Question 1.

$$\mathrm{AR}(3): \mathrm{Y}_t = 0.2 + 0.4\mathrm{Y}_{t-1} + 0.2\mathrm{Y}_{t-2} + \alpha_3 \mathrm{Y}_{t-3} + \varepsilon_t$$

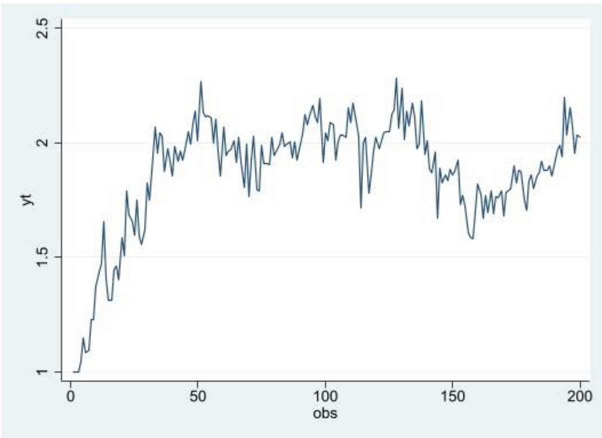
Stability condition:

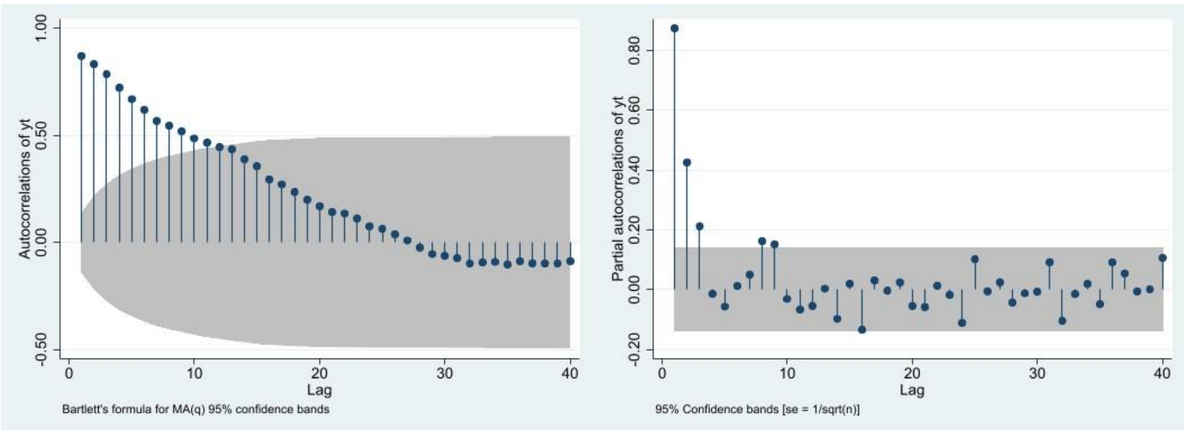
$$\mathrm{AR}(3): Y_t = 0.2 + 0.4Y_{t-1} + 0.2Y_{t-2} + \alpha_3 Y_{t-3} + \varepsilon_t$$

Stability condition:
$$\begin{cases} 1 - 0.4 - 0.2 - \alpha_3 > 0 \\ 1 + 0.4 - 0.2 + \alpha_3 > 0 \\ 1 - 0.4\alpha_3 + 0.2 - \alpha_3^2 > 0 \\ \left[\begin{array}{l} 3 + 0.4 + 0.2 - 3\alpha_3 > 0 \\ 3 - 0.4 + 0.2 + 3\alpha_3 > 0 \end{array}\right. \end{cases}$$

$$\Leftrightarrow -1.2 < \alpha_3 < 0.4$$

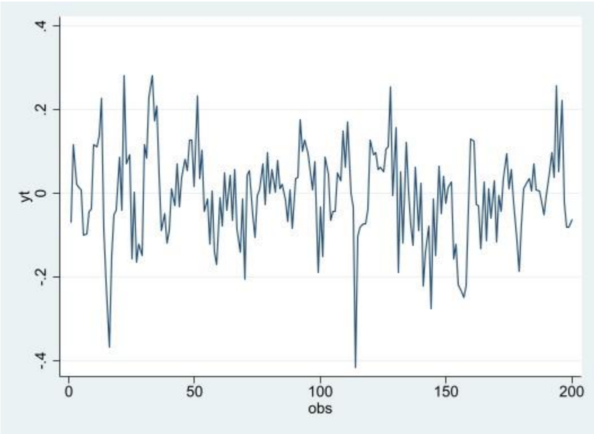
Choose $\alpha_3 = 0.3$

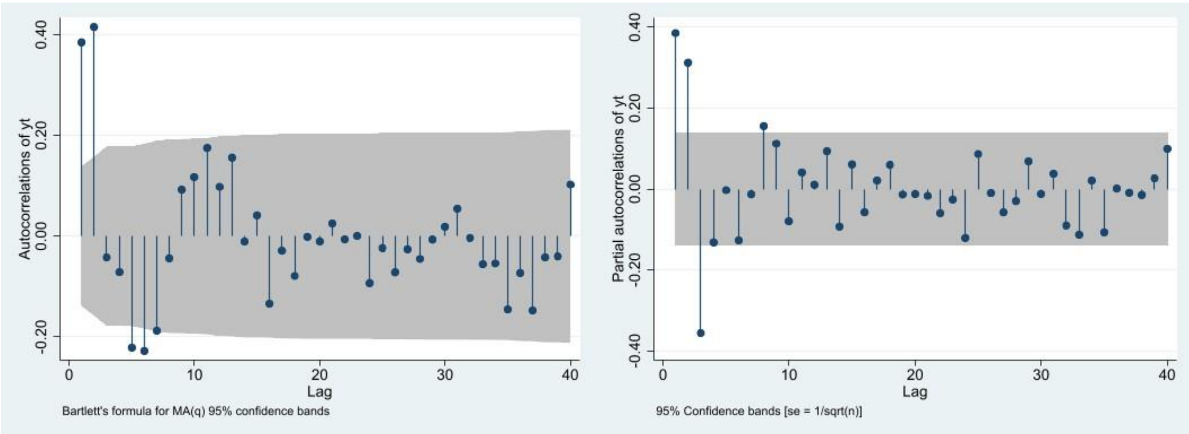




The time plot shows a stationary series with moderate persistence. The ACF gradually decays toward 0, and the PACF cuts off after lag 3 — both patterns are consistent with the theoretical expectations for an AR(3) process.

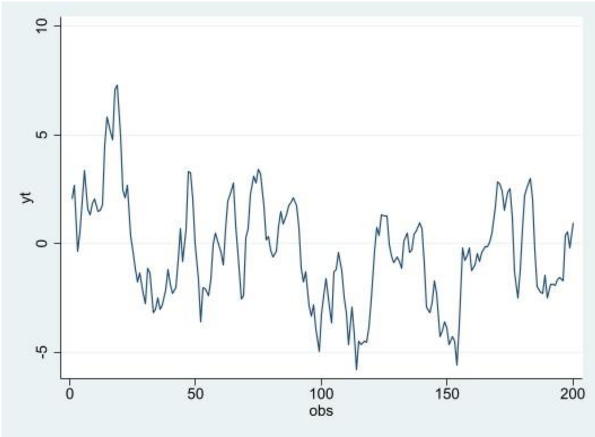
$$\mathrm{MA(2)}: \mathrm{Y}_t \;=\; 0.4\varepsilon_{t-1} + 0.5\varepsilon_{t-2} + \;\varepsilon_t$$

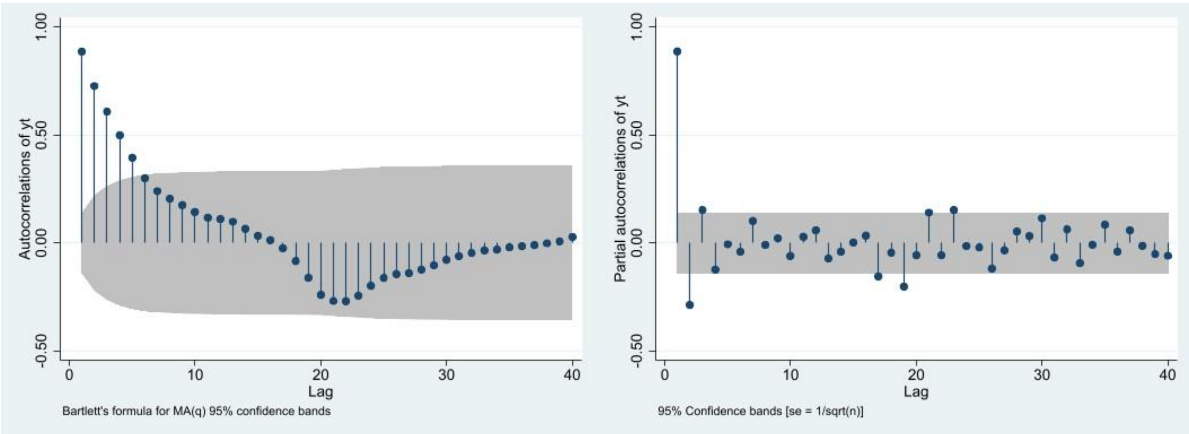




The time plot shows a stationary series with moderate persistence. The PACF oscillating decays toward 0, and the ACF cuts off after lag 2 — both patterns are consistent with the theoretical expectations for an MA(2) process.

$$\text{ARMA(2,1)}: Y_t = \alpha_0 + 0.5Y_{t-1} + 0.3Y_{t-2} + 0.8\varepsilon_{t-1} + \varepsilon_t$$





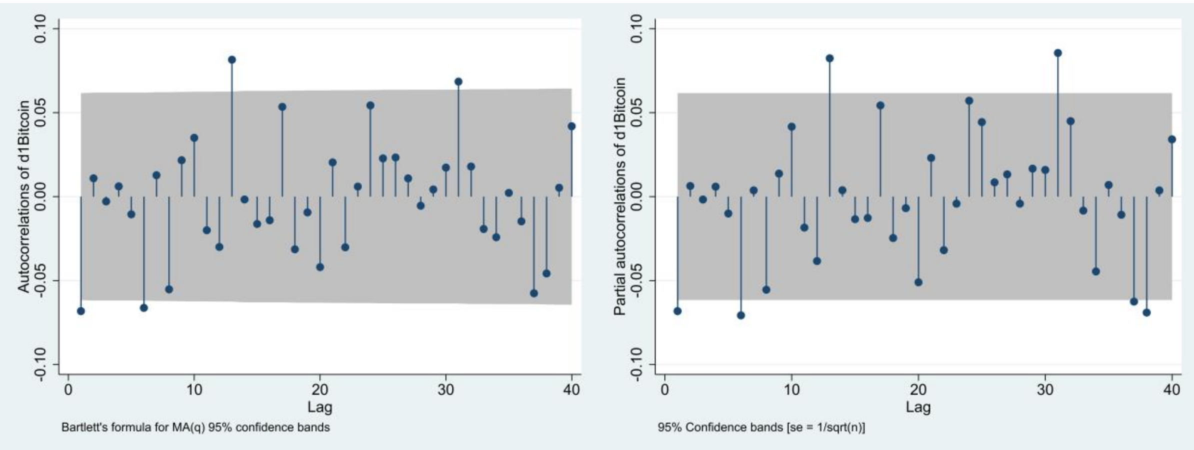
The time plot shows a stationary series with moderate persistence. The ACF decays after lag 1, while the PACF cuts off after about lag 2, aligning with the theoretical behavior of an ARMA(2,1) model. Overall, the plots match the expected dynamics for this process type.

Question 2.

```
. dfuller Bitcoin, lags(7)

Augmented Dickey-Fuller test for unit root

Variable: Bitcoin                       Number of obs  = 1,006
                                        Number of lags =     7

H0: Random walk without drift, d = 0

                                       Dickey-Fuller
                   Test      ────────── critical value ──────────
              statistic           1%           5%            10%
─────────────────────────────────────────────────────────────────
 Z(t)            -0.598       -3.430       -2.860         -2.570
─────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.8714.
```

```
. dfuller d1Bitcoin, lags(7)

Augmented Dickey-Fuller test for unit root

Variable: d1Bitcoin                        Number of obs  = 1,005
                                           Number of lags =     7

H0: Random walk without drift, d = 0

                                       Dickey-Fuller
                   Test          -------- critical value ---------
              statistic           1%           5%          10%
------------------------------------------------------------------
 Z(t)           -12.255       -3.430       -2.860       -2.570
------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000.
```

The Bitcoin series is non-stationary at level I(0) since the ADF test fails to reject the null hypothesis (p = 0.8371). However, after first differencing, the series becomes stationary at I(1), meaning it is integrated of order one.



The series is non-stationary and became stationary after first differencing, so we set d=1. The ACF and PACF of the first-differenced series contain only a few isolated significant lags and otherwise lie within the 95% confidence bands, showing no clear cutoff in either plot. This suggests no strong AR or MA structure; therefore an ARIMA(0,1,0) is a reasonable baseline. To confirm, we compared ARIMA(0,1,0) to ARIMA(1,1,0), and ARIMA(0,1,1) using AIC/BIC and white noise check for the disturbance. Finally, we adopt the most parsimonious model that yields white-noise residuals and competitive information criteria.

Model	N	ll(null)	ll(model)	df	AIC	BIC
arima_010	**1,013**	**.**	**-8938.833**	**2**	**17881.67**	**17891.51**
arima_110	**1,013**	**.**	**-8936.475**	**3**	**17878.95**	**17893.71**
arima_011	**1,013**	**.**	**-8936.513**	**3**	**17879.03**	**17893.79**

ARIMA(1,1,0) is the best specification due to the lowest AIC, while ARIMA(0,1,0) is the best specification due to the lowest BIC.

Model **arima_010**

ARIMA regression

Sample: **1/2/2023** thru **10/10/2025**

Number of obs = **1013**
Wald chi2(.) = **.**
Prob > chi2 = **.**

Log likelihood = **-8938.833**

D.Bitcoin	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Bitcoin						
_cons	**94.65177**	**51.76127**	**1.83**	**0.067**	**-6.798456**	**196.102**
/sigma	**1644.482**	**20.59111**	**79.86**	**0.000**	**1604.125**	**1684.84**

In the ARIMA(0,1,0) model, the estimated constant (drift term) is 94.65 with a p-value of 0.067. Because this p-value is greater than the 5% significance level, the constant (drift) is statistically insignificant.

Model **arima_110**

ARIMA regression

Sample: **1/2/2023** thru **10/10/2025**

Number of obs = **1013**
Wald chi2(**1**) = **9.45**
Prob > chi2 = **0.0021**

Log likelihood = **-8936.475**

D.Bitcoin	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Bitcoin						
_cons	**94.71455**	**48.4629**	**1.95**	**0.051**	**-.2709818**	**189.7001**
ARMA						
ar						
L1.	**-.0681239**	**.0221637**	**-3.07**	**0.002**	**-.1115639**	**-.0246839**
/sigma	**1640.674**	**20.52975**	**79.92**	**0.000**	**1600.437**	**1680.912**

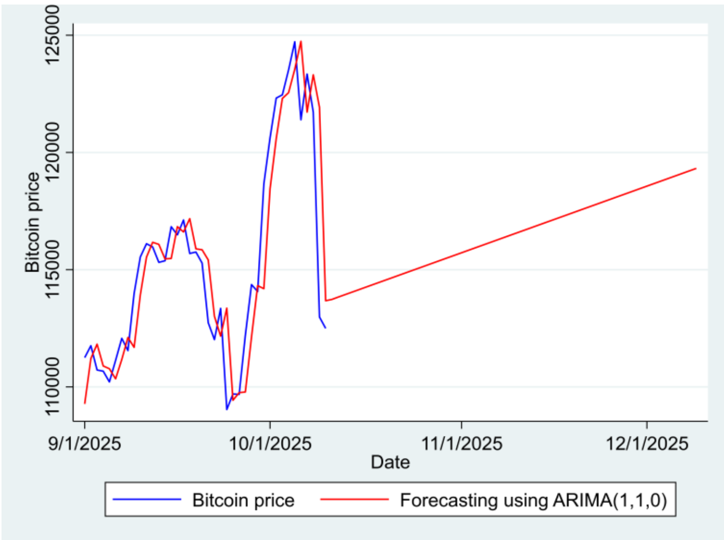
The ARIMA(1,1,0) model shows that the estimated constant is statistically insignificant with p-value of 0.051. However, the estimated Bitcoin lag 1 is statistically significant with p-value of 0.002.

```
. wntestq resid_arima_010

Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    49.1785
 Prob > chi2(40)           =     0.1515
```

```
. wntestq resid_arima_110

Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    45.8510
 Prob > chi2(40)           =     0.2424
```

The white noise test also shows that disturbance term of both ARIMA(0,1,0) and ARIMA(1,1,0) follow the white noise process.

Conclusion: ARIMA(1,1,0) is the best specification with the lowest AIC, statistically significant coefficients ($p < 0.05$), and residuals is a white noise.



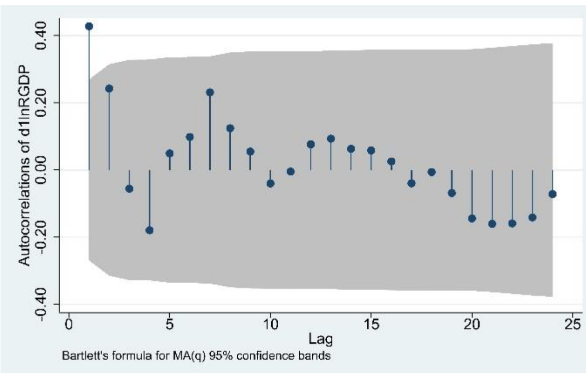
The point forecast from the ARIMA(1,1,0) model indicates that Bitcoin prices are expected to rise steadily over the next 60 days, reaching around $120,000 by the end of the forecast period. However, the forecast follows a linear trend and fails to capture the high volatility observed in actual Bitcoin prices.

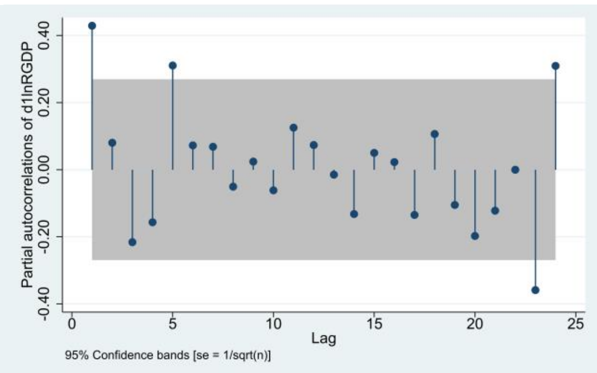
Question 3.

Dickey-Fuller Test Summary

Variable	Test Stat.	1%	5%	10%	p-value
RGDP	12.719	-3.577	-2.928	-2.599	1.0000
d1RGDP	-1.786	-3.577	-2.928	-2.599	0.3875
lnRGDP	1.894	-3.577	-2.928	-2.599	0.9985
d1lnRGDP	-4.495	-3.577	-2.928	-2.599	0.0002

The Dickey–Fuller test results show that both RGDP and lnRGDP are non-stationary in levels, as their p-values are close to 1. After first differencing, the log-transformed series (d1lnRGDP) becomes stationary with a p-value of 0.0002. Therefore, RGDP achieves stationarity at the first difference of its logarithmic form, indicating it is integrated of order one, I(1).





The ACF plot shows a sharp decline after lag 1, suggesting an MA(1) component, while the PACF shows a similar pattern with a significant spike at lag 1, indicating an AR(1) process. Since the Dickey–Fuller test confirms that RGDP becomes stationary at the first difference of its logarithmic form (I=1), this implies an integrated order of one. Thus, the suitable ARIMA candidates are ARIMA(1,1,0), ARIMA(0,1,1), and ARIMA(1,1,1), with the final choice determined by the lowest AIC/BIC values for best model fit.

Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
arima_110	53	.	114.7194	3	-223.4388	-217.528
arima_011	53	.	113.247	3	-220.4939	-214.5831
arima_111	53	.	114.8025	4	-221.605	-213.7239

ARIMA(1,1,0) is the best specification due to the lowest AIC and BIC, we now then check the residual of this model.

Portmanteau test for white noise

Portmanteau (Q) statistic	=	14.0810
Prob > chi2(24)	=	0.9448

The residual ACF and PACF show that there is no autocorrelation. In addition, Portmanteau test for white noise fails to reject the null hypothesis (white noise process)

=> Residual follow a white noise process

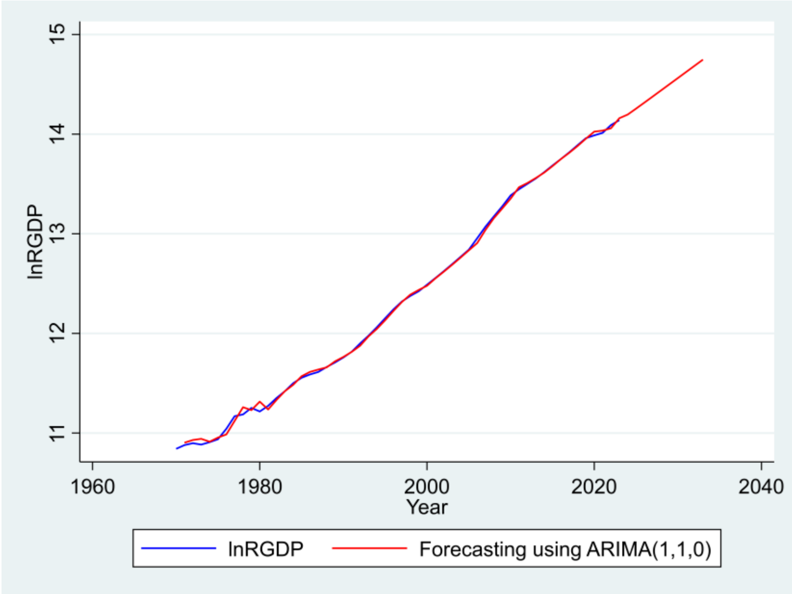
ARIMA regression

Sample: 1971 thru 2023	Number of obs	=	53
	Wald chi2(1)	=	17.78
Log likelihood = 114.7194	Prob > chi2	=	0.0000

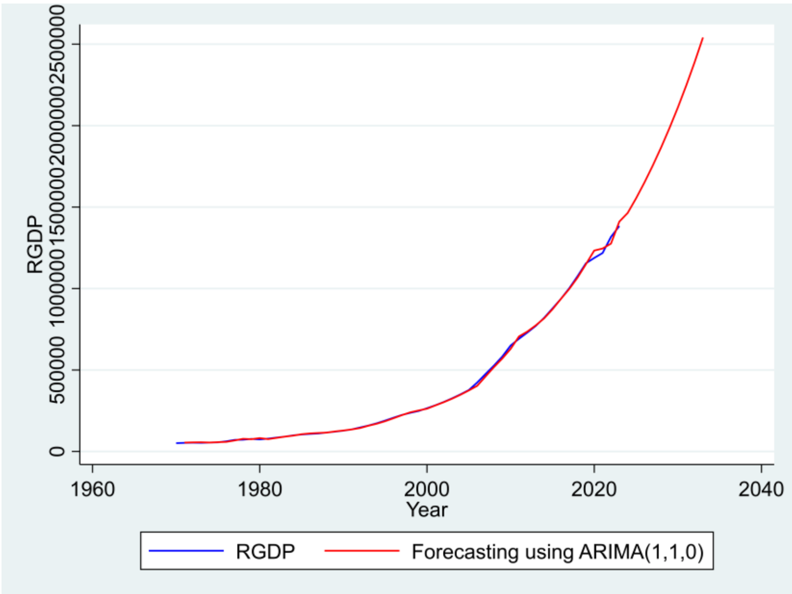
D.lnRGDP	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
lnRGDP						
_cons	.0617105	.0074289	8.31	0.000	.0471502	.0762708
ARMA						
ar L1.	.4265448	.101151	4.22	0.000	.2282925	.6247971
/sigma	.0277222	.0021246	13.05	0.000	.0235581	.0318863

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

Conclusion: ARIMA(1,1,0) is the best specification with the lowest AIC and BIC, statistically significant coefficients (p = 0.000), and residuals is a white noise.

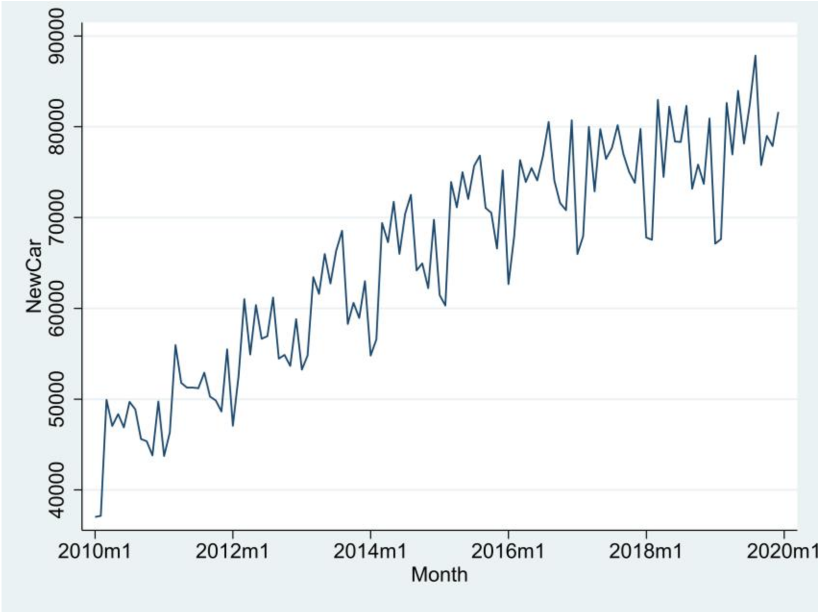


Since the ARIMA model is estimated using the log-transformed RGDP, the fitted values lnRGDP_hat are in logarithmic form. To obtain the predicted real GDP (RGDP_hat) in its original scale, we simply exponentiate the predicted log values: RGDP_hat = exp(lnRGDP_hat)



The forecast from the ARIMA(1,1,0) model using the logarithm of real GDP follows the historical upward trend, implying a geometric (exponential) growth pattern. This occurs because the model assumes that past growth rates persist into the future, resulting in continuously rising forecasts over time.

Question 4:



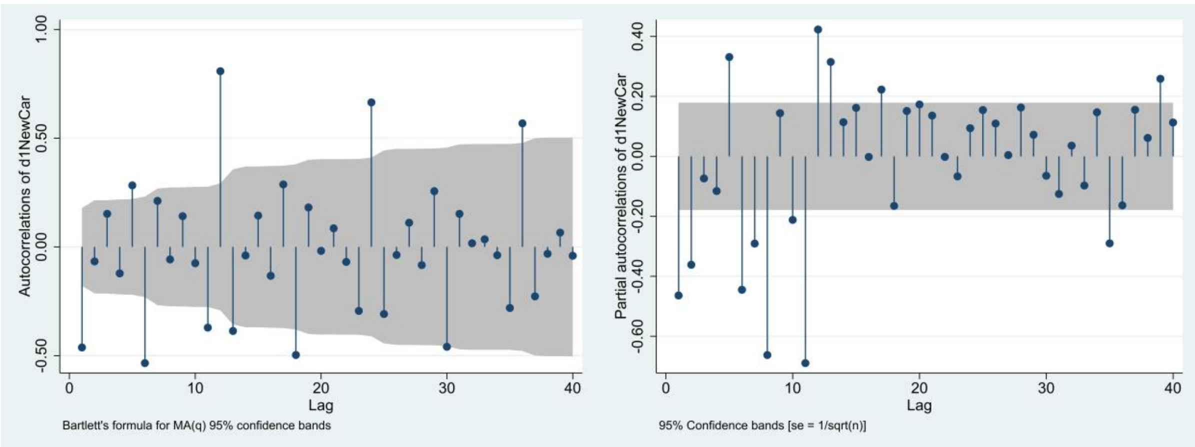
The time series plot of NewCar from 2010 to 2020 shows a clear upward trend and regular seasonal fluctuations. This indicates that the data is non-stationary due to both trend and seasonality. Therefore, to achieve stationarity, we take first order differences to remove the trend, and seasonal differences (with a seasonal period of 12 months) to eliminate the seasonal pattern.

Dickey-Fuller Test Summary

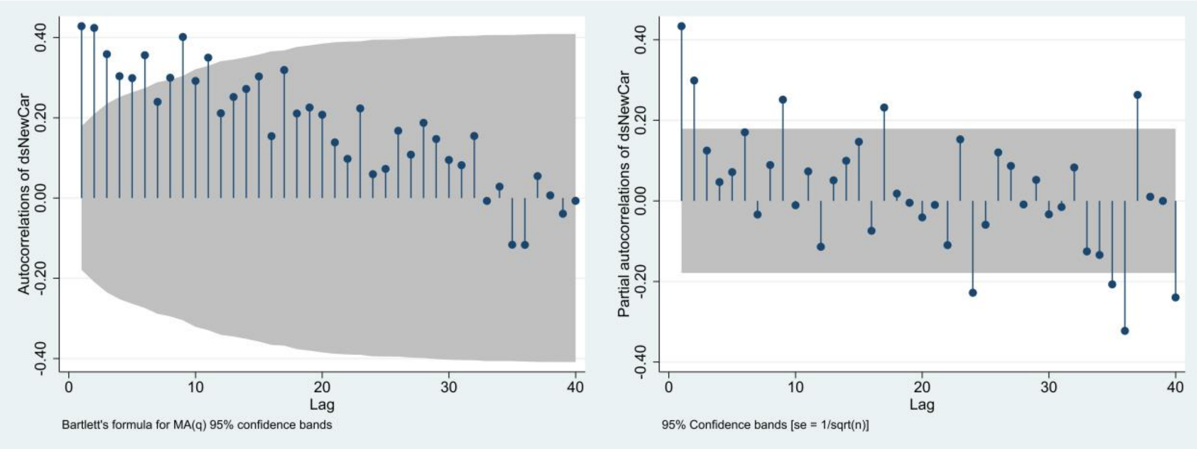
Variable	Test Stat.	1%	5%	10%	p-value
NewCar	-3.003	-3.504	-2.889	-2.579	0.0346
d1NewCar	-18.071	-3.504	-2.889	-2.579	0.0000
dsNewCar	-6.768	-3.504	-2.889	-2.579	0.0000

Do Dicky – Fuller Test => NewCar stationary at the first order differences I(1) and seasonal differences.

SARIMA(p,d,q)(P,D,Q)m



- ACF and PACF seems like there is no stationary for first order differences I(1) => p = d = q = 0

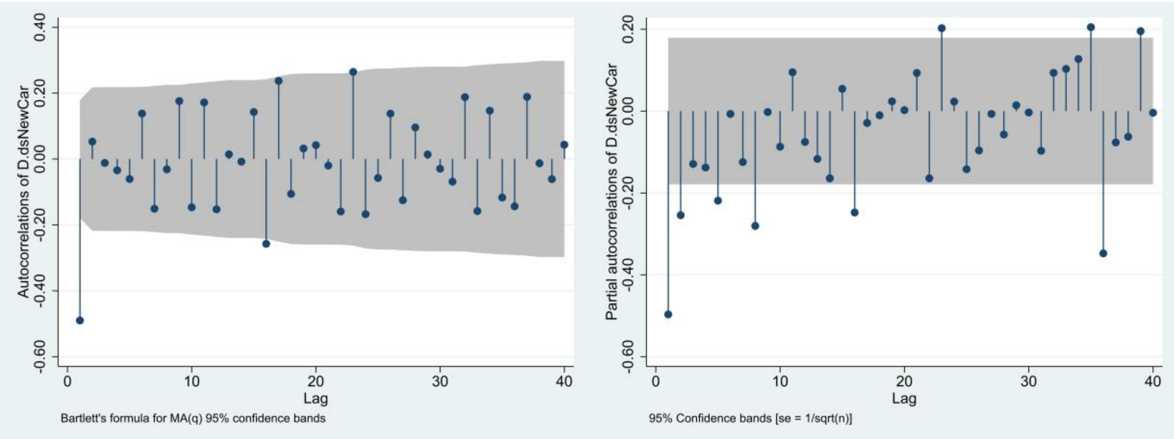


- ACF and PACF of the seasonal difference shows that the data is now stationary for the seasonal differences => choose D = 1

PACF of dsNewCar spikes at 1 => choose P = 1

ACF of dsNewCar spikes at 1, and 2 => choose Q = 1,2

Choose: SARIMA(0,0,0)(1,1,1)12, and SARIMA(0,0,0)(1,1,2)12



- ACF and PACF of the first order difference of seasonal difference shows that the data is now stationary for the seasonal differences => choose d = 1

PACF of d1dsNewCar spikes at 1 => choose p = 1

ACF of d1dsNewCar spikes at 1 => choose q = 1

Choose: SARIMA(1,1,1)(1,1,1)12, and SARIMA(1,1,1)(1,1,2)12

Thus, the suitable ARIMA candidates are SARIMA(0,0,0)(1,1,1)12, SARIMA(0,0,0)(1,1,2)12, and SARIMA(0,1,0)(1,0,0)12, with the final choice determined by the lowest AIC/BIC values for best model fit.

Model	N	ll(null)	ll(model)	df	AIC	BIC
s000_111_12	120	.	-1107.77	4	2223.54	2234.69
s000_112_12	120	.	-1106.229	5	2222.459	2236.396
s111_111_12	120	.	-1083.793	6	2179.586	2196.311
s111_112_12	120	.	-1083.138	7	2180.276	2199.789
s000_011_12	120	.	-1108.292	3	2222.583	2230.946
s000_012_12	120	.	-1106.892	4	2221.785	2232.935
s000_110_12	120	.	-1107.785	3	2221.569	2229.932
s111_011_12	120	.	-1086.603	5	2183.206	2197.143
s111_012_12	120	.	-1083.274	6	2178.548	2195.273
s111_110_12	120	.	-1088.498	5	2186.997	2200.934

SARIMA(1,1,1)(0,1,2)12 is the best specification due to the lowest AIC, we now then check the residual of this model.

```
Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    43.3600
 Prob > chi2(40)           =     0.3301
```

The white noise test for residual implies that the disturbance follows white noise process

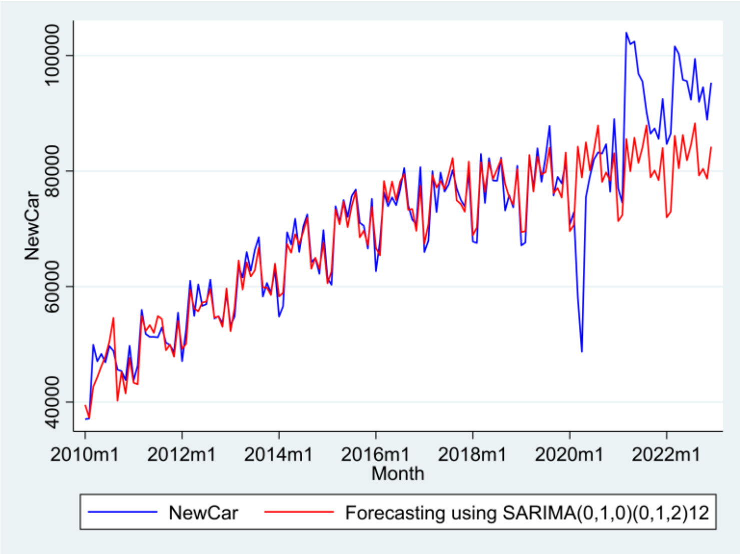
SARIMA(1,1,1)(0,1,2)12

```
ARIMA regression

Sample: 2010m1 thru 2019m12                     Number of obs     =        120
                                                Wald chi2(4)      =     272.16
Log likelihood = -1083.274                      Prob > chi2       =     0.0000
```

DS12.NewCar	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
NewCar						
_cons	-35.88054	12.98636	-2.76	0.006	-61.33333	-10.42775
ARMA						
ar L1.	.1422145	.0980532	1.45	0.147	-.0499662	.3343951
ma L1.	-.9064999	.0581881	-15.58	0.000	-1.020546	-.7924534
ARMA12						
ma L1.	-.3460355	.1248887	-2.77	0.006	-.5908127	-.1012582
L2.	-.2325926	.1072559	-2.17	0.030	-.4428104	-.0223748
/sigma	1966.737	135.5239	14.51	0.000	1701.115	2232.359

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

The SARIMA(1,1,1)(0,1,2)12 model fits the data well, capturing both short-term and seasonal effects. The MA(1) and seasonal MA terms are statistically significant, indicating strong short-term and yearly seasonal adjustments. The AR(1) term is insignificant, suggesting limited persistence after differencing. Overall, the model effectively accounts for seasonality and short-term fluctuations in new car sales.



The SARIMA(0,1,0)(0,1,2)12 model captures the overall upward trend and seasonal pattern of the NewCar series quite well. However, its forecasts for 2020–2022 deviate from the actual data, especially during periods of sudden fluctuation. Overall, the model provides a reasonable long-term trend forecast but lacks accuracy in capturing short-term shocks.