Price Elasticity of Demand

Let $j = 1, \ldots, M = 88$ denote each store, and $i = 1, \ldots, n_j$ the individual measurements within each store. Also, let $N = \sum_{j=1}^{M} n_j$. We will write the model as

$$y_{ij} = \beta_{1j} + \beta_{2j}x_{1,ij} + \beta_{3j}x_{2,ij} + \beta_{4j}(x_{1,ij} \times x_{2,ij}) + \varepsilon_{ij},$$

where y_{ij} is the log of the sales volume (in thousands of units sold), $x_{1,ij}$ is the log of the price (in USD), and $x_{2,ij}$ is an indicator denoting whether or not advertising was present in the store; i.e., $x_{2,ij} = 1$ if the ij^{th} observation was measured when an advertisement display was present, and 0 otherwise.

In words,

$$\log \text{Quantity}_{ij} = \beta_{1j} + \beta_{2j} \log \text{Price}_{ij} + \beta_{3j}\text{Display}_{ij} + \beta_{4j}\left(\log \text{Price}_{ij} \times \log \text{Display}_{ij}\right) + \varepsilon_{ij},$$

or

$$\log Q_{ij} = \beta_{1j} + \beta_{2j} \log P_{ij} + \beta_{3j}D_{ij} + \beta_{4j}\left(\log P_{ij} \times D_{ij}\right) + \varepsilon_{ij},$$

where $Q_{ij} = \text{Quantity}_{ij}$, $P_{ij} = \text{Price}_{ij}$, and $D_{ij} = \text{Display}_{ij}$.

Also, we assume the ε_{ij} are iid $N(0, \sigma^2)$. We will denote $\boldsymbol{\beta}_j = \left(\beta_{1j}, \beta_{2j}, \beta_{3j}, \beta_{4j}\right)^T$ and $\boldsymbol{x}_{ij} = \left(1, x_{1,ij}, x_{2,ij}, x_{1,ij}x_{2,ij}\right)^T$, so that the model can also be written as

$$y_{ij} = \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} + \varepsilon_{ij}.$$

We further assume that $\boldsymbol{\beta}_j$ are random effects, deriving from a $\mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma})$ distribution. We then place (conditionally) conjugate prior distributions on $\boldsymbol{\theta}$, $\boldsymbol{\Sigma}$, and $\lambda = \frac{1}{\sigma^2}$. The hierarchical nature of the model can thus be specified as:

$$y_{ij} | \boldsymbol{\beta}_j, \lambda \sim N\left(\boldsymbol{\beta}_j^T \boldsymbol{x}_{ij}, \frac{1}{\lambda}\right)$$

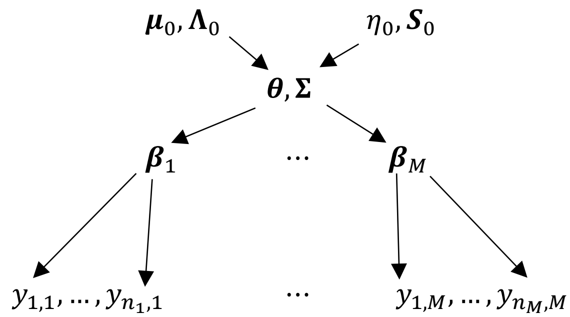
$$\boldsymbol{\beta}_j | \boldsymbol{\theta}, \boldsymbol{\Sigma} \sim \mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0)$$

$$\boldsymbol{\Sigma} \sim \text{IW}(\eta_0, \boldsymbol{S}_0)$$

$$\lambda \sim \text{Gamma}(a, b).$$

We can represent the model graphically as follows:



The full posterior is then

$$p(\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M, \boldsymbol{\theta}, \boldsymbol{\Sigma}, \lambda | \text{Data}) \propto p(\text{Data} | \boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M, \boldsymbol{\theta}, \boldsymbol{\Sigma}, \lambda) p(\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_M) p(\boldsymbol{\theta}) p(\boldsymbol{\Sigma}) p(\lambda)$$

$$= \left(\prod_{j=1}^{M} \prod_{i=1}^{n_j} N\left(y_{ij} \middle| \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij}, \frac{1}{\lambda} \right) \right) \left(\prod_{j=1}^{M} \mathcal{N}\left(\boldsymbol{\beta}_j \middle| \boldsymbol{\theta}, \boldsymbol{\Sigma} \right) \right) \times \mathcal{N}(\boldsymbol{\theta} | \boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0) \times \text{IW}(\boldsymbol{\Sigma} | \eta_0, \boldsymbol{S}_0) \times \text{Gamma}(\lambda | a, b)$$

$$\propto \left(\prod_{j=1}^{M} \prod_{i=1}^{n_j} \sqrt{\lambda} \exp\left\{ -\frac{\lambda}{2} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \right) \times \left(\prod_{j=1}^{M} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \right)$$

$$\times \left(\exp\left\{ -\frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\mu}_0)^T \boldsymbol{\Lambda}_0^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu}_0) \right\} \right) \times |\boldsymbol{\Sigma}|^{-\frac{\eta_0 + 4 + 1}{2}} \exp\left\{ -\frac{1}{2} \text{tr}(\boldsymbol{S}_0 \boldsymbol{\Sigma}^{-1}) \right\}$$

$$\times \lambda^{a-1} \exp\{-b\lambda\}$$

From here we can derive the full conditionals for the $\boldsymbol{\beta}_j$, $\boldsymbol{\theta}$, $\boldsymbol{\Sigma}$, and λ, so that we may then run a Gibbs sampler.

$$\begin{aligned}
p(\boldsymbol{\beta}_j | -) &\propto \left[\prod_{i=1}^{n_j} \exp\left\{ -\frac{\lambda}{2} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \right] \times \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \\
&\propto \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n_j} \left(y_{ij} - \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \right)^2 \right\} \exp\left\{ -\frac{1}{2} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\beta}_j - \boldsymbol{\theta} \right) \right\} \\
&= \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n_j} \left(y_{ij}^2 - 2 y_{ij} \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j + \boldsymbol{\beta}_j^T \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T \boldsymbol{\beta}_j \right) - \frac{1}{2} \left(\boldsymbol{\beta}_j^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\beta}_j - 2 \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\beta}_j + \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\theta} \right) \right\} \\
&\propto \exp\left\{ -\frac{1}{2} \left[\boldsymbol{\beta}_j^T \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right) \boldsymbol{\beta}_j^T - 2 \left(\lambda \sum_{i=1}^{n_j} y_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\theta}^T \boldsymbol{\Sigma}^{-1} \right) \boldsymbol{\beta}_j \right] \right\} \\
&\propto \mathcal{N}\left(\boldsymbol{\beta}_j \middle| \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right)^{-1} \left(\boldsymbol{\Sigma}^{-1} \boldsymbol{\theta} + \lambda \sum_{i=1}^{n_j} y_{ij} \boldsymbol{x}_{ij} \right); \left(\lambda \sum_{i=1}^{n_j} \boldsymbol{x}_{ij} \boldsymbol{x}_{ij}^T + \boldsymbol{\Sigma}^{-1} \right)^{-1} \right)
\end{aligned}$$

$$
\begin{aligned}
p(\boldsymbol{\theta}|-) &\propto \left(\prod_{j=1}^{M}|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\right\}\right)\times\left(\exp\left\{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)^T\boldsymbol{\Lambda}_0^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)\right\}\right)\\
&\propto \exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)^T\boldsymbol{\Lambda}_0^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu}_0)\right\}\\
&\propto \exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(-2\boldsymbol{\beta}_j^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\theta}+\boldsymbol{\theta}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\theta}\right)-\frac{1}{2}\left(\boldsymbol{\theta}^T\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\theta}-2\boldsymbol{\mu}_0^T\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\theta}\right)\right\}\\
&= \exp\left\{-\frac{1}{2}\left[\boldsymbol{\theta}^T(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})\boldsymbol{\theta}-2\left(\boldsymbol{\mu}_0^T\boldsymbol{\Lambda}_0^{-1}+\sum_{j=1}^{M}\boldsymbol{\beta}_j^T\boldsymbol{\Sigma}^{-1}\right)\boldsymbol{\theta}\right]\right\}\\
&\propto \mathcal{N}\left(\boldsymbol{\theta}\middle|(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})^{-1}\left(\boldsymbol{\Lambda}_0^{-1}\boldsymbol{\mu}_0+\boldsymbol{\Sigma}^{-1}\sum_{j=1}^{M}\boldsymbol{\beta}_j\right);(M\boldsymbol{\Sigma}^{-1}+\boldsymbol{\Lambda}_0^{-1})^{-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
p(\boldsymbol{\Sigma}|-) &\propto \left(\prod_{j=1}^{M}|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\right\}\right)\times|\boldsymbol{\Sigma}|^{-\frac{\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left[\sum_{j=1}^{m}\left(\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\Sigma^{-1}\right)\right]-\frac{1}{2}\mathrm{tr}(\boldsymbol{S}_0\boldsymbol{\Sigma}^{-1})\right\}\\
&\propto |\boldsymbol{\Sigma}|^{-\frac{M+\eta_0+4+1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left[\left(\boldsymbol{S}_0+\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\right)\boldsymbol{\Sigma}^{-1}\right]\right\}\\
&\propto \mathrm{IW}\left(\boldsymbol{\Sigma}\middle|\eta_0+M;\boldsymbol{S}_0+\textstyle\sum_{j=1}^{M}\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)\left(\boldsymbol{\beta}_j-\boldsymbol{\theta}\right)^T\right)
\end{aligned}
$$

$$
\begin{aligned}
p(\lambda|-) &\propto \left(\prod_{j=1}^{M}\prod_{i=1}^{n_j}\sqrt{\lambda}\exp\left\{-\frac{\lambda}{2}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right\}\right)\times\lambda^{a-1}\exp\{-b\lambda\}\\
&= \lambda^{N+a-1}\exp\left\{-\frac{\lambda}{2}\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2-b\lambda\right\}\\
&= \lambda^{N+a-1}\exp\left\{-\lambda\left[b+\frac{1}{2}\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right]\right\}\\
&\propto \mathrm{Gamma}\left(\lambda\middle|a+N;b+\frac{1}{2}\textstyle\sum_{j=1}^{M}\sum_{i=1}^{n_j}\left(y_{ij}-\boldsymbol{\beta}_j^T\boldsymbol{x}_{ij}\right)^2\right).
\end{aligned}
$$

Summary Statistics

	$\beta_{1,j}$				$\beta_{2,j}$				$\beta_{3,j}$				$\beta_{4,j}$			
Store j	Mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%	mean	mode	2.5%	97.5%
1	8.40	8.40	7.36	9.43	-1.74	-1.73	-2.70	-0.76	3.92	3.93	2.80	5.07	-3.58	-3.57	-4.69	-2.50
2	10.05	10.05	9.47	10.64	-1.56	-1.55	-2.09	-1.03	0.51	0.51	-0.26	1.28	-0.44	-0.45	-1.18	0.30
3	9.84	9.90	8.49	11.20	-1.88	-1.94	-3.26	-0.50	-0.39	-0.38	-2.57	1.76	0.30	0.21	-1.87	2.50
4	12.21	12.20	11.14	13.30	-2.90	-2.88	-3.73	-2.09	1.12	1.14	-0.25	2.49	-0.83	-0.82	-1.89	0.24
5	11.07	11.08	10.48	11.67	-2.10	-2.11	-2.54	-1.66	1.00	1.01	-0.08	2.08	-0.56	-0.56	-1.35	0.23
6	10.85	10.84	10.27	11.42	-2.84	-2.83	-3.29	-2.40	2.02	2.02	0.38	3.72	-1.19	-1.13	-2.53	0.10
7	10.57	10.57	8.81	12.33	-1.83	-1.85	-3.49	-0.16	0.18	0.16	-1.81	2.17	0.05	0.02	-1.81	1.91
8	8.40	8.41	7.91	8.89	-1.00	-1.00	-1.56	-0.44	0.31	0.30	-0.48	1.10	-0.33	-0.34	-1.23	0.57
9	9.99	9.96	8.65	11.30	-1.94	-1.92	-3.46	-0.41	0.60	0.62	-0.79	2.00	-0.45	-0.47	-2.06	1.14
10	10.14	10.12	8.77	11.59	-2.58	-2.53	-4.21	-1.04	0.57	0.61	-1.37	2.44	-0.23	-0.29	-2.23	1.84
11	11.06	11.05	10.47	11.66	-3.97	-3.97	-4.51	-3.44	-1.59	-1.58	-2.66	-0.52	2.14	2.16	1.17	3.12
12	11.19	11.20	10.62	11.76	-3.64	-3.64	-4.27	-3.01	-0.26	-0.31	-2.97	2.37	0.59	0.51	-1.77	3.01
13	8.90	8.94	7.71	10.05	-1.07	-1.12	-2.06	-0.04	4.57	4.55	3.39	5.79	-3.83	-3.83	-4.88	-2.81
14	10.92	10.90	9.16	12.67	-2.65	-2.63	-4.10	-1.19	2.67	2.66	0.91	4.46	-2.07	-2.10	-3.55	-0.61
15	8.67	8.73	7.23	10.07	-1.55	-1.52	-3.17	0.11	0.54	0.50	-1.10	2.18	-0.41	-0.38	-2.30	1.47
16	9.94	9.92	9.01	10.86	-1.73	-1.72	-2.64	-0.82	-0.32	-0.33	-1.39	0.75	0.38	0.37	-0.69	1.46
17	8.65	8.65	7.63	9.65	-1.34	-1.33	-2.44	-0.23	0.34	0.36	-1.06	1.76	-0.31	-0.35	-1.86	1.23
18	9.10	9.12	7.78	10.39	-1.49	-1.53	-2.98	0.02	0.07	0.03	-1.26	1.41	0.13	0.13	-1.41	1.65
19	11.15	11.15	9.97	12.33	-2.11	-2.09	-3.16	-1.07	1.20	1.23	0.00	2.39	-0.97	-0.99	-2.03	0.09
20	13.27	13.21	11.92	14.69	-3.23	-3.19	-4.44	-2.09	-0.89	-0.85	-2.33	0.47	1.01	0.97	-0.16	2.25
21	9.74	9.78	8.53	10.92	-1.72	-1.74	-2.92	-0.48	3.54	3.52	2.28	4.84	-3.31	-3.30	-4.64	-2.01
22	10.02	10.06	8.81	11.20	-1.35	-1.35	-2.53	-0.13	1.33	1.33	0.00	2.68	-1.27	-1.30	-2.62	0.05
23	10.04	10.02	9.06	11.04	-1.88	-1.86	-2.81	-0.95	0.03	0.04	-1.10	1.15	-0.02	0.01	-1.09	1.06
24	10.25	10.24	9.57	10.93	-2.53	-2.53	-3.10	-1.95	0.45	0.45	-0.33	1.22	-0.34	-0.35	-1.03	0.34
25	9.11	9.10	8.61	9.61	-1.06	-1.07	-1.58	-0.53	2.25	2.24	1.61	2.88	-2.04	-2.03	-2.70	-1.39
26	10.17	10.18	9.78	10.57	-1.79	-1.80	-2.16	-1.43	0.02	0.03	-1.10	1.13	0.17	0.18	-0.79	1.14
27	9.41	9.40	8.76	10.05	-1.39	-1.40	-1.92	-0.86	1.48	1.48	0.73	2.24	-1.03	-1.02	-1.69	-0.37
28	10.04	10.04	9.69	10.38	-1.77	-1.78	-2.08	-1.47	0.24	0.24	-0.23	0.71	-0.18	-0.18	-0.61	0.25
29	8.22	8.22	6.42	9.94	-1.81	-1.85	-3.58	0.02	3.07	3.06	1.22	5.00	-2.27	-2.25	-4.23	-0.37
30	10.76	10.74	9.23	12.28	-2.03	-2.05	-3.24	-0.82	0.99	1.03	-0.56	2.55	-0.58	-0.60	-1.83	0.65
31	11.00	11.00	9.48	12.54	-2.47	-2.46	-3.79	-1.19	1.44	1.43	-0.15	3.00	-1.09	-1.01	-2.43	0.27
32	10.37	10.37	9.87	10.87	-1.96	-1.95	-2.44	-1.49	0.43	0.43	-0.53	1.39	-0.19	-0.16	-1.14	0.75
33	8.91	8.94	7.34	10.47	-1.69	-1.65	-3.31	-0.06	0.15	0.15	-1.48	1.77	-0.09	-0.07	-1.77	1.59
34	11.83	11.82	11.49	12.17	-3.63	-3.63	-3.93	-3.34	-0.38	-0.33	-3.03	2.21	0.66	0.67	-1.65	3.03
35	9.67	9.73	8.38	10.88	-0.63	-0.67	-1.87	0.68	1.16	1.11	-0.14	2.52	-1.06	-1.00	-2.44	0.27
36	10.33	10.35	9.76	10.89	-1.77	-1.78	-2.24	-1.30	0.43	0.42	-0.24	1.11	-0.17	-0.16	-0.76	0.41
37	9.80	9.75	8.23	11.36	-1.65	-1.66	-3.07	-0.23	0.14	0.16	-1.60	1.86	-0.05	-0.02	-1.67	1.57
38	8.55	8.58	6.71	10.38	-1.42	-1.43	-3.15	0.32	-0.86	-0.86	-2.80	1.08	0.83	0.76	-0.99	2.65
39	9.34	9.33	8.53	10.15	-1.89	-1.86	-2.66	-1.13	0.11	0.13	-1.01	1.23	-0.09	-0.10	-1.17	1.00
40	9.54	9.57	8.88	10.20	-1.59	-1.61	-2.29	-0.89	0.57	0.55	-0.69	1.85	-0.51	-0.49	-1.90	0.87
41	10.48	10.47	9.36	11.65	-2.91	-2.93	-4.05	-1.83	-1.25	-1.22	-2.64	0.08	1.29	1.24	-0.02	2.66
42	11.35	11.36	9.55	13.15	-2.00	-1.95	-3.69	-0.31	0.63	0.65	-1.19	2.46	-0.30	-0.29	-2.02	1.41
43	10.70	10.69	8.94	12.46	-1.88	-1.92	-3.52	-0.22	0.36	0.35	-1.41	2.14	-0.10	-0.05	-1.78	1.56
44	12.24	12.22	11.61	12.87	-2.75	-2.74	-3.26	-2.24	0.17	0.20	-0.61	0.95	-0.09	-0.10	-0.73	0.55
45	9.86	9.87	8.53	11.17	-1.97	-2.00	-3.34	-0.58	1.60	1.56	0.19	3.04	-1.61	-1.58	-3.11	-0.15
46	8.77	8.77	7.53	9.97	-1.38	-1.46	-2.67	-0.07	0.93	0.91	-0.47	2.36	-0.90	-0.93	-2.44	0.60
47	10.78	10.75	9.01	12.55	-1.85	-1.88	-3.51	-0.18	0.16	0.22	-1.71	2.03	0.08	0.11	-1.67	1.83
48	10.52	10.55	9.14	11.86	-1.65	-1.67	-3.11	-0.15	0.34	0.39	-1.46	2.16	-0.31	-0.25	-2.28	1.64
49	9.66	9.68	8.57	10.75	-1.96	-1.94	-2.92	-0.99	0.55	0.48	-1.50	2.58	-0.09	-0.05	-1.91	1.74
50	9.72	9.74	8.10	11.32	-1.62	-1.66	-3.18	-0.05	-0.07	-0.04	-1.71	1.57	0.02	0.01	-1.58	1.61
51	10.29	10.28	9.48	11.11	-2.38	-2.37	-3.42	-1.36	0.34	0.37	-0.54	1.21	-0.04	-0.08	-1.14	1.09
52	10.70	10.70	9.95	11.45	-2.43	-2.42	-3.18	-1.67	-0.45	-0.45	-1.67	0.77	0.51	0.51	-0.77	1.80
53	10.64	10.64	10.32	10.97	-1.74	-1.74	-1.98	-1.50	-0.17	-0.17	-0.89	0.55	0.50	0.50	-0.06	1.07
54	11.95	11.96	10.49	13.42	-2.66	-2.66	-3.67	-1.66	1.32	1.32	-0.15	2.78	-0.77	-0.74	-1.77	0.25
55	10.33	10.33	9.87	10.78	-1.97	-1.98	-2.29	-1.66	1.40	1.35	-0.73	3.56	-0.88	-0.85	-2.97	1.18
56	9.33	9.36	7.61	11.03	-1.88	-1.88	-3.35	-0.39	0.72	0.71	-1.00	2.45	-0.50	-0.58	-2.01	1.00
57	8.42	8.39	7.21	9.64	-1.80	-1.80	-3.11	-0.52	-0.91	-0.88	-2.29	0.43	1.08	1.07	-0.36	2.56
58	10.24	10.22	9.24	11.23	-1.71	-1.72	-2.67	-0.74	0.31	0.32	-0.93	1.56	-0.28	-0.25	-1.51	0.94
59	9.71	9.72	8.38	11.01	-1.50	-1.55	-2.92	-0.04	0.12	0.15	-1.30	1.56	-0.01	-0.04	-1.58	1.54
60	11.25	11.22	9.78	12.75	-2.45	-2.46	-3.62	-1.30	1.54	1.56	0.02	3.05	-1.17	-1.16	-2.35	0.03
61	11.18	11.18	10.75	11.62	-2.92	-2.92	-3.29	-2.56	0.78	0.80	-0.27	1.83	-0.55	-0.59	-1.40	0.30
62	9.61	9.62	8.54	10.68	-2.23	-2.23	-3.10	-1.36	0.54	0.54	-0.59	1.68	-0.10	-0.09	-1.04	0.83
63	9.28	9.27	8.07	10.51	-1.89	-1.87	-2.94	-0.85	-0.85	-0.81	-2.15	0.43	0.95	0.95	-0.15	2.07
64	8.62	8.63	8.23	9.00	-0.67	-0.67	-1.05	-0.30	1.38	1.38	0.89	1.88	-1.05	-1.03	-1.52	-0.57
65	9.99	9.99	8.89	11.09	-1.64	-1.63	-2.72	-0.55	0.15	0.09	-1.08	1.38	-0.06	-0.04	-1.30	1.17
66	9.82	9.88	8.27	11.39	-2.13	-2.15	-3.75	-0.51	0.63	0.67	-1.00	2.26	-0.70	-0.73	-2.39	0.99
67	11.40	11.37	10.03	12.84	-2.89	-2.80	-4.24	-1.62	-1.01	-1.00	-2.55	0.46	1.03	1.02	-0.37	2.50
68	8.72	8.73	7.99	9.45	-1.02	-1.03	-1.77	-0.27	0.48	0.49	-0.37	1.33	-0.29	-0.26	-1.15	0.56
69	9.67	9.68	8.79	10.54	-1.78	-1.77	-2.63	-0.94	-0.19	-0.20	-1.19	0.81	0.30	0.30	-0.68	1.29
70	10.32	10.28	9.22	11.43	-2.01	-2.00	-3.18	-0.85	0.53	0.51	-0.77	1.84	-0.53	-0.49	-1.92	0.84
71	9.80	9.82	8.27	11.33	-1.88	-1.87	-3.06	-0.70	0.77	0.75	-0.78	2.32	-0.54	-0.50	-1.73	0.65
72	9.05	9.02	7.26	10.84	-1.52	-1.58	-3.21	0.17	-0.63	-0.61	-2.44	1.17	0.67	0.65	-1.04	2.37
73	11.54	11.51	9.73	13.36	-1.91	-1.90	-3.55	-0.28	0.33	0.34	-1.53	2.18	-0.06	-0.01	-1.73	1.61
74	10.46	10.44	8.76	12.14	-1.87	-1.88	-3.30	-0.43	1.12	1.07	-0.58	2.84	-0.88	-0.90	-2.34	0.57
75	9.87	9.85	8.35	11.44	-2.06	-2.06	-3.54	-0.61	-0.05	0.05	-1.63	1.50	0.11	0.05	-1.36	1.62
76	10.98	10.99	9.37	12.60	-1.92	-1.94	-3.21	-0.63	0.30	0.32	-1.33	1.93	-0.03	-0.02	-1.33	1.29
77	10.37	10.40	8.59	12.17	-1.73	-1.71	-3.43	0.01	-0.25	-0.22	-2.95	2.33	0.42	0.41	-1.97	2.92
78	9.67	9.65	8.78	10.56	-1.57	-1.53	-2.51	-0.63	-0.90	-0.91	-1.97	0.16	1.14	1.14	0.02	2.29

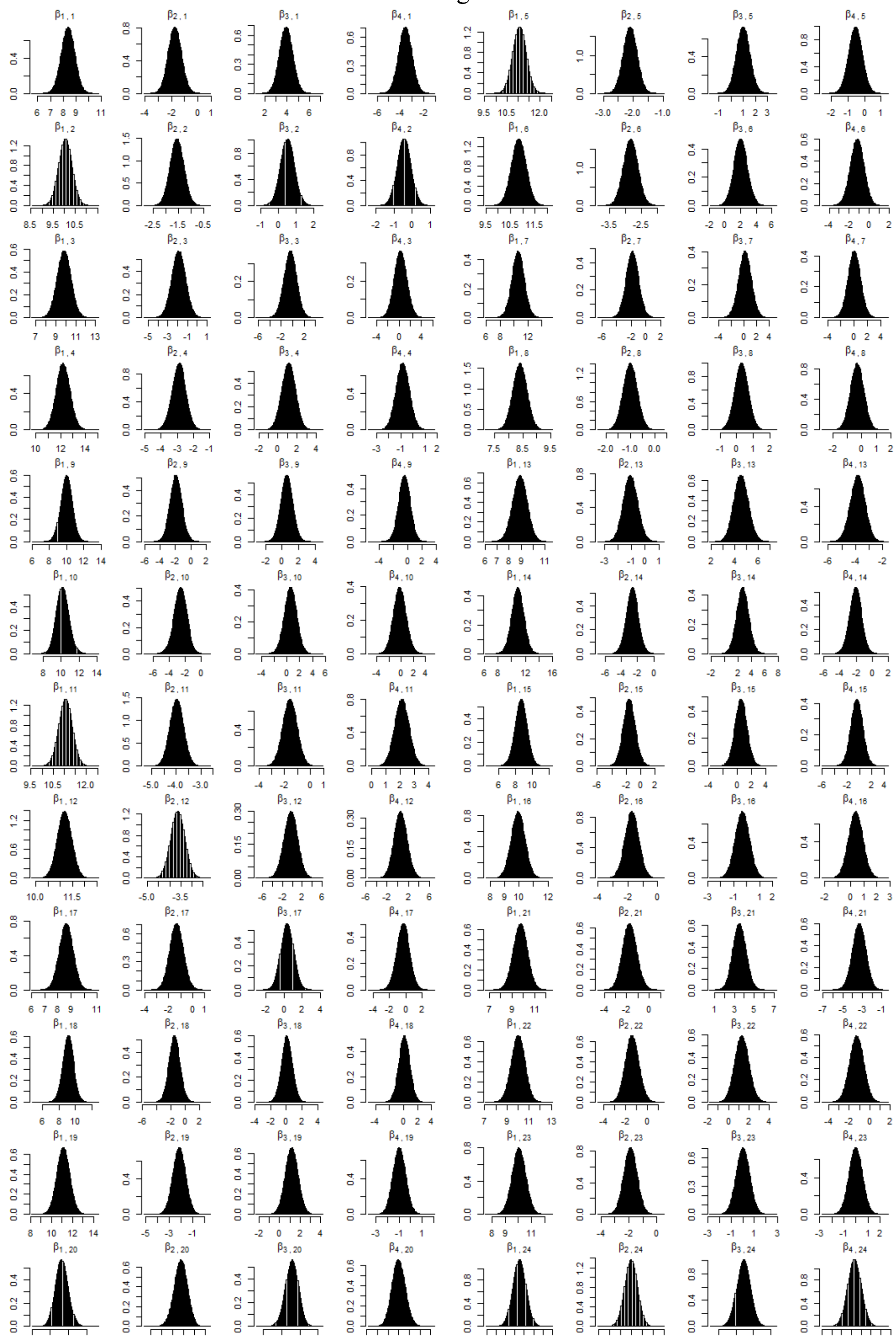
79	9.63	9.62	8.18	11.06	-1.87	-1.88	-3.52	-0.22	0.89	0.87	-0.73	2.53	-0.65	-0.63	-2.52	1.20
80	10.40	10.45	9.24	11.54	-2.24	-2.24	-3.14	-1.31	3.42	3.41	2.17	4.69	-2.63	-2.61	-3.66	-1.61
81	10.53	10.60	8.90	12.13	-1.79	-1.81	-3.16	-0.39	1.74	1.71	0.13	3.40	-1.44	-1.39	-2.87	-0.05
82	8.57	8.56	8.07	9.07	-1.67	-1.67	-2.12	-1.21	1.47	1.47	0.88	2.06	-1.00	-1.00	-1.55	-0.46
83	8.37	8.38	7.02	9.70	-2.25	-2.25	-3.51	-0.97	3.14	3.14	1.75	4.55	-2.44	-2.45	-3.79	-1.11
84	9.78	9.82	8.04	11.49	-2.73	-2.77	-4.13	-1.32	2.25	2.29	0.53	3.99	-1.59	-1.54	-3.02	-0.18
85	9.38	9.40	8.69	10.06	-1.11	-1.13	-1.76	-0.46	0.64	0.69	-0.48	1.77	-0.49	-0.54	-1.55	0.57
86	10.66	10.71	9.45	11.88	-2.07	-2.11	-3.25	-0.90	0.25	0.25	-1.11	1.60	-0.25	-0.22	-1.55	1.07
87	10.12	10.13	9.04	11.19	-1.70	-1.68	-2.78	-0.61	0.49	0.48	-0.95	1.94	-0.41	-0.40	-1.89	1.06
88	8.99	8.98	8.24	9.74	-1.57	-1.58	-2.18	-0.95	-0.08	-0.05	-1.14	0.97	0.15	0.17	-0.72	1.02

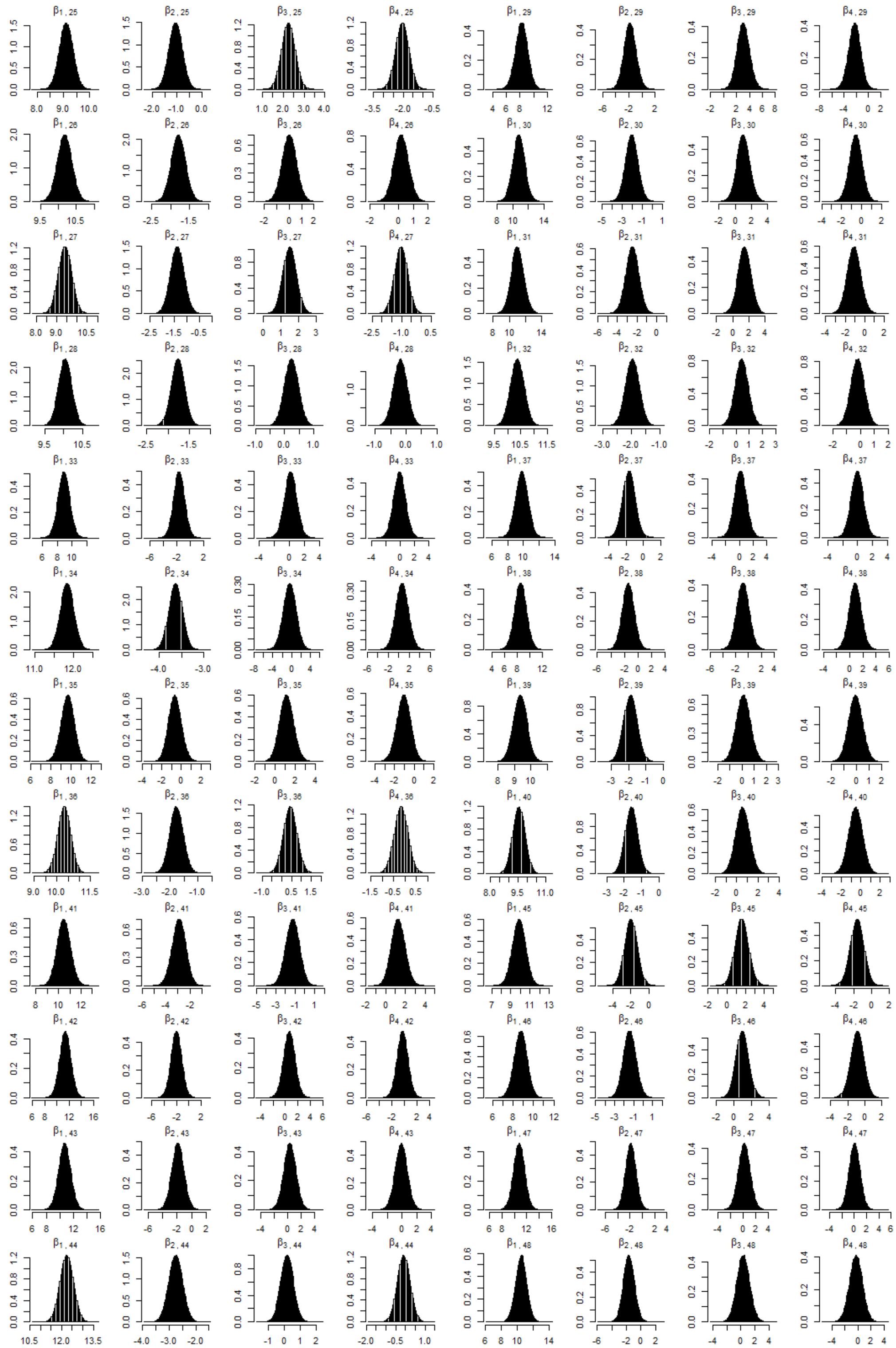
	Post Mean	Post Mode	95% Post CI
θ_1	9.884	9.896	(9.563,10.19)
θ_2	-1.858	-1.875	(-2.102, -1.599)
θ_3	0.711	0.706	(0.365, 1.064)
θ_4	-0.484	-0.486	(-0.813, -0.162)

	Post Mean	Post Mode	95% Post CI
σ_1^2	1.481	1.351	(0.917, 2.293)
σ_{12}	-0.818	-0.731	(-1.371, -0.438)
σ_{13}	-0.627	-0.522	(-1.343, -0.094)
σ_{14}	0.614	0.530	(0.129, 1.270)
σ_2^2	0.762	0.694	(0.460, 1.209)
σ_{23}	0.476	0.401	(0.0561, 1.032)
σ_{24}	-0.499	-0.440	(-1.018, -0.113)
σ_3^2	1.940	1.797	(1.192, 2.987)
σ_{34}	-1.735	-1.612	(-2.689, -1.054)
σ_4^2	1.621	1.499	(0.991, 2.507)

	Post Mean	Post Mode	95% Post CI
λ	30.46	30.45	(29.65, 31.27)

Histograms

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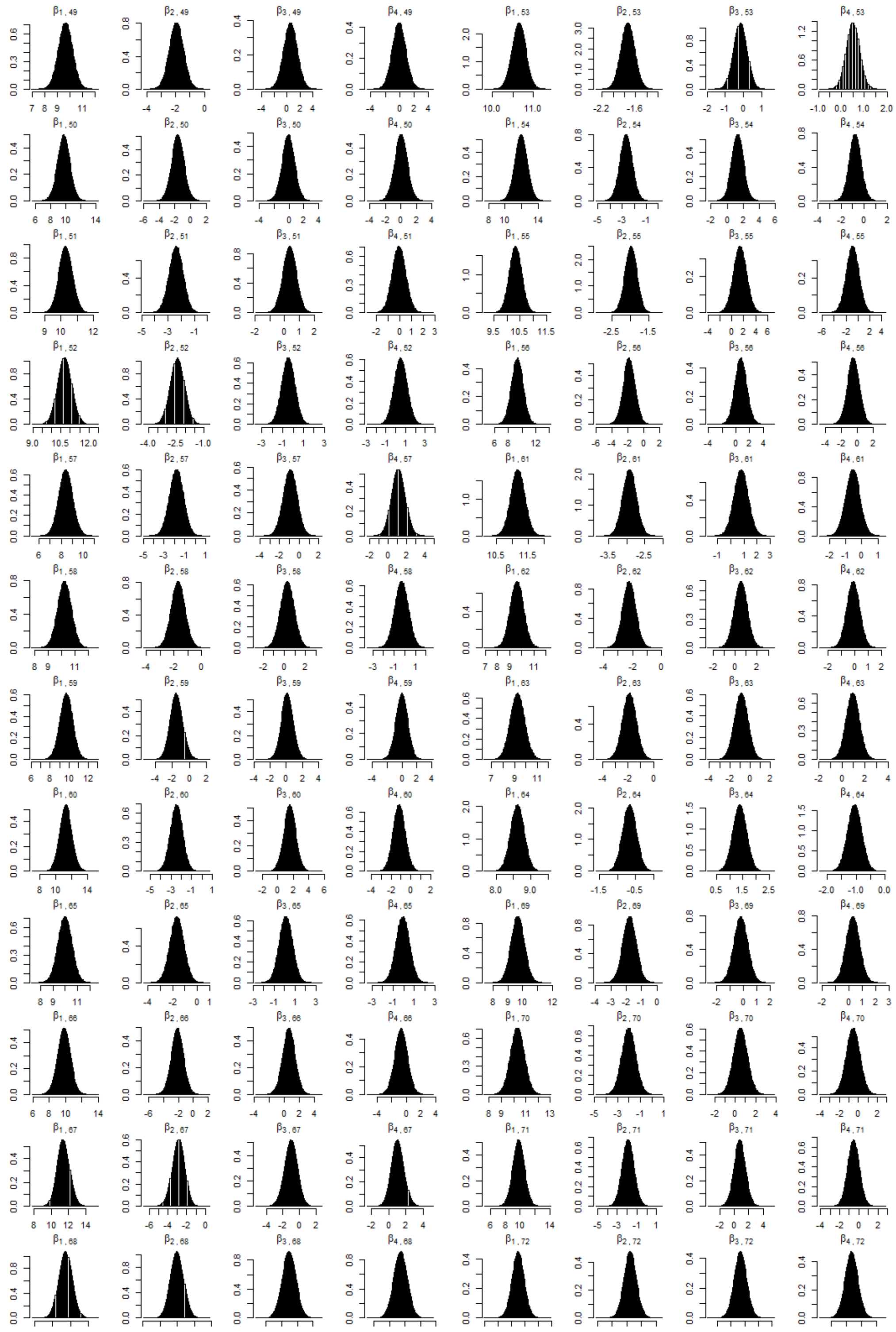

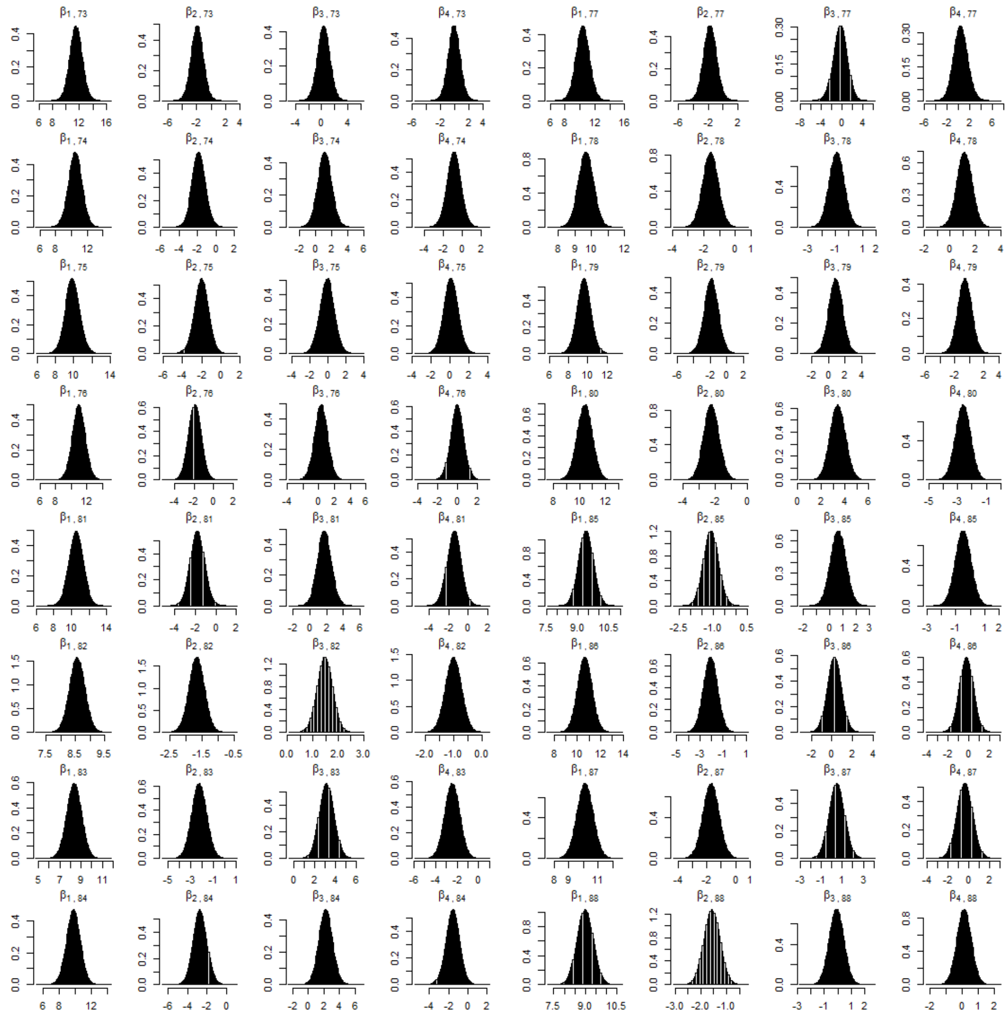

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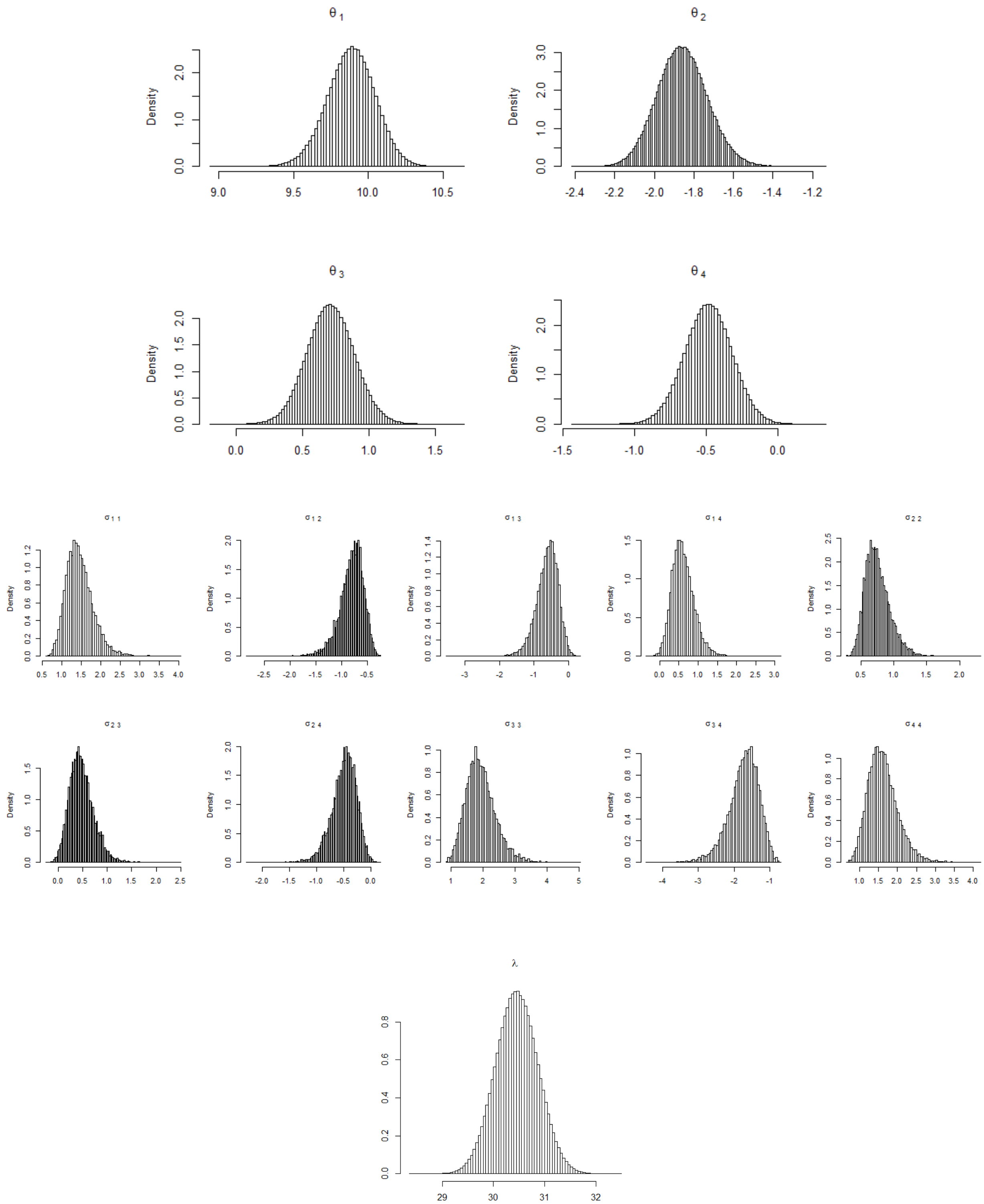
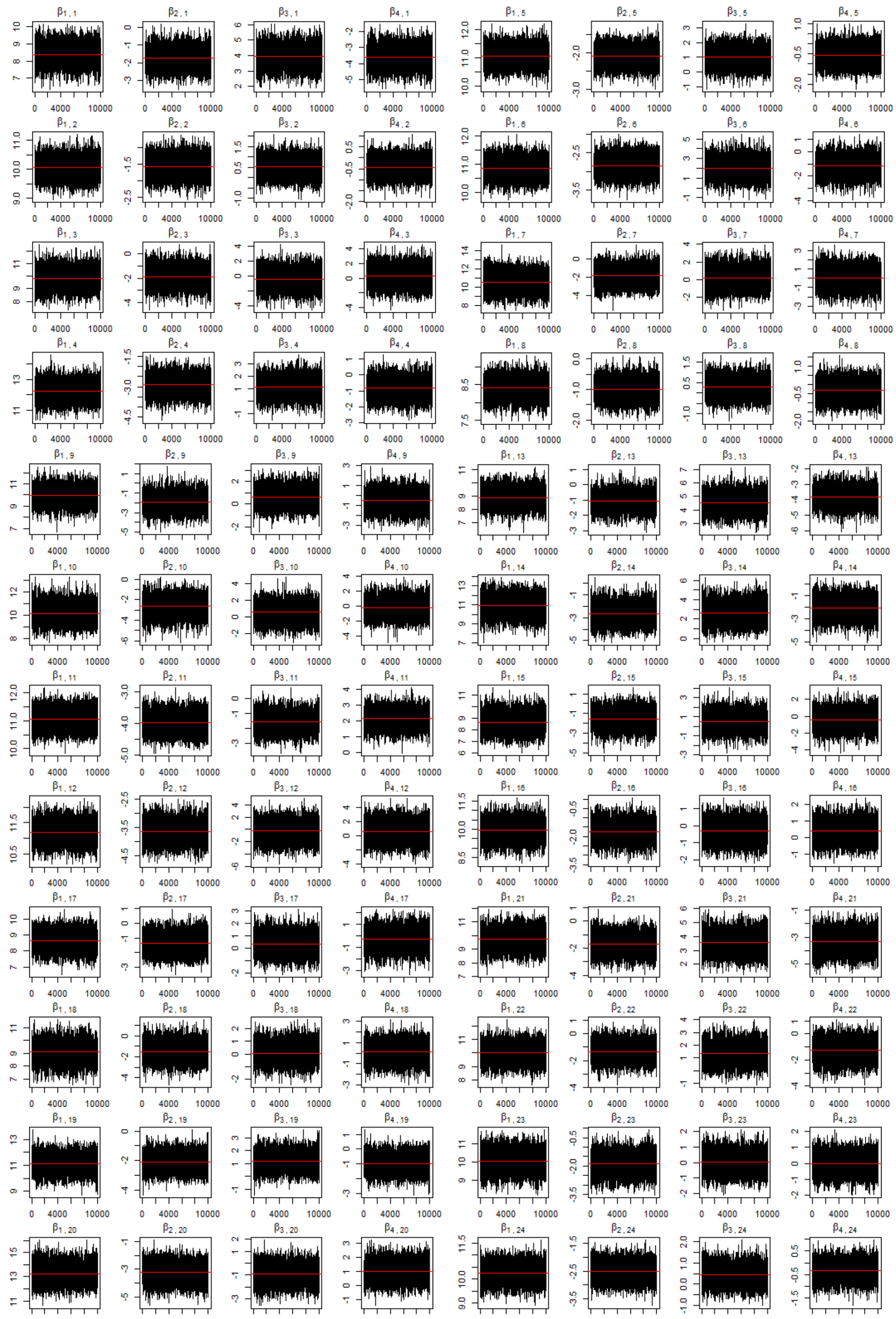
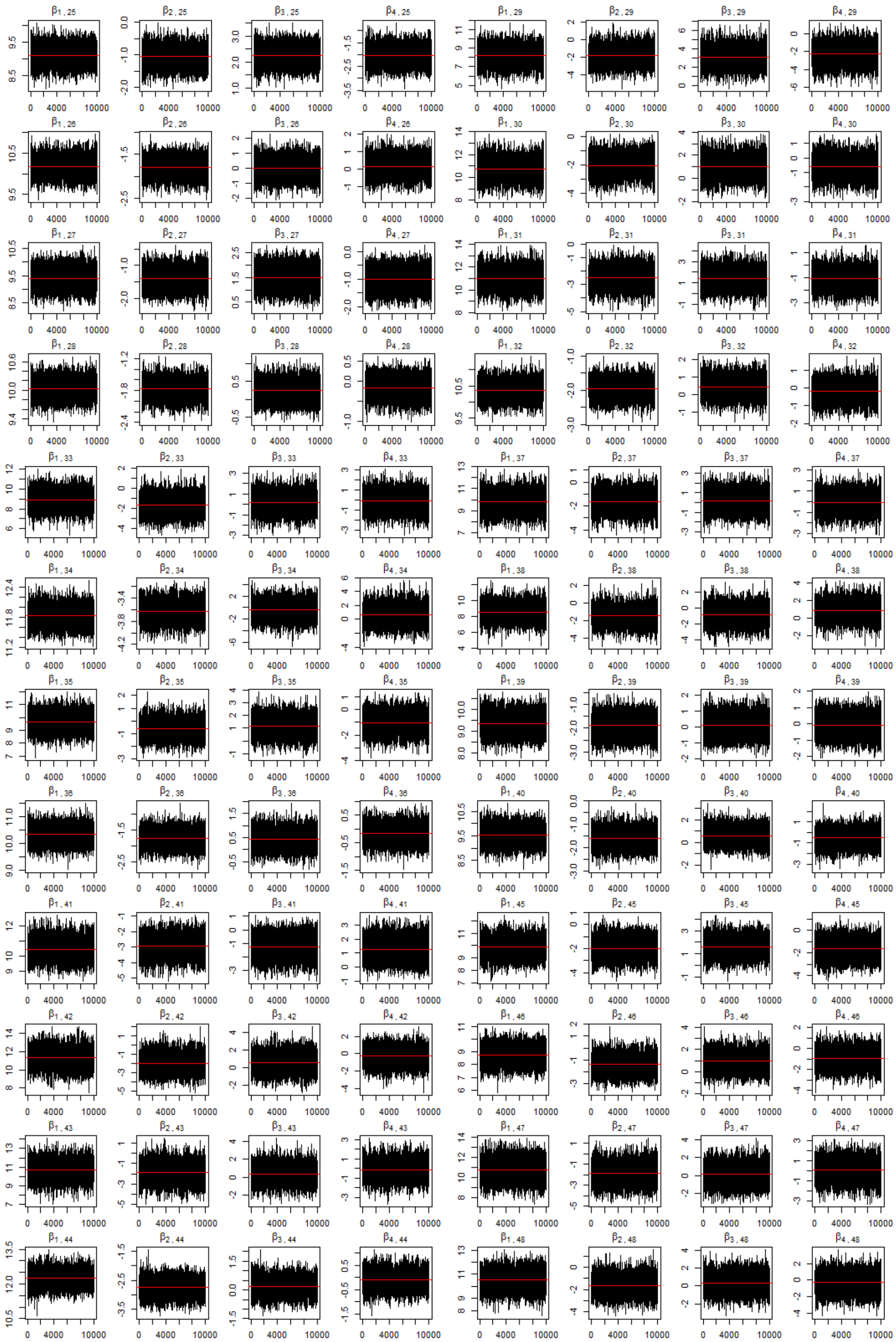

θ 1
θ 2
θ 3
θ 4
Density
σ 1 1
σ 1 2
σ 1 3
σ 1 4
σ 2 2
σ 2 3
σ 2 4
σ 3 3
σ 3 4
σ 4 4
λ

Trace Plots





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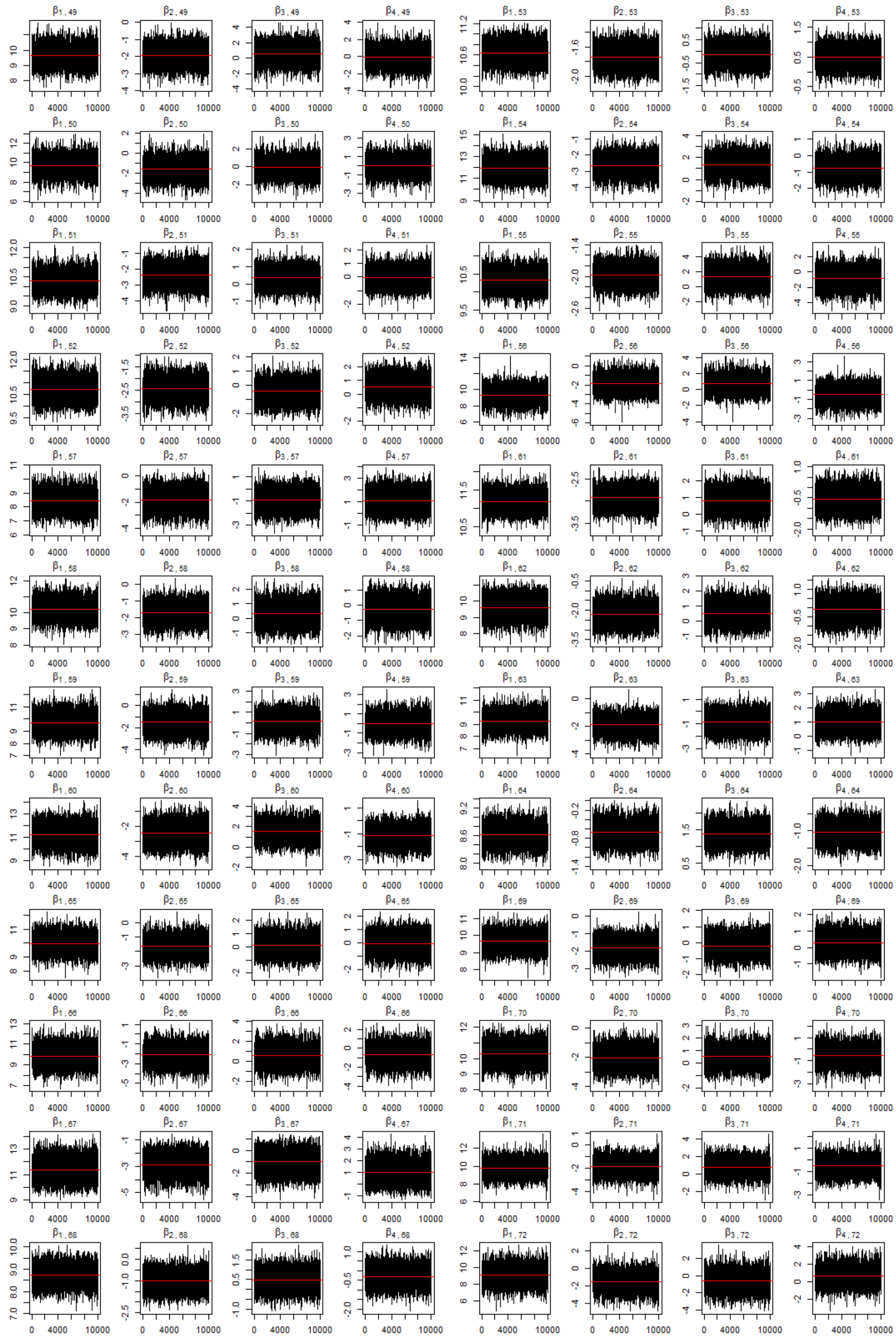
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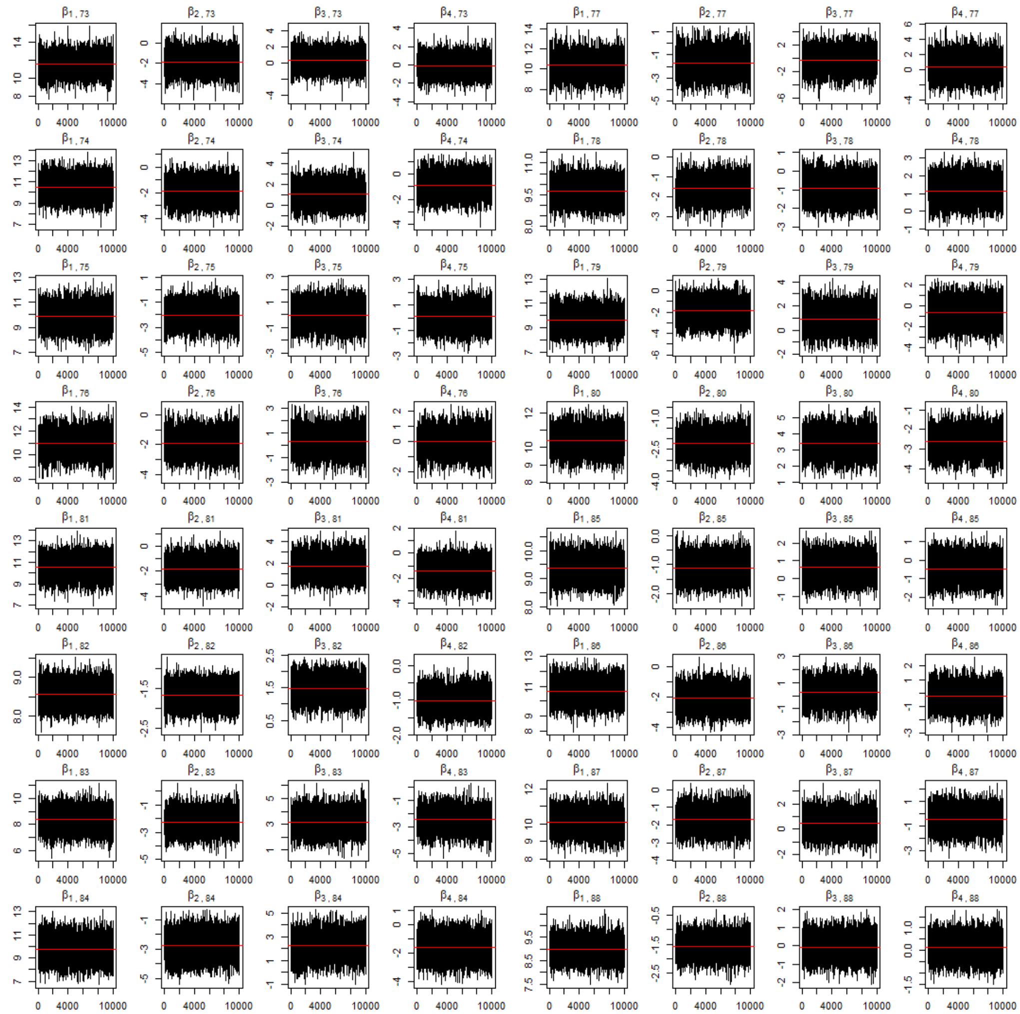
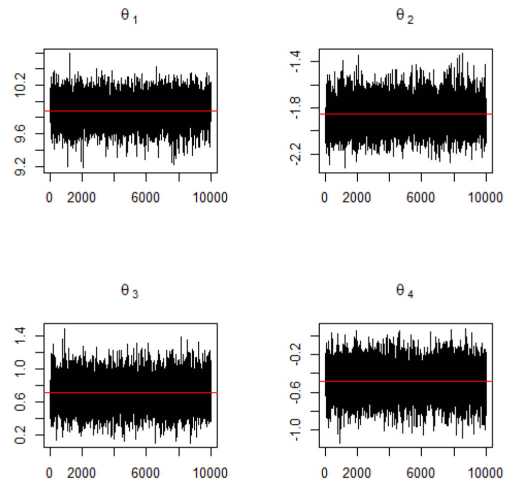
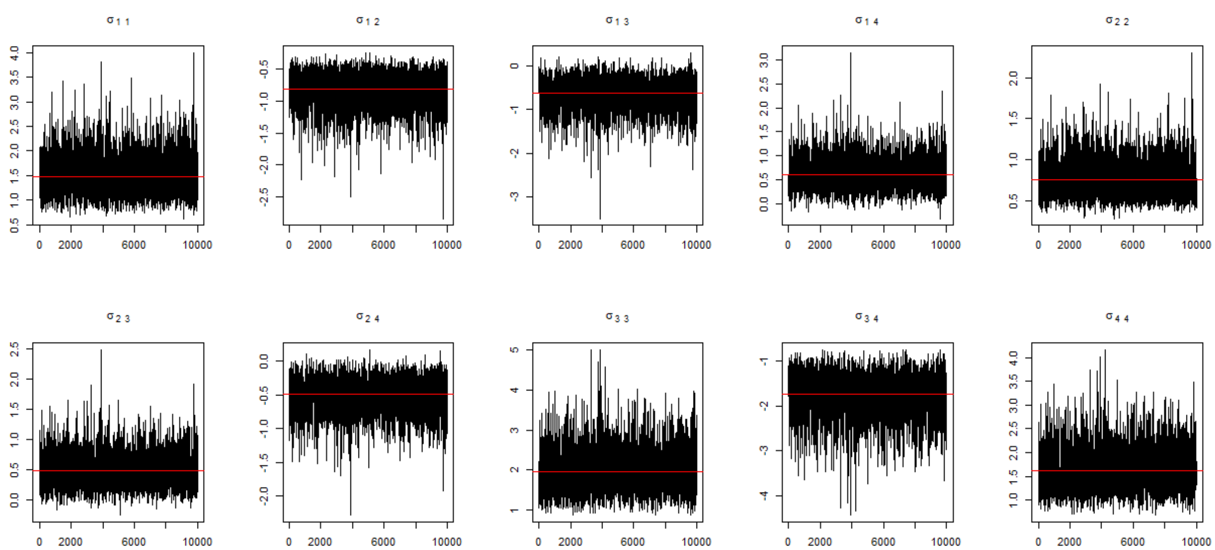



θ_1
θ_2
θ_3
θ_4


σ_{11}
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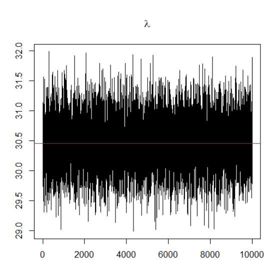
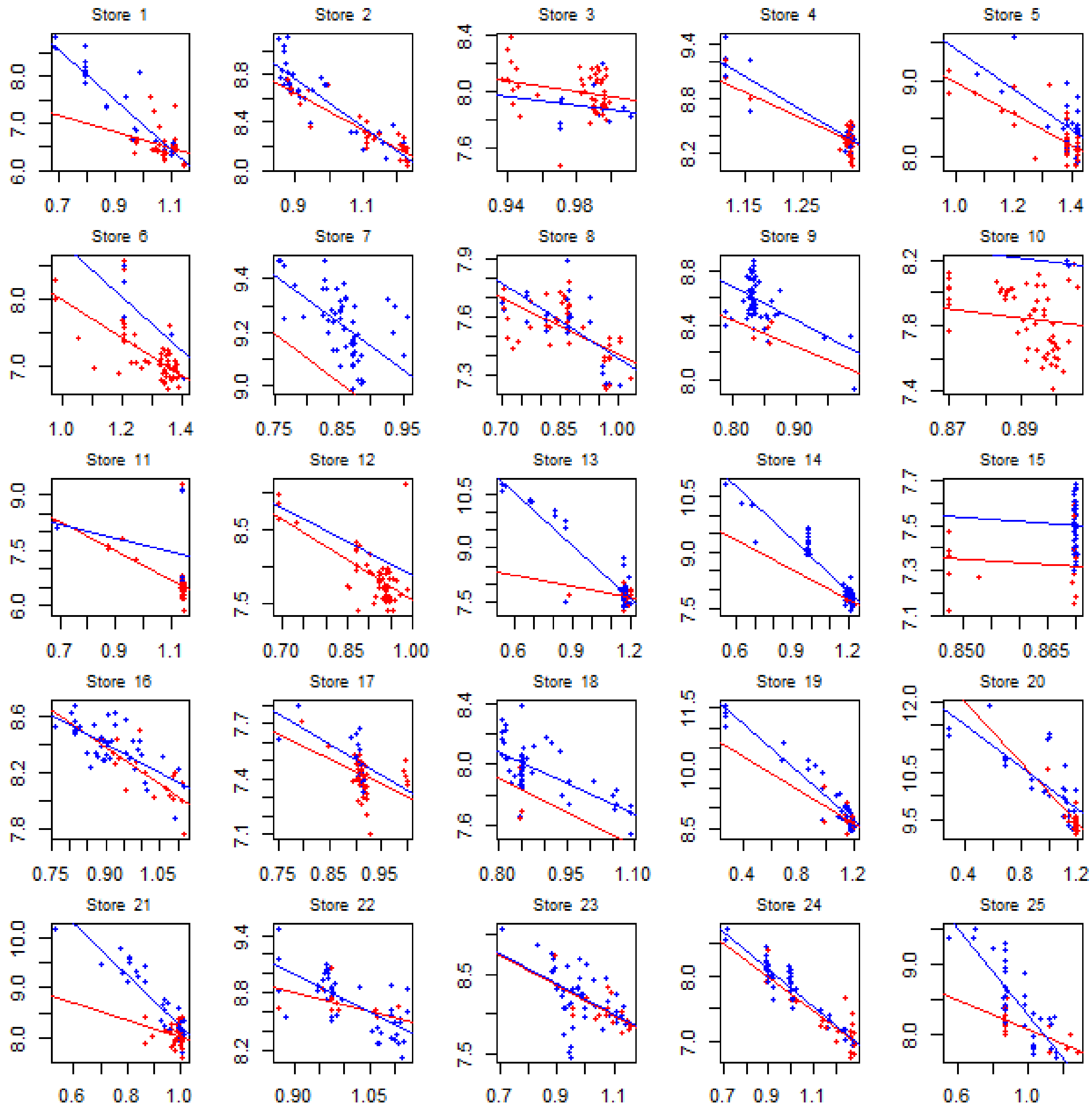
σ_{13}
σ_{14}
σ_{22}
σ_{23}
σ_{24}
σ_{33}
σ_{34}
σ_{44}

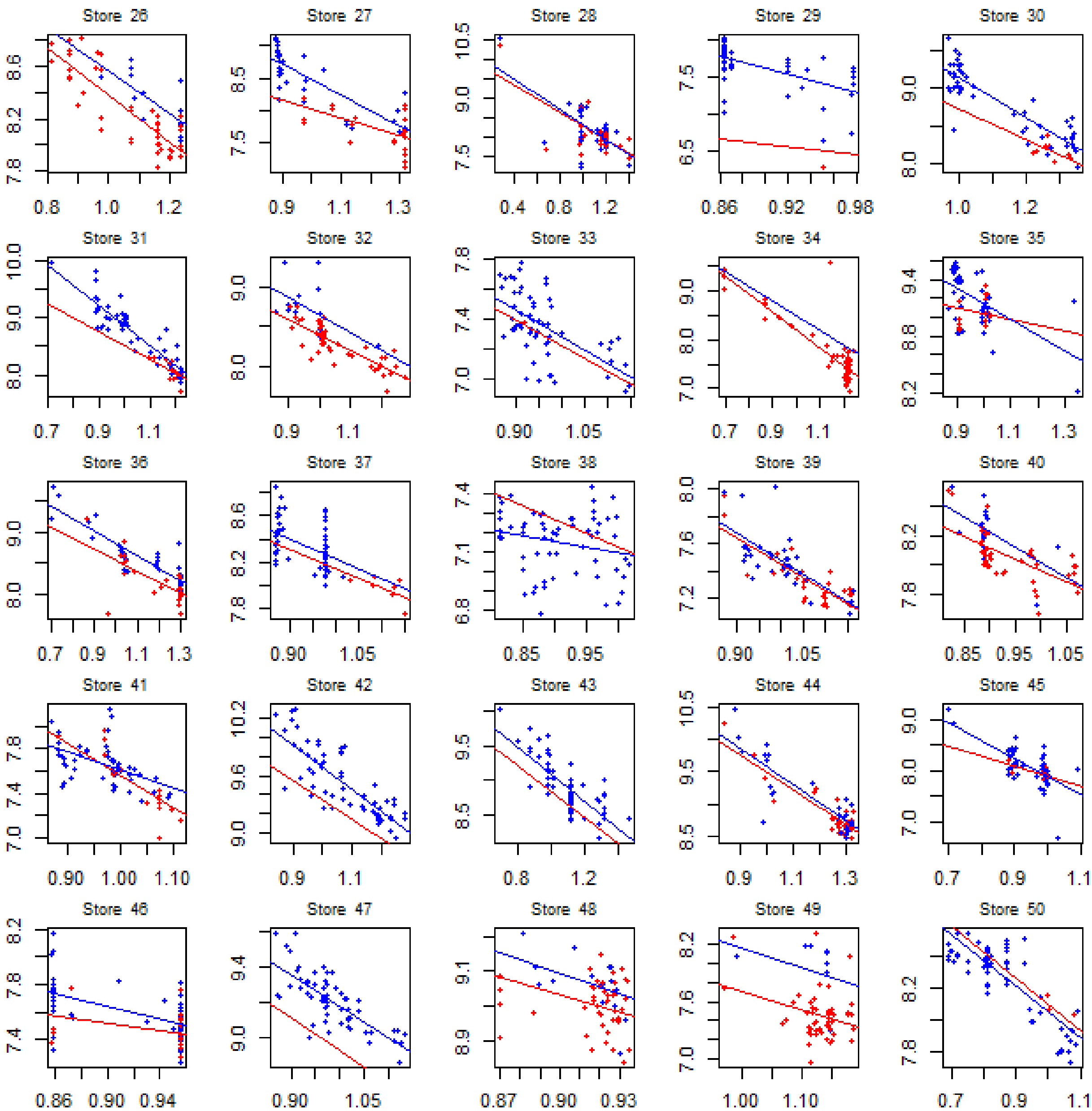

λ

Graphs with Fitted Lines
Log Quantity vs. Log Price Per Store
Display in Blue, Non-Display in Red

$$\log \hat{Q}_{ij} = \hat{\beta}_{1j} + \hat{\beta}_{2j} \log P_{ij}$$

$$\log \hat{Q}_{ij} = \left(\hat{\beta}_{1j} + \hat{\beta}_{3j}\right) + \left(\hat{\beta}_{2j} + \hat{\beta}_{4j}\right) \log P_{ij}$$

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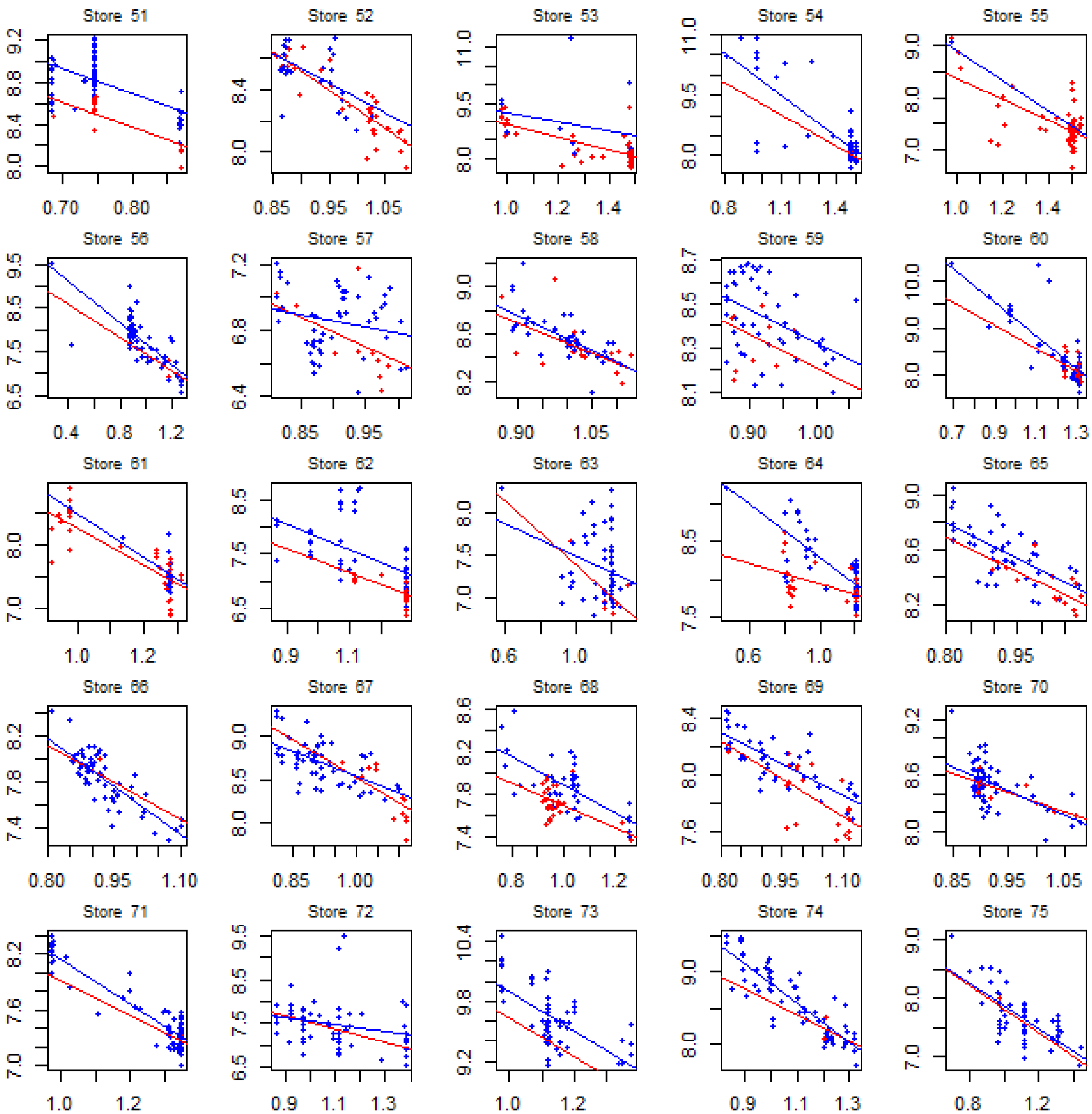
Store 50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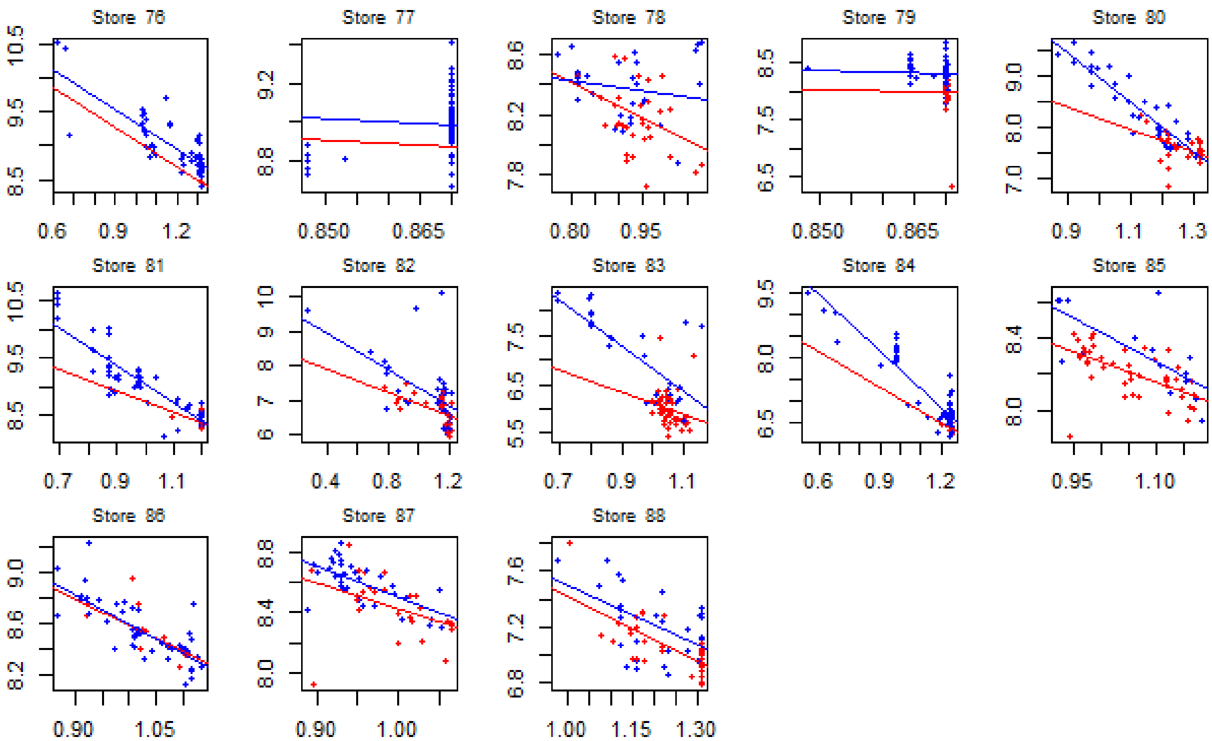
Store 76
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Store 78
Store 79
Store 80
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Store 82
Store 83
Store 84
Store 85
Store 86
Store 87
Store 88

```r
# Problem 2: Price Elasticity of Demand

# Packages
library(lattice)
library(MASS)
library(MCMCpack)

# Load data
cheese <- read.csv("C:\\Users\\trlil\\Google Drive\\UT Austin\\SDS 383D
Statistical Modeling II\\Homework\\Exercises 4\\cheese.csv", header = T)

# Add log-transformed variables
cheese <- cbind(cheese, logv = log(cheese$vol), logp = log(cheese$price))

# Number of groups
M <- length(table(cheese$store))

# Total observations
N <- dim(cheese)[1]

# Observations per group
n <- as.vector(table(cheese$store))

# Group indicator
grp <- rep(1:M, n)

# Design matrix:
# Column of 1's for intercept
# Main effect of Log Price
# Main effect of Display
# Interaction of Price and Display
X <- cbind(1, cheese$logp,
              cheese$disp,
              cheese$logp*cheese$disp)

# Number of predictors
p <- dim(X)[2]

# Response
y <- cheese$logv

# Number of Gibbs samples
T <- 10^6

# Prior parameters
Lambda0 <- diag(1, p)
Mu0 <- rep(0, p)
eta0 <- 1
S0 <- diag(1, p)
a <- 1
b <- 1

# Objects to hold parameters
Beta <- array(NA, dim = c(M, p, T))
Theta <- matrix(NA, nrow = T, ncol = p)
Sigma <- array(NA, dim = c(p, p, T))
lambda <- rep(NA, T)
```

```r
# Initialize
Beta[, , 1] <- cbind(1,
                     tapply(X[, 2], grp, mean),
                     tapply(X[, 3], grp, mean),
                     tapply(X[, 4], grp, mean))

Theta[1, ] <- colMeans(Beta[, , 1])
Sigma[, , 1] <- cov(Beta[, , 1])
Sigma[1, 1, 1] <- 1
lambda[1] <- 1/var(tapply(y, grp, mean))

#########################
##### Gibbs sampler #####
#########################

for(t in 1:(T-1)){

    # Track loop progress
    if(t %% (T/100) == 0){print(t)}

    # Update Betas
    for(j in 1:M){
        Xj <- X[grp == j, ]
        S1 <- t(Xj) %*% Xj
        S2 <- t(Xj) %*% y[grp == j]

        m_star <- solve(lambda[t]*S1 + solve(Sigma[,,t])) %*%
                (solve(Sigma[,,t]) %*% Theta[t,] + lambda[t]*S2)

        s_star <- solve(lambda[t]*S1 + solve(Sigma[,,t]))

        Beta[j,,t+1] <- mvrnorm(1, m_star, s_star)
    }

    # Update Theta
    m_star <- solve(M*solve(Sigma[,,t])+solve(Lambda0)) %*%
            (solve(Lambda0) %*% Mu0 + solve(Sigma[,,t]) %*%
             apply(Beta[,,t+1], 2, sum))

    s_star <- solve(M*solve(Sigma[,,t])+solve(Lambda0))

    Theta[t+1,] <- mvrnorm(1, m_star, s_star)

    # Update Sigma
    S3 <- 0

    for(j in 1:M){
        S3 <- S3 + (Beta[j,,t+1]-Theta[t+1,]) %*%
                 t(Beta[j,,t+1]-Theta[t+1,])
    }

    Sigma[,,t+1] <- riwish(eta0 + M, S0 + S3)

    # Update lambda
    SSR <- 0
```

```r
    for(j in 1:M){
        SSR <- SSR +
            sum((y[grp==j] - X[grp==j,] %*% Beta[j,,t+1])^2)
    }

    lambda[t+1] <- rgamma(1, a+N, b+1/2*SSR)
}

# Burn-in 10%
Beta <- Beta[,,(T/10):T]
Theta <- Theta[(T/10):T,]
Sigma <- Sigma[,,(T/10):T]
lambda <- lambda[(T/10):T]

###########################
##### Examine results #####
###########################

##### Beta #####

# Trace plots of last 10,000 samples
par(mfrow = c(4, 4))
par(mai = rep(0.25, 4))
tt <- (9*T/10)
for(j in 1:M){
    for(k in 1:p){
        plot(Beta[j, k, (tt-10000):tt],
             main = bquote(beta[.(k)~","~.(j)]),
             xlab = NULL,
             ylab = NULL,
             typ = "l")

        abline(h = mean(Beta[j, k, ]),
             col = "red")
    }
    par(ask = T)
}

# Graphs of posteriors
par(mfrow = c(4, 4))
par(mai = rep(0.25, 4))
for(j in 1:M){
    for(k in 1:p){
        hist(Beta[j, k, ],
             breaks = 100,
             prob = T,
             main = bquote(beta[.(k)~","~.(j)]),
             xlab = NULL)
    }
    par(ask = T)
}

# Make table to output summary stats
Beta_out <- matrix(NA, nrow = M, ncol = p)

md <- NULL
for(j in 1:M){
```

```r
    for(k in 4:4){
        md <- c(md,
              density(Beta[j,k,])$x[which.max(density(Beta[j,k,])$y)])
    }
}

Beta_out <- rbind(mean = apply(Beta[, 4, ], 1, mean),
                  apply(Beta[, 4, ], 1, quantile, c(0.025, 0.975)),
                  mode = md)

write.csv(Beta_out, "C:\\Users\\trlil\\Google Drive\\UT Austin\\SDS 383D
Statistical Modeling II\\Homework\\Exercises 4\\out.csv")

##### Theta #####

# Trace Plots
par(mfrow = c(2, 2))
for(k in 1:p){
    plot(Theta[(tt-10000):tt, k],
         main = bquote(theta[~.(k)]),
         xlab = "",
         ylab = "",
         typ = "l")
    abline(h = mean(Theta[, k]),
         col = "red")
}

# Posteriors
par(mfrow = c(2, 2))
for(k in 1:p){
    hist(Theta[, k],
         breaks = 100,
         prob = T,
         main = bquote(theta[~.(k)]),
         xlab = "")
}

# Summary statistics
apply(Theta, 2, mean)
apply(Theta, 2, quantile, c(0.025, 0.975))

for(k in 1:p){
    print(density(Theta[, k])$x[which.max(density(Theta[, k])$y)])
}

##### Sigma #####

# Trace plots
par(mfrow = c(2, 5))
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            plot(Sigma[i, j, (tt-10000):tt],
                 main = bquote(sigma[~.(i)~""~.(j)]),
                 xlab = "",
                 ylab = "",
                 typ = "l")
```

```
            abline(h = mean(Sigma[i, j, ]),
                 col = "red")
        }
    }
}

# Posteriors
par(mfrow = c(2, 5))
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            hist(Sigma[i, j, (tt-10000):tt],
                 main = bquote(sigma[~.(i)~""~.(j)]),
                 breaks = 100,
                 prob = T,
                 xlab = "")
        }
    }
}

# Summary statistics
for(i in 1:p){
    for(j in 1:p){
        if(j >= i){
            print(mean(Sigma[i, j, ]))
            print(quantile(Sigma[i, j, ], c(0.025, 0.975)))
            print(density(Sigma[i, j, ])$
                  x[which.max(density(Sigma[i, j, ])$y)])
        }

    }
}

##### Lambda #####

# Trace plot
plot(lambda[(tt-10000):tt],
     typ = "l",
     main = bquote(lambda),
     xlab = "",
     ylab = "")
abline(h = mean(lambda),
     col = "red")

# Posterior
hist(lambda,
     prob = T,
     breaks = 100,
     main = bquote(lambda),
     xlab = "",
     ylab = "")

# Summary statistics
mean(lambda)
quantile(lambda, c(0.025, 0.975))
density(lambda)$x[which.max(density(lambda)$y)]
```

```r
####################################
##### Graphs with fitted lines #####
####################################

##### For each store, plot the log quantity
##### vs. log price. Distinguish between the
##### points with display and not display.
##### Plot lines using posterior means of
##### beta coefficients with display in blue,
##### and without display in red.

# 5x5 panel of graphs
par(mfrow = c(5, 5))

# Narrow margins
par(mai = c(0.25, 0.25, 0.25, 0.25))

# Make plot for each store
for(j in 1:M){

    # Blue points are display
    clr <- rep("red", sum(grp == j))
    clr[which(X[grp == j, 3] == 1)] <- "blue"

    # Plot points and indicate
    # which are display
    plot(X[grp == j, 2], y[grp == j],
         col = clr,
         pch = 20,
         xlab = "",
         ylab = "",
         main = bquote("Store "~.(j)),
         cex.main = .9)

    # Fitted line for no display
    abline(a = mean(Beta[j, 1, ]),
         b = mean(Beta[j, 2, ]),
         col = "red")

    # Fitted line for display
    abline(a = mean(Beta[j, 1, ]) + mean(Beta[j, 3, ]),
           b = mean(Beta[j, 2, ]) + mean(Beta[j, 4, ]),
           col = "blue")

    # Pause before next graph
    par(ask = T)
}
```