

Benchmarking CMA-ES with Basic Integer Handling on a Mixed-Integer Test Problem Suite

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ABSTRACT

We compare the performances of one implementation of CMA-ES (pycma version 3.3.0) for optimizing functions with both continuous and integer variables. The implementation incorporate a lower bound on the variance along the integer coordinates to keep the optimization from stalling. This benchmark will serve as a baseline for further works on pycma. Results shows substantial improvement since the last benchmarked version of pycma. Also this implementation is competitive with other mixed integer algorithms.

CCS CONCEPTS

•Computing methodologies → Continuous space search;

KEYWORDS

Benchmarking, Black-box optimization, Mixed-integer optimization

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1 INTRODUCTION

Mixed optimization problems are of great importance in real-world applications. Typically, variables include real-valued degrees of freedom that correspond, directly or not, to physical measurements (eg angles, frequencies, forces, etc.) as well as discrete (integer or binary) settings that correspond to categorical choices (eg materials) or service activations (eg radar modes). Therefore, developing algorithmic variants that can handle several types of optimization variable at the same time without impairing the search process will be very useful in practice.

In the context of black box optimization for problems with continuous variables, evolutionary algorithms which update a covariance

matrix, namely CMA-ES, are studied from more than 20 years [10]. While the CMA-ES was originally not designed to handle also integer variables, it can be employed with integer variables by using the integer part of a continuous variable in the fitness evaluation. This is however expected to lead to integer variables getting stuck in the resulting fitness plateaus, and several versions have been proposed to improve the algorithm in this situation [4, 5].

In order to study mixed integer problems within the COCO framework [9], the bboob-mixint testbed was proposed in [13]. This testbed is based on the 24 functions of the bboob testbed with 80% of the variables to be integer with an arity between 2 and 16.

In order to prevent variables getting stuck on the same integer level, for example in CMA-ES, a lower bound on the variance can be set such that there remains a reasonable probability for an integer variable to “jump” to another integer level, as for example in CMA-ES with margin (CMA-ESwM) [4]. The same principle is implemented in the pycma code with Integer Handling (cma-IH in the benchmark) with a more simplistic approach.

The purpose of this paper is to measure the performance of the current implementation of the widely distributed cma-IH code (version 3.3.0) [6] with different options and compare its performance with previous results and enhancements.

2 ALGORITHM PRESENTATION

The general idea behind cma-IH is to set a lower bound for the variance of the integer variables. It prevents integer variables from being trapped in one value. We formulate the bounding using the covariance matrix decomposition from the CMA-ES with diagonal decoding [1]. At each new iteration $t + 1$, new candidate solutions, $x^{(t+1)}$, are sampled according to a multivariate normal distribution.

$$\begin{aligned} x^{(t+1)} &\sim \mathcal{N}(m, \sigma^{(t)^2} C^{(t)}) \\ &\sim m + \sigma^{(t)} \sqrt{C^{(t)}} \mathcal{N}(0, \mathbf{I}), \end{aligned}$$

where C is a covariance matrix and σ the step size. In order to bound the standard deviation of each integer variable, we introduce the diagonal scaling matrix D ,

$$x^{(t+1)} \sim m + \sigma^{(t)} D^{(t)} \sqrt{C^{(t)}} \mathcal{N}(0, \mathbf{I}),$$

where $D^{(0)}$ is the identity. The update of σ and C are done as in the original CMA-ES algorithm. Then, the diagonal matrix D is updated based on the current standard deviation in each coordinate. The standard deviation of the i^{th} -th coordinate of x , $\sigma_{\text{std}(i)}^{(t)}$, is computed with the diagonal elements of C and D :

*Submission deadline: to be announced (likely late March/early April).

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```

1 import cocoex
2 import cma
3 suite = cocoex.Suite('bbob-mixint', '', '')
4 for problem in suite:
5     up_bound = list(problem.upper_bounds[
6         :problem.number_of_integer_variables]) +
7         [None]
8     lo_bound = list(problem.lower_bounds[
9         :problem.number_of_integer_variables]) +
10        [None]
11     cma.fmin2(problem, problem.initial_solution, 1,
12              {'bounds': [lo_bound, up_bound]
13               'CMA_std': (problem.upper_bounds
14                           - problem.lower_bounds)/5,
15               'integer_variables': list(range(
16                   problem.number_of_integer_variables)),
17               restarts = 9)})

```

Figure 1: Calling code to benchmark `cma.fmin2`.

$$\sigma_{\text{std}(i)}^{(t)} = \sigma^{(t)} D_{i,i}^{(t)} \sqrt{C_{i,i}^{(t)}}$$

This standard deviation has to be compared with a lower bound σ_{LB} . If the standard deviation in the i^{th} coordinate is smaller than the threshold value, $\sigma_{\text{std}(i)}^{(t)} < \sigma_{LB}$, then the D matrix is scaled up in this coordinate.

$$D_{i,i}^{(t+1)} \leftarrow \begin{cases} \frac{\sigma_{LB}}{\sigma^{(t)} \sqrt{C_{i,i}^{(t)}}} & \text{if } 1 \leq i \leq N_{\text{int}} \text{ and } \sigma_{\text{std}(i)}^{(t)} \leq \sigma_{LB} \\ D_{i,i}^{(t)} & \text{otherwise} \end{cases} \quad (1)$$

where N_{int} is the number of integer variables. In the version 3.3.0 of pycma with integer handling (cma-IH), the lower bound is set to:

$$\sigma_{LB} = \frac{1}{2N_{\text{int}} + 1} \quad (2)$$

3 EXPERIMENTAL PROCEDURE

Figure 1 shows the calling code and options used to run the benchmark for cma-IH. A maximum of nine restarts may be performed and each restart doubles the population size (IPOP-CMA-ES) [2]. The initial step size is chosen to sample solutions mainly within the upper and the lower bound of the problem. The bounds are enforced on the integer variables only. The maximum number of function evaluations allowed per problem is set to 10^4 times the number of dimension of the problem.

This current implementation cma-IH is compared to performance data from the bboob-mixint COCO archive¹. We show results of the algorithms given in Table 1.

The experiments were performed and the plots were produced with COCO [9], version 2.6.3.

4 CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the cma-IH algorithm with restarts on the bboob-mixint test

¹See <https://numbbio.github.io/data-archive>

Table 1: Compared algorithms

cma-IH	pycma version 3.3 (current) with the default integer handling option
CMA-ES-pycma	previous integer handling of pycma benchmarked in [13]
CMA-ESwM	CMA-ES with margin benchmarked in [3]
cma-NoIH	pycma version 3.3 (current) without any integer handling option
cmaIH1e-1	as cma-IH with a larger coordinate-wise lower bound on the sample standard deviation of $\sigma_{LB} = 10^{-1}$
DE-scipy	Differential Evolution implemented in scipy benchmarked in [13]
RANDOMSEARCH	Random Search algorithm for baseline

suite [13] with a maximum budget of $10^4 \times$ dimensions function evaluations according to [11].

The Python code was run on a Linux machine with 32 cores, Intel® Xeon® E7 to T3 v4 processor. The time per function evaluation averaged over all functions in the test bed for dimensions 5, 10, 20, 40, 80 and 160 equals 3.7, 4.9, 5.1, 5.6, 6.4 and 8.4×10^{-4} seconds respectively.

5 RESULTS

Results from experiments according to [11] and [8] on the benchmark functions given in [13]. Figures 2 and 3 show summary results for the different function groups. Figure 4 shows the empirical distribution of function evaluations to reach 51 target values on the single functions [9]. Tables 2 and 3 present expected runtimes and statistical tests.²

The **expected runtime (ERT)**, used in the tables, depends on a given target precision, $I_{\text{target}} = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_i , summed over all trials and divided by the number of trials that actually reached f_i [11, 12]. **Statistical significance** is tested with the rank-sum test for a given target Δf_i using, for each trial, either the number of needed function evaluations to reach f_i (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Comparing two integer handling versions of pycma. We compare the performance of cma-IH versus CMA-ES-pycma. In comparison, the cma-IH performs increasingly better with increasing dimension and is overall about ten times faster than CMA-ES-pycma in dimension 80 (see Figure 3, lower right). The largest differences we observe on the ill-conditioned and multi-modal functions. The difference on the linear slope function is mainly due to the boundary handling which was applied also to the continuous variables in the CMA-ES-pycma experiment, as we confirmed with additional experiments. The cma-IH performs statistically significantly better on 6, 8, 10, 16 and 15 functions in dimension 5, 10, 20, 40 and 80,

²The entire data can be found at <https://trmarty.github.io/MixedIntegerData/>

respectively. The CMA-ES-pycma performs better on the separable ellipsoid function f_2 in dimension 20, 40 and 80.

Comparing pycma with and without the integer handling. The cma-IH and CMA-ES-pycma perform very similar overall. The only statistically significant difference we observed is on the separable ellipsoid function f_2 in dimension 5 (not shown), where cma-IH is about three times faster than CMA-ES-pycma, probably due to preventing additional restarts before to reach better targets. The default lower bound on the sample standard deviations for integer variables has overall little impact on the performance on this test suite.

A larger threshold for the standard deviations lower bound. In order to understand the impact of the choice of the threshold, we have run experiments with a few different thresholds for the standard deviations and show the results for the lower bound set to 10^{-1} . While in the smaller dimensions the effects are minor, in larger dimension the cmaIH1e-1 performs overall best and is three times faster than cma-IH on the ill-conditioned functions in larger dimension for budgets above 100 times dimension. On the 80-dimensional *sphere* function, cmaIH1e-1 solves the last target ten times quicker than cma-IH and also two times quicker than CMA-ESwM. The cmaIH1e-1 also solves the *Ellipsoid separable* whereas cma-IH gets stuck at around 20% of targets. The naive cmaIH1e-1 algorithm is slower than CMA-ESwM by a factor of approximately 3 on functions 5, 13 and 14, however it is significantly better than the current version cma-IH on functions 2, 5 or 12 while not being worse on all other functions. For this range of dimensions, bounding the standard deviation to 10^{-1} is a simple yet effective solution.

6 CONCLUSION

We have assessed the integer handling implemented in the current pycma module [7] (version 3.3.0). The performance with integer handling is overall superior to a previously benchmarked version, however it behaves quite similar to the version without integer handling. We attribute the latter observation to a suboptimal parameter setting: tests with a lower bound for the sample standard deviations of 1/10, which is considerably larger than the original

setting, reveal a remarkably improved performance on a number of functions. In particular in larger dimension, this variant performs overall best.

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Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1									f13								
cma-IH	818(133)	2349(137)	7769(8446)	8453(8858)	9602(9016)	1.2e4(9556)	1.5e4(1e4)	15/15	cma-IH	7429(8778)	2.2e4(2e4)	2.6e4(2e4)	2.9e4(2e4)	3.2e4(3e4)	3.9e4(3e4)	4.6e4(3e4)	15/15
CMA-ES	1.5e4(2e4)	4.2e4(5e4)	4.2e4(6e4)	4.5e4(5e4)	5.3e4(5e4)	7.0e4(5e4)	8.7e4(5e4)	15/15	CMA-ES	7.1e4(7e4)	9.3e4(9e4)	1.1e5(9e4)	1.2e5(8e4)	1.4e5(8e4)	1.9e5(7e4)	<i>oo des</i>	0/15
CMA-ESw	2239(849)	3993(2022)	4522(2228)	5079(2058)	5992(1916)	8350(1843)	1.1e4(1698)	15/15	CMA-ESw	1.2e4(1e4)	1.4e4(1e4)	1.6e4(1e4)	1.9e4(1e4)	2.1e4(1e4)	2.6e4(1e4)	3.2e4(1e4)	15/15
DE-scip	8636(2819)	4.5e4(5327)	8.9e4(2e5)	9.2e4(2e5)	9.6e4(2e5)	1.0e5(2e5)	1.1e5(2e5)	13/15	DE-scip	8.9e4(2e5)	9.9e4(2e5)	1.1e5(2e5)	1.2e5(2e5)	1.3e5(2e5)	1.4e5(2e5)	1.6e5(2e5)	13/15
cma-Nol	859(114)	1405(122)	8689(8511)	9428(8770)	1.1e4(8925)	1.4e4(9846)	1.7e4(1e4)	15/15	cma-Nol	1.0e4(1e4)	1.8e4(2e4)	2.1e4(3e4)	2.4e4(3e4)	2.8e4(3e4)	3.4e4(3e4)	4.1e4(3e4)	15/15
cmaIHie	866(140)	1532(493)	3957(3556)	4159(3350)	4477(3065)	5727(2316)*	6979(1526)*2	15/15	cmaIHie	3558(3540)	7104(7991)	1.2e4(8631)	1.7e4(9380)	2.6e4(9920)	2.9e4(1e4)	3.3e4(1e4)	15/15
f2									f14								
cma-IH	4.4e4(2e4)	2.4e5(7e4)	3.6e5(3e5)	5.6e5(3e5)	5.6e5(6e5)	7.2e5(5e5)	1.2e6(7e5)	2/15	cma-IH	578(136)	7193(8184)	1.5e4(2e4)	2.3e4(2e4)	2.8e4(2e4)	3.6e4(2e4)	4.2e4(3e4)	15/15
CMA-ES	1.1e4(1751)	6.4e4(2e4)	2.9e5(2e5)	4.5e5(3e5)	4.5e5(5e5)	4.5e5(4e5)	4.5e5(5e5)	8/15	CMA-ES	588(200)	6.2e4(6e4)	6.8e4(7e4)	1.1e5(1e5)	1.2e5(1e5)	1.5e5(9e4)	1.9e5(9e4)	11/15
CMA-ESw	3.5e4(2e4)	3.7e4(2e4)	4.3e4(2e4)	5.6e4(6e4)	5.8e4(6e4)	6.1e4(7e4)	6.6e4(6e4)	15/15	CMA-ESw	1643(629)	3018(658)	7015(1e4)	9529(1e4)	1.2e4(1e4)	1.9e4(1e4)	2.5e4(1e4)	15/15
DE-scip	1.0e4(1948)	1.7e4(2474)	5.3e4(5931)	8.9e4(2e5)	9.4e4(2e5)	1.0e5(2e5)	1.1e5(2e5)	13/15	DE-scip	4850(1211)	2.3e4(6259)	3.1e5(4e5)	4.0e5(5e5)	4.2e5(4e5)	4.8e5(4e5)	5.7e5(4e5)	8/15
cma-Nol	4.2e4(2e4)	2.0e5(1e5)	3.7e5(3e4)	9.8e5(1e6)	1.2e6(8e5)	2.0e6(1e6)	6.0e6(7e6)	0/15	cma-Nol	652(182)	5705(8030)	2.0e4(4e4)	2.7e4(4e4)	3.1e4(4e4)	3.8e4(5e4)	4.6e4(5e4)	15/15
cmaIHie	1.2e4(3940)	3.6e4(3e4)	5.1e4(3e4)	5.4e4(4e4)	5.5e4(4e4)	5.7e4(4e4)	6.0e4(4e4)	15/15	cmaIHie	620(134)	1959(938)	5783(4326)	1.3e4(1e4)	2.2e4(1e4)	4.9e4(4e4)	5.6e4(4e4)	15/15
f3									f15								
cma-IH	1623(444)	1.2e5(9e4)	5.7e6(8e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-IH	2.3e5(6e4)	2.3e5(6e4)	2.3e5(6e4)	2.4e5(6e4)	2.5e5(6e4)	2.6e5(6e4)	2.7e5(7e4)	14/15
CMA-ES	4467(4746)	6.0e5(5e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ES	2.9e6(3e6)	2.9e6(4e6)	2.9e6(4e6)	2.9e6(3e6)	2.9e6(2e6)	2.9e6(3e6)	2.9e6(4e6)	2/15
CMA-ESw	6105(1148)	2.1e5(3e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	<i>oo des</i>	0/15
DE-scip	1.7e4(4340)	6.7e5(7e5)	5.7e6(6e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	DE-scip	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	<i>oo des</i>	0/15
cma-Nol	3292(1500)	1.2e5(9e4)	2.9e6(4e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-Nol	2.3e5(2e5)	2.5e5(2e5)	2.6e5(2e5)	2.6e5(2e5)	2.7e5(2e5)	2.7e5(2e5)	2.8e5(3e5)	12/15
cmaIHie	1558(431)	2.5e4(2e4)*	5.6e6(8e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cmaIHie	1.4e5(6e4)	1.4e5(6e4)	1.4e5(6e4)	1.5e5(6e4)	1.5e5(6e4)	1.6e5(7e4)	1.7e5(7e4)	15/15
ppdata/f4									f16								
cma-IH	8499(1e4)	2.0e5(2e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-IH	4.0e4(5e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
CMA-ES	1.2e4(1e4)	5.9e6(5e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ES	1.8e4(2e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
CMA-ESw	2.5e4(2e4)	3.8e5(3e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	8.7e5(8e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
DE-scip	2.2e4(3909)	5.2e5(5e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	DE-scip	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cma-Nol	1.2e4(1e4)	2.3e5(2e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-Nol	3.0e4(4e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cmaIHie	2921(654)	1.6e5(2e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cmaIHie	2.8e4(2e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
f5									f17								
cma-IH	1270(184)	3429(2040)	3429(2040)	3429(2040)	3429(2040)	3429(2040)	3429(2040)	15/15	cma-IH	9577(1e4)	1.5e4(3e4)	1.7e4(3e4)	2.0e4(3e4)	2.4e4(3e4)	2.9e4(3e4)	3.6e4(4e4)	15/15
CMA-ES	1.8e4(2e4)	7.5e4(8e4)	7.5e4(8e4)	7.6e4(8e4)	7.9e4(8e4)	8.8e4(6e4)	1.0e5(5e4)	15/15	CMA-ES	1.0e5(1e5)	1.7e5(1e5)	2.0e5(1e5)	2.2e5(1e5)	2.4e5(1e5)	2.8e5(1e5)	<i>oo des</i>	0/15
CMA-ESw	887(370)*2	1451(780)	1458(792)	1458(792)	1458(792)	1458(792)	1458(792)	15/15	CMA-ESw	3.1e4(3e4)	1.1e5(2e5)	1.1e5(2e5)	1.2e5(2e5)	1.2e5(2e5)	1.2e5(2e5)	1.3e5(2e5)	13/15
DE-scip	4.2e4(7880)	1.8e5(2e5)	2.0e5(2e5)	2.2e5(2e5)	2.4e5(2e5)	2.8e5(2e5)	3.3e5(2e5)	12/15	DE-scip	7.0e4(3e4)	2.5e5(3e5)	2.6e5(4e5)	2.7e5(4e5)	2.8e5(3e5)	3.0e5(4e5)	3.2e5(2e5)	10/15
cma-Nol	1429(230)	4171(4692)	4171(4692)	4171(4692)	4171(4692)	4171(4692)	4171(4692)	15/15	cma-Nol	1.4e4(1e4)	2.5e4(3e4)	2.8e4(3e4)	3.2e4(3e4)	3.6e4(3e4)	4.2e4(3e4)	5.1e4(3e4)	15/15
cmaIHie	1318(226)	3429(4218)	3429(4218)	3429(4218)	3429(4218)	3429(4218)	3429(4218)	15/15	cmaIHie	6473(8852)	2.5e4(2e4)	2.9e4(2e4)	3.1e4(2e4)	3.4e4(2e4)	3.8e4(2e4)	4.4e4(2e4)	15/15
f6									f18								
cma-IH	1283(816)	5.2e5(6e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-IH	1434(678)	2.7e4(3e4)	4.7e4(4e4)	5.2e4(4e4)	5.6e4(4e4)	6.5e4(4e4)	7.5e4(4e4)	15/15
CMA-ES	4.5e4(7e4)	2.9e6(4e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ES	1533(335)	4.0e5(3e5)	4.1e5(3e5)	4.5e5(3e5)	4.5e5(3e5)	4.9e5(3e5)	<i>oo des</i>	0/15
CMA-ESw	9.6e4(2e5)	2.6e6(2e6)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	2825(1204)	6.3e5(7e5)	8.2e5(1e6)	8.3e5(8e5)	8.3e5(1e6)	8.4e5(8e5)	8.4e5(1e6)	5/15
DE-scip	1.7e4(6864)	5.9e5(6e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	DE-scip	1.7e6(2e6)	1.7e6(2e6)	1.7e6(2e6)	1.7e6(2e6)	1.7e6(2e6)	1.7e6(2e6)	1.7e6(2e6)	3/15
cma-Nol	1316(620)	4.3e5(8e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-Nol	1260(378)	4.0e4(5e4)	5.2e4(6e4)	5.6e4(6e4)	6.0e4(6e4)	6.9e4(7e4)	7.7e4(8e4)	15/15
cmaIHie	4796(7773)	6.2e5(6e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cmaIHie	1451(568)	4.2e4(5e4)	5.2e4(6e4)	5.6e4(7e4)	6.0e4(7e4)	6.8e4(7e4)	7.7e4(8e4)	15/15
f7									f19								
cma-IH	5.2e5(4e5)	8.2e5(6e5)	8.3e5(8e5)	8.4e5(1e6)	8.4e5(5e5)	8.4e5(8e5)	8.4e5(1e6)	6/15	cma-IH	2.9e6(3e6)	6.0e6(5e6)	6.0e6(5e6)	6.0e6(9e6)	6.0e6(6e6)	6.0e6(7e6)	6.0e6(7e6)	0/15
CMA-ES	4.8e5(4e5)	1.4e6(2e6)	1.4e6(1e6)	1.4e6(1e6)	1.4e6(1e6)	1.4e6(1e6)	1.4e6(2e6)	4/15	CMA-ES	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
CMA-ESw	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
DE-scip	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	DE-scip	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cma-Nol	4.0e5(4e5)	5.2e5(6e5)	6.0e5(4e5)	6.0e5(8e5)	6.0e5(7e5)	6.0e5(4e5)	6.0e5(6e5)	7/15	cma-Nol	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cmaIHie	4.0e5(5e5)	4.1e5(3e5)	4.2e5(4e5)	4.2e5(4e5)	4.2e5(4e5)	4.2e5(4e5)	4.2e5(5e5)	9/15	cmaIHie	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
f8									f20								
cma-IH	2340(2820)	4.0e5(4e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-IH	867(340)	1012(274)	1.4e5(8e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
CMA-ES	5.3e4(6e4)	1.9e5(3e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ES	979(370)	2.0e4(2e4)	4.4e5(4e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
CMA-ESw	6995(4116)	1.5e5(2e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	4501(6348)	4864(6336)	1.8e5(2e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
DE-scip	4.1e4(4962)	3.9e5(6e5)	5.6e6(9e6)	5.7e6(5e6)	5.7e6(8e6)	5.8e6(4e6)	5.8e6(7e6)	1/15	DE-scip	4935(1332)	6037(2072)	1.6e5(4e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cma-Nol	3682(8742)	3.8e5(4e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-Nol	872(187)	994(242)	2.5e5(1e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
cmaIHie	1487(282)	6.0e4(9e4)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cmaIHie	875(148)	997(238)	9.3e4(1e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15
f9									f21								
cma-IH	1.4e4(9322)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	cma-IH	4.4e4(4e4)	9.4e4(2e5)	1.3e5(2e5)	1.4e5(3e5)	1.9e5(2e5)	1.9e5(2e5)	1.9e5(3e5)	11/15
CMA-ES	6.9e5(8e5)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ES	1.4e5(1e5)	3.4e5(4e5)	7.4e5(6e5)	7.4e5(6e5)	7.5e5(6e5)	7.5e5(7e5)	7.5e5(7e5)	6/15
CMA-ESw	4.3e4(4112)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	CMA-ESw	1.4e5(2e5)	1.9e5(2e5)	2.0e5(2e5)	2.2e5(2e5)	2.2e5(3e5)	2.2e5(3e5)	2.2e5(3e5)	11/15
DE-scip	8627(9834)	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo</i>	<i>oo des</i>	0/15	DE-scip	6.3e5(3882)	7.9e4(2e5)	6.3e5(6e5)	1.1e6(1e6)	1.1e6(8e5)	1.2e6(2e		

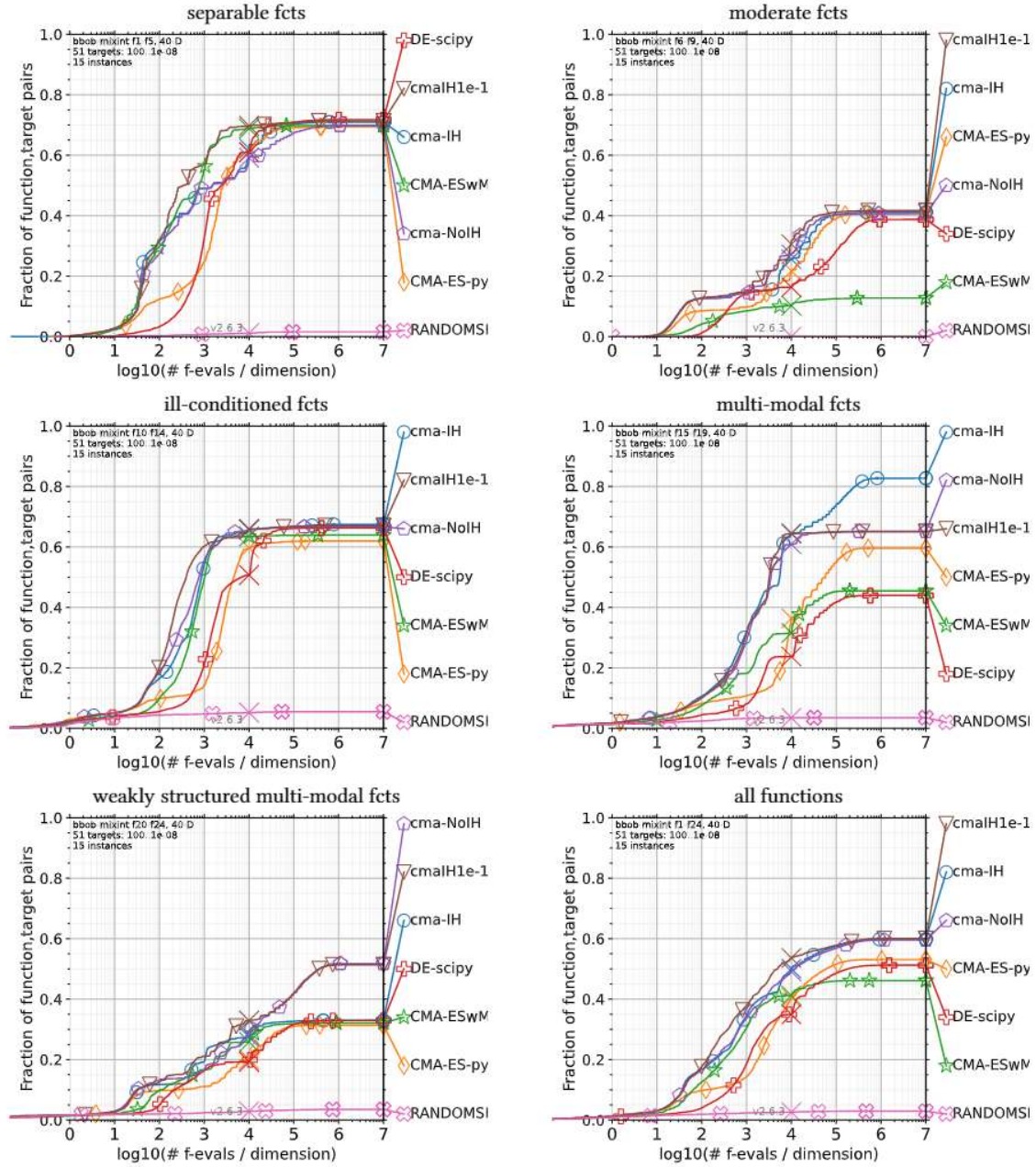


Figure 2: Bootstrapped empirical cumulative distribution of the number of f -evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 40-D.

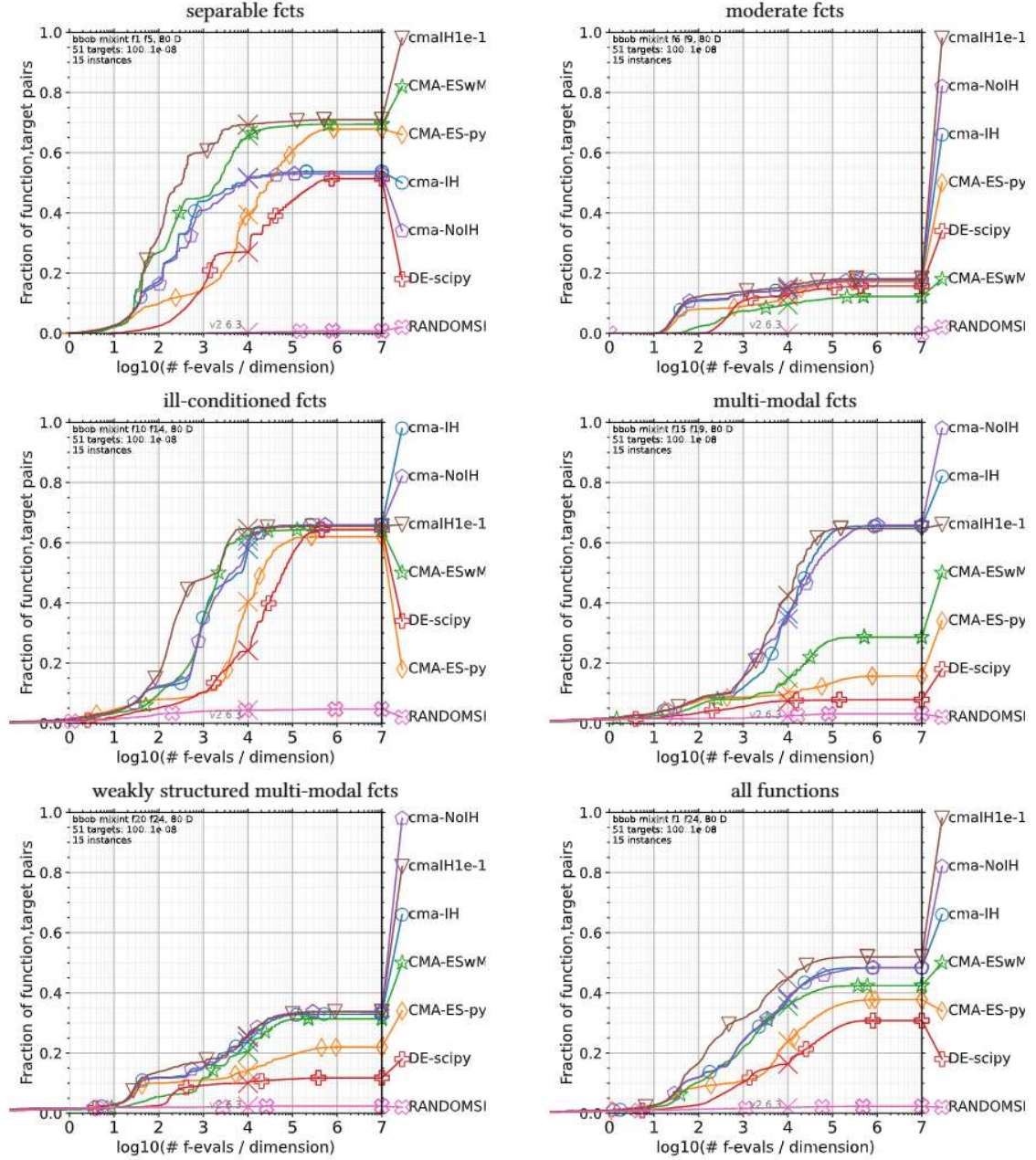


Figure 3: Bootstrapped empirical cumulative distribution of the number of f -evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 80-D.

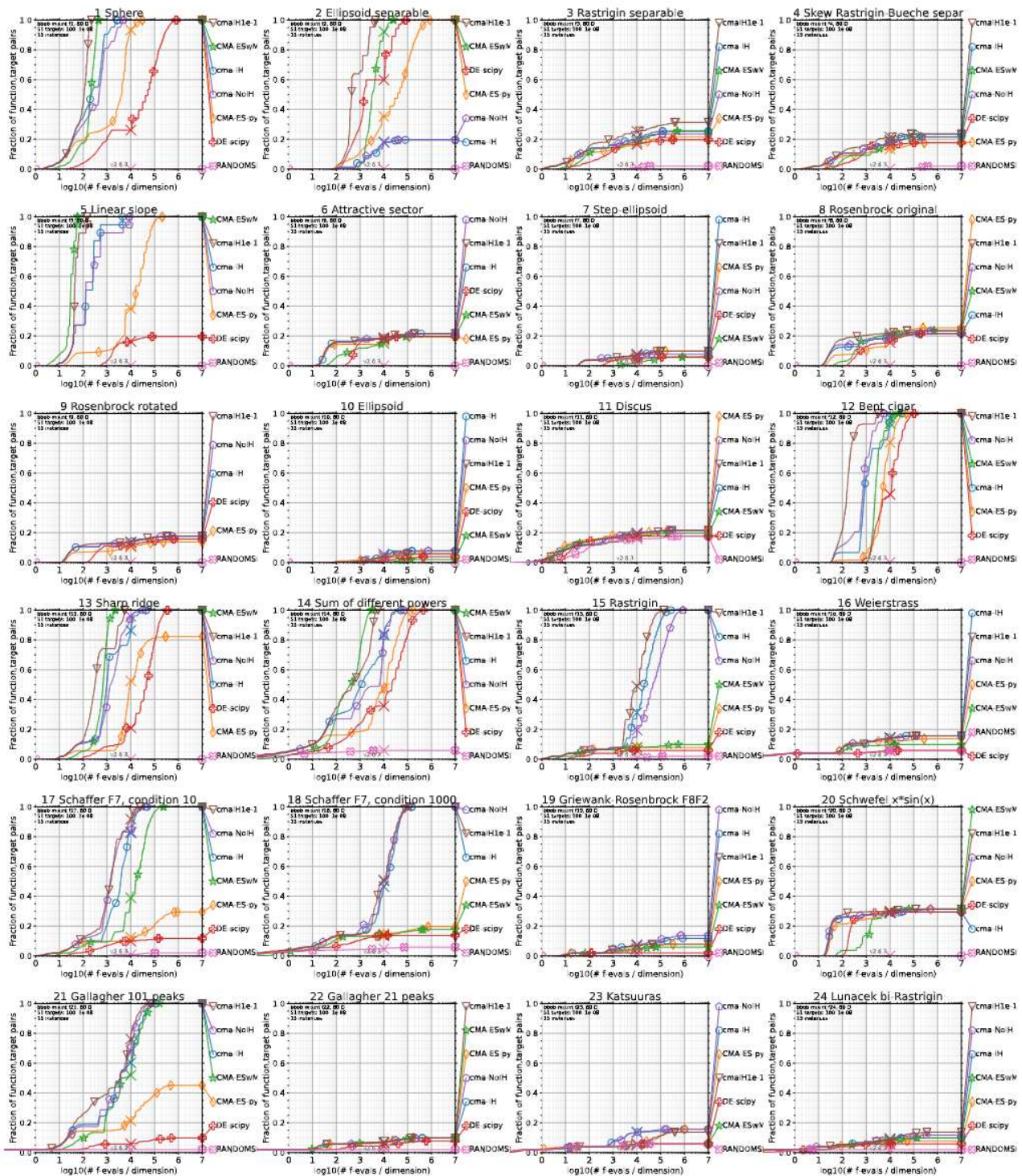


Figure 4: Empirical cumulative distribution of simulated (bootstrapped) runtimes, measured in number of f -evaluations, divided by dimension (FEvals/DIM) for the 51 targets $10^{[-8..2]}$ in dimension 80.