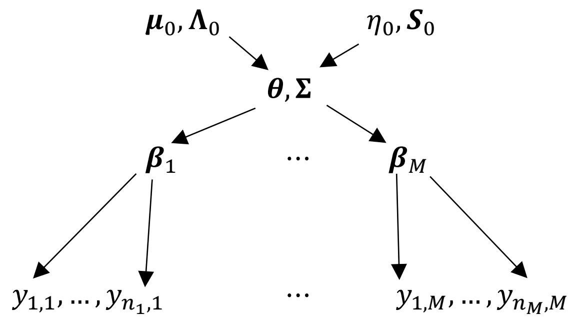
$$\boldsymbol{\beta}_j | \boldsymbol{\theta}, \boldsymbol{\Sigma} \sim \mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0)$$

$$\boldsymbol{\Sigma} \sim \text{IW}(\eta_0, \boldsymbol{S}_0)$$

$$\lambda \sim \text{Gamma}(a, b).$$

We can represent the model graphically as follows:



The full posterior is then

$$p(\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M, \boldsymbol{\theta}, \boldsymbol{\Sigma}, \lambda | \text{Data}) \propto p(\text{Data} | \boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M, \boldsymbol{\theta}, \boldsymbol{\Sigma}, \lambda) p(\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M) p(\boldsymbol{\theta}) p(\boldsymbol{\Sigma}) p(\lambda)$$

$$= \left(\prod_{j=1}^{M} \prod_{i=1}^{n_j} N\left(y_{ij} \middle| \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij}, \frac{1}{\lambda} \right) \right) \left(\prod_{j=1}^{M} \mathcal{N}(\boldsymbol{\beta}_j | \boldsymbol{\theta}, \boldsymbol{\Sigma}) \right) \times \mathcal{N}(\boldsymbol{\theta} | \boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0) \times \text{IW}(\boldsymbol{\Sigma} | \eta_0, \boldsymbol{S}_0) \times \text{Gamma}(\lambda | a, b)$$

$$\propto \left(\prod_{j=1}^{M} \prod_{i=1}^{n_j} \sqrt{\lambda} \exp\left\{ -\frac{\lambda}{2} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \right) \times \left(\prod_{j=1}^{M} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \right)$$

$$\times \left(\exp\left\{ -\frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\mu}_0)^T \boldsymbol{\Lambda}_0^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu}_0) \right\} \right) \times |\boldsymbol{\Sigma}|^{-\frac{\eta_0 + 4 + 1}{2}} \exp\left\{ -\frac{1}{2} \text{tr}(\boldsymbol{S}_0 \boldsymbol{\Sigma}^{-1}) \right\}$$

$$\times \lambda^{a-1} \exp\{-b\lambda\}$$

From here we can derive the full conditionals for the $\boldsymbol{\beta}_j$, $\boldsymbol{\theta}, \boldsymbol{\Sigma}$, and λ, so that we may then run a Gibbs sampler.

$$\begin{aligned}
p(\boldsymbol{\beta}_j | -) &\propto \left[\prod_{i=1}^{n_j} \exp\left\{ -\frac{\lambda}{2} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \right] \times \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \\
&\propto \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n_j} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \\
&= \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n_j} \left(y_{ij}^2 - 2 y_{ij} \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j + \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j \right) - \frac{1}{2} \left(\boldsymbol{\beta}_j^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\beta}_j - 2 \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\beta}_j + \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\theta} \right) \right\} \\
&\propto \exp\left\{ -\frac{1}{2} \left[\boldsymbol{\beta}_j^T \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right) \boldsymbol{\beta}_j^T - 2 \left(\lambda \sum_{i=1}^{n_j} y_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \right) \boldsymbol{\beta}_j \right] \right\} \\
&\propto \mathcal{N}\left(\boldsymbol{\beta}_j \ \middle| \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right)^{-1} \left(\boldsymbol{\Sigma}^{-1} \boldsymbol{\theta} + \lambda \sum_{i=1}^{n_j} y_{ij} \boldsymbol{x}_{ij} \right); \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right)^{-1} \right)
\end{aligned}$$

$$
\begin{aligned}
p(\boldsymbol{\theta}|-) &\propto \left(\prod_{j=1}^{M}|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\right\}\right)\times\left(\exp\left\{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)^T\boldsymbol{\Lambda}_0^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)\right\}\right)\\
&\propto \exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)^T\boldsymbol{\Lambda}_0^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)\right\}\\
&\propto \exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(-2\boldsymbol{\beta}_j^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\theta}+\boldsymbol{\theta}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\theta}\right)-\frac{1}{2}\left(\boldsymbol{\theta}^T\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\theta}-2\boldsymbol{\mu}_0^T\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\theta}\right)\right\}\\
&= \exp\left\{-\frac{1}{2}\left[\boldsymbol{\theta}^T(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})\boldsymbol{\theta}-2\left(\boldsymbol{\mu}_0^T\boldsymbol{\Lambda}_0^{-1}+\sum_{j=1}^{M}\boldsymbol{\beta}_j^T\boldsymbol{\Sigma}^{-1}\right)\boldsymbol{\theta}\right]\right\}\\
&\propto \mathcal{N}\left(\boldsymbol{\theta}\,\middle|\,(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})^{-1}\left(\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\mu}_0+\boldsymbol{\Sigma}^{-1}\sum_{j=1}^{M}\boldsymbol{\beta}_j\right);(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})^{-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
p(\boldsymbol{\Sigma}|-) &\propto \left(\prod_{j=1}^{M}|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\right\}\right)\times|\boldsymbol{\Sigma}|^{-\frac{\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left[\sum_{j=1}^{m}\left(\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\Sigma^{-1}\right)\right]-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left[\left(\boldsymbol{S}_0+\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\right)\boldsymbol{\Sigma}^{-1}\right]\right\}\\
&\propto \mathrm{IW}\left(\boldsymbol{\Sigma}\middle|\eta_0+M;\boldsymbol{S}_0+\textstyle\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\right)
\end{aligned}
$$

$$
\begin{aligned}
p(\lambda|-) &\propto \left(\prod_{j=1}^{M}\prod_{i=1}^{n_j}\sqrt{\lambda}\exp\left\{-\frac{\lambda}{2}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right\}\right)\times\lambda^{a-1}\exp\{-b\lambda\}\\
&= \lambda^{N+a-1}\exp\left\{-\frac{\lambda}{2}\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2-b\lambda\right\}\\
&= \lambda^{N+a-1}\exp\left\{-\lambda\left[b+\frac{1}{2}\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right]\right\}\\
&\propto \mathrm{Gamma}\left(\lambda\middle|a+N;b+\frac{1}{2}\textstyle\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right).
\end{aligned}
$$

Summary Statistics

	$\beta_{1,j}$				$\beta_{2,j}$				$\beta_{3,j}$				$\beta_{4,j}$			
Store *j*	Mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%
1	8.40	8.40	7.36	9.43	-1.74	-1.73	-2.70	-0.76	3.92	3.93	2.80	5.07	-3.58	-3.57	-4.69	-2.50
2	10.05	10.05	9.47	10.64	-1.56	-1.55	-2.09	-1.03	0.51	0.51	-0.26	1.28	-0.44	-0.45	-1.18	0.30
3	9.84	9.90	8.49	11.20	-1.88	-1.94	-3.26	-0.50	-0.39	-0.38	-2.57	1.76	0.30	0.21	-1.87	2.50
4	12.21	12.20	11.14	13.30	-2.90	-2.88	-3.73	-2.09	1.12	1.14	-0.25	2.49	-0.83	-0.82	-1.89	0.24
5	11.07	11.08	10.48	11.67	-2.10	-2.11	-2.54	-1.66	1.00	1.01	-0.08	2.08	-0.56	-0.56	-1.35	0.23
6	10.85	10.84	10.27	11.42	-2.84	-2.83	-3.29	-2.40	2.02	2.02	0.38	3.72	-1.19	-1.13	-2.53	0.10
7	10.57	10.57	8.81	12.33	-1.83	-1.85	-3.49	-0.16	0.18	0.16	-1.81	2.17	0.05	0.02	-1.81	1.91
8	8.40	8.41	7.91	8.89	-1.00	-1.00	-1.56	-0.44	0.31	0.30	-0.48	1.10	-0.33	-0.34	-1.23	0.57
9	9.99	9.96	8.65	11.30	-1.94	-1.92	-3.46	-0.41	0.60	0.62	-0.79	2.00	-0.45	-0.47	-2.06	1.14
10	10.14	10.12	8.77	11.59	-2.58	-2.53	-4.21	-1.04	0.57	0.61	-1.37	2.44	-0.23	-0.29	-2.23	1.84
11	11.06	11.05	10.47	11.66	-3.97	-3.97	-4.51	-3.44	-1.59	-1.58	-2.66	-0.52	2.14	2.16	1.17	3.12
12	11.19	11.20	10.62	11.76	-3.64	-3.64	-4.27	-3.01	-0.26	-0.31	-2.97	2.37	0.59	0.51	-1.77	3.01
13	8.90	8.94	7.71	10.05	-1.07	-1.12	-2.06	-0.04	4.57	4.55	3.39	5.79	-3.83	-3.83	-4.88	-2.81
14	10.92	10.90	9.16	12.67	-2.65	-2.63	-4.10	-1.19	2.67	2.66	0.91	4.46	-2.07	-2.10	-3.55	-0.61
15	8.67	8.73	7.23	10.07	-1.55	-1.52	-3.17	0.11	0.54	0.50	-1.10	2.18	-0.41	-0.38	-2.30	1.47
16	9.94	9.92	9.01	10.86	-1.73	-1.72	-2.64	-0.82	-0.32	-0.33	-1.39	0.75	0.38	0.37	-0.69	1.46
17	8.65	8.65	7.63	9.65	-1.34	-1.33	-2.44	-0.23	0.34	0.36	-1.06	1.76	-0.31	-0.35	-1.86	1.23
18	9.10	9.12	7.78	10.39	-1.49	-1.53	-2.98	0.02	0.07	0.03	-1.26	1.41	0.13	0.13	-1.41	1.65
19	11.15	11.15	9.97	12.33	-2.11	-2.09	-3.16	-1.07	1.20	1.23	0.00	2.39	-0.97	-0.99	-2.03	0.09
20	13.27	13.21	11.92	14.69	-3.23	-3.19	-4.44	-2.09	-0.89	-0.85	-2.33	0.47	1.01	0.97	-0.16	2.25
21	9.74	9.78	8.53	10.92	-1.72	-1.74	-2.92	-0.48	3.54	3.52	2.28	4.84	-3.31	-3.30	-4.64	-2.01
22	10.02	10.06	8.81	11.20	-1.35	-1.35	-2.53	-0.13	1.33	1.33	0.00	2.68	-1.27	-1.30	-2.62	0.05
23	10.04	10.02	9.06	11.04	-1.88	-1.86	-2.81	-0.95	0.03	0.04	-1.10	1.15	-0.02	0.01	-1.09	1.06
24	10.25	10.24	9.57	10.93	-2.53	-2.53	-3.10	-1.95	0.45	0.45	-0.33	1.22	-0.34	-0.35	-1.03	0.34
25	9.11	9.10	8.61	9.61	-1.06	-1.07	-1.58	-0.53	2.25	2.24	1.61	2.88	-2.04	-2.03	-2.70	-1.39
26	10.17	10.18	9.78	10.57	-1.79	-1.80	-2.16	-1.43	0.02	0.03	-1.10	1.13	0.17	0.18	-0.79	1.14
27	9.41	9.40	8.76	10.05	-1.39	-1.40	-1.92	-0.86	1.48	1.48	0.73	2.24	-1.03	-1.02	-1.69	-0.37
28	10.04	10.04	9.69	10.38	-1.77	-1.78	-2.08	-1.47	0.24	0.24	-0.23	0.71	-0.18	-0.18	-0.61	0.25
29	8.22	8.22	6.42	9.94	-1.81	-1.85	-3.58	0.02	3.07	3.06	1.22	5.00	-2.27	-2.25	-4.23	-0.37
30	10.76	10.74	9.23	12.28	-2.03	-2.05	-3.24	-0.82	0.99	1.03	-0.56	2.55	-0.58	-0.60	-1.83	0.65
31	11.00	11.00	9.48	12.54	-2.47	-2.46	-3.79	-1.19	1.44	1.43	-0.15	3.00	-1.09	-1.01	-2.43	0.27
32	10.37	10.37	9.87	10.87	-1.96	-1.95	-2.44	-1.49	0.43	0.43	-0.53	1.39	-0.19	-0.16	-1.14	0.75
33	8.91	8.94	7.34	10.47	-1.69	-1.65	-3.31	-0.06	0.15	0.15	-1.48	1.77	-0.09	-0.07	-1.77	1.59
34	11.83	11.82	11.49	12.17	-3.63	-3.63	-3.93	-3.34	-0.38	-0.33	-3.03	2.21	0.66	0.67	-1.65	3.03
35	9.67	9.73	8.38	10.88	-0.63	-0.67	-1.87	0.68	1.16	1.11	-0.14	2.52	-1.06	-1.00	-2.44	0.27
36	10.33	10.35	9.76	10.89	-1.77	-1.78	-2.24	-1.30	0.43	0.42	-0.24	1.11	-0.17	-0.16	-0.76	0.41
37	9.80	9.75	8.23	11.36	-1.65	-1.66	-3.07	-0.23	0.14	0.16	-1.60	1.86	-0.05	-0.02	-1.67	1.57
38	8.55	8.58	6.71	10.38	-1.42	-1.43	-3.15	0.32	-0.86	-0.86	-2.80	1.08	0.83	0.76	-0.99	2.65
39	9.34	9.33	8.53	10.15	-1.89	-1.86	-2.66	-1.13	0.11	0.13	-1.01	1.23	-0.09	-0.10	-1.17	1.00
40	9.54	9.57	8.88	10.20	-1.59	-1.61	-2.29	-0.89	0.57	0.55	-0.69	1.85	-0.51	-0.49	-1.90	0.87
41	10.48	10.47	9.36	11.65	-2.91	-2.93	-4.05	-1.83	-1.25	-1.22	-2.64	0.08	1.29	1.24	-0.02	2.66
42	11.35	11.36	9.55	13.15	-2.00	-1.95	-3.69	-0.31	0.63	0.65	-1.19	2.46	-0.30	-0.29	-2.02	1.41
43	10.70	10.69	8.94	12.46	-1.88	-1.92	-3.52	-0.22	0.36	0.35	-1.41	2.14	-0.10	-0.05	-1.78	1.56
44	12.24	12.22	11.61	12.87	-2.75	-2.74	-3.26	-2.24	0.17	0.20	-0.61	0.95	-0.09	-0.10	-0.73	0.55
45	9.86	9.87	8.53	11.17	-1.97	-2.00	-3.34	-0.58	1.60	1.56	0.19	3.04	-1.61	-1.58	-3.11	-0.15
46	8.77	8.77	7.53	9.97	-1.38	-1.46	-2.67	-0.07	0.93	0.91	-0.47	2.36	-0.90	-0.93	-2.44	0.60
47	10.78	10.75	9.01	12.55	-1.85	-1.88	-3.51	-0.18	0.16	0.22	-1.71	2.03	0.08	0.11	-1.67	1.83
48	10.52	10.55	9.14	11.86	-1.65	-1.67	-3.11	-0.15	0.34	0.39	-1.46	2.16	-0.31	-0.25	-2.28	1.64
49	9.66	9.68	8.57	10.75	-1.96	-1.94	-2.92	-0.99	0.55	0.48	-1.50	2.58	-0.09	-0.05	-1.91	1.74
50	9.72	9.74	8.10	11.32	-1.62	-1.66	-3.18	-0.05	-0.07	-0.04	-1.71	1.57	0.02	0.01	-1.58	1.61
51	10.29	10.28	9.48	11.11	-2.38	-2.37	-3.42	-1.36	0.34	0.37	-0.54	1.21	-0.04	-0.08	-1.14	1.09
52	10.70	10.70	9.95	11.45	-2.43	-2.42	-3.18	-1.67	-0.45	-0.45	-1.67	0.77	0.51	0.51	-0.77	1.80
53	10.64	10.64	10.32	10.97	-1.74	-1.74	-1.98	-1.50	-0.17	-0.17	-0.89	0.55	0.50	0.50	-0.06	1.07
54	11.95	11.96	10.49	13.42	-2.66	-2.66	-3.67	-1.66	1.32	1.32	-0.15	2.78	-0.77	-0.74	-1.77	0.25
55	10.33	10.33	9.87	10.78	-1.97	-1.98	-2.29	-1.66	1.40	1.35	-0.73	3.56	-0.88	-0.85	-2.97	1.18
56	9.33	9.36	7.61	11.03	-1.88	-1.88	-3.35	-0.39	0.72	0.71	-1.00	2.45	-0.50	-0.58	-2.01	1.00
57	8.42	8.39	7.21	9.64	-1.80	-1.80	-3.11	-0.52	-0.91	-0.88	-2.29	0.43	1.08	1.07	-0.36	2.56
58	10.24	10.22	9.24	11.23	-1.71	-1.72	-2.67	-0.74	0.31	0.32	-0.93	1.56	-0.28	-0.25	-1.51	0.94
59	9.71	9.72	8.38	11.01	-1.50	-1.55	-2.92	-0.04	0.12	0.15	-1.30	1.56	-0.01	-0.04	-1.58	1.54
60	11.25	11.22	9.78	12.75	-2.45	-2.46	-3.62	-1.30	1.54	1.56	0.02	3.05	-1.17	-1.16	-2.35	0.03
61	11.18	11.18	10.75	11.62	-2.92	-2.92	-3.29	-2.56	0.78	0.80	-0.27	1.83	-0.55	-0.59	-1.40	0.30
62	9.61	9.62	8.54	10.68	-2.23	-2.23	-3.10	-1.36	0.54	0.54	-0.59	1.68	-0.10	-0.09	-1.04	0.83
63	9.28	9.27	8.07	10.51	-1.89	-1.87	-2.94	-0.85	-0.85	-0.81	-2.15	0.43	0.95	0.95	-0.15	2.07
64	8.62	8.63	8.23	9.00	-0.67	-0.67	-1.05	-0.30	1.38	1.38	0.89	1.88	-1.05	-1.03	-1.52	-0.57
65	9.99	9.99	8.89	11.09	-1.64	-1.63	-2.72	-0.55	0.15	0.09	-1.08	1.38	-0.06	-0.04	-1.30	1.17
66	9.82	9.88	8.27	11.39	-2.13	-2.15	-3.75	-0.51	0.63	0.67	-1.00	2.26	-0.70	-0.73	-2.39	0.99
67	11.40	11.37	10.03	12.84	-2.89	-2.80	-4.24	-1.62	-1.01	-1.00	-2.55	0.46	1.03	1.02	-0.37	2.50
68	8.72	8.73	7.99	9.45	-1.02	-1.03	-1.77	-0.27	0.48	0.49	-0.37	1.33	-0.29	-0.26	-1.15	0.56
69	9.67	9.68	8.79	10.54	-1.78	-1.77	-2.63	-0.94	-0.19	-0.20	-1.19	0.81	0.30	0.30	-0.68	1.29
70	10.32	10.28	9.22	11.43	-2.01	-2.00	-3.18	-0.85	0.53	0.51	-0.77	1.84	-0.53	-0.49	-1.92	0.84
71	9.80	9.82	8.27	11.33	-1.88	-1.87	-3.06	-0.70	0.77	0.75	-0.78	2.32	-0.54	-0.50	-1.73	0.65
72	9.05	9.02	7.26	10.84	-1.52	-1.58	-3.21	0.17	-0.63	-0.61	-2.44	1.17	0.67	0.65	-1.04	2.37
73	11.54	11.51	9.73	13.36	-1.91	-1.90	-3.55	-0.28	0.33	0.34	-1.53	2.18	-0.06	-0.01	-1.73	1.61
74	10.46	10.44	8.76	12.14	-1.87	-1.88	-3.30	-0.43	1.12	1.07	-0.58	2.84	-0.88	-0.90	-2.34	0.57
75	9.87	9.85	8.35	11.44	-2.06	-2.06	-3.54	-0.61	-0.05	0.05	-1.63	1.50	0.11	0.05	-1.36	1.62
76	10.98	10.99	9.37	12.60	-1.92	-1.94	-3.21	-0.63	0.30	0.32	-1.33	1.93	-0.03	-0.02	-1.33	1.29
77	10.37	10.40	8.59	12.17	-1.73	-1.71	-3.43	0.01	-0.25	-0.22	-2.95	2.33	0.42	0.41	-1.97	2.92
78	9.67	9.65	8.78	10.56	-1.57	-1.53	-2.51	-0.63	-0.90	-0.91	-1.97	0.16	1.14	1.14	0.02	2.29

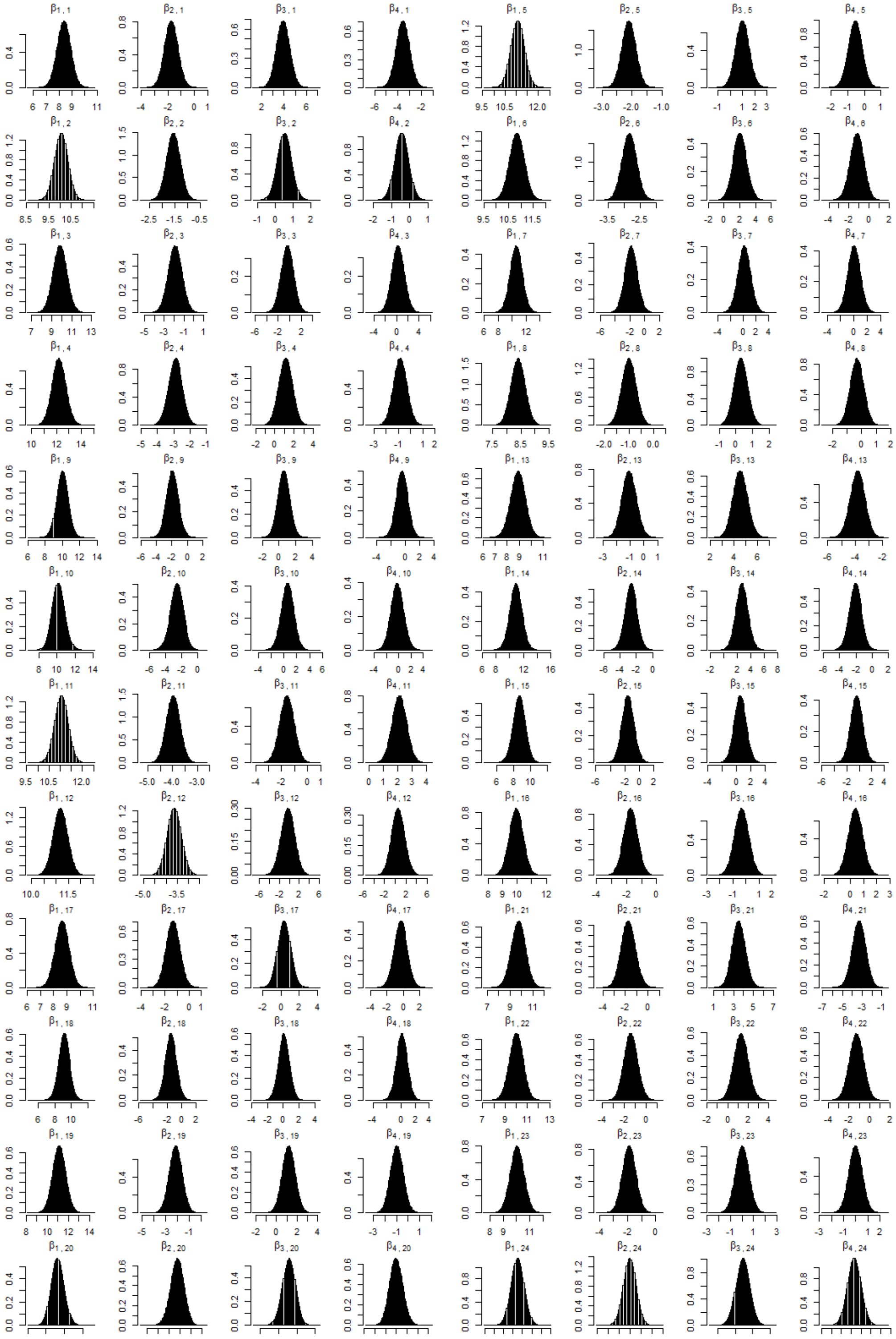
79	9.63	9.62	8.18	11.06	-1.87	-1.88	-3.52	-0.22	0.89	0.87	-0.73	2.53	-0.65	-0.63	-2.52	1.20
80	10.40	10.45	9.24	11.54	-2.24	-2.24	-3.14	-1.31	3.42	3.41	2.17	4.69	-2.63	-2.61	-3.66	-1.61
81	10.53	10.60	8.90	12.13	-1.79	-1.81	-3.16	-0.39	1.74	1.71	0.13	3.40	-1.44	-1.39	-2.87	-0.05
82	8.57	8.56	8.07	9.07	-1.67	-1.67	-2.12	-1.21	1.47	1.47	0.88	2.06	-1.00	-1.00	-1.55	-0.46
83	8.37	8.38	7.02	9.70	-2.25	-2.25	-3.51	-0.97	3.14	3.14	1.75	4.55	-2.44	-2.45	-3.79	-1.11
84	9.78	9.82	8.04	11.49	-2.73	-2.77	-4.13	-1.32	2.25	2.29	0.53	3.99	-1.59	-1.54	-3.02	-0.18
85	9.38	9.40	8.69	10.06	-1.11	-1.13	-1.76	-0.46	0.64	0.69	-0.48	1.77	-0.49	-0.54	-1.55	0.57
86	10.66	10.71	9.45	11.88	-2.07	-2.11	-3.25	-0.90	0.25	0.25	-1.11	1.60	-0.25	-0.22	-1.55	1.07
87	10.12	10.13	9.04	11.19	-1.70	-1.68	-2.78	-0.61	0.49	0.48	-0.95	1.94	-0.41	-0.40	-1.89	1.06
88	8.99	8.98	8.24	9.74	-1.57	-1.58	-2.18	-0.95	-0.08	-0.05	-1.14	0.97	0.15	0.17	-0.72	1.02

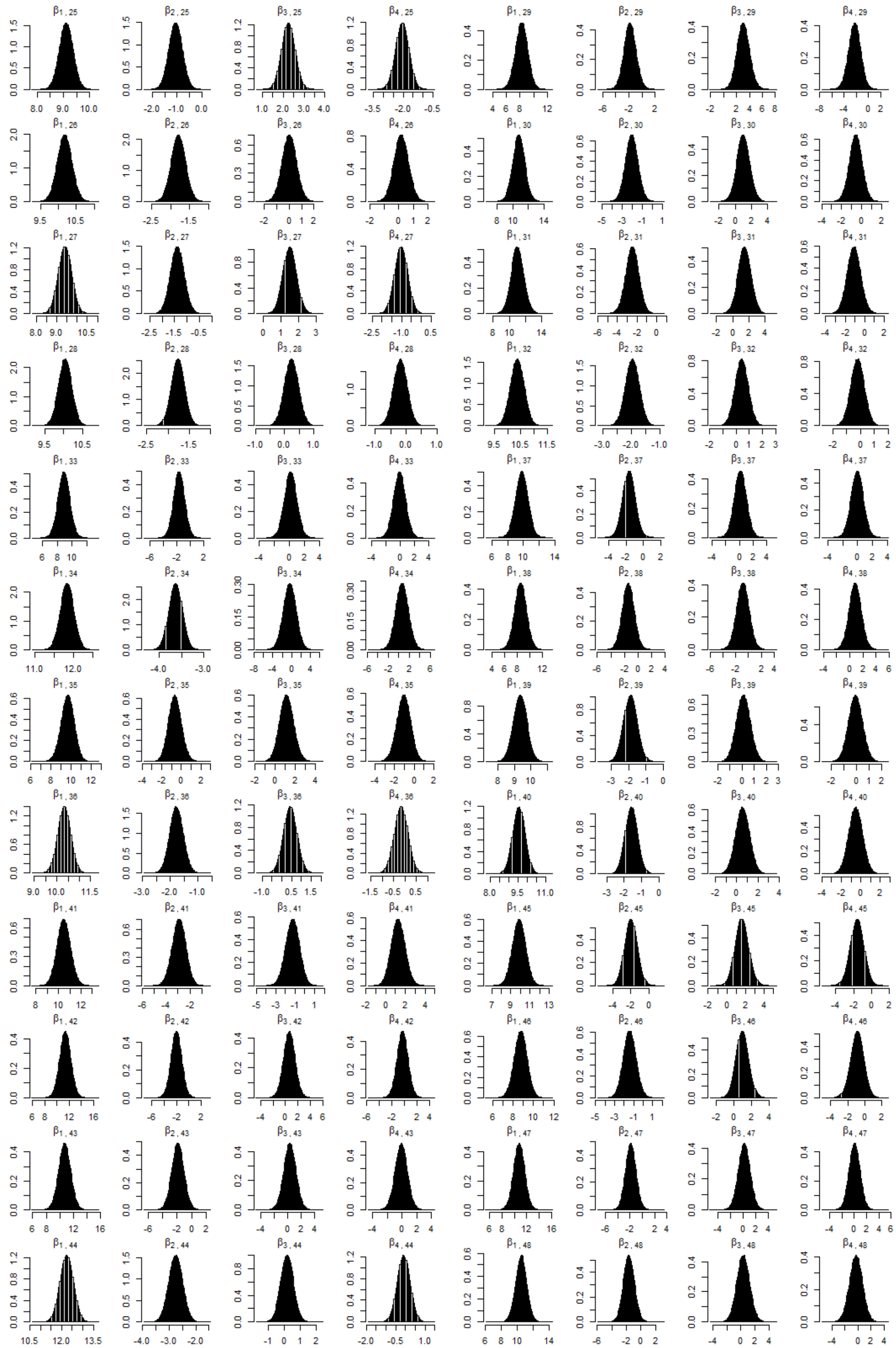
	Post Mean	Post Mode	95% Post CI
θ_1	9.884	9.896	(9.563,10.19)
θ_2	-1.858	-1.875	(-2.102, -1.599)
θ_3	0.711	0.706	(0.365, 1.064)
θ_4	-0.484	-0.486	(-0.813, -0.162)

	Post Mean	Post Mode	95% Post CI
σ_1^2	1.481	1.351	(0.917, 2.293)
σ_{12}	-0.818	-0.731	(-1.371, -0.438)
σ_{13}	-0.627	-0.522	(-1.343, -0.094)
σ_{14}	0.614	0.530	(0.129, 1.270)
σ_2^2	0.762	0.694	(0.460, 1.209)
σ_{23}	0.476	0.401	(0.0561, 1.032)
σ_{24}	-0.499	-0.440	(-1.018, -0.113)
σ_3^2	1.940	1.797	(1.192, 2.987)
σ_{34}	-1.735	-1.612	(-2.689, -1.054)
σ_4^2	1.621	1.499	(0.991, 2.507)

	Post Mean	Post Mode	95% Post CI
λ	30.46	30.45	(29.65, 31.27)

Histograms

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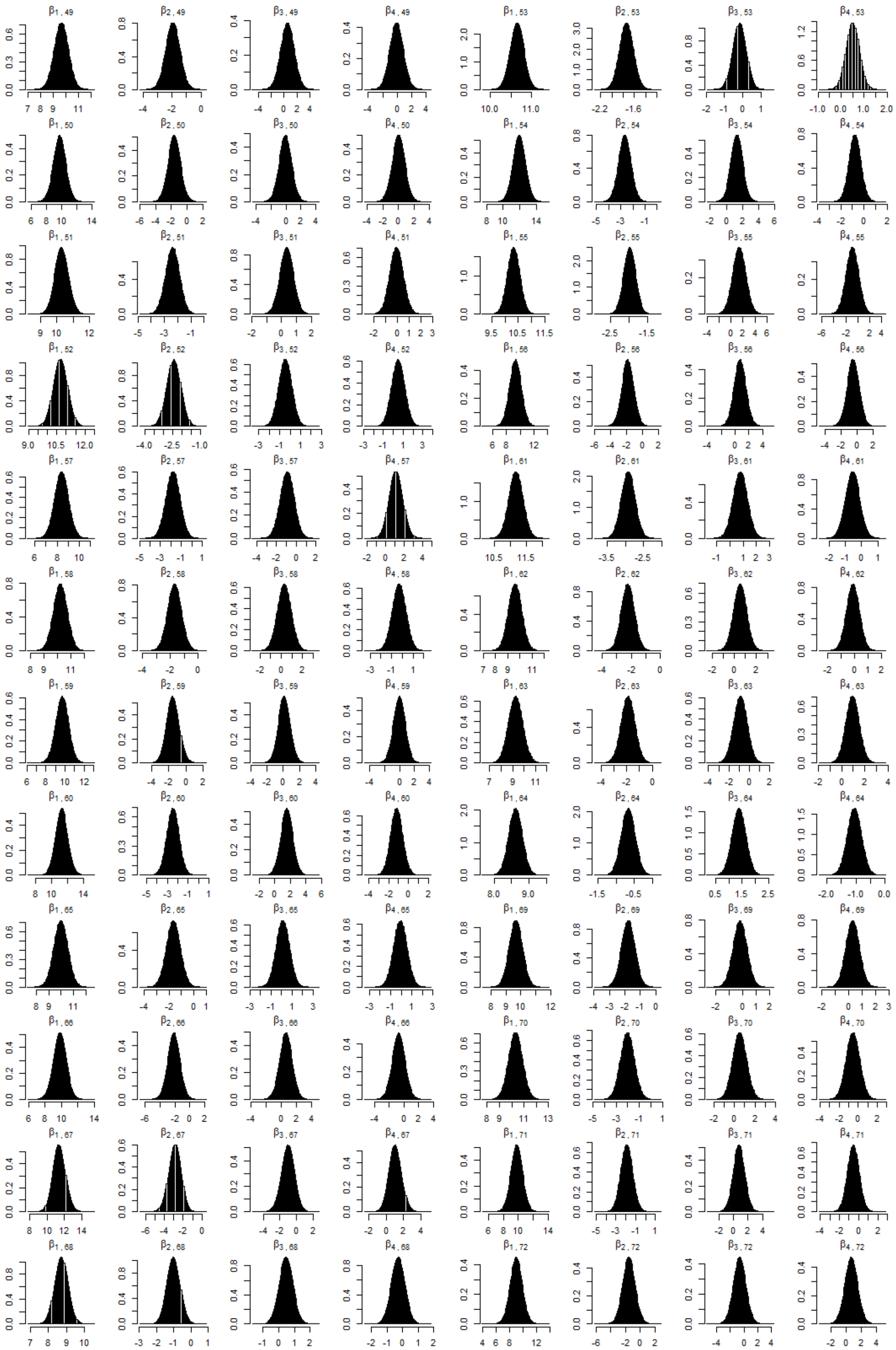
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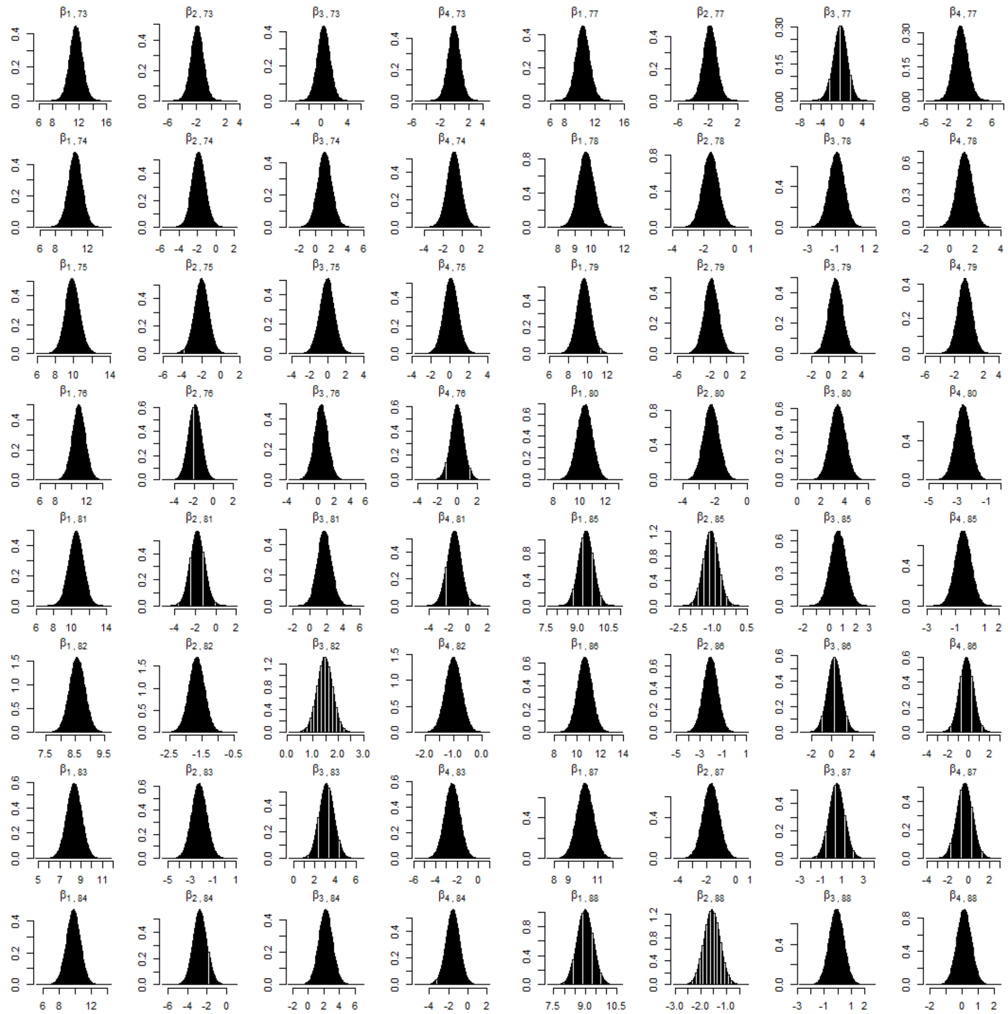

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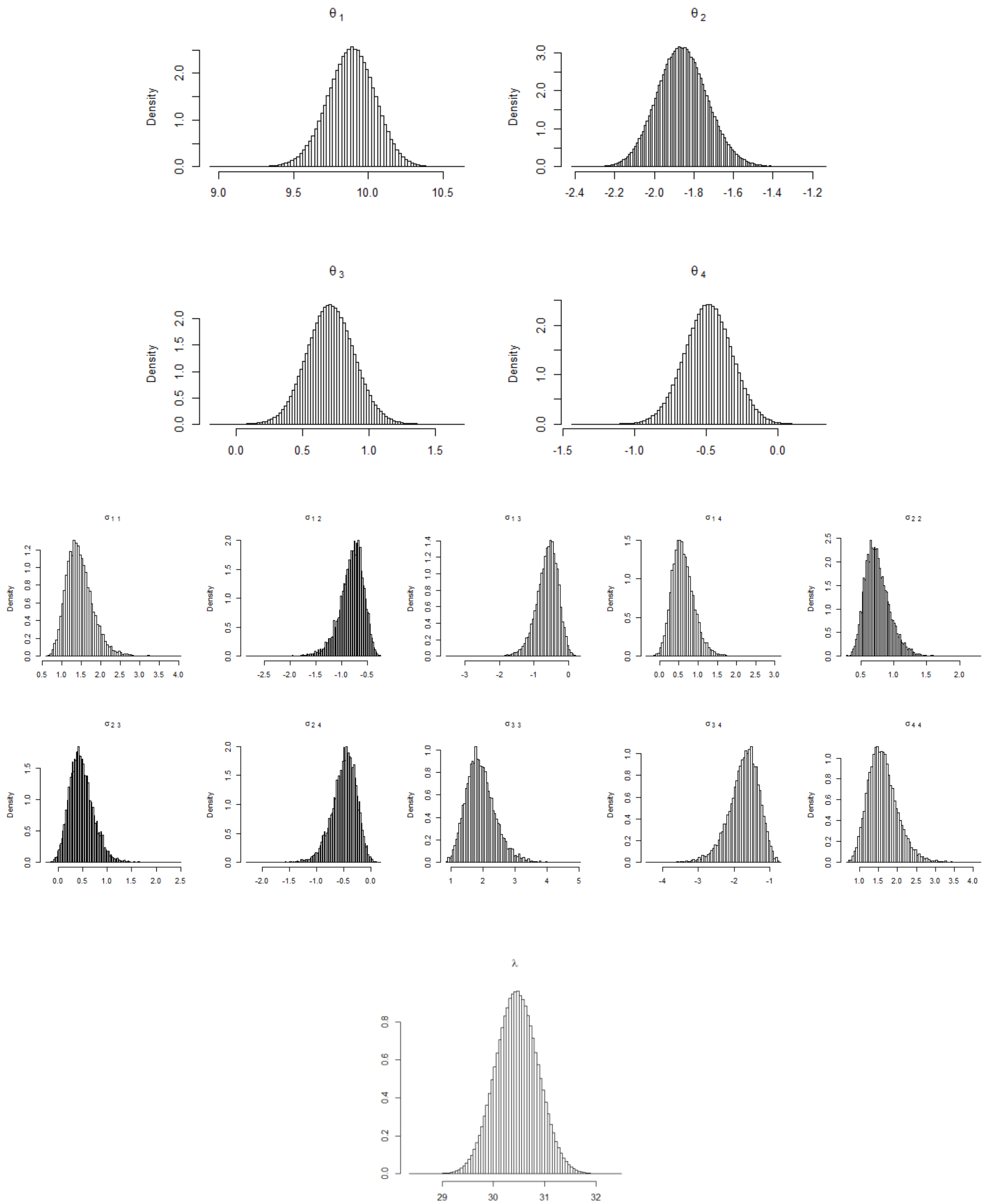
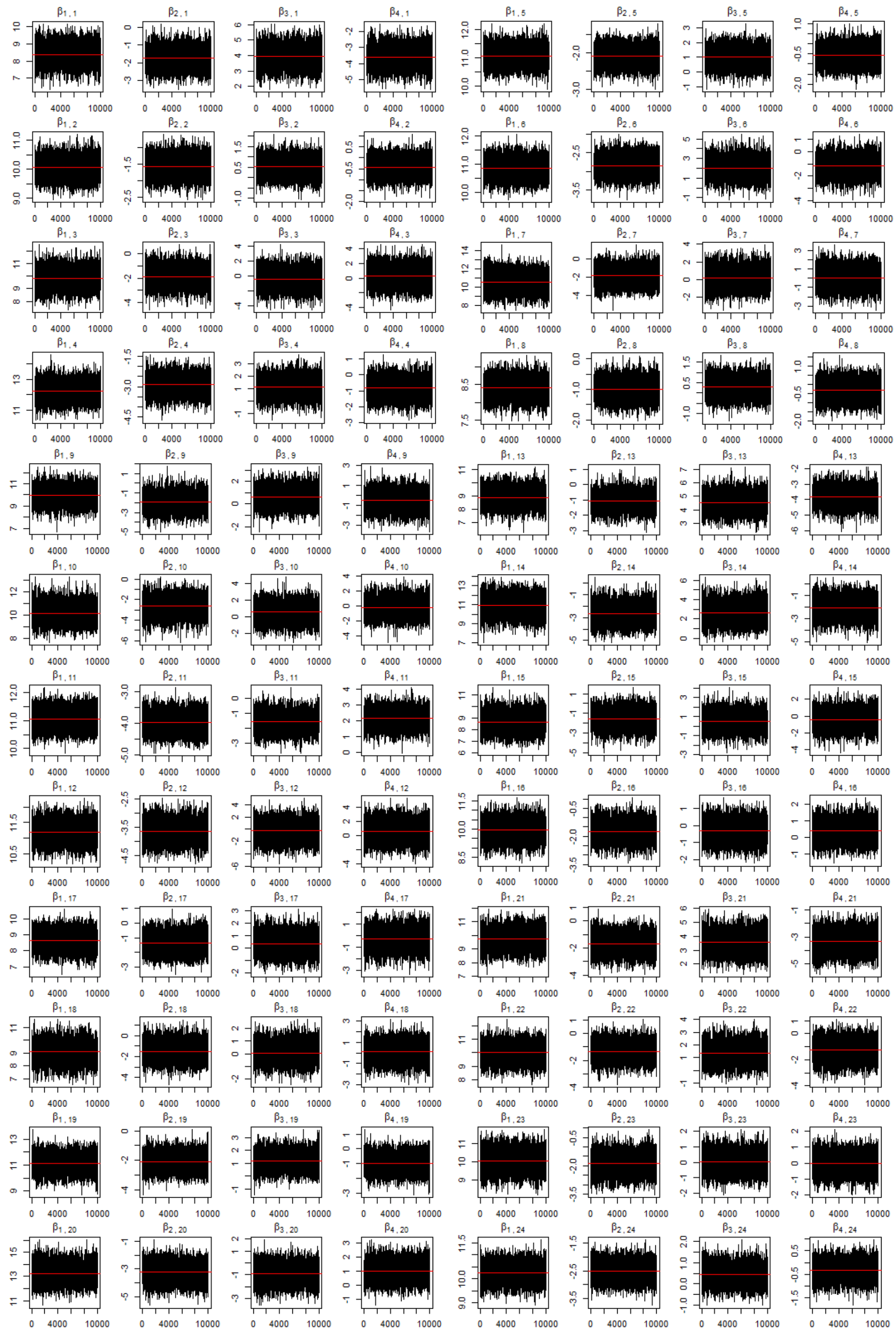
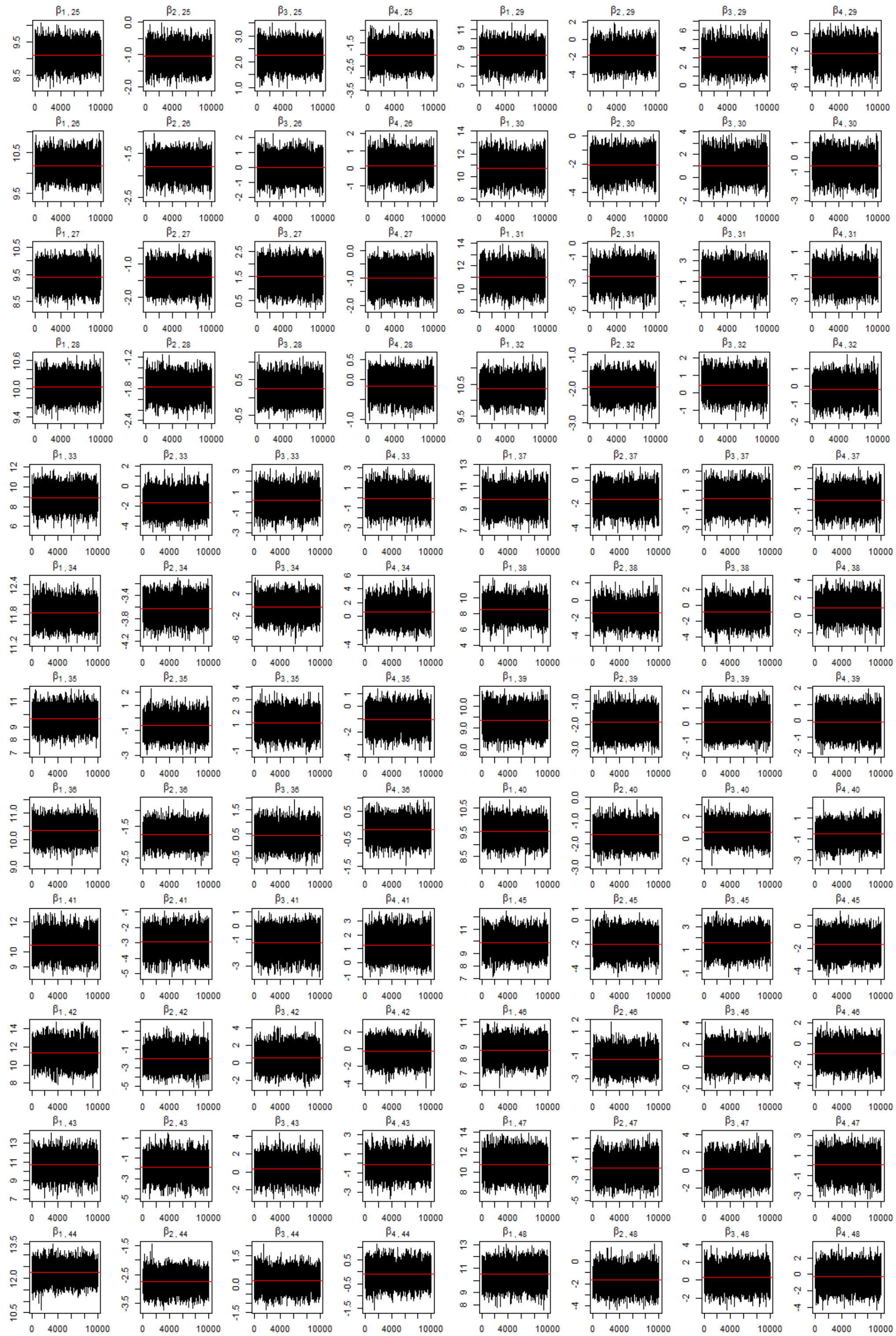

θ1
θ2
θ3
θ4
σ11
σ12
σ13
σ14
σ22
σ23
σ24
σ33
σ34
σ44
λ
Density

Trace Plots

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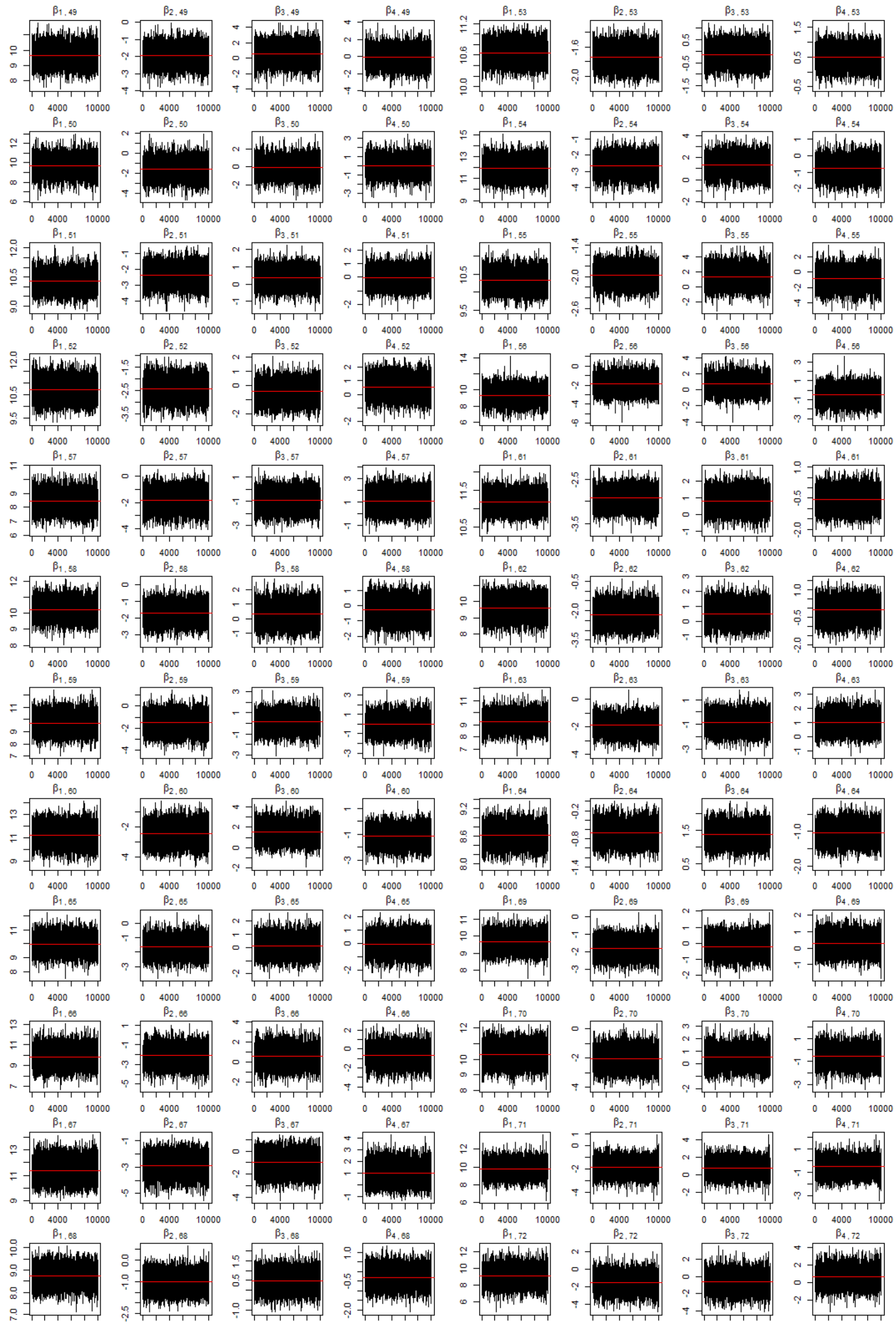

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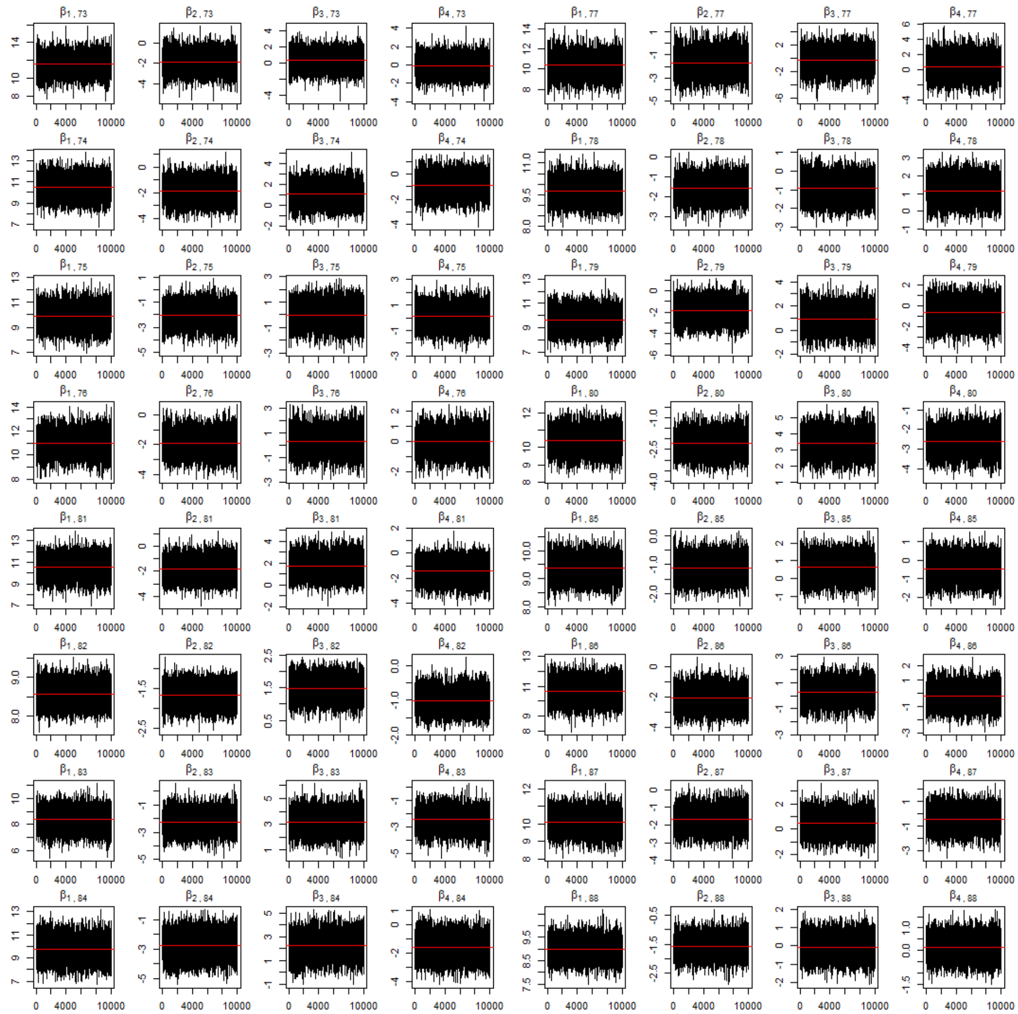
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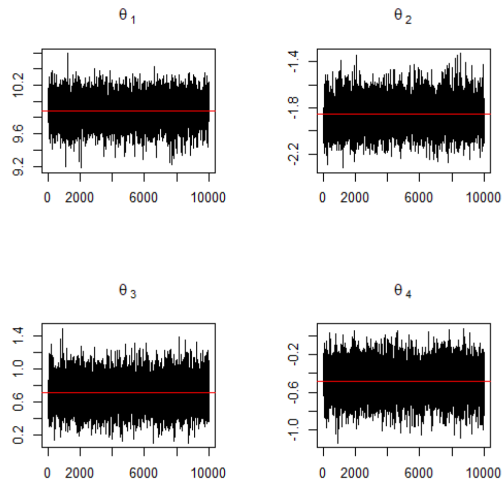

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θ_2
θ_3
θ_4
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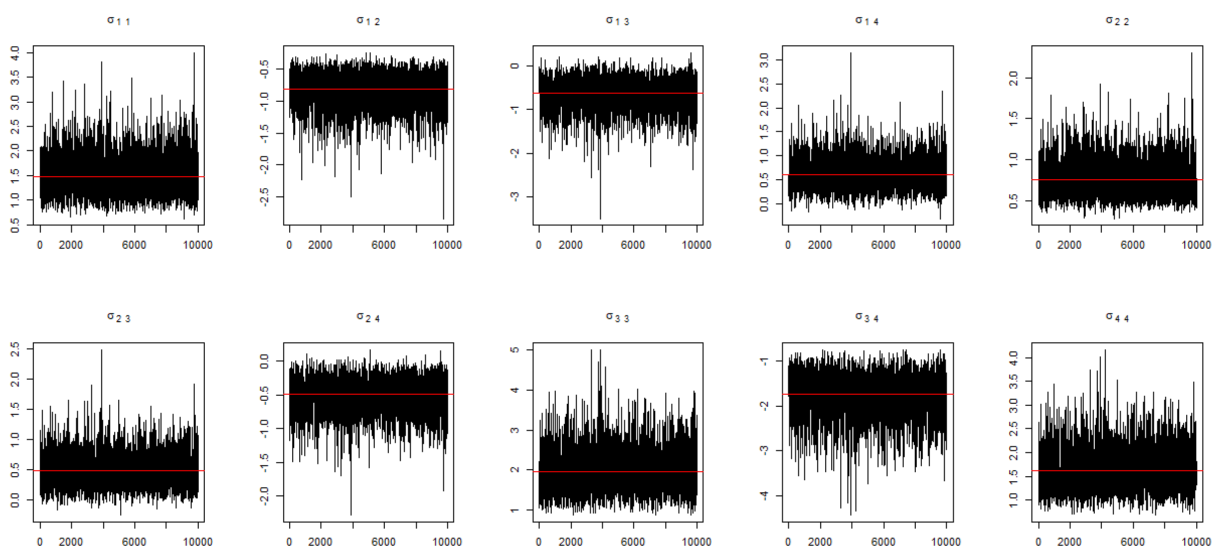

σ_{11}
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σ_{13}
σ_{14}
σ_{22}
σ_{23}
σ_{24}
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σ_{44}
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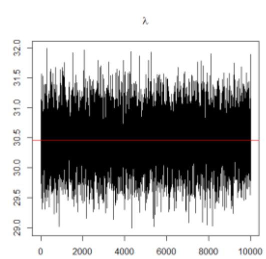
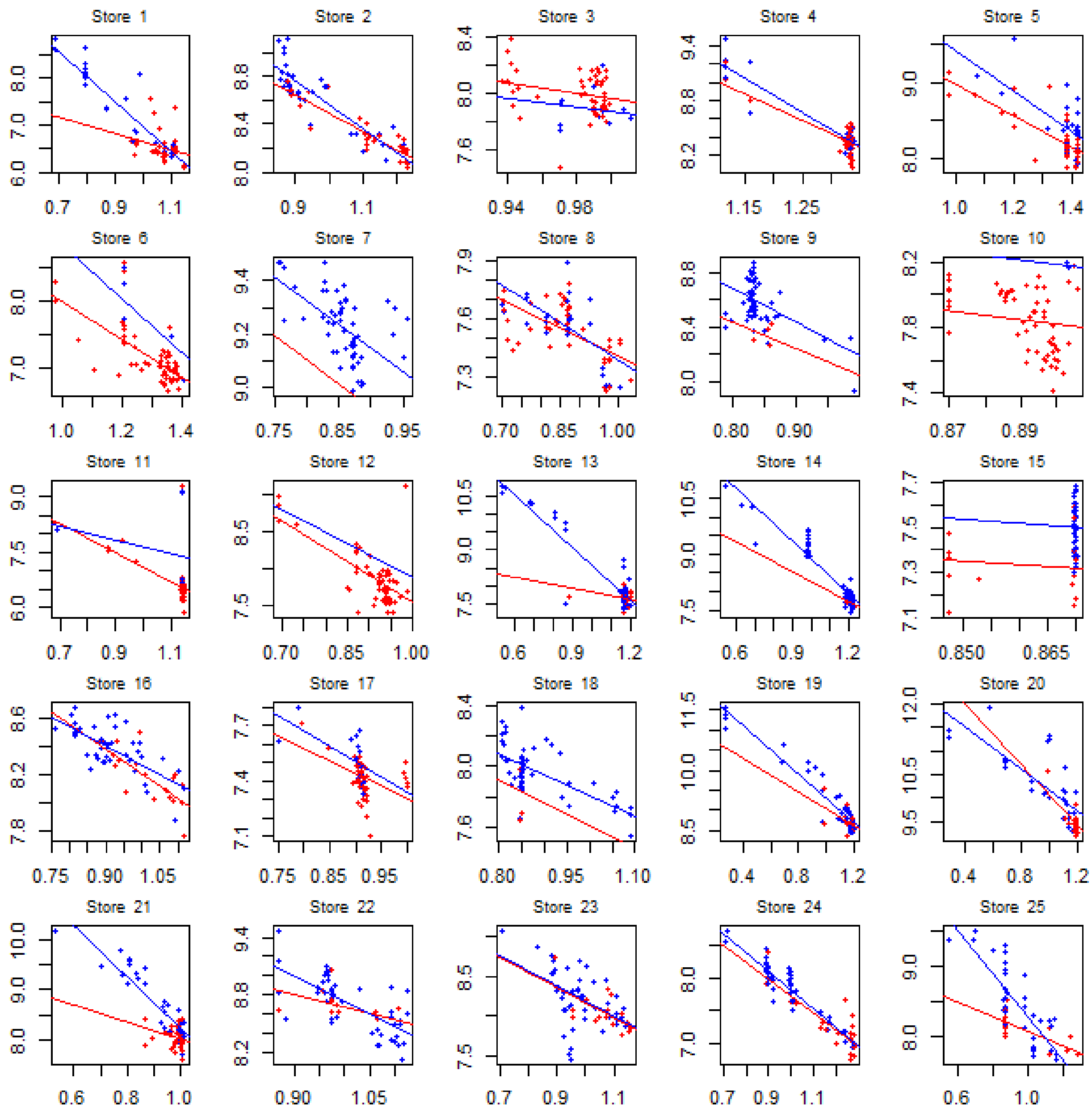
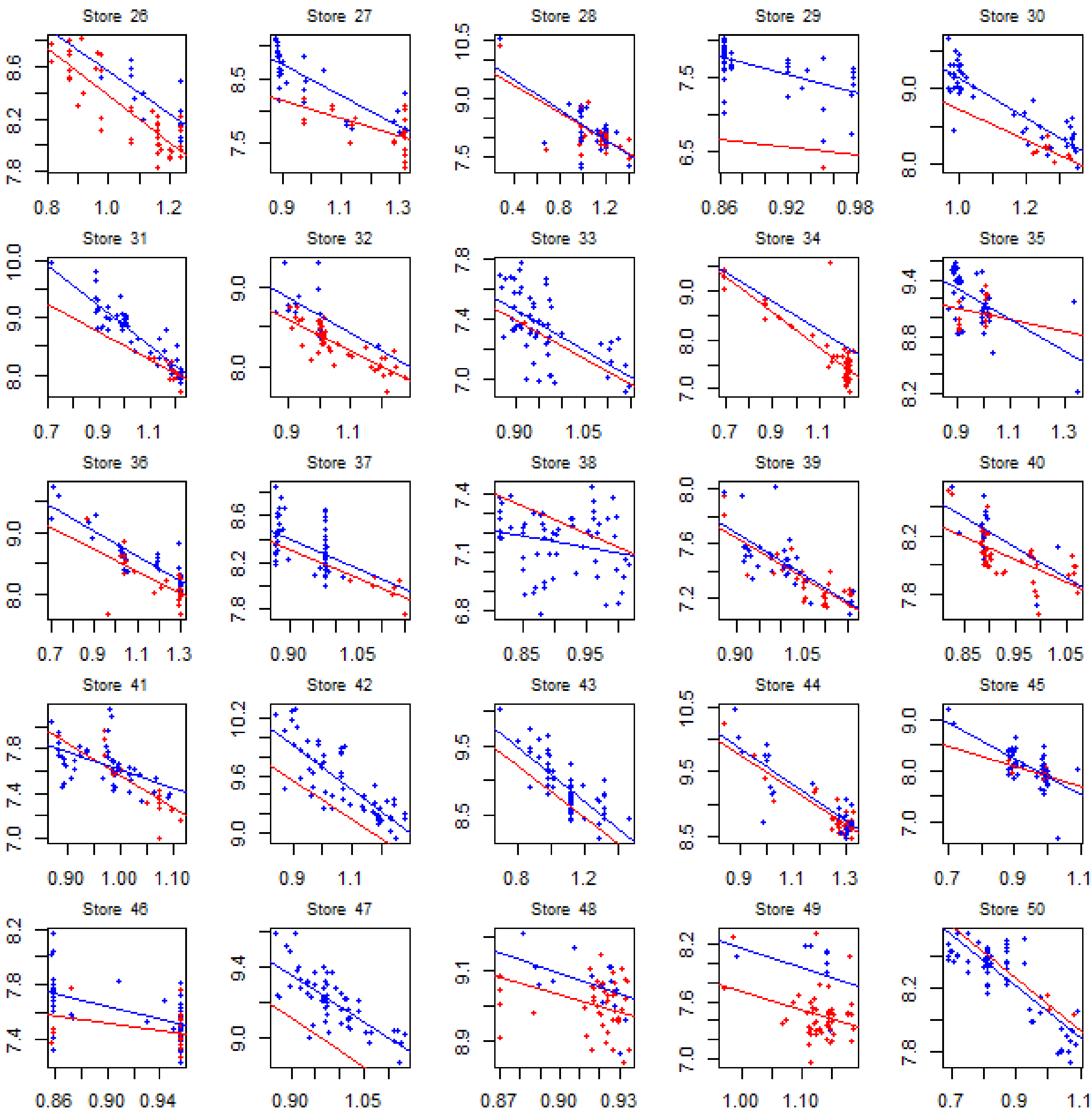


λ
0 2000 4000 6000 8000 10000

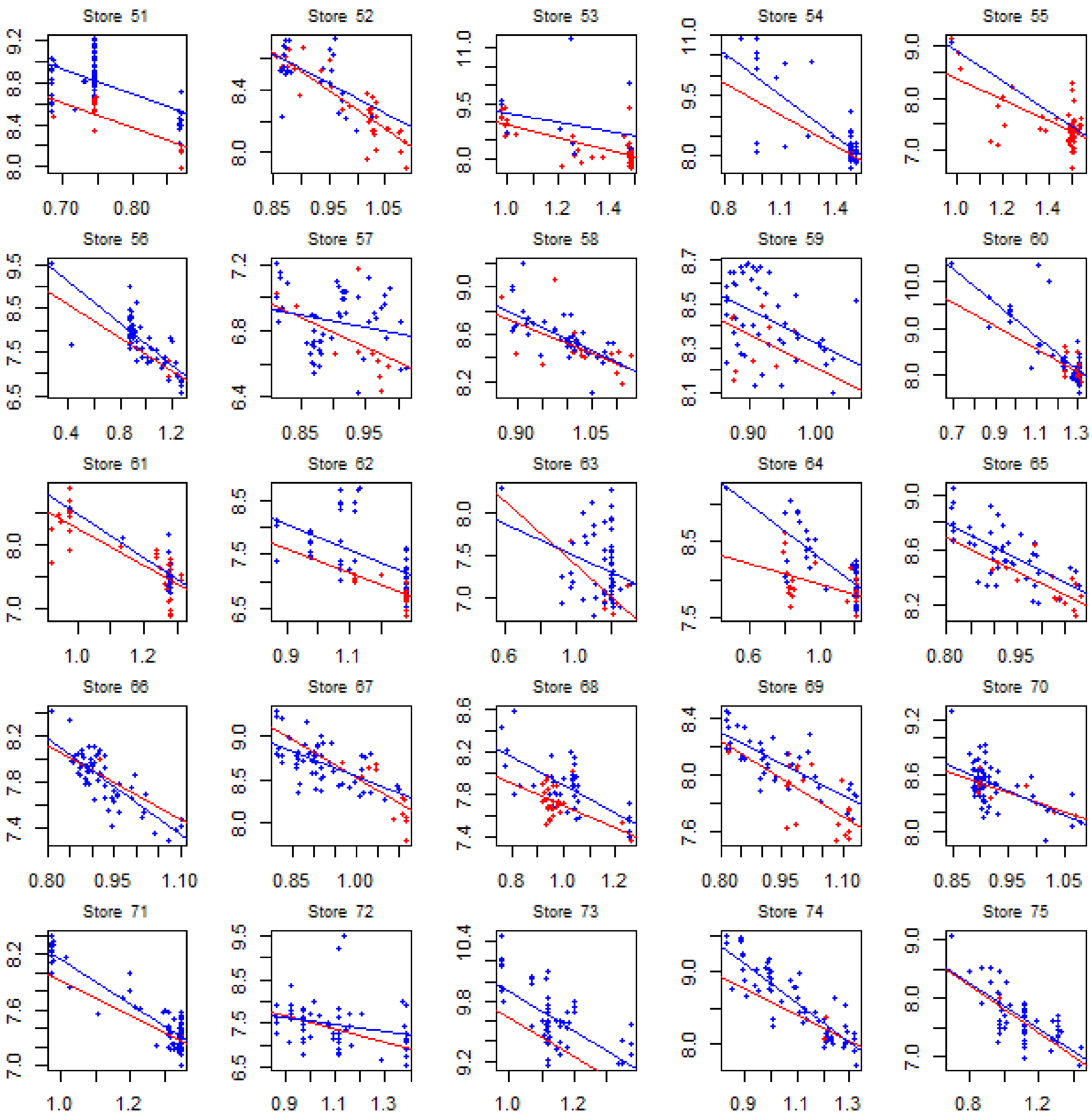
Graphs with Fitted Lines
Log Quantity vs. Log Price Per Store
Display in Blue, Non-Display in Red

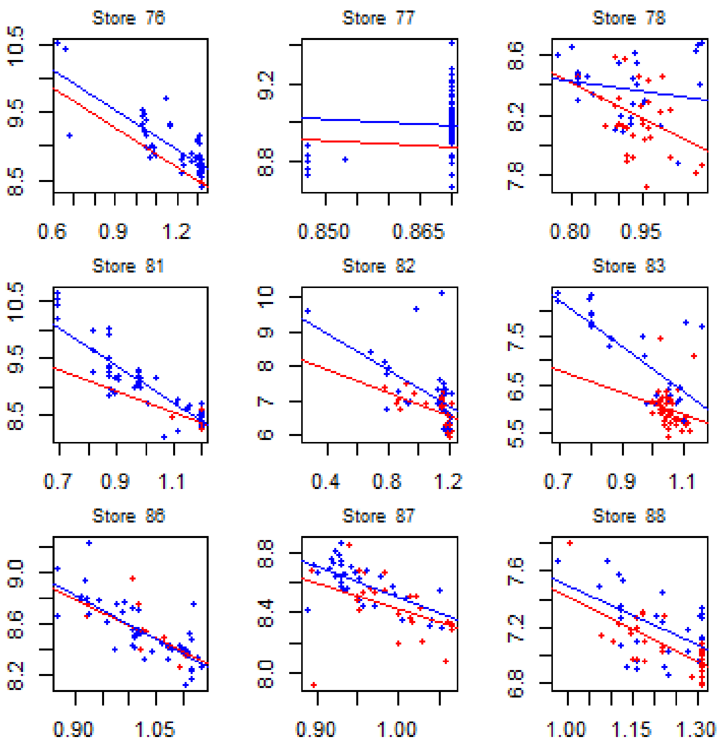
$$\log \hat{Q}_{ij} = \hat{\beta}_{1j} + \hat{\beta}_{2j} \log P_{ij}$$
$$\log \hat{Q}_{ij} = (\hat{\beta}_{1j} + \hat{\beta}_{3j}) + (\hat{\beta}_{2j} + \hat{\beta}_{4j}) \log P_{ij}$$





Store 26
Store 27
Store 28
Store 29
Store 30
Store 31
Store 32
Store 33
Store 34
Store 35
Store 36
Store 37
Store 38
Store 39
Store 40
Store 41
Store 42
Store 43
Store 44
Store 45
Store 46
Store 47
Store 48
Store 49
Store 50


Store 51
Store 52
Store 53
Store 54
Store 55
Store 56
Store 57
Store 58
Store 59
Store 60
Store 61
Store 62
Store 63
Store 64
Store 65
Store 66
Store 67
Store 68
Store 69
Store 70
Store 71
Store 72
Store 73
Store 74
Store 75


Store 76
Store 77
Store 78
Store 79
Store 80
Store 81
Store 82
Store 83
Store 84
Store 85
Store 86
Store 87
Store 88

```r
# Problem 2: Price Elasticity of Demand

# Packages
library(lattice)
library(MASS)
library(MCMCpack)

# Load data
cheese <- read.csv("C:\\Users\\trlil\\Google Drive\\UT Austin\\SDS 383D
Statistical Modeling II\\Homework\\Exercises 4\\cheese.csv", header = T)

# Add log-transformed variables
cheese <- cbind(cheese, logv = log(cheese$vol), logp = log(cheese$price))

# Number of groups
M <- length(table(cheese$store))

# Total observations
N <- dim(cheese)[1]

# Observations per group
n <- as.vector(table(cheese$store))

# Group indicator
grp <- rep(1:M, n)

# Design matrix:
# Column of 1's for intercept
# Main effect of Log Price
# Main effect of Display
# Interaction of Price and Display
X <- cbind(1, cheese$logp,
              cheese$disp,
              cheese$logp*cheese$disp)

# Number of predictors
p <- dim(X)[2]

# Response
y <- cheese$logv

# Number of Gibbs samples
T <- 10^6

# Prior parameters
Lambda0 <- diag(1, p)
Mu0 <- rep(0, p)
eta0 <- 1
S0 <- diag(1, p)
a <- 1
b <- 1

# Objects to hold parameters
Beta <- array(NA, dim = c(M, p, T))
Theta <- matrix(NA, nrow = T, ncol = p)
Sigma <- array(NA, dim = c(p, p, T))
lambda <- rep(NA, T)
```

```
# Initialize
Beta[, , 1] <- cbind(1,
                tapply(X[, 2], grp, mean),
                tapply(X[, 3], grp, mean),
                tapply(X[, 4], grp, mean))

Theta[1, ] <- colMeans(Beta[, , 1])
Sigma[, , 1] <- cov(Beta[, , 1])
Sigma[1, 1, 1] <- 1
lambda[1] <- 1/var(tapply(y, grp, mean))

#########################
##### Gibbs sampler #####
#########################

for(t in 1:(T-1)){

    # Track loop progress
    if(t %% (T/100) == 0){print(t)}

    # Update Betas
    for(j in 1:M){
        Xj <- X[grp == j, ]
        S1 <- t(Xj) %*% Xj
        S2 <- t(Xj) %*% y[grp == j]

        m_star <- solve(lambda[t]*S1 + solve(Sigma[,,t])) %*%
                (solve(Sigma[,,t]) %*% Theta[t,] + lambda[t]*S2)

        s_star <- solve(lambda[t]*S1 + solve(Sigma[,,t]))

        Beta[j,,t+1] <- mvrnorm(1, m_star, s_star)
    }

    # Update Theta
    m_star <- solve(M*solve(Sigma[,,t])+solve(Lambda0)) %*%
            (solve(Lambda0) %*% Mu0 + solve(Sigma[,,t]) %*%
             apply(Beta[,,t+1], 2, sum))

    s_star <- solve(M*solve(Sigma[,,t])+solve(Lambda0))

    Theta[t+1,] <- mvrnorm(1, m_star, s_star)

    # Update Sigma
    S3 <- 0

    for(j in 1:M){
        S3 <- S3 + (Beta[j,,t+1]-Theta[t+1,]) %*%
                t(Beta[j,,t+1]-Theta[t+1,])
    }

    Sigma[,,t+1] <- riwish(eta0 + M, S0 + S3)

    # Update lambda
    SSR <- 0
```

```
    for(j in 1:M){
        SSR <- SSR +
             sum((y[grp==j] - X[grp==j,] %*% Beta[j,,t+1])^2)
    }

    lambda[t+1] <- rgamma(1, a+N, b+1/2*SSR)
}

# Burn-in 10%
Beta <- Beta[,,(T/10):T]
Theta <- Theta[(T/10):T,]
Sigma <- Sigma[,,(T/10):T]
lambda <- lambda[(T/10):T]

###########################
##### Examine results #####
###########################

##### Beta #####

# Trace plots of last 10,000 samples
par(mfrow = c(4, 4))
par(mai = rep(0.25, 4))
tt <- (9*T/10)
for(j in 1:M){
    for(k in 1:p){
        plot(Beta[j, k, (tt-10000):tt],
             main = bquote(beta[.(k)~","~.(j)]),
             xlab = NULL,
             ylab = NULL,
             typ = "l")

        abline(h = mean(Beta[j, k, ]),
             col = "red")
    }
    par(ask = T)
}

# Graphs of posteriors
par(mfrow = c(4, 4))
par(mai = rep(0.25, 4))
for(j in 1:M){
    for(k in 1:p){
        hist(Beta[j, k, ],
             breaks = 100,
             prob = T,
             main = bquote(beta[.(k)~","~.(j)]),
             xlab = NULL)
    }
    par(ask = T)
}

# Make table to output summary stats
Beta_out <- matrix(NA, nrow = M, ncol = p)

md <- NULL
for(j in 1:M){
```

```
    for(k in 4:4){
        md <- c(md,
              density(Beta[j,k,])$x[which.max(density(Beta[j,k,])$y)])
    }
}

Beta_out <- rbind(mean = apply(Beta[, 4, ], 1, mean),
            apply(Beta[, 4, ], 1, quantile, c(0.025, 0.975)),
            mode = md)

write.csv(Beta_out, "C:\\Users\\trlil\\Google Drive\\UT Austin\\SDS 383D
Statistical Modeling II\\Homework\\Exercises 4\\out.csv")

##### Theta #####

# Trace Plots
par(mfrow = c(2, 2))
for(k in 1:p){
    plot(Theta[(tt-10000):tt, k],
         main = bquote(theta[~.(k)]),
         xlab = "",
         ylab = "",
         typ = "l")
    abline(h = mean(Theta[, k]),
         col = "red")
}

# Posteriors
par(mfrow = c(2, 2))
for(k in 1:p){
    hist(Theta[, k],
         breaks = 100,
         prob = T,
         main = bquote(theta[~.(k)]),
         xlab = "")
}

# Summary statistics
apply(Theta, 2, mean)
apply(Theta, 2, quantile, c(0.025, 0.975))

for(k in 1:p){
    print(density(Theta[, k])$x[which.max(density(Theta[, k])$y)])
}

##### Sigma #####

# Trace plots
par(mfrow = c(2, 5))
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            plot(Sigma[i, j, (tt-10000):tt],
                 main = bquote(sigma[~.(i)~""~.(j)]),
                 xlab = "",
                 ylab = "",
                 typ = "l")
```

```
            abline(h = mean(Sigma[i, j, ]),
                 col = "red")
        }
    }
}

# Posteriors
par(mfrow = c(2, 5))
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            hist(Sigma[i, j, (tt-10000):tt],
                 main = bquote(sigma[~.(i)~""~.(j)]),
                 breaks = 100,
                 prob = T,
                 xlab = "")
        }
    }
}

# Summary statistics
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            print(mean(Sigma[i, j, ]))
            print(quantile(Sigma[i, j, ], c(0.025, 0.975)))
            print(density(Sigma[i, j, ])$
                  x[which.max(density(Sigma[i, j, ])$y)])
        }

    }
}

##### Lambda #####

# Trace plot
plot(lambda[(tt-10000):tt],
     typ = "l",
     main = bquote(lambda),
     xlab = "",
     ylab = "")
abline(h = mean(lambda),
     col = "red")

# Posterior
hist(lambda,
     prob = T,
     breaks = 100,
     main = bquote(lambda),
     xlab = "",
     ylab = "")

# Summary statistics
mean(lambda)
quantile(lambda, c(0.025, 0.975))
density(lambda)$x[which.max(density(lambda)$y)]
```

```r
###################################
##### Graphs with fitted lines #####
###################################

##### For each store, plot the log quantity
##### vs. log price. Distinguish between the
##### points with display and not display.
##### Plot lines using posterior means of
##### beta coefficients with display in blue,
##### and without display in red.

# 5x5 panel of graphs
par(mfrow = c(5, 5))

# Narrow margins
par(mai = c(0.25, 0.25, 0.25, 0.25))

# Make plot for each store
for(j in 1:M){

    # Blue points are display
    clr <- rep("red", sum(grp == j))
    clr[which(X[grp == j, 3] == 1)] <- "blue"

    # Plot points and indicate
    # which are display
    plot(X[grp == j, 2], y[grp == j],
         col = clr,
         pch = 20,
         xlab = "",
         ylab = "",
         main = bquote("Store "~.(j)),
         cex.main = .9)

    # Fitted line for no display
    abline(a = mean(Beta[j, 1, ]),
         b = mean(Beta[j, 2, ]),
         col = "red")

    # Fitted line for display
    abline(a = mean(Beta[j, 1, ]) + mean(Beta[j, 3, ]),
           b = mean(Beta[j, 2, ]) + mean(Beta[j, 4, ]),
           col = "blue")

    # Pause before next graph
    par(ask = T)
}
```