

Demographics, Capital Efficiency, and the Labor Share of Income

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Abstract

When the supply of one factor of production is constrained, what are the consequences on factor shares of income? This paper examines the common but optimistic assumption that if the supply of labor in an economy is constrained, returns to capital can be sustained or increased by substituting capital for labor. We offer a simple two-factor model that shows formally that such substitution would only increase the capital share of income where the marginal product of capital (MPK) exceeds the marginal product of labor (MPL). We test our theoretical expectations with a cross-national panel dataset and a Japanese industry dataset, finding evidence supporting our hypothesis. These results imply that any productivity gains from factor substitution will not be evenly distributed because elasticity of substitution varies within and across economies. Moreover, unless an economy's MPK is greater than its MPL, additional units of capital will lead to a declining capital share of income. Further work includes validation and application at the industry or firm level. Over the coming decades, the relationship between productivity and relative factor supply could become more important as countries with shrinking and aging populations face labor constraints.

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1 Introduction

1.1 Background

Many countries with advanced economies today have aging and shrinking populations, so firms in those countries face constrained labor input. To maintain or increase economic production under these pressures, firms have three options: find more labor, increase productivity, or export capital to places with more labor. This paper analyzes the second option.

When the supply of one factor of production is constrained, what are the consequences on the rate of growth when shifting input to another factor? Sustained constraints on labor input could put downward pressures on economic growth, both potential and actual – unless countries are able to make up for it by finding labor from other structural sources. This paper takes as a starting point the difficulty of finding structural sources of new labor (higher labor force participation, more immigration, later effective retirement ages, or more hours worked per worker). Assuming the labor supply is sticky, we look elsewhere for cues about economic growth.

This paper examines the common but optimistic assumption that if the supply of labor in an economy is constrained, the rate of growth can be sustained by substituting capital for labor. This assumption rests on two premises: first, adding additional capital will result in higher output growth via higher productivity and second, additional capital will generate positive returns on these additional input units (i.e., increasing marginal returns).

These premises confront two economic intuitions. The first, that output should grow more slowly for every unit of additional input, is rooted in any production process exhibiting diminishing marginal returns. The second, that any productivity gains from factor substitution are unlikely to be evenly distributed within and across economies – because otherwise the economy must both have uniform marginal product of capital (MPK) and marginal product of labor (MPL), and have an elasticity of substitution that is greater than one.

Over the coming decades, the relationship between productivity and relative factor supply could become more important: countries with shrinking and aging populations may face labor constraints,

and therefore shift their attention to maximizing productivity.¹ Given the variation in elasticity of substitution across industries, firms, and jobs, some firms and sectors are better placed to do so than others.

1.2 Argument

This paper offers a simple two-factor model of an economy exhibiting Constant Elasticity of Substitution (CES). It shows formally that the aforementioned substitution would only have the assumed effects where the marginal product of capital exceeds the marginal product of labor. Our theory implies that any productivity gains from factor substitution will not be evenly distributed because elasticity of substitution varies within and across economies. Consequently, noticing the relationship between MPK and MPL is vital to how additional factor inputs affect both productivity (and therefore output) and the labor-capital split (via diminishing marginal returns).

1.3 Implications for Firms

Variation in MPK arises because some kinds of capital investment convert into output more effectively than others. What economists call “capital” can take the form of a share of equity or a die-stamping machine or a server farm, and much more besides. The way the input capital is turned into a sustainable cash flow – its “capitalization” – contributes to the productivity of that capital. A die-stamping machine is less likely to be well-capitalized than a server farm, for instance – the latter might permit the development of intellectual property that generates long-term returns.

Firms facing scarce production factors regularly assess the relative productivity of capital and labor. If capital is more productive than labor, it makes sense to allocate a marginal dollar to capital investment rather than to labor. This is, of course, subject to the prices of labor and capital (wages and real rates, respectively). By design, cheap credit encourages increased investment – with the side effect of permitting firms that use factors less effectively to continue to exist, and is thus associated with lower productivity (Acharya et al., 2020). Under low or negative real rates, capital investment is broadly

¹The opposite is true for countries that have growing demographics but are capital constrained: a tempting solution to make up for capital scarcity is to “throw more bodies at the problem,” but an efficient solution must consider relative factor productivities.

accessible to wide swaths of the economy. But at any time, if capital is less productive than labor, a substitution will by definition be net negatively productive – and substituting more capital will decrease productivity by more than substituting only a little.

1.4 Contributions

Our findings have implications for the labor-capital split of income. The labor share affects the split of wages, standard of living, and inequality; its obverse, the capital share, informs the return on capital that an economy may invest to supplant scarce labor. Further work could include careful analysis of company and/or industry factor substitution. How is a firm's capital expenditure split between real assets and research and development, and how are different kinds of investment and labor expenses capitalized? What is the relationship between labor availability and capital efficiency (Keohane and Inagaki, 2024)?

To economists, this finding may seem like an algebraic artifact of production functions. But the conclusions this paper reaches were not at all obvious to Japanese economic actors – especially policymakers or companies – for at least the last 30 years. Japanese firms facing labor shortages applied vast amounts of capital input in an attempt to solve a problem of labor shortage – and the capital was not all efficiently transformed into output.

So much capital was deployed ineffectively that the aggregate price-to-book ratio of Nikkei 225 companies has steadily decreased from approximate three in 1999 to hover just above one since 2008.² Capital services contribution to GDP growth has also shrunk to almost zero in Japan since 2008.³ In comparison, aggregate price-to-book of the S&P 500 has grown from approximately two in 2008 to almost five now, and capital services continues to make stable contributions to US GDP growth.⁴ As of April 9 2024, ten percent of S&P 500 listed companies trade below book; the same figure is 32% for the Nikkei 225.⁵ In the autumn of 2023, the Tokyo Stock Exchange went so far as to adopt a very public name-and-shame regime incentivizing listed firms to make their operations more capital-

²Source: S&P Capital IQ.

³OECD productivity statistics.

⁴Sources: S&P Capital IQ, OECD productivity statistics.

⁵Source: WorldScope.

efficient (Keohane and Lewis, 2023); even the largest Japanese firms have felt pressure to explain and improve their capital allocations (Keohane and Inagaki, 2024).

Despite its incarnation in Japan, this paper’s straightforward finding remains obscured from policy in other countries soon facing the same tight labor conditions as Japan did in the 1990s and 2000s. The Korean Stock Exchange is considering a similar policy as the Tokyo Stock Exchange (Lee, 2024), and other countries will soon face demographic pressures as severe as Japan and Korea. In the context of transnational capital-intensive projects such as climate change and Artificial Intelligence, a strong understanding of the concepts at play – including their historical embodiments – is a key building block for thoughtful capital allocation.

1.5 Roadmap

The following section reviews existing literature on the determinants of labor productivity and the labor share of income. The third section walks through the theoretical framework and concludes with hypotheses. The fourth section describes our approach towards testing the theory empirically and the following section describes the results of the analysis. The final section concludes and discusses implications.

2 Background

2.1 An Economy’s Production

A common starting point for modeling economic output is a standard Cobb-Douglas production function. This approach, popularized by Solow (1956), has inspired numerous theoretical evolutions and underlies the reporting and bureaucratic collection of economic data today (Miller, 2008; Shackleton, 2018). Assuming diminishing marginal returns to capital, non-zero rates of labor growth, and non-zero technological progress, the Solow approach models economic output Q as a function of input factors labor L and capital K as well as the proportion of overall income that goes to each factor of production (α and $1 - \alpha$). Factors not included in the model are described by the Solow Residual A , which commonly is referred to as “multi-factor productivity”, and is usually defined as a measure of

how efficiently production turns inputs into outputs.

$$Q = AK^{1-\alpha}L^\alpha \quad (1)$$

Implicit in Equation 1 is that substituting one factor for another will not change overall output. In economics jargon, an economy with this characteristic would exhibit “unity elasticity of substitution”, meaning each factor of production is equally productive. It is not generally accepted that modern economies exhibit elasticity of substitution of one.⁶ That is to say, capital and labor are not necessarily perfect substitutes. It is easy to intuit why – no matter the level of technical advancement or labor skill, people can do some things that machines cannot and vice versa. A more general set of production functions, called Constant Elasticity of Substitution (CES), are the generalized version of Cobb-Douglas production and allow for non-unity elasticity of substitution (Arrow et al., 1961):

$$Q = A \left[\beta K^r + \alpha L^r \right]^{\frac{1}{r}}. \quad (2)$$

Hirakata et al. (2018) empirically analyze substitution in the US and Japanese economies and estimate that Japanese elasticity is approximately 0.2, while US elasticity is close to 1.5. These drastically different estimates suggest that the two economies are structurally different and have varying productivities of factor inputs. For this reason we base our theoretical analysis on economies exhibiting CES production.

2.2 Drivers of Elasticity of Substitution

The literature has not reached a clear consensus on why elasticity of substitution varies in the first place. One prominent strand of the literature finds that different sectors of the economy use capital and labor differently, and therefore find the two substitutable to different extents (Watanabe, 1992; Judzik and Sala, 2015; Fukao and Perugini, 2018; Fukao et al., 2019; Jerbashian, 2022; Manu, 2022).

⁶Although it is not clear that from a measurement point of view Cobb-Douglas specifications describe economic output worse than CES specifications do (Miller, 2008).

Institutional factors could also drive elasticity of substitution. Some analysts have found that more globalized countries have higher elasticity of substitution (Sala and Trivín, 2018; Saam, 2008; Boehm et al., 2019). Political institutional factors could also have an effect: trade unions affect labor market regulations, which could drop elasticity of substitution in the US and Canada – but empirical support for this hypothesis is mixed (Freeman and Medoff, 1982; Maki and Meredith, 1987; Shahiri and Osman, 2016). Economists have also pointed to political intervention that artificially lowers the price of capital (Yuhn, 1991) and the necessity of a well-functioning financial system (Klump and Preissler, 2000) as key drivers, though empirical evidence substantiating the relationship between these factors and elasticity of substitution is sparse.

Even the accurate measurement of elasticity of substitution is an area for scholarly debate (Miller et al., 2019). To illustrate the lack of consensus on what elasticity of substitution is, Knoblach and Stöckl (2020) collect estimates of elasticity of substitution for the US economy over 49 studies between 1961 and 2017. They find the majority of results fall in the 0.3 to 1.0 range, with peaks at 0.3 and 0.9. Globally, a few recent studies using cross-country data find estimates of 1.25 (Karabarbounis and Neiman, 2014), between 1.3 and 1.6 (Piketty, 2014), and 0.3 for both select high-income countries and a set of 90 countries (Mallick, 2012). Given the wide range of outcomes, often well below unity, and the many factors that influence the elasticity of substitution, it has been suggested the frequently used Cobb-Douglas production function be reconsidered in theoretical models (Sala and Trivín, 2018; Knoblach and Stöckl, 2020; Paul, 2019; Bellocchi and Travaglini, 2023; Chirinko and Mallick, 2017).

2.3 Drivers of the Labor Share of Income

With non-unity elasticity, substitution affects output – but it can also affect the labor-capital split of income. The labor share is an element of the aggregate production function of economies, and can be defined as the portion of an economy’s aggregate income that goes towards remunerating labor through wages, benefits, and other social benefits. Its downstream implications affect the distribution of income across society.

Its obverse, the capital share of income, is what remains with firms after paying labor expenses. Firms

then decide how much of the capital share to save (return to shareholders, pay down debt, etc.) versus invest in growing the business. Figure 1 shows the general, decades-long decline in the labor share of income in advanced economies, leaving the capital share to correspondingly increase. This has been a tailwind for capital markets investors but made life harder for workers.

Interestingly, changes in the labor share of income move inversely to gains in labor productivity (output per hour worked). Figure 1 shows this phenomenon across three major economies since 2000. This stylized fact motivates our analysis of the connection between the two concepts.

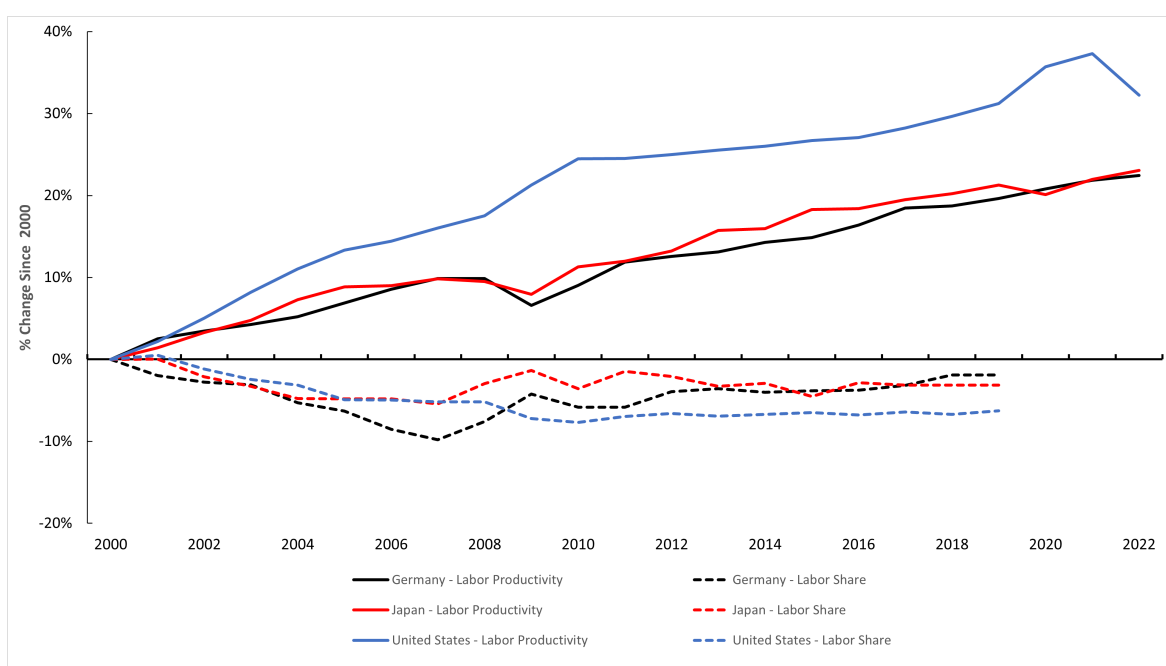


Figure 1: Labor Productivity and Labor Share of Income in the US, Germany, and Japan Since 2000. Source: Penn World Tables 10.01, OECD National Accounts, as of 21 November 2023.

A large body of literature has analyzed the movements in the labor-capital split of income to determine its drivers. For our purposes, this literature falls into three general categories: studies based on the macro bargaining mechanism, studies investigating its relationship with relative factor supplies, and microeconomic studies focusing on input factor heterogeneity.

Existing analyses of the factors driving the labor-capital split of income mostly exclude labor supply pressures, instead focusing on supercycle and boom-bust periods, rising and faster depreciation,

market concentration, globalization, and declining labor bargaining power (Manyika et al., 2019).

2.3.1 Bargaining and the Labor Share

Labor bargaining power is the mechanism underlying most analyses of movements in the labor share, though most research has concluded its direct impact is minimal relative to the other causes mentioned. Manyika et al. (2019) suggest that the erosion of labor market institutions such as unions weakens the bargaining power of workers; while a higher minimum wage or stronger bargaining power can raise labor share in the short run, it drives faster capital substitution in the longer term. Other research finds the effect of declining labor bargaining power on the labor share to be inconclusive (Elsby et al., 2013; OECD and OECD, 2018).

Some firms face strong local labor bargaining power, but other firms go abroad to get around it. Globalization and the resulting increase in offshoring is another often-cited driver of the decline in labor share. Industrialized countries, such as the US, have been offshoring the labor-intensive components of the supply chain over the last 25 years, leaving behind the capital-intensive parts of production, and putting upward pressure on the capital share of income (Elsby et al., 2013).

Some analysts argue that an increasingly financialized world economy affects the capital-labor split of income, through a bargaining channel. Increased rentier income in the US and Germany could push the labor share downward because increasingly financialized economies prioritized shareholder value, diminishing worker bargaining power (Dünhaupt, 2012). Even the composition of the capital share itself could shift more towards pure profits because of increased market concentration (Barkai, 2020).

2.3.2 Relative Factor Supply and the Labor Share

Economic historians have long considered the relationship between relative factor supply and the capital-labor split of income – but most of the literature relies on the same bargaining mechanism and does not mention substitution. In economies with a relative surplus of one factor of production over another, the relative price of the scarce factor will rise so high that producers can choose to impose its presence by force instead of paying a market rate (Domar, 1970). Goodhart and Pradhan

(2020) connect this analytical tradition to modern demographic pressures. They argue that changing demographics, specifically aging societies and a decline of the labor supply, will return bargaining power to workers and therefore drive up real wages and the labor share of income in developed economies.

But both analyses rely on bargaining power as a mechanism. Neither Domar nor Goodhart and Pradhan mention factor substitution – and because of this, neither model can explain the decline in the Japanese labor share of income over the last several decades.

Only d’Albis et al. (2021) posit substitution as a connection between relative factor surplus and the labor-capital split of income. The authors determine that migration shocks boost the labor share of income, but natural increases in population growth are soon followed by drops in the labor share. But the authors focus more on robust empirical analysis than a convincing theoretical framework.⁷

2.3.3 Input Factor Heterogeneity and the Labor Share

In a CES economy, substitution self-evidently affects output (via productivity), but its effects on the labor-capital split of income are more subtle. Not all capital or labor is equally suitable for substitution – and a growing body of research focuses on how firm-level capital allocation strategies drive the decline of the labor share of income.

One prominent driver is technology (Manu, 2022). As technological improvements have driven down the relative price of investment goods, companies have shifted investment from labor to capital, focusing on developing intellectual property – contributing to about half the decline in labor share since the early 1990s (Karabarbounis and Neiman, 2014; Koh et al., 2020; Dao et al., 2017; Manyika et al., 2019).⁸ Information Communications Technology (ICT) capital and Intellectual Property (IP) capital together have an elasticity of substitution well above one, while traditional capital (transport,

⁷As d’Albis et al. mention, the additional literature on the relationship between demographics and the labor share of income has mainly focused on population structure and the disparity in income distribution among the population (Lam, 1989, 1997) and on the relationship between immigration and wage inequalities (Borjas et al., 1997; Lerman and Schmidt, 1999; Card, 2009; Dustmann et al., 2013; Edo and Toubal, 2015). The literature connecting the labor share to inequality is a separate body of work – we leave it aside from this analysis because it is distributional rather than aggregate.

⁸Dao et al. (2017) also concluded that economies with a higher exposure to the displacement of labor by information technology experienced about four times the decline in labor income shares than those with low exposure.

machinery, equipment, residential and non-residential structures) has an elasticity of substitution below one (Jerbashian, 2022). Thus places like the US, with rapidly progressing ICT and IP capital accumulation, have seen a large decline in the labor share of income in recent decades.

Recent scholarship has posited several theoretical frameworks of how various kinds of technology could affect the labor-capital split of income. Building on the distinction between technology that augments labor and technology that augments capital (Hicks, 1963; Uzawa, 1961), Acemoglu and Restrepo (2018) zoom in on the macro and micro reasons that labor-substituting automation drives down the labor share. Lawrence (2015) presents an analysis based on industry heterogeneity, arguing that labor and capital are gross complements, and labor-augmenting technical progress has been so fast as to partially offset the measured gains in the US' capital-labor ratio since 1980. Our paper takes inspiration from this approach and generalizes from automation and labor-augmenting technical progress to total substitutability between capital and labor.

Phelps et al. (2020) offer a careful aggregate theoretical models of the wage and growth effects of economies with “additive” and “multiplicative” robots – that is, labor automating and labor augmenting technology. Their approach arrives at similar findings to ours, but they focus on the causes of innovation rather than on its effect on the split of income, as we do. Such an approach fits well with their thoughtful interrogation of the causes of productivity itself.

Moreover, there is further heterogeneity in the kinds of labor that technology capital investments can substitute for. ICT and IP technological advancements traditionally displaced low-skilled labor while complementing high-skilled labor, having a net positive effect on labor share. It remains an open question in academic and industry analysis whether new generative AI technology such as large language models may exhibit the same pattern or the opposite (displacing high-skill labor while complementing low skill labor).

Recent research has begun to explore this topic by comparing the US and Japan, which, given their different sector compositions, have differing economy-wide elasticities of substitution. The US has an elasticity of substitution greater than one, while Japan's is less than one (Hirakata et al., 2018; Miyake and Osumi, 2020). Capital-augmenting technologies have contributed to over 80% of the decline in

labor share in the US, while they have exerted upward pressure on the labor share in Japan (Hirakata et al., 2018: Figure 5b). The authors do not explain this phenomenon, but it is possible that it arises from the relative substitutability of capital and labor in the Japanese economy. Our theoretical model suggests that adding relatively unproductive capital would push the labor share up.

More granular sector analysis has found that more rapid advancement of new ICT technologies in the US than Japan has been a key driver of the relatively higher elasticity of demand in the United States (Miyake and Osumi, 2020). Outside the US, the decline in the labor share is concentrated in the real estate sector – but inside the US, the labor share has declined steeply outside the real estate sector (Gutierrez, 2017).

Although this literature focuses on the heterogeneity of within-factor productivity, it fails to look beyond capital to consider labor productivity. Our theoretical model expands on the literature’s dominant bargaining mechanism, suggesting that labor bargaining power is strongest when capital is not substitutable for labor.

3 The Theoretical Model

To examine the effect of labor supply changes on the labor share of income, we turn to a model of an economy’s production. Using a CES production function, we model the effect of the labor supply on the labor share of income. We then decompose the labor supply into cyclical and structural factors and examine how structural demographic changes could affect the labor share.

3.1 A Formal Model for a CES Economy

Perhaps a more realistic depiction of modern economies is a CES production function. Consider instead an economy with production Q that follows a CES form:

$$Q = A \left[\beta K^r + \alpha L^r \right]^{\frac{1}{r}},$$

where $r = \frac{\sigma-1}{\sigma}$ is the substitution parameter and σ is the elasticity of substitution between capital and

labor. The economy's cost function, with wages w and cost of capital c , is defined by

$$C = wL + cK.$$

Its revenue function R , with price level p and quantity of output q , is

$$R = pq = pA \left[\beta K^r + \alpha L^r \right]^{\frac{1}{r}}.$$

Its profit function is defined by

$$\pi = pq - C = pA \left[\beta K^r + \alpha L^r \right]^{\frac{1}{r}} - (wL + cK).$$

3.1.1 Deriving the Labor Share

The next step in analyzing the effect of structural demographic trends on the price of labor is to define the labor share of income:

$$\alpha = wL.$$

The price of labor W is given by its marginal revenue product MRP_L , defined as

$$w = MRP_L = \frac{\partial R}{\partial Q} * \frac{\partial Q}{\partial L}.$$

In this economy,

$$\frac{\partial R}{\partial Q} = p,$$

$$\frac{\partial Q}{\partial L} = \alpha L^{r-1} \left[\beta K^r + \alpha L^r \right]^{\frac{1-r}{r}}$$

When reduced, this takes the form

$$MRP_L = p \alpha L^{r-1} \left[\beta K^r + \alpha L^r \right]^{\frac{1-r}{r}}.$$

$$\alpha = WL = L * MRP_L = L * \left[p \alpha L^{r-1} \left[\beta K^r + \alpha L^r \right]^{\frac{1-r}{r}} \right].$$

$$\alpha = p \alpha L^r \left[\beta K^r + \alpha L^r \right]^{\frac{1-r}{r}}.$$

$$\frac{1}{p L^r} = \left[\beta K^r + \alpha L^r \right]^{\frac{1-r}{r}}.$$

$$\ln \frac{1}{p L^r} = \frac{1-r}{r} \ln \left[\beta K^r + \alpha L^r \right]$$

$$\frac{r}{1-r} \ln \frac{1}{p L^r} = \ln \left[\beta K^r + \alpha L^r \right]$$

For $\beta = 1 - \alpha$ (constant returns to scale), we have:

$$\left[\frac{1}{p L^r} \right]^{\frac{r}{1-r}} = \left[(1 - \alpha) K^r + \alpha L^r \right]$$

$$\left[\frac{1}{p L^r} \right]^{\frac{r}{1-r}} = \alpha (L^r - K^r) + K^r$$

$$\alpha = \frac{\left[\frac{1}{pL^r} \right]^{\frac{r}{1-r}} - K^r}{L^r - K^r}$$

$$\alpha = \frac{(pL^r)^{-\frac{r}{1-r}} - K^r}{L^r - K^r}$$

$$\alpha = \frac{p^{-\frac{r}{1-r}} L^{-\frac{r}{1-r}} L^r - K^r}{L^r - K^r}$$

We re-write with $r = \frac{\sigma-1}{\sigma}$, where σ is the elasticity of substitution of capital for labor:

$$\alpha = \frac{\left[\frac{1}{pL^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\frac{\sigma-1}{\sigma}}{1-\frac{\sigma-1}{\sigma}}} - K^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} - K^{\frac{\sigma-1}{\sigma}}}$$

$$\alpha = \frac{\left[\frac{L^{\frac{(1-\sigma)}{\sigma}}}{p} \right]^{\sigma-1} - K^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} - K^{\frac{\sigma-1}{\sigma}}} \quad (3)$$

$$\alpha = \frac{\frac{L^{\frac{(1-\sigma)(\sigma-1)}{\sigma}}}{p^{\sigma-1}} - K^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} - K^{\frac{\sigma-1}{\sigma}}} \quad (4)$$

$$\alpha = \frac{\frac{L^{1-\sigma}}{p^{\sigma-1}} L^{\frac{\sigma-1}{\sigma}} - K^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} - K^{\frac{\sigma-1}{\sigma}}} \quad (5)$$

This theoretical formation of α suggests that for $\sigma > 1$, incremental increases in L result in lower α (but at a diminishing pace). For $\sigma < 1$, incremental increases in L result in higher α (but at a diminishing pace). The implications for the $\frac{K}{L}$ ratio are that for a given level of K , equal L means 50/50 factor split of income. As the level of K increases, it takes more L to extend beyond a given α .

The dynamics of price level are also interesting, if secondary to our theoretical framework. For $\sigma < 1$,

incremental increases in p result in higher α ; for $\sigma > 1$, incremental increases in p result in lower α .

Figure 2 shows the way that the labor share changes with increases in the capital supply for different elasticities of substitution. The effect of increases in capital on the labor share of income vary across the elasticity of substitution.

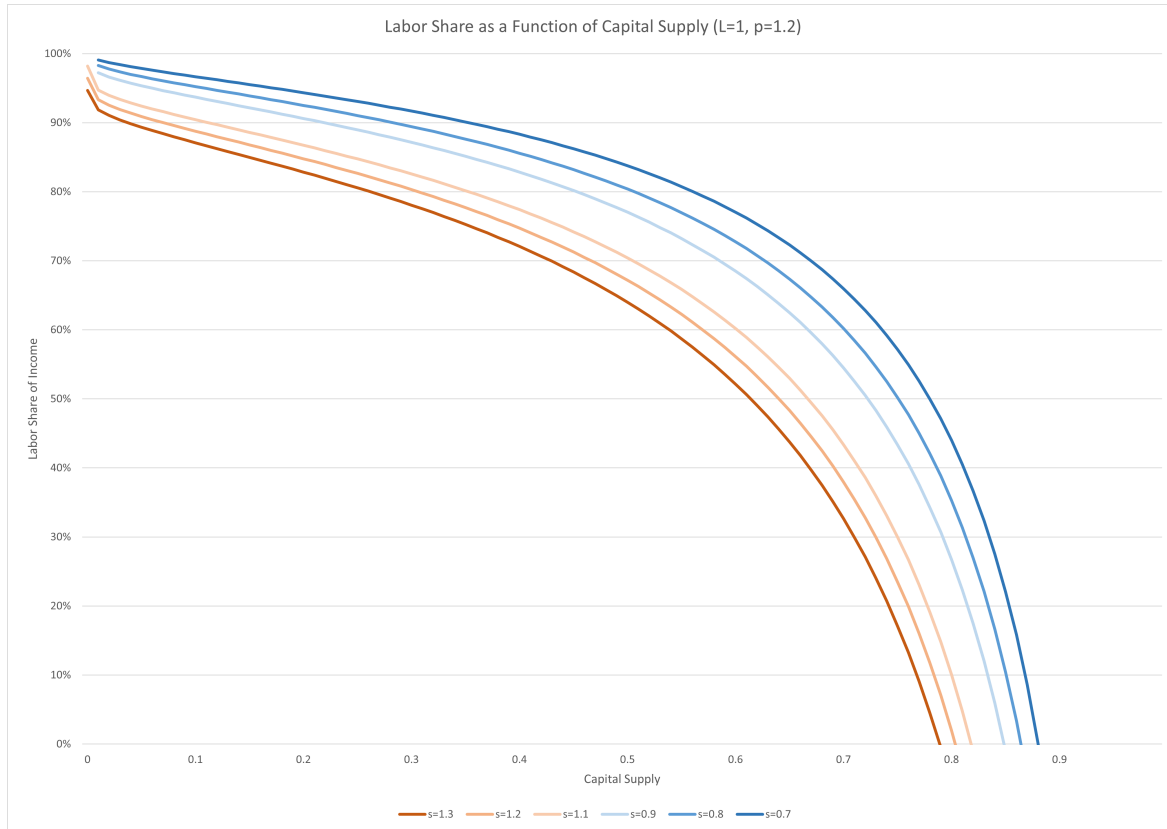


Figure 2: CES labor share of income as a function of Capital Input. Source: author calculations, as of 27 June 2024.

3.1.2 Comparative Statics

By holding cyclical factors constant (or, in future work, projecting them), we use the CES model as a basis for observing how movements in certain structural and cyclical demographic factors affect the labor share of income. We can then test this relationship in empirical analysis later in the paper.

3.2 Hypothesis

- With MPK higher than MPL, capital is more productive than labor, so increases in capital services (higher k) should correspond to α decreasing. This effect becomes stronger as sigma increases (MPK increases relative to MPL).
- Correspondingly, with MPL higher than MPK, labor is more productive than capital, so increases in capital services (higher k) should correspond to α increasing. This effect becomes stronger as sigma decreases (MPL increases relative to MPK).

4 Empirics

We test our hypothesis with two sets of analysis. The first is a cross-national regression testing these relationships over space and time. The second is a more in-depth analysis of these trends in Japan using Japanese Economic Census data.

4.1 Cross-National Empirics

4.1.1 Cross-National Data Description

We first test this theory across countries over time by performing several time-series cross-sectional regressions.

Our dependent variable is the labor share of income α , sourced from Penn World Tables (Feenstra et al., 2015). Labor income is hourly wages times the number of hours worked in an economy. More specifically, it includes both hourly wages and social contributions that employers make on behalf of employees. Capital income is gross operating surplus and gross mixed income, including paid or received interest, rents, or changes on financial or tangible non-produced assets. Both incomes are computed on a real per capita basis. Total income is the sum of these two, corresponding to GDP minus taxes less subsidies on production and imports. The labor share of income is labor expenditures divided by total income (d’Albis et al., 2021).

To relate this framework to our theory more explicitly, we separate out marginal increase in capital

services (which is investment rather than savings, and is made with part of the capital share of income) from an increase in marginal hiring (which comes from the labor share of income). Increasing capital services – all else equal – would put downward pressure on the capital share of income, and increasing hiring would put downward pressure on the labor share. This analysis studies how these decisions are made in aggregate across economies over time. But if investment and hiring remain stable, firms have the additional option of increasing savings (i.e., profit) via mechanisms such as dividends.

We have two main independent variables. The first is a measure of capital services (KSI), sourced from Penn World Tables, a measure of the “services” a capital asset provides during a given year. It is the typical measure of capital in GDP calculations (Shackleton, 2018). The second is the “labor supply”, which can broadly be defined as the number of hours worked in an economy in a year. We calculate this data from Penn World Tables data (average hours worked per employee times number of employed persons).

Our theory suggests that the relationship between the labor supply depends on the economy’s elasticity of substitution between capital and labor (σ), which depends on the marginal product of capital (MPK) and the marginal product of labor (MPL). We calculate MPL as an arithmetic average of an economy’s change in GDP from five years in the past to five years in the future divided by its change in labor input (Hours Worked Index) over the same time period. We calculate the marginal product of capital (MPK) similarly and the elasticity of substitution σ as the quotient MPK/MPL (before one was subtracted from each). Year over year growth rates are denoted by AGR (annual growth rate). We subtract one from σ for ease of inference. With this adjustment, values of σ between negative one and zero correspond to situations where a one-unit increase in labor input results in an output increases of more than one unit; values greater than zero correspond to situations where a one-unit increase in labor input results in an output increases of less than one unit.

Our empirical approach relies on sourcing macro data from international organizations, compiled using Haver Analytics’ API. We source national accounts variables from the OECD. Various measures of labor cost and contributions to GDP growth come from OECD National Accounts data. Effective retirement age, labor force participation, union density, and unemployment data come from the OECD

Table 1: Descriptive Statistics: Cross-National Analysis

	count	mean	std	min	25%	50%	75%	max
Delta_FD	928.0	-0.09	1.79	-10.65	-0.95	0.11	0.97	12.94
MPL	1431.0	-327.64	12668.32	-479206.77	-1.02	3.17	7.80	2040.42
CBPR	958.0	1.29	33.50	-0.01	0.02	0.05	0.08	1034.84
Unemp	1077.0	0.07	0.05	0.00	0.04	0.07	0.09	0.29
CPI	1499.0	47.52	36.88	0.00	10.22	45.33	81.55	160.50
Sigma_new_norm	1130.0	-0.88	7.18	-152.48	-1.12	-0.91	-0.67	128.92
Tariff_Mean	786.0	0.06	0.06	0.00	0.03	0.04	0.09	0.81
USD_LC	897.0	1939.02	13237.05	0.00	0.19	0.93	1.34	110864.75
Labor_Advanced_Educ	543.0	0.80	0.05	0.61	0.78	0.81	0.84	0.91
Unions	930.0	0.31	0.19	0.06	0.16	0.26	0.38	0.87
HW_CALC	1780.0	0.77	0.25	0.13	0.59	0.84	0.98	1.37

Labor Force Survey. Inflation and exchange rates data come from OECD Main Economic Indicators. We source tariff measures and the labor force's skilled/unskilled mix from the World Bank's World Development Indicators. We pull central bank policy rates from the Bank of International Settlements, and all population data from the United Nations Population database.

Table 1 shows the mean, standard deviation, and inter-quartile range of the variables used in the analysis.

4.1.2 Cross-National Empirical Specification

To predict the labor share of income using input factors and elasticity of substitution, we regress the capital services index (KSI, k) on the labor share of income α . But because our theoretical model shows that the relationship between factor supply and labor share depends on elasticity of substitution, we interact k with σ .

We run several empirical specifications. The base specification includes only σ as an explanatory variable, the next also includes some measure of the labor supply, and the third interacts the two. The fourth specification includes all the aforementioned variables as well as a vector ζ of measures of other possible explanations of changes in α : tariffs, exchange rates, the skilled/unskilled profile of the labor force, HWI growth, and prevalence of trade unions. The fourth specification follows the below form:

$$\alpha_{c,t+1} = \alpha + \gamma_0 \sigma_{c,t} + \gamma_1 k_{c,t} + \gamma_2 \sigma_{c,t} k_{c,t} + \vec{\gamma}_3 \zeta_{c,t} + \varepsilon_{c,t}. \quad (6)$$

The coefficients in these analyses should correspond with the figures in the comparative statics section above. Our theoretical model predicts that $\gamma_1 < 0$ and $\gamma_2 < 0$.

We also run numerous robustness checks to ensure that our results are not merely a coincidence of model specification. We run the same analysis testing our model's predictive power by leading the dependent variable α by one year: this year's economic conditions predict next year's labor share. Substantively, statistically significant results for such a test would suggest a time lag for firms and the economy to adjust their income statements to demographic pressures. We also test our hypotheses using alternate measures of the labor supply.

4.1.3 Cross-National Results: Capital Supply

Table 2 shows the results of our primary empirical specifications. These results generally support our hypotheses. The baseline coefficient for KSI is statistically insignificant, meaning an undetermined relationship between capital services and the labor share of income when MPK equals MPL. The negative and statistically significant interaction term shows that with a higher σ , increases in capital services correspond to a lower α .

A different way of looking at this effect is in relation to a given level of σ . The statistically significant small positive coefficient for σ implies a small baseline effect of σ on the labor share: higher σ (MPK > MPL) is associated with very slightly higher labor shares. But because our regression includes an interaction term, this negative relationship only holds if capital services is zero. If KSI takes on a positive value – adding capital – the statistically significant negative coefficient for the interaction term means that the net effect of σ on α decreases. Substantively, this means that even though higher values of σ correspond to higher labor shares, adding capital slows this effect; adding enough capital could reverse the sign of the cumulative effect, instead increasing the capital share.

Table 2: Demographic Predictors of Labor Share of Income

	<i>Dependent variable: Labor Share</i>			
	(1)	(2)	(3)	(4)
Intercept	0.090*** (0.020)	0.091*** (0.020)	0.092*** (0.020)	0.317*** (0.060)
Lag DV	0.850*** (0.035)	0.845*** (0.036)	0.844*** (0.036)	0.611*** (0.059)
CPI	-0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	-0.001*** (0.000)
USD/LC				-0.011 (0.007)
CBPR	-0.004 (0.013)	0.001 (0.013)	-0.000 (0.013)	-0.230*** (0.065)
Skilled Labor				0.003 (0.026)
Unions				-0.034 (0.042)
Tariffs				-0.050 (0.067)
KSI		-0.003 (0.003)	-0.003 (0.003)	0.011 (0.007)
Unemp.	-0.102*** (0.021)	-0.104*** (0.021)	-0.103*** (0.021)	-0.185*** (0.040)
HWI				-0.044* (0.025)
σ	0.000 (0.000)	0.000 (0.000)	0.001 (0.001)	0.002*** (0.001)
KSI: σ			-0.001 (0.000)	-0.001** (0.001)
Country FE	Yes	Yes	Yes	Yes
Lag DV	Yes	Yes	Yes	Yes
Observations	461	461	461	231
R^2	0.984	0.985	0.985	0.989
Adjusted R^2	0.984	0.984	0.984	0.988
Residual Std. Error	0.010	0.010	0.010	0.009
F Statistic	87113.607***	84404.507***	81341.908***	65660.904***

Note:

*p<0.1; **p<0.05; ***p<0.01

4.1.4 Cross-National Results: Labor Supply

To predict the labor share of income directly using demographic variables, we regress the labor supply ℓ on the labor share of income α . But because our theoretical model shows that the relationship between the labor supply and labor share depends on elasticity of substitution, we interact ℓ with σ .

We run several empirical specifications. The base specification includes only σ as an explanatory

variable, the next also includes some measure of the labor supply, and the third interacts the two. The fourth specification includes all the aforementioned variables as well as a vector ζ of measures of other possible explanations of changes in α : tariffs, exchange rates, the skilled/unskilled profile of the labor force, KSI growth, and prevalence of trade unions. The fourth specification follows the below form:

$$\alpha_{c,t+1} = \alpha + \gamma_0 \sigma_{c,t} + \gamma_1 \ell_{c,t} + \gamma_2 \sigma_{c,t} \ell_{c,t} + \vec{\gamma}_3 \zeta_{c,t} + \varepsilon_{c,t}. \quad (7)$$

Our theoretical model predicts that results show the opposite sign from the capital regression coefficients: $\gamma_1 > 0$ and $\gamma_2 > 0$.

Table 3 shows the results of this analysis. The hypothesized effects do not show in the results.

There are a few potential explanations for this finding. First, one of measurement: capital services is inherently measured in currency, and therefore attaches a price to goods. Hours worked, on the other hand, is the raw labor input without a price (i.e., wages) attached. There is a mismatch of units between the two.

Second, labor could have a systematically different marginal product than capital. Figure 3 shows the distribution of MPK and MPL in our dataset (via Penn World Tables). The two have markedly different distributions, with MPL showing a fatter right tail. Perhaps the fatter left tail on the MPK distribution is indicative of the cyclical nature of investment spending, whereas the fatter right tail on the MPL distribution can be attributed to the stickiness of wages.⁹

The general logic behind this explanation would be that investment spending can change very easily in reaction to market conditions. When rates go up, financing responds quickly – and when rates drop, financing picks up. But firm expenditures on labor rely on information that is much less public and much less constantly available than public information on interest rates. Information about labor expenses comes in through candidate-by-candidate negotiation over compensation or by quarterly payroll surveys, not minute-by-minute ticker reports. Firms face regulatory restrictions on revoking

⁹The latter is a well-studied concept, perhaps most famously by Keynes (1936) and Phelps (1965, 1967, 1968).

Table 3: Demographic Predictors of Labor Share of Income

	<i>Dependent variable: Labor Share</i>			
	(1)	(2)	(3)	(4)
Intercept	0.090*** (0.020)	0.139*** (0.026)	0.138*** (0.026)	0.298*** (0.063)
Lag DV	0.850*** (0.035)	0.817*** (0.037)	0.818*** (0.037)	0.645*** (0.061)
CPI	-0.000 (0.000)	0.000** (0.000)	0.000** (0.000)	-0.001** (0.000)
USD/LC				-0.004 (0.006)
CBPR	-0.004 (0.013)	-0.006 (0.014)	-0.006 (0.014)	-0.181** (0.083)
Skilled Labor				-0.005 (0.028)
Unions				-0.042 (0.043)
Tariffs				0.039 (0.073)
KSI				0.000 (0.000)
Unemp.	-0.102*** (0.021)	-0.157*** (0.026)	-0.156*** (0.026)	-0.179*** (0.040)
HWI		-0.046*** (0.013)	-0.046*** (0.013)	-0.042 (0.027)
σ	0.000 (0.000)	0.000* (0.000)	0.001*** (0.000)	0.005 (0.006)
HWI: σ			-0.001** (0.000)	-0.005 (0.006)
Country FE	Yes	Yes	Yes	Yes
Lag DV	Yes	Yes	Yes	Yes
Observations	461	461	461	211
R^2	0.984	0.985	0.985	0.975
Adjusted R^2	0.984	0.984	0.984	0.971
Residual Std. Error	0.010	0.010	0.010	0.009
F Statistic	87113.607***	83272.278***	85145.481***	80311.590***

Note:

*p<0.1; **p<0.05; ***p<0.01

offers of employment or laying off employees and worry about the consequences of doing so on their business operations. Furthermore, firms cannot unilaterally pay their existing workers less – but they may be able to refinance a debt obligation. Due to these factors and more, labor expenses therefore end up changing more slowly than capital investment, and are less correlated with real-time output.

Figure 4 shows this phenomenon in a different way. The first panel shows a strong positive correlation

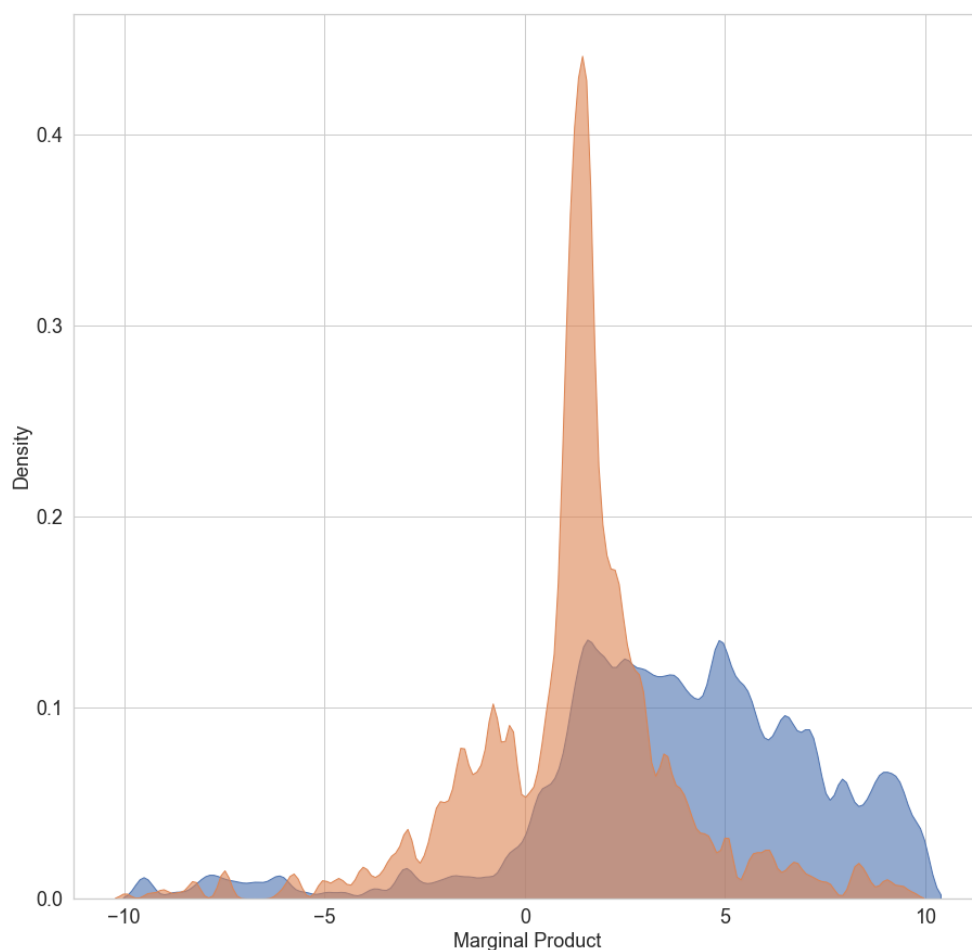


Figure 3: The distribution of marginal products of labor (blue) and capital (orange), as calculated in our data set. Source: author calculations, as of 7 February 2024.

between average change in capital services and average GDP growth. The second panel shows that the same relationship does not exist for labor. Again, this could be interpreted as evidence of the “wage stickiness” hypothesis (or the “incomplete information” hypothesis). Capital services changes procyclically (positive slope on the left panel of Figure 4), but labor is much slower to adjust to conditions of the economy.



Figure 4: The joint distribution of average changes in GDP and capital services (top) and hours worked (bottom). Color denotes country. Source: author calculations, as of 7 February 2024.

4.1.5 Cross-National Robustness

To check our results, we run several robustness tests. Our main results test for a contemporaneous relationship between the independent and dependent variables and assess a linear (non-logarithmic) relationship between the variables in question. In the appendix, we present results that allow for a one-year lag to permit for time effects. Such an approach results in a loss of statistical significance, perhaps because the relationship we test is a mechanical one, not a micro-founded pattern that takes time to manifest.

The hours worked in an economy do not appear out of thin air. It is affected by the size of the working age population (WAP) and net migration (NM), the percentage of the WAP that participates in the labor force by being employed or actively seeking employment (LFPR), the effective retirement age of males and females (ERA_M and ERA_F), and the percentage of the labor force that is unemployed (Unemp). The unemployment rate is a measure of cyclical economic conditions, and the others are measures of structural economic conditions. Governments and economies do not merely take these economic conditions as a given, however – they have agency to adapt to the demographic cards they have been dealt. It is also possible to measure the adaptations a country has undertaken to make the most efficient use of its labor market, given by the difference between a country's hours worked index and its working age population index. Although we omit an empirical study here, further analysis could investigate the relationship between these variables when measuring particular aspects of the labor supply. The same procedure is applicable to capital.

4.2 Micro Empirics: Japanese Industry and Firm Data

4.2.1 Micro Empirical Setting

Testing this hypothesis at a more micro level is difficult because of a lack of firm-level data about the labor share of income and elasticity of substitution. But we know that elasticity of substitution varies across industry (Manu, 2022). So rather than testing our hypothesis directly, we instead test the micro effects of factor input expenditures on profit, capturing variation across industries as a proxy for elasticity of substitution.

We first use Japanese aggregated firm production expense data to provide evidence that expenditures on capital and labor do indeed have systematic relationships with output (as measured by profit: sales divided by expenses). We next repeat the analysis but instead use labor and capital expenditures to predict the labor share of income. Lastly, we analyze the coefficients of industry fixed effects to find how cross-sectional industry variation relates to both profit and the labor share. We interpret these results relative to our hypotheses from earlier in the paper.

We analyze data from the Japanese Bureau of Statistics Economic Census for Business Activity.¹⁰ This data set is available for 2012, 2016, and 2021, and describes the production structure of firms: their cost structure (production inputs and taxes), sales, and value added. It is available with values reported across firm size and industry.

For each year, we group the data by the first aggregate of industry (e.g., Agriculture, Forestry and Fisheries; Manufacturing; Information and Communications, etc.). We next group the data by firm size.¹¹ It is a 2021 snapshot of the Japanese economy. We calculate three major measures of factor inputs: labor, depreciation, and capital expenditure.

Labor is the fraction of total expenses spent on wages/salaries and health and welfare. We offer two measures of capital spending. The first is depreciation as a fraction of total expenses, measuring how much a firm spends every year on its existing capital stock, regardless of whether the depreciating capital asset is tabulated under COGS or Selling Cost & General Expenses. The second measure is Capital Expenditure (CapEx) as a fraction of total expenses, a tabulation of the total spending on net new capital, tangible and intangible, during the reporting period.

Figure 5 shows how expenditures on these three items varies across Japanese industries.

Models of economic production suggest that the use of these three factors are related: a three-dimensional relationship between Labor, COGS, and Depreciation.

¹⁰The 2021 Economic Census for Business Activity Tabulation of Enterprises, etc. Tabulation across Industries Financial Items, etc.: Enterprises, etc., Establishments, Persons Engaged, Sales (Income), Total Expenses, Major Break-down of Expenses, Gross Value Added, Net Value Added and Amount of Capital Investment by Industry Major Groups of Enterprises and Size of Regular Employees of Enterprises of Enterprises (7 Groups) - Japan (link: <https://www.e-stat.go.jp/en/dbview?sid=0004006335>).

¹¹Size categories are 0-4, 5-9, 10-19, 20-29, 30-49, 50-99, and 100+ employees.

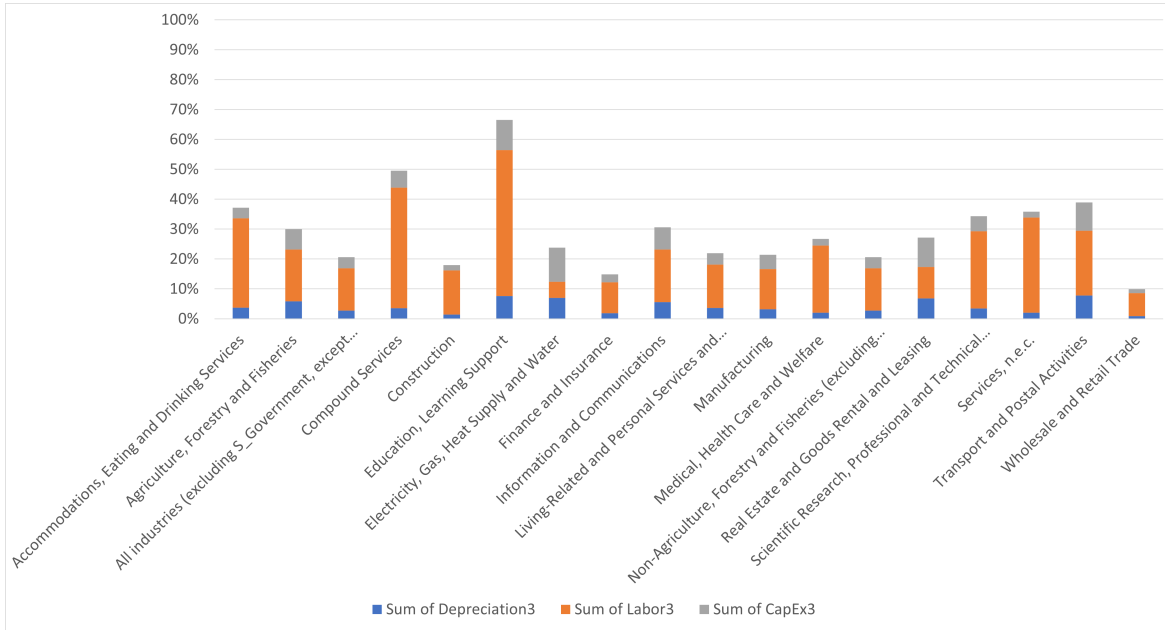


Figure 5: Spending on input factors by industry in Japan. Source: author calculations, as of 13 May 2024.

There is a strong relationship between Depreciation and CapEx: firms with large capital stocks tend to increase it further with more CapEx. Firms with larger capital stocks also tend to spend less on Labor – although the third panels of Figures 6 and 10 shows that there tends to be a general floor of capital stock per worker.

But these relationships do not appear against Profit (sales minus total expenses). Figures 9 and 7 show the relationship between Profit, plotted on the y-axes, and the three factor inputs, plotted on the x-axis. In some cases, there industry clustering across the factor input – for example, CapEx – but the factor inputs do not generally exhibit a relationship with Profit.

4.2.2 Micro Empirical Models

To further understand these three factors of production, we run an ordinary least squares regression on this cross-sectional data set. Because of the three-dimensional relationships depicted above, our empirical models use three-way interactions to parse the interactive relationships that Depreciation (δ), Labor (λ), and CapEx (κ) have with Profit (ρ).

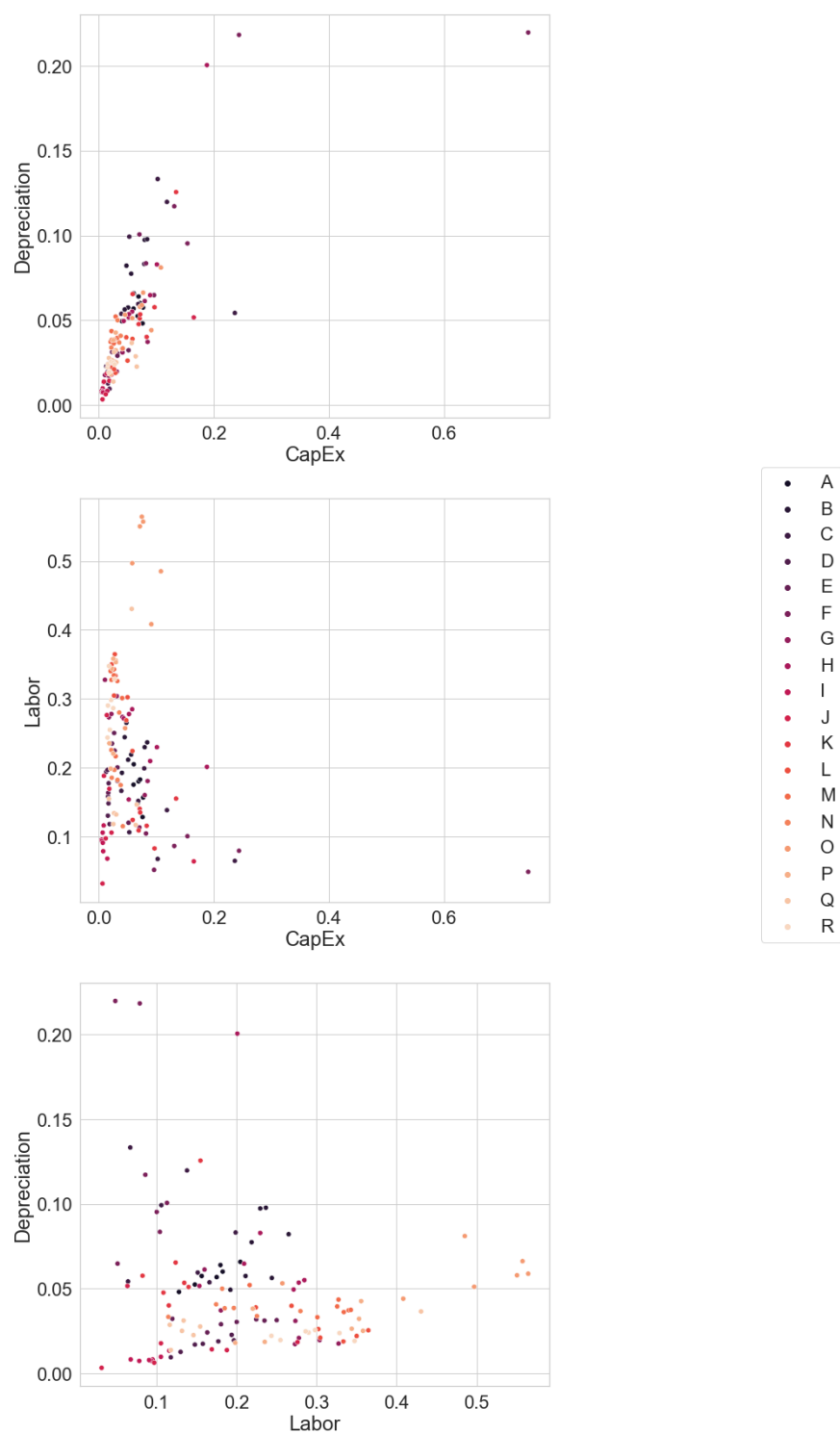


Figure 6: The joint distribution of factor inputs in Japan. Color denotes industry. Source: author calculations, as of 29 April 2024.

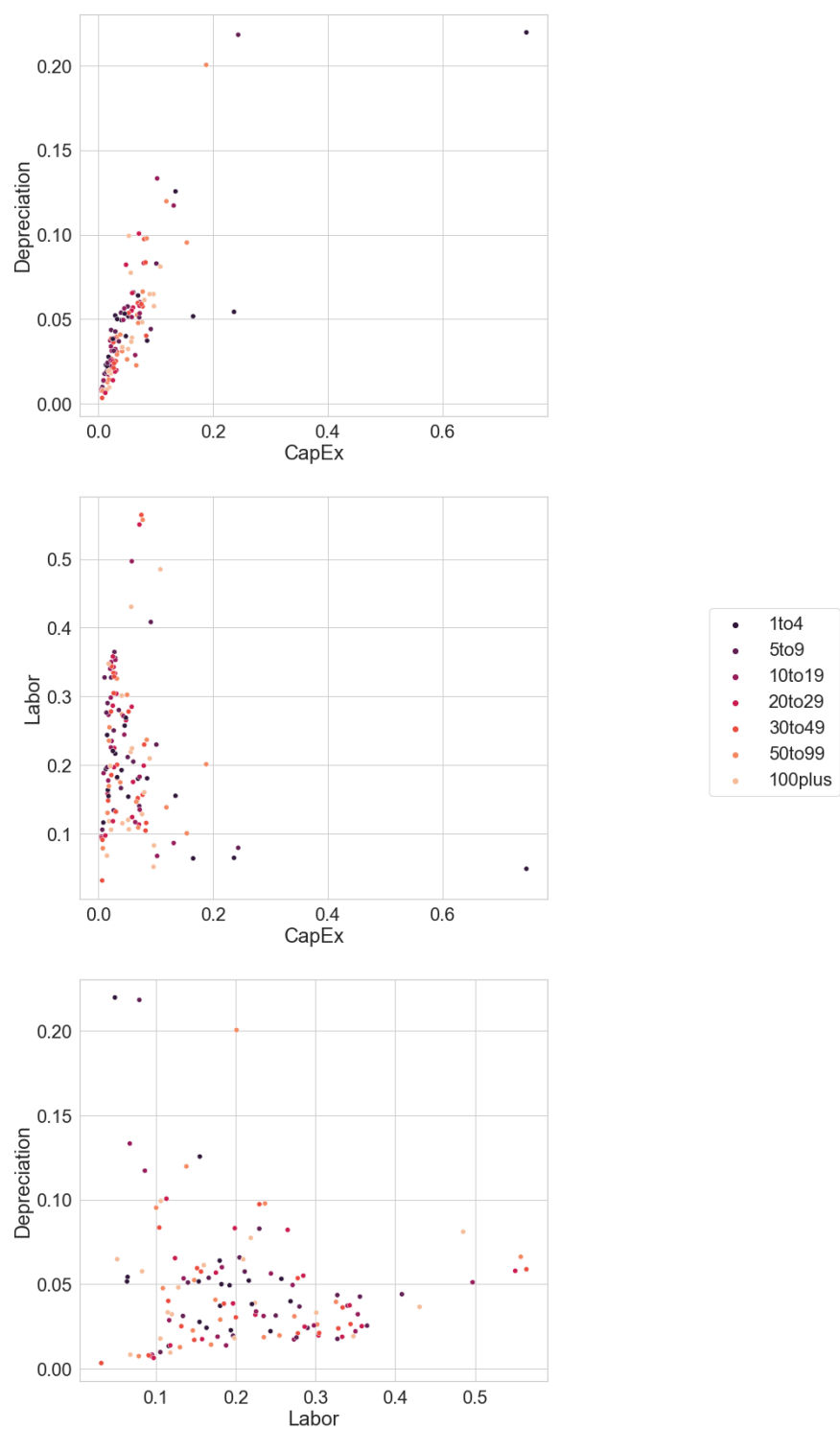


Figure 7: The joint distribution of factor inputs in Japan. Color denotes firm size. Source: author calculations, as of 29 April 2024.

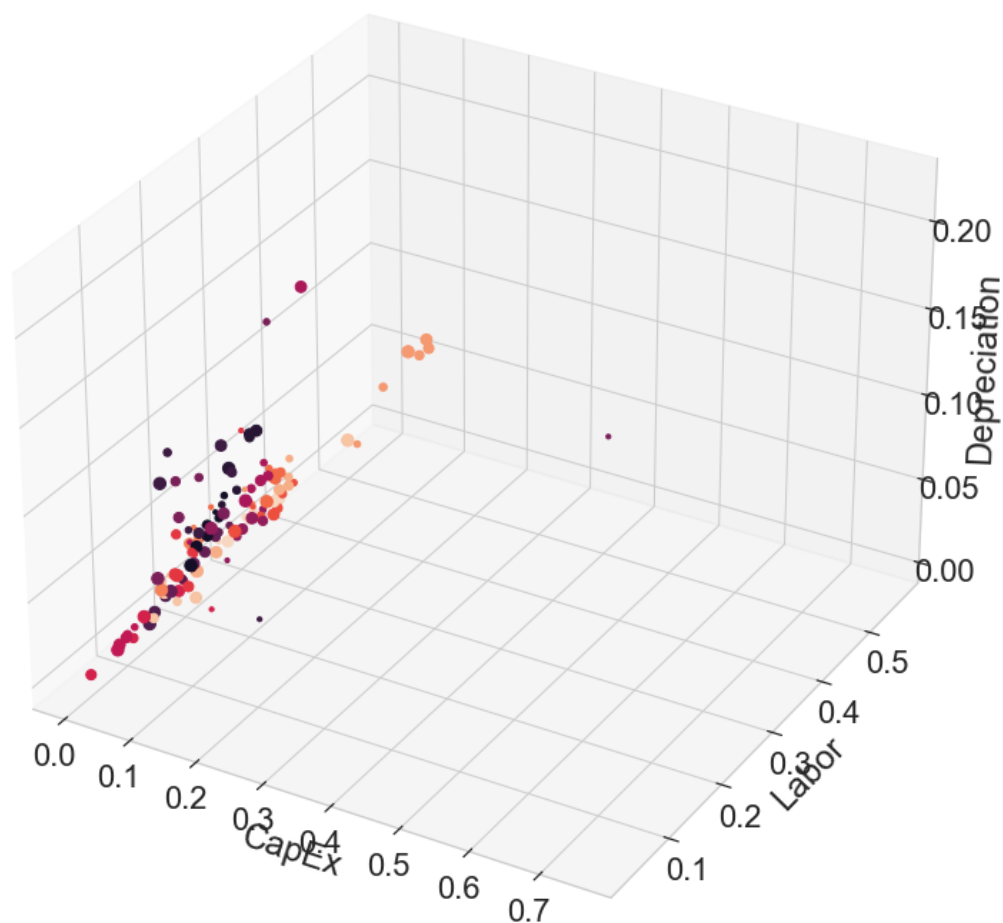


Figure 8: The joint distribution of factor inputs in Japan. Color denotes industry. Source: author calculations, as of 29 April 2024.

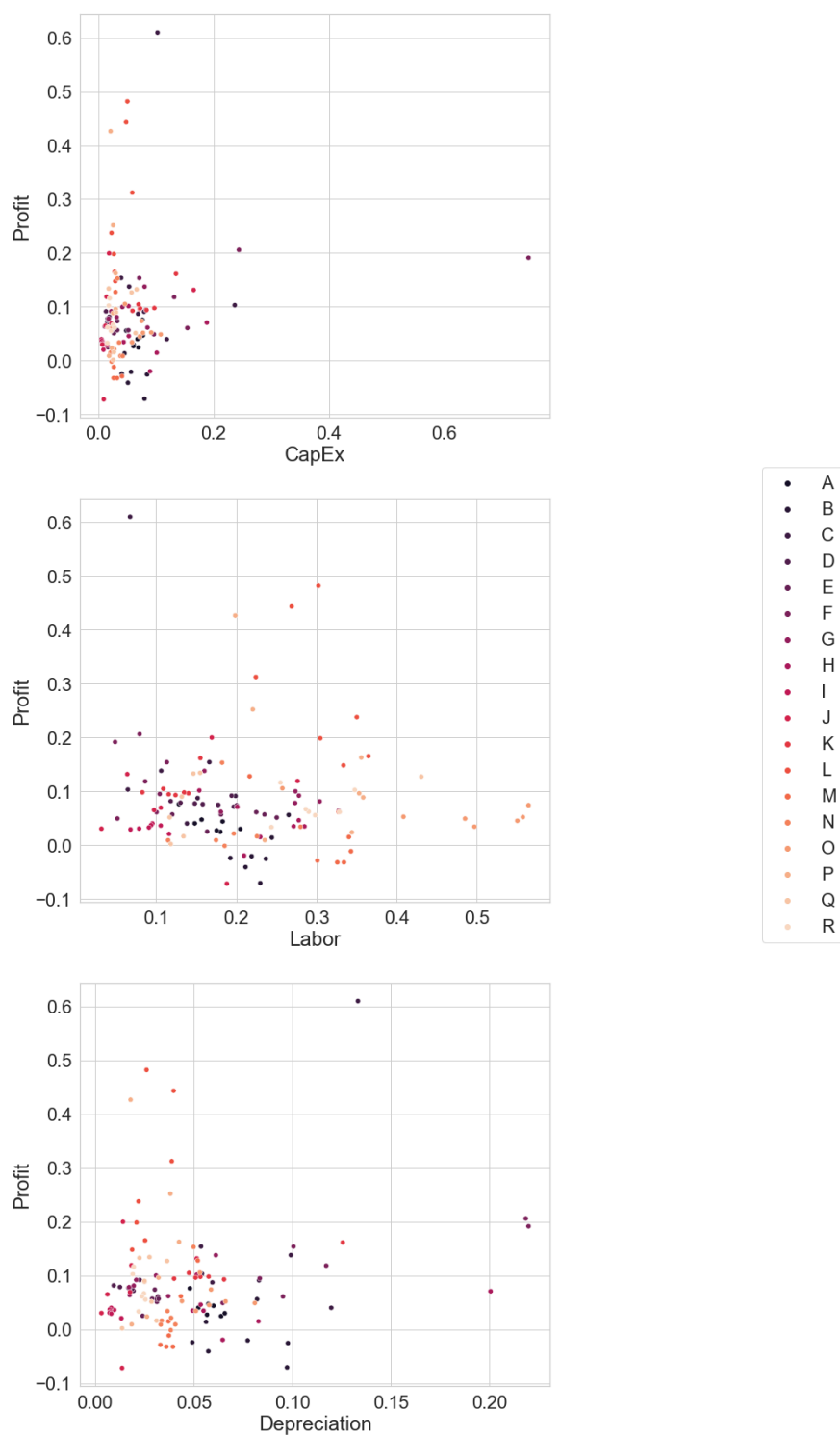


Figure 9: The distribution of factor inputs in Japan with profits. Color denotes industry. Source: author calculations, as of 29 April 2024.

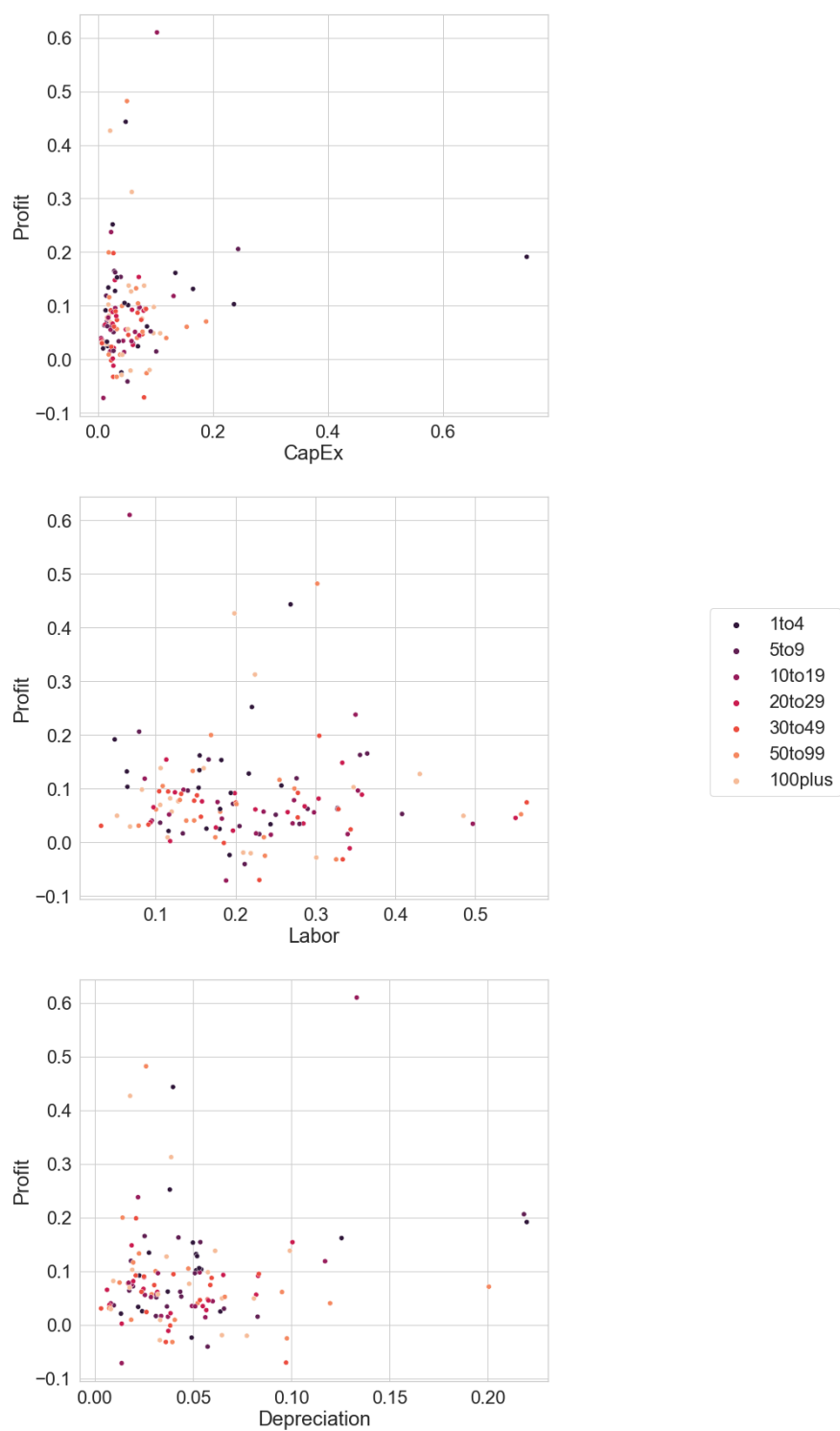


Figure 10: The distribution of factor inputs in Japan with profits. Color denotes firm size. Source: author calculations, as of 29 April 2024.

We analyze four specifications: the first includes no fixed effects; the second includes firm size fixed effects (ϕ_{fs}), the third includes industry fixed effects (ϕ_i), and the fourth includes fixed effects for both firm size and industry. Showing the results of all four specifications helps us identify where the variation in the system comes from. While all instructive, we suspect that the second specification will capture the most interesting variation because our above scatterplots show production patterns vary more by industry than by firm size. The fourth specification follows the below form:

$$\begin{aligned} \rho = & \alpha + \beta_0 \delta + \beta_1 \lambda + \beta_2 \kappa + \\ & \beta_3 \delta \lambda + \beta_4 \lambda \kappa + \beta_5 \delta \kappa + \\ & \beta_6 \delta \lambda \kappa + \vec{\beta}_7 \phi_{fs} + \vec{\beta}_8 \phi_i + \varepsilon. \end{aligned} \tag{8}$$

Our theoretical model predicts that results show the opposite sign from the capital regression coefficients: $\beta_0 > 0$ and $\beta_3 < 0$.

4.2.3 Micro Empirical Results

We conduct separate analyses for the three years the Economic Census of Business Activity was run: 2012, 2016, and 2021. Results are broadly consistent across all three years, and support our hypothesis.

Table 4 shows the results from running the same analysis in 2021. An interesting change took place in the five years since 2016: the pattern begins to take different forms depending on whether the specification uses firm size or industry fixed effects. Specification two uses firm size fixed effects to show how these relationships vary across industry. Industries with higher spending on Labor have a baseline higher Profit. But industries that must also spend more on a depreciating capital stock find this effect to be reversed, as indicated by a large negative interaction coefficient. If their existing capital base is small enough to not accrue significant depreciation spending, industries can claw back some of this profit by building new capital (CapEx). These relationships look different when varied across firm size (using industry fixed effects, as in specification three). The same baseline

relationship with Depreciation remains: firms with high spending on existing capital stocks tend to be more profitable, and even remain profitable while expanding their capital stock via CapEx. But once those firms start spending on Labor, they become strongly unprofitable.

Table 4: Production Factor Predictors of Profit: 2021

	<i>Dependent variable: Profit</i>			
	(1)	(2)	(3)	(4)
Intercept	-0.011 (0.013)	-0.013 (0.012)	0.019 (0.030)	0.016 (0.026)
Labor	0.556*** (0.139)	0.603*** (0.149)	-0.328 (0.239)	-0.259 (0.220)
Depreciation	3.354*** (1.266)	3.640*** (1.261)	2.989* (1.557)	2.875* (1.604)
CapEx	-0.760 (0.640)	-1.086 (0.681)	-0.695 (0.514)	-0.907* (0.507)
Labor:CapEx	8.887* (5.231)	9.883* (5.546)	6.863 (4.356)	7.193 (4.496)
Labor:Depreciation	-23.195*** (7.145)	-24.477*** (7.115)	-1.000 (7.007)	-1.313 (6.554)
Depreciation:CapEx	-0.597 (2.026)	0.305 (2.104)	4.349** (2.015)	5.057** (2.130)
Labor:Depreciation:CapEx	2.732 (28.912)	-0.893 (29.988)	-99.283*** (38.088)	-97.422** (40.807)
Firm Size FE	No	Yes	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	126	126	126	126
R^2	0.208	0.251	0.603	0.617
Adjusted R^2	0.161	0.164	0.509	0.496
Residual Std. Error	0.089	0.089	0.068	0.069
F Statistic	25.119***	19.461***	61.899***	34.872***

Note:

*p<0.1; **p<0.05; ***p<0.01

The full tables for the 2016 and 2012 analyses are listed in the appendix, but we will discuss them briefly here.

Table 7 shows the results of this analysis for the 2012 time cohort. The positive and statistically significant coefficient for CapEx suggests that firms that spend more on tangible and intangible capital make higher profits. Moreover, because the interaction term between CapEx and Labor is negative, firms that have high capital costs but still spend high fractions of their total expenses on labor tend to make lower profits. The positive effect from spending on CapEx is also drowned out by also spending

on Depreciation. These results suggest that industries with bigger capital endowments (i.e., the ones “spending” more on depreciation) have found themselves able to adapt to tight Japanese labor markets – if they have simultaneously been able to reduce their labor expenditure. These results persist across all four specifications, meaning that the same patterns persist across industries and firm sizes.

Table 8 shows a transition taking place in 2016. In a change from 2012, CapEx has a baseline positive correlation with Profit and Depreciation has a baseline negative effect. This negative effect of Depreciation on Profit can be reversed by spending either on Labor or CapEx. That is, firms do not profit from merely sitting on a depreciating capital stock – they must either do something to it (via Labor) or add to it via continued CapEx. If the firm chooses CapEx, it enjoys an overall positive effect on profit. If the firm chooses Labor, it also can see an overall positive effect on profit. But once the firm spends on both new capital (CapEx) and Labor, its profits decrease by an order of magnitude. These patterns persist across all four specifications, suggesting that the same relationships exist across industries and firm sizes.

It’s worth taking a moment to consider how these relationships evolved over the three sampling periods. The results suggest that there was a limit of the amount of capital firms could tolerate while still being profitable. In general, the existence or build-up of capital is positively associated with profit margins. Higher amounts of capital only become detrimental to profits when firms also spend on labor. This suggests that firms that could automate away labor found higher profits, but firms that increased their capital despite still needing labor found dropping profits. Perhaps more interestingly, the measure of capital that has a negative relationship with profit seems to be shifting over time from CapEx to Depreciation. This could signal that companies had consistently deployed capital inefficiently, and that the capital stock is reaching saturation. Regardless of the measure of capital, the persistence of the negative labor interaction effect shows that our hypothesized mechanism has found empirical support throughout the time period.

4.2.4 Industry Fixed Effect Analysis

Specifications three and four of these results include industry fixed effects. Because elasticity of substitution varies by industry, we expect that these coefficients, which capture the industry-specific

baseline effect on production, should also vary in the cross-section.

Figure 11 shows the magnitudes of industry fixed effect coefficients as predictors of Profit after allowing for variation on Labor, Depreciation, and CapEx expenditures (as a proportion of total expenditure) and their interactions. The different colors denote different years of data collection.

These coefficients capture residual variation accounted for by industry after regressing on labor and capital expenditures. These results suggest that although many industries have no statistically significant baseline relationship with profits, some do. Because our regressions have already accounted for expenditures on labor and capital stock and flows, the remaining variation describes is one of two things. The first possibility is that variation in profits comes from certain industries more productively putting together the same input factors (the Solow residual). The second possibility is that certain industries can more effectively substitute one factor for another (elasticity of substitution). Higher values for the fixed effect coefficients in research and human-capital sensitive industries suggests that it corresponds to lower elasticities of substitution; lower values for capital-intensive industries (ag, fish, mining, transport, etc.) suggests a correspondence with higher elasticity of substitution industries.

We next use the same variables to predict the labor share of income, with results shown in figure 12. These results show relationships that are similar in magnitude but reverse in sign. Skilled-labor-heavy industries (e.g., finance, education, real estate) have higher-valued fixed effect coefficients, and more capital-heavy industries (e.g., research/tech, healthcare) have higher-valued fixed effect coefficients. This suggests that industries less able to substitute capital for labor have a baseline higher labor share of income, all else equal.

Moreover, several industries exhibit an interesting shift in their coefficients over the nine year span for which data is available. Services, IC, Research, Construction, and Mining, for example, see their coefficients decreasing dramatically in successive waves of the survey. This pattern might suggest a change in production process over time, perhaps reflective of effectively deployed capital. But for other industries, the coefficients hover in the same area for all three data waves.

The general segmentation of industry fixed effect coefficients is similar but inverse among regressions

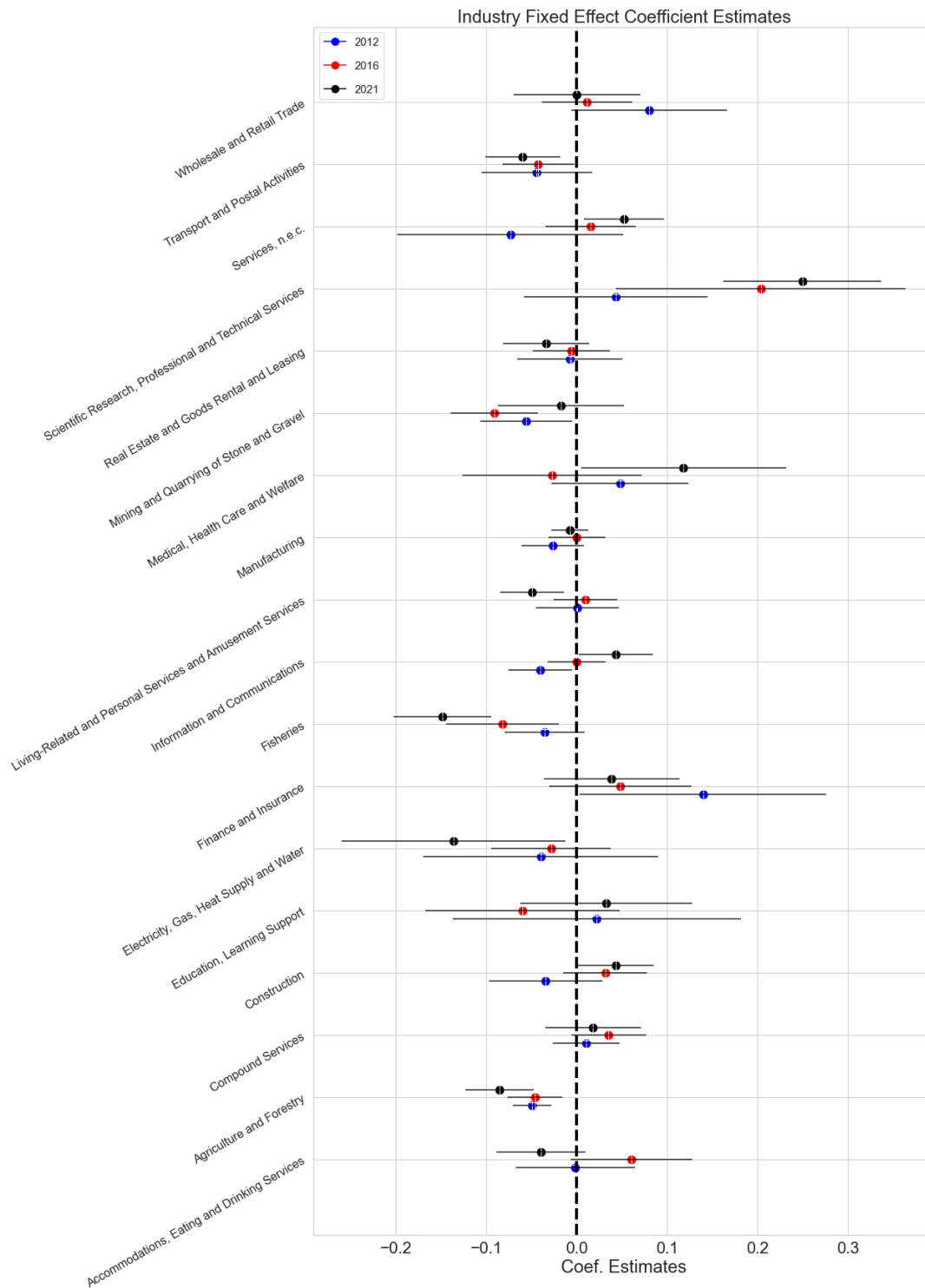


Figure 11: Fixed effect coefficients for Japanese industries predicting profit. Source: author calculations, as of 21 May 2024.

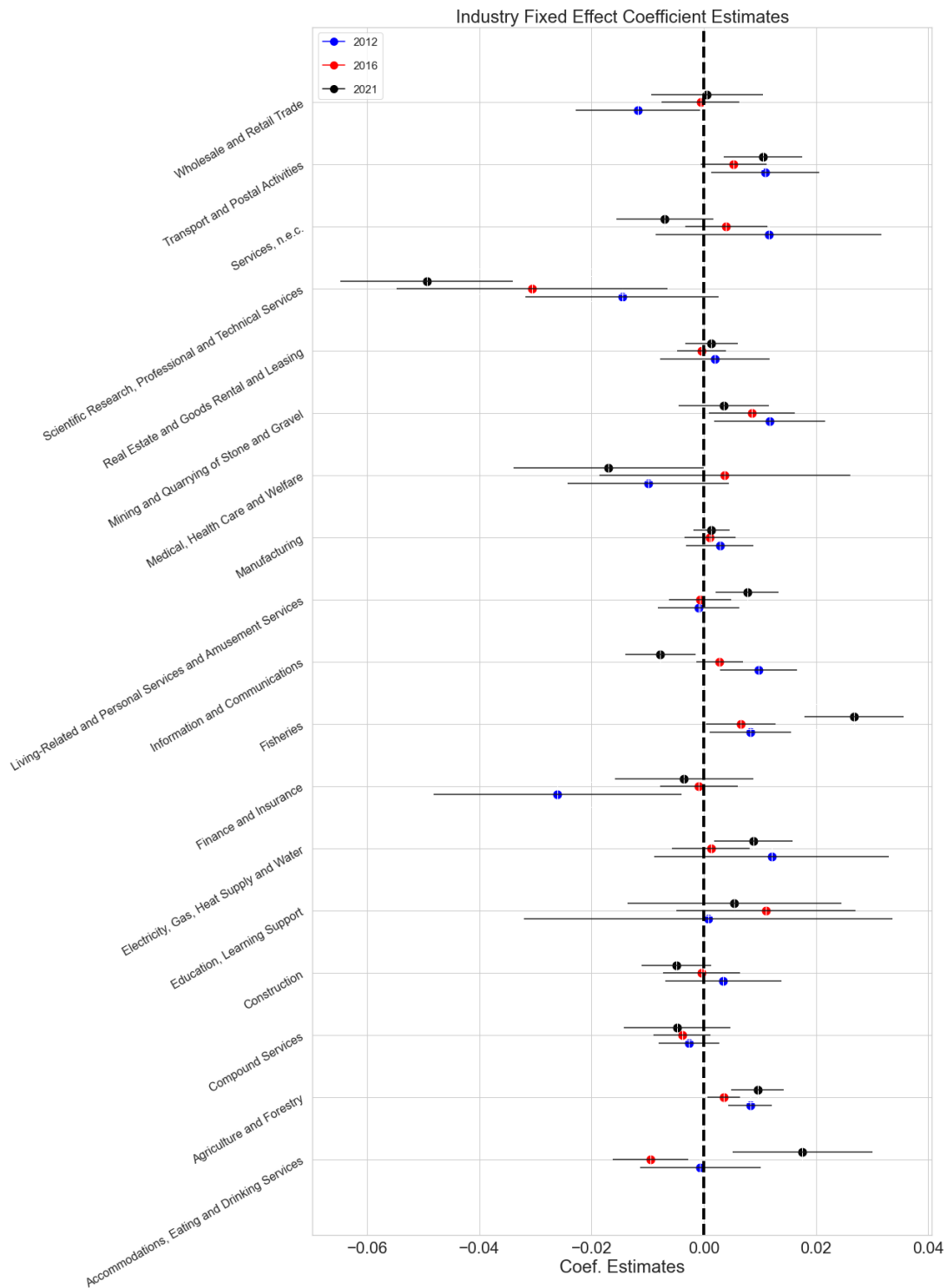


Figure 12: Fixed effect coefficients for Japanese industries predicting the labor share of income. Source: author calculations, as of 21 May 2024.

predicting profit and the labor share. These results suggest that industry production processes vary in the cross-section, and this variation affects profit and the labor share in opposing ways. These results correspond to the predictions of our theoretical framework, which hypothesized that those firms that can more effectively deploy capital will find twin effects: lower profits and lower capital share of income. ‘‘

5 Conclusion

This paper offers a simple two-factor model of a CES economy that shows formally that substituting capital for labor can sustain the rate of growth only where the marginal product of capital exceeds the marginal product of labor.¹² Moreover, our model shows that only when $MPK > MPL$ will the return on additional units of capital stay stable or increase.

Our theoretical framework is more an attribute of a neoclassical economic model than a micro-founded model of choices made by individual actors in an economy. Nonetheless, our findings are more salient now as countries with advanced economies find themselves with a relative surplus of capital over labor. Countries with advanced economies have aging and shrinking populations, so firms in those countries face constrained labor input. Countries with developing economies more commonly find themselves with more labor than they can productively employ, leading to concerns about institutional strength and capital availability. To maintain or increase economic production under these pressures, firms have three options: find more factor inputs, increase productivity, or export an abundant factor to somewhere the scarce factor is more abundant. This paper analyzes the second option.

Our paper cautions against the assumption that capital investment can naturally always substitute for scarce labor. It also implies that any productivity gains from factor substitution will not be evenly distributed because elasticity of substitution varies across industries, sectors, and firms. Noticing the relationship between MPK and MPL is vital to how additional factor inputs affect both productivity (and therefore output) and the labor-capital split (via diminishing marginal returns). More generally,

¹²It is worth noting that our model shows that the reverse is also true – substituting labor for capital is only productive when $MPL > MPK$.

our paper suggests that labor bargaining power – whose relationship with the labor-capital split of income is highly studied – is strongest when capital is not substitutable for labor.

Fruitful areas for future work include thoughtful analysis of the way firms facing scarce production factors regularly assess the relative productivity of capital and labor. Managers make decisions about where to allocate marginal resources – how do they choose between maximizing their use of existing factor supply, finding net new factor inputs, or substituting between factors?

A Appendix

A.1 Robustness: Lagged Capital Supply

Table 5: Demographic Predictors of Labor Share of Income

	<i>Dependent variable: Labor Share, $t+1$</i>			
	(1)	(2)	(3)	(4)
Intercept	0.099*** (0.020)	0.099*** (0.020)	0.101*** (0.020)	0.266*** (0.055)
Lag DV	0.826*** (0.034)	0.826*** (0.035)	0.825*** (0.035)	0.617*** (0.062)
CPI	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	-0.000** (0.000)
USD/LC				-0.034*** (0.008)
CBPR	0.020* (0.011)	0.020 (0.012)	0.019 (0.012)	-0.039 (0.057)
Skilled Labor				-0.033 (0.025)
Unions				0.019 (0.040)
Tariffs				-0.038 (0.063)
KSI		-0.000 (0.003)	-0.001 (0.003)	0.025*** (0.009)
Unemp.	-0.142*** (0.024)	-0.142*** (0.024)	-0.141*** (0.024)	-0.185*** (0.042)
HWI				0.009 (0.022)
σ	0.000 (0.000)	0.000 (0.000)	0.001* (0.001)	0.004*** (0.001)
KSI: σ			-0.001* (0.001)	-0.003*** (0.001)
Country FE	Yes	Yes	Yes	Yes
Lag DV	Yes	Yes	Yes	Yes
Observations	461	461	461	231
R^2	0.985	0.985	0.985	0.989
Adjusted R^2	0.984	0.984	0.984	0.988
Residual Std. Error	0.010	0.010	0.010	0.009
F Statistic	92816.433***	90985.226***	88003.540***	55254.194***

Note:

*p<0.1; **p<0.05; ***p<0.01

A.2 Robustness: Lagged Labor Supply

Table 6: Demographic Predictors of Labor Share of Income

	<i>Dependent variable: Labor Share, $t+1$</i>			
	(1)	(2)	(3)	(4)
Intercept	0.099*** (0.020)	0.122*** (0.026)	0.122*** (0.026)	0.272*** (0.058)
Lag DV	0.826*** (0.034)	0.811*** (0.036)	0.811*** (0.036)	0.650*** (0.062)
CPI	0.000 (0.000)	0.000* (0.000)	0.000* (0.000)	-0.000 (0.000)
USD/LC				-0.015*** (0.005)
CBPR	0.020* (0.011)	0.019* (0.011)	0.019* (0.011)	0.077 (0.068)
Skilled Labor				-0.061** (0.027)
Unions				-0.022 (0.042)
Tariffs				-0.028 (0.082)
KSI				0.000 (0.000)
Unemp.	-0.142*** (0.024)	-0.168*** (0.029)	-0.167*** (0.029)	-0.167*** (0.042)
HWI		-0.021* (0.012)	-0.021* (0.012)	-0.005 (0.029)
σ	0.000 (0.000)	0.000 (0.000)	0.001*** (0.000)	0.012 (0.011)
HWI: σ			-0.001*** (0.000)	-0.012 (0.011)
Country FE	Yes	Yes	Yes	Yes
Lag DV	Yes	Yes	Yes	Yes
Observations	461	461	461	211
R^2	0.985	0.985	0.985	0.974
Adjusted R^2	0.984	0.984	0.984	0.971
Residual Std. Error	0.010	0.010	0.010	0.009
F Statistic	92816.433***	88999.493***	89757.292***	57754.919***

Note:

*p<0.1; **p<0.05; ***p<0.01

B Appendix: Micro Results

Table 7: Production Factor Predictors of Profit: 2012

	<i>Dependent variable: Profit</i>			
	(1)	(2)	(3)	(4)
Intercept	-0.019 (0.034)	-0.020 (0.035)	-0.063 (0.055)	-0.066 (0.056)
Labor	0.827 (0.593)	0.860 (0.607)	1.104 (0.815)	1.183 (0.815)
Depreciation	-0.259 (1.323)	-0.463 (1.260)	1.689 (2.153)	0.932 (2.085)
CapEx	5.204*** (1.848)	5.308*** (1.961)	6.127** (2.949)	6.482** (2.923)
Labor:CapEx	-28.987** (12.248)	-28.893** (12.777)	-31.169* (16.416)	-32.018** (15.908)
Labor:Depreciation	-3.075 (12.283)	-3.017 (11.401)	-8.723 (15.844)	-6.720 (14.007)
Depreciation:CapEx	-42.254** (20.729)	-41.305** (20.820)	-54.287* (29.834)	-54.695** (27.855)
Labor:Depreciation:CapEx	273.449 (188.309)	271.641 (185.511)	301.173 (221.348)	306.960 (200.934)
Firm Size FE	No	Yes	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	160	160	160	160
R^2	0.140	0.216	0.358	0.437
Adjusted R^2	0.100	0.134	0.244	0.296
Residual Std. Error	0.094	0.092	0.086	0.083
F Statistic	37.870***	30.396***	30.283***	21.115***

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 8: Production Factor Predictors of Profit: 2016

	<i>Dependent variable: Profit</i>			
	(1)	(2)	(3)	(4)
Intercept	0.034** (0.015)	0.034*** (0.013)	0.034** (0.016)	0.038** (0.018)
Labor	-0.151 (0.254)	-0.137 (0.239)	-0.555** (0.231)	-0.576** (0.282)
Depreciation	-2.110*** (0.606)	-2.007*** (0.632)	-2.257*** (0.512)	-2.166*** (0.531)
CapEx	1.435*** (0.279)	1.190*** (0.266)	1.866*** (0.289)	1.529*** (0.335)
Labor:CapEx	0.881 (3.359)	4.790 (4.073)	-0.080 (3.070)	4.613 (4.796)
Labor:Depreciation	18.333*** (6.483)	14.606** (6.846)	29.199*** (5.790)	24.932*** (5.804)
Depreciation:CapEx	10.514*** (3.731)	10.747*** (3.997)	10.336*** (3.463)	10.761*** (3.796)
Labor:Depreciation:CapEx	-204.089*** (42.310)	-209.121*** (44.771)	-236.739*** (42.351)	-245.009*** (47.419)
Firm Size FE	No	Yes	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	162	162	162	162
R^2	0.405	0.449	0.549	0.580
Adjusted R^2	0.378	0.393	0.470	0.476
Residual Std. Error	0.108	0.106	0.099	0.099
F Statistic	82.582***	61.159***	45.136***	38.273***

Note:

*p<0.1; **p<0.05; ***p<0.01

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