Introductory Seminar Econometrics

Single-depot vehicle scheduling problem with electric buses

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Abstract

The single-depot vehicle scheduling problem (SDVSP) is the problem of assigning vehicles to a set of predetermined trips with fixed starting and ending times and a single depot, while minimizing capital and operating costs. This paper aims to provide guidance on executing a transition from regular (fossil fuel) to electric buses by looking into the scheduling problem with regular and electric buses, with multiple vehicle types, and with an additional charging station. The SDVSP is formulated using the single-commodity flow and set covering formulations. We conclude that more buses and capital are needed when regular buses are replaced by electric buses, and that it is not optimal to operate a bus fleet of a single type. Besides that, a shorter battery life can render the SDVSP infeasible, and an increased battery life can reduce costs significantly. Finally, the single-commodity flow formulation outperforms the set covering formulation for the SDVSP.

1 Introduction

In this paper we examine the single-depot vehicle scheduling problem applied to (electric) buses, which is a specific case of the vehicle scheduling problem that involves one depot at which all buses have to depart and end at. More specifically, given a set of trips with fixed starting and ending times, we assign vehicles to the scheduled trips with the objective of minimizing capital and operating costs. Additionally, we analyze the scenario where on four bus lines regular buses are replaced by electric buses. Lastly, we extend the single-depot vehicle scheduling problem by considering multiple vehicle types and by finding the optimal location for an additional charging station.

The main question that we want to answer is whether public transport companies should invest in electric buses from an economic perspective. It seems that public transport companies are dedicated to invest in an electric bus fleet in the future (Nix, 2022). This will reduce greenhouse gases, from which society benefits as a whole. However, the question remains whether the public transport companies themselves profit from the transition of regular to electric buses. Therefore, we will study the SDVSP problem in this paper for electric and regular buses, using a single-commodity flow and set covering formulation.

The SDVSP is already extensively studied in academic literature. In this paper, we build mainly upon two research papers. The first paper is Freling et al. (2001), in which an auction algorithm for the quasi-assignment formulation of the SDVSP is proposed. We use Freling et al. (2001) mainly to define the SDVSP and refer to it when defining our single-commodity flow formulation. The second paper we reviewed is Ferland and Michelon (1988), which extends the vehicle scheduling problem by considering multiple vehicle types. We add value to this literature by applying the problem of Ferland and Michelon (1988) to buses and a practical case. Overall, we have seen that there is a lack of literature on the problem for electric buses, and specifically on the optimal transition from regular to electric buses. Therefore, since we do study these topics, we add significant value to existing literature.

The paper is organized as follows. In section 2 we give a comprehensive problem description and discuss some of the basic assumptions we make. In section 3 we provide mathematical models for the single-commodity flow formulation, the set covering formulation, and the extensions of multiple vehicle types and an additional charging station. Finally, we discuss the numerical results in section 4 and conclude this paper in section 5.

2 Problem description

In this section we describe the *single-depot vehicle scheduling problem* (SDVSP) applied to regular and electric buses. More specifically, we consider the situation with four bus lines of which the ending and starting location are indicated by A, B, C, and D. Furthermore, there are three different timetables with trips. Namely, we have a schedule for the weekdays, Saturday, and Sunday. The line between B and D only operates on the weekdays, and on Sunday there are higher frequencies on the lines A-B and B-C. First, we provide a problem description of the SDVSP for regular buses, and extent it to electric buses. After that, we will describe the SDVSP with multiple vehicle types and an additional charging station.

2.1 Single-depot vehicle scheduling problem

First, we consider the general SDVSP. We adhere to problem description as discussed in Freling et al. (2001). Therefore, we define the problem as follows: Given a set of trips, with fixed start and end times and a single depot, and given fixed travelling times between all pairs of locations, the problem is about finding a feasible minimum-cost schedule such that (1) each trip is performed by exactly one vehicle, and (2) the sequence of trips assigned to a vehicle is feasible. We define a *duty* as a sequence of trips that starts with a departure from the depot and ends with a trip back to the depot. In this way, a schedule for a vehicle consists of a sequence of feasible duties.

We assume that there are two *types* of vehicles: regular (fossil fuel) buses and electric buses. In both instances, the costs in the objective function consists of a fixed component for every bus that is used and a variable component corresponding to idle and/or travel time. More specifically, there are costs per kilometer driven and wage costs per hour. Part of this idle and travel time could consist of *deadhead trips*, which are movements of the bus between two locations without transporting passengers. The objective for the SDVSP is to minimize the total costs of buses and bus driver. Nevertheless, the specific set of assumptions that we consider for the SDVSP depends on the vehicle type that we use. Therefore, we will now consider the problem for regular and electric buses, separately.

2.1.1 Regular buses

When considering regular buses, we solve the SDVSP under the following assumptions:

1. We will assume that all buses start and end at the same depot. So after all trips

have been performed, all buses should be parked at the depot. Besides that, the number of vehicles at and the capacity of the depot are unlimited.

2. A bus and bus driver always stay together. So, once a regular bus is parked at the depot, it cannot be used by a new bus driver.

3. There are no restricting labour rules, which means that there are no constraints on the maximum number of working hours, or required breaks, for the drivers.

4. A bus can operate on multiple bus lines during the day.

5. There can be at most 50 minutes between the arrival of a bus at a bus stop and its next departure. This means that there should be at most 50 minutes between the ending time of the current trip and the starting time of the next trips.

The objective function for the SDVSP with regular buses consists of a fixed and variable component, as mentioned before. More specifically, the variable costs consist of two euros per kilometer driven and a wage cost of 30 euros per hour. The fixed component, on the other hand, amounts to 100 euros per bus. With this problem description, we will specify a single-commodity flow and set covering formulation in section 3.

2.1.2 Electric buses

Apart from regular buses, we also consider the SDVSP with electric buses. We make the same assumptions as for regular buses, mentioned in section 2.1.1, but relax assumption 5. Besides that, we also introduce the following assumptions for this problem:

6. The maximum working time of a driver is eight hours.

7. The maximum driving time of a bus without recharging is eight hours, and recharging is only possible at the depot.

8. Recharging of a bus at the depot takes several hours, thereby prohibiting the bus from leaving the depot that day after it has entered.

For this problem, we use the same objective function as for the SDVSP with regular buses (see section 2.1.1). Ultimately, in section 4.2.2, we will consider a new problem where we change assumption 7, such that the recharging time at the depot will be one hour, and assumption 8 such that the bus can be used by a second driver after recharging. This relaxation will be solved using a two-step heuristic. Besides that, both of these problems will be formulated using the set-covering formulation in section 3.

2.2 Multiple vehicle types

In this section we consider an extension of the SDVSP with multiple vehicles, also known as the *multiple vehicle types vehicle scheduling problem* (MVT-VSP). We will assume that there are two types of buses: regular and electric buses. Electric buses need to be recharged at the depot after eight hours and the bus drivers for both bus types should not work more than eight hours. This means that both buses need to be back at the depot within eight hours. After the bus is back at the depot, buses of both types cannot be used on the same day anymore. Moreover, for any bus there is no constraint on the duration between the ending time of a trip and the beginning time of the next trip. Similar as before, we want to find a feasible minimum-cost schedule such that (1) each trip is performed by exactly one vehicle and (2) the sequence of trips assigned to a vehicle is feasible.

In the MVT-VSP, we assume that electric buses have higher fixed costs and lower operational costs than regular buses. Therefore, we will consider the situation in which regular and electric buses have a capital cost of 100 and 150 euros per bus, respectively. Moreover, the operational cost of an electric bus is two euros per kilometer, compared to six euros per kilometer for a regular bus. Labour costs are 30 euros per hour for both types of buses. We motivate this assumption by referring to Mathieu (2018), who shows that the purchase price of electric buses was typically twice as high as the price of regular buses in 2018. Additionally, Mathieu (2018) shows that electric buses have lower operational costs, since they rely on electricity to run, which is cheaper than fuel and more efficient.

2.3 Additional charging station

Lastly, we study the optimal placement of an additional charging station for the SDVSP with electric buses. In this extension we assume that recharging at this location will take one hour for an electric bus, and we have to decide on which of the four start/end locations to optimally place this charging station. In this setting, we make the same assumptions as mentioned in section 2.1.2 for electric buses. However, we relax assumption 6, change assumption 7 (in that recharging is possible at the depot and at the recharging location), and assume that a bus remains fully charged at the recharging location if it waits at that recharging location before departing for its next trip. Note that the modification of assumption 7 means that a duty can be longer than eight hours. As before, we optimize for the set covering formulation with the same data as in section 2.1.1.

3 Methodology

In this section, we provide the notation and formulation for the SDVSP. First, we describe the single-commodity flow and set covering formulation. Afterwards, we will formulate the SDVSP with multiple vehicle types and an additional charging station. Finally, we discuss the solution methods that we used.

3.1 Variables

In our formulations for the SDVSP, we will follow the notation as in Freling et al. (2001). As mentioned in section 2.1, we assume a set of trips to be given, each with a fixed starting and ending time, and starting and ending location. Therefore, we define b_i and e_i as the beginning and ending location of trips i, respectively. Similarly, let bt_i be the beginning time and et_i be the ending time of trip i. Trip i is said to be *compatible* with trip j if trip j and trip i can be covered by the same bus, which means that $et_i + trav(e_i, b_j) \leq bt_j$, where $trav(e_j, b_i)$ is the deadhead travel time from location e_j to location b_i in minutes. The deadhead travel time consists of idle and travel time, where idle time is the time a bus waits at a location other than the depot. Furthermore, $dist(e_i, b_j)$ is the distance between e_i and b_j. Throughout this paper, we will assume the distance to be expressed in kilometers and travel time in minutes.

3.2 Single-Commodity Flow Formulation

We first construct the set of trips $N = \{1, 2, ..., n\}$ in which trips are ordered by their starting time in ascending order. Besides that, we define a set of compatible trips which is denoted by $E = \{(i, j) : i < j;\ i,\ j\ \textit{compatible};\ trav(e_i, b_j) \leq 50;\ i, j \in N\}$. With these sets a directed network, $G = (V, A)$, is constructed with source s and sink t both representing the depot. Moreover, $V = N \cup \{s, t\}$ and $A = E \cup (s \times N) \cup (N \times t)$.

For the single-commodity flow formulation, we define a decision variable m representing the number of buses that is used. To decide which trips are covered by the same bus, for each element of compatible trips $(i, j) \in A$ we introduce a binary decision variable y_{ij}, which is defined as follows

$$y_{ij} = \begin{cases} 1, & \text{if the same bus covers trip } j \text{ directly after trip } i \\ 0, & \text{otherwise.} \end{cases}$$

The problem can then be formulated as

$$
\begin{aligned}
\min \quad & 100m + F + \sum_{(i,j)\in A} \left[\frac{30}{60}\, trav(e_i, b_j) + 2\, dist(e_i, b_j)\right] y_{ij} && && (1)\\
\text{s.t.} \quad & \sum_{i:(i,j)\in A} y_{ij} = 1 && \forall j \in N && (2)\\
& \sum_{j:(i,j)\in A} y_{ij} = 1 && \forall i \in N && (3)\\
& \sum_{j:(s,j)\in A} y_{sj} = \sum_{i:(i,t)\in A} y_{it} = m && && (4)\\
& y_{ij} \in \{0,1\} && \forall (i,j) \in A && (5)\\
& m \in \mathbb{N} && && (6)
\end{aligned}
$$

The objective function (1) minimizes the fixed and variable costs described in section 2.1.1. Component F represents the cost of covering all the scheduled trips exactly once, hence is known beforehand.

Constraint (2) ensures that trip j has at least one predecessor which can either be another trip or the depot. Similarly, constraint (3) makes sure that each trip i has at least one successor. The constraints in (4) make sure that the number of buses that leave and enter the depot are equal.

3.3 Set Covering Formulation

Next, we define the set covering formulation of the SDVSP. We define the set of feasible duties $D = \{1, 2, ..., d\}$. A duty is considered feasible if the maximum time between two consecutive compatible trips in the duty is at most 50 minutes and, where necessary, it satisfies a maximum duration. We construct the duties based on the *Depth-First Search* (DFS) algorithm (see section 3.6). Since each feasible duty j consists of a sequence of trips, we can calculate the total distance travelled, d_j, and the the duration, t_j, of duty j. Similar as in the previous formulation, let $N = \{1, 2, ..., n\}$ be the set of trips ordered in ascending starting time. Finally, let a_{ij} equal 1 if trip i is covered by duty j and 0 otherwise.

For each feasible duty $j \in D$, we introduce the following binary decision variable to track which feasible duty is chosen

$$x_j = \begin{cases} 1, & \text{if a bus performs duty } j \\ 0, & \text{otherwise.} \end{cases}$$

This gives the following formulation

$$\min \quad \sum_{j \in D} \left(2\, d_j + \frac{30}{60} t_j + 100 \right) x_j \tag{7}$$

$$\text{s.t.} \quad \sum_{j \in D} a_{ij} x_j = 1 \qquad \forall i \in N \tag{8}$$

$$x_j \in \{0, 1\} \qquad \forall j \in D \tag{9}$$

Objective function (7) minimizes the fixed and variable costs described in section 2.1.1. Constraint (8) makes sure that every trip i is covered exactly once by the set of chosen duties.

3.4 Multiple Vehicle Types

We will extend the SDVSP to consider multiple types of vehicles that can be used jointly to cover all the trips, the MVT-VSP. We use the assumptions as mentioned in the problem description. Besides that, we use similar notation as in Ferland and Michelon (1988).

There are two types of buses: regular and electric buses. Let $K = \{1, 2\}$ be the set of bus types, where $k = 1$ indicates regular buses and $k = 2$ electric buses. Each vehicle has a speed, v_k, in kilometers per hour. We assume that electric buses have a speed of 5 and regular buses a speed of 8. Furthermore, let f_k be the fixed cost of using bus k. As we motivated before in section 2.2, we assume that electric buses are more expensive than regular buses, with $f_1 = 100$ and $f_2 = 150$. The wage costs stay the same, however, the cost per kilometer is 3 times higher for the regular buses – 2 euro for electric buses and 6 euro for regular buses. Again d_j and t_{kj} are the total distance and duration of duty j, respectively. In the MVT-VSP, however, the duration of a duty depends also on the type $k \in K$

Again, let N be the set of ordered trips. Let D_k be the set of feasible duties for each type $k \in K$, so $D_k = \{1, 2, ..., d_k\}$ where d_k is the total number of feasible trips for type k. In section 3.6 we will discuss how these duties are constructed. Furthermore, we define a_{kij} such that $a_{kij} = 1$ when bus type k covers trip i in feasible duty $j \in D_k$, and 0 otherwise. Finally, we have to decide which duties are performed, and which bus type corresponds to that duty, while minimizing the total costs. Therefore, the binary decision variable $z_{k,j}$ is constructed, where

$$z_{kj} = \begin{cases} 1, & \text{if a bus of type } k \text{ performs duty } j \\ 0, & \text{otherwise.} \end{cases}$$

Now, we can formulate the MVT-VSP as follows

$$\begin{aligned} \min \quad & \sum_{k=1}^{K} \sum_{j \in D_k} c_{kj}\, z_{kj} && && (10) \\ \text{s.t.} \quad & \sum_{k=1}^{K} \sum_{j \in D_k} a_{kij}\, z_{kj} = 1 && \forall i \in N && (11) \\ & z_{kj} \in \{0,1\} && 1 \leq k \leq K \qquad \forall j \in D_k && (12) \end{aligned}$$

$$c_{kj} = \begin{cases} 6\, d_j + \frac{30}{60} t_{kj} + 100, & \text{if } k = 1 \\ 2\, d_j + \frac{30}{60} t_{kj} + 150, & \text{otherwise.} \end{cases} \tag{13}$$

The objective function (10) minimizes the total costs of buses and bus drivers. The exact specification of the cost function is provided in (13). Moreover, every trip i should be covered by exactly one of the duties that is selected, this is formulated with constraint (11).

3.5 Additional Charging Station

In this section we analyze at which location $i \in \{A, B, C, D\}$ it is optimal to place an additional recharging station where it takes one hour to recharge a bus. Furthermore, let $N = \{1, 2, ..., n\}$ be the *original* set of trips, meaning that these trips do not recharge at their end location. For a given value of i, we order the elements of set N such that the $K = \{1, 2, ..., k\}$ is the set of the first k trips, which end at location i where there is a recharging station, but the bus does *not* recharge at the ending location. To this set N, we add the set $P = \{n+1, n+2, ..., n+k\}$ which are k *recharging* trips that have the same starting times as the the trips in set K, but in these trips the bus *does* decide to recharge. We can see the k trips in P as *recharging* versions of the k original trips in K. Let $M = N \cup P$ denote the set containing both the original and recharging trips. Finally, let D be the set of feasible duties, using all the trips in set M.

Furthermore, to decide which duty the buses take we define the following decision variable

$$x_j = \begin{cases} 1, & \text{if a bus performs duty } j \\ 0, & \text{otherwise.} \end{cases}$$

We define (a_{ij}) as the matrix that indicates whether trip i is covered by duty j. Hence a_{ij} equals 1 if trip i is covered by duty j and 0 otherwise. This will give the following formulation

$$\min \quad \sum_{j \in D} \left(2\, d_j + \frac{30}{60} t_j + 100 \right) x_j \tag{14}$$

$$\text{s.t.} \sum_{j \in D} a_{ij} x_j = 1 \qquad \forall i \in N \setminus K \tag{15}$$

$$\sum_{j \in D} (a_{ij} + a_{i+n,j})\, x_j = 1 \qquad \forall i \in K \tag{16}$$

The objective function (14) is the same in section 3.3. Furthermore, constraint (15) makes sure that each trip $i \in N \setminus K = \{k+1, k+2, ..., n\}$, meaning a trip that does *not* end at recharging location i, is covered exactly once by the set of chosen duties. Moreover, constraint (16) makes sure that the chosen duties cover either the *original* or *recharging* version of the first k trips.

3.6 Solution methods

This section provides the solution methods used for the vehicle scheduling models. More specifically, we will explain the procedures used to construct the set of compatible trips and feasible duties. Once these sets are constructed, the formulations of the models in the previous section can be implemented using a commercial solver, such as *CPLEX*.

3.6.1 Generating the set of compatible trips

In Algorithm 1, a pseudo-code is provided that shows how we constructed the set of compatible trips E. Each element (i, j) in this set indicates that trip j can be performed by the same bus that finished trip i. This set is used in the single-commodity flow formulation of the SDVSP with regular buses. We constructed the sets of compatible trips in Python and *AIMMS*.

Algorithm 1 Construction of sets with compatible trips

Require: $N = \{1, 2, ..., n\}$

$E \leftarrow \emptyset$
for $i = 1$ to $n - 1$ **do**
  for $j = i + 1$ to n **do**
    if $et_i + trav(e_i, b_j) \leq bt_j$ and $bt_j - et_i \leq 50$ **then**
      $E \leftarrow E \cup (i, j)$
    end if
  end for
end for

3.6.2 Generating feasible duties

Besides that, we constructed the set of feasible duties, D. Recall that a duty is composed of a sequence of trips, such that each trip in this sequence is compatible with its successor and predecessor in the sequence. The definition of *feasible*, however, is different for the problems that we consider. For example, for the SDVSP with regular buses we do not impose additional constraints on the definition of feasible besides that the trips in the feasible duty should be compatible and the time between two trips should nopt exceed 50 minutes. For the SDVSP with electric buses, however, we assume that a duty is feasible when its total duration is at most eight hours. Furthermore, for the MVT-VSP, we also require the duration of a feasible duty to be at most eight hours, both for regular and electric bus types. Note that we need the set of feasible duties for all these formulations in order to solve the SDVSP with the set covering formulation.

In order to construct the sets of feasible duties, we use the Pandas library for Python. This library allows us to store and access the start and ending times of the trips, as well as their start and end locations. Besides that, we will use Algorithm 1 to construct the sets of compatible trips, which will be needed to determine the sets of feasible duties. Together with the set of trips, N, the set of compatible trips will be used as input for a directed network objected that we have constructed in Python. The nodes of this network will be the elements from the set of trips, and a directed arc between two nodes exists when those nodes represent compatible trips - the arc will always be directed towards the trip that has a later starting time.

The directed network will be used, together with the *Depth-First Search* (DFS) algorithm, to construct the feasible duties. The algorithm starts with a sequence that consists only of the depot, node s, and moves through the network finding all possible compatible trips that can be added to this sequence such that all duties end at the depot, node t. Finally, we end up with a set of duties. If we define duties to be feasible when its duration is at most eight hours, we will make a new set of feasible duties that consist of the duties that satisfy this constraint. Otherwise, we have constructed a set of feasible duties, which can be used in the set covering formulations.

3.6.3 A simple two-step heuristic

As mentioned in section 2.1.2, ultimately we will relax some of the assumptions of the SDVSP with electric buses. More specifically, we will reduce the recharging time at the depot to one hour enabling the buses to be used by a second driver after recharging. In order to solve this problem we will use the answer of the original SDVSP with electric buses, meaning without the reduced charging time in place. This answer contains a set of duties that optimize the set covering formulation, which we use to create *duty chains*. Duty chains are sequences of feasible duties that can be performed by the same bus. For the feasibility of duty chains, we need to take into account that an electric bus needs to stay at the depot for at least one hour to recharge. The two-step heuristic is about selecting the least number of feasible duty chains such that all duties are covered exactly once. By selecting the minimum number of duty chains, we make sure that the fixed cost of 100 euros per bus are minimized, since each selected duty chain is assigned to a new bus. The set of feasible duty chains will, therefore, be implemented in *AIMMS* using the set covering formulation.

4 Numerical results

This section provides the numerical results we obtain by optimizing the different problems. For this research, we use timetable data of four bus lines for three different parts of the week: weekdays, Saturday, and Sunday. The data consists of a start and end time, and a beginning and ending location. We also used data on the deadhead time and distance between two locations, and data on the corresponding costs. To solve the optimization problems, we used the commercial *CPLEX* solver that is integrated in *AIMMS* version 4.85 on a computer with Intel Core i7-9750H CPU and 16GB of RAM.

4.1 Single-commodity flow and set covering formulations

First, we look at the basic problem of the SDVSP that only considers regular buses and compare the computation times of the single-commodity flow and set covering formulation described in sections 3.2 and 3.3. In Table 1, we see the total costs per day, amount of buses and computation times obtained for both formulations. We observe that the total costs are higher on weekdays compared to the working days in weekends. Besides that, also more buses are used on weekdays.

Table 1 shows that the single-commodity flow formulation has a lower computation time for every part of the week compared to the set covering formulation. Since the number of nodes that the commercial solver visited is the same for both formulations, namely 0, this could not be the reason for the lower computation time of the single-commodity flow formulation. The main reason for the difference in computations times is that the single-commodity flow model is formulated as a network flow problem, while the set covering model is not. There are some efficient network flow algorithms which, therefore, allow the commercial solver to solve the single-commodity flow models faster than the set covering problems.

Table 1 Results single-commodity flow and set covering optimization

	Single Flow formulation			Set covering Formulation		
	Weekdays	Saturday	Sunday	Weekdays	Saturday	Sunday
Total cost	29038.63	16468.45	27999.41	29038.63	16468.45	27999.41
Amount of buses	45	23	38	45	23	38
Solving time (seconds)	0.02	0.02	0.02	0.05	0.08	0.28

4.2 Results with electric buses

4.2.1 Set covering optimization

Second, we replace all regular buses by electric buses as explained in section 2.1. We use the set covering formulation for the SDVSP with electric buses for two reasons. First, the constraint of eight hours can be naturally implemented in the duty generation without altering the problem's formulation. Secondly, due to the eight hours constraint, we have significantly less feasible duties (Weekdays: 242, Saturday: 210, Sunday: 408) as compared to the problem with regular buses. Because the set covering formulation is less complex, we anticipate that it will be more efficient in this specific case.

The results are presented in Table 2 in a similar way to Table 1. First of all, looking at the solving times for the electric bus case, our suspicion on increased efficiency in the set covering optimization appears to be correct. Moreover, the results show that when deploying electric buses, we need 130 buses on weekdays, 78 buses on Saturdays and 143 buses on Sunday in order to cover all trips. Provided that a bus and its driver always stay together, this increases the number of drivers needed by 85 on weekdays, 55 during Saturdays and 105 during Sundays compared to the SDVSP with regular buses presented in Table 1. This does not come unexpectedly as electric buses can service less trips due to the eight hour time restriction, and thus more duties and therefore buses are needed to cover all trips.

Table 2 Results set covering optimization with electric buses

	Weekdays	Saturday	Sunday
Total costs	48744.55	29299.29	53411.3
Number of buses	130	78	143
Solving time (in seconds)	0.02	0.00	0.03

4.2.2 Two-step heuristic optimization

In Table 3 we have denoted the results for the two-step heuristic with electric buses. We included the number of buses and solving time in seconds. Looking at the number of buses needed in Table 2 and Table 3, we observe that the number of buses saved by reducing the recharging time to one hour and, after that, can be used by a second driver is 47 for weekdays, 28 for Saturdays and 52 for Sundays which is about 50 percent of the total increase when deploying electric buses instead of regular buses.

Table 3 Results two-step heuristic for electric buses

	Weekdays	Saturday	Sunday
Number of buses	83	50	91
Solving time (in seconds)	0.03	0.02	0.02

4.2.3 Sensitivity analysis

In order to see how sensitive the above results are to the data we use and assumptions we make, we conduct sensitivity analyses. In specific, we we will focus on battery life of nine till seven hours. Our motivation for looking at different battery life is to make claims about the importance of technological developments regarding batteries for electric buses. The results are presented in Table 4, which shows the total costs for different parts of the week.

In Table 4, we see that the larger the battery life the lower the costs. If the battery life is increased from eight to ten hours in the original problem, the total costs can be reduced by about 21 percent for weekdays, 20 percent for Saturdays, and 22 percent for Sundays. This results indicates that technologically advancements regarding the battery life for electric buses can be important for the total costs made by public transport companies. When performing the sensitivity analysis for a battery life of six and seven hours, the SDVSP with electric buses was infeasible. The reason for this is that there are trips that cannot be performed with a battery life of six or seven hours. This shows that the SDVSP is relatively sensitive to battery life. For this reason, we consider the placement of an additional charging station at one of the bus stops, which we do in section 4.4.

Table 4 Results sensitivity analysis with different battery life

	Total Costs		
Charging interval	8 hours	9 hours	10 hours
Weekdays	48744.55	45444.60	37810.72
Saturday	29299.29	27095.67	23327.33
Sunday	53411.30	48725.96	41895.46

4.3 Results with multiple vehicle types

In Table 5, the results are presented for the SDVSP with multiple bus types. This table shows the total costs, the number of regular and electric buses used and the solving time.

We solved the MVT-VSP for three time schedules: the time schedule on weekdays, on Saturdays, and on Sundays. As is also the case for the results of the basic problem, the total costs for the buses and bus drivers are lower on Saturdays compared to weekdays and Sundays (see Table 1 and Table 5).

Moreover, we observe that in none of the three time periods we construct bus schedules with only electric buses or only regular buses. This result is especially relevant in practice, since it indicates that it is not optimal to invest in a complete electric bus fleet nor a complete regular bus fleet. Since we motivated in section 2.2 that electric buses have a lower speed and higher fixed cost per bus, this result indicates that public transport companies should carefully decide how to renew their bus fleet.

Table 5 Results for the multiple vehicle types vehicle scheduling problem

	Weekdays	Saturday	Sunday
Total costs	52013	30478	55586
Regular buses	46	36	70
Electric buses	61	24	38
Solving time	0.03	0.03	0.05

4.4 Results with an additional charging station

In Table 6, the results regarding the optimal placement of an additional charging station are presented. The table shows the total costs, number of buses and solving time for two locations. Not all results are presented, due to lack of time. In Table 6 we see that the total costs and number of buses used are higher for location A than location D. Therefore, we can conclude that location D is the optimal location for the additional charging station, if we were to choose between location A or D.

Table 6 Recharging station for location A and D

	Location A			Location D		
	Total Cost	Buses	Solving Time	Total Cost	Buses	Solving Time
Weekdays	48141.12	125	0.02	44648.98	101	0.05
Saturday	28338.57	71	0.02	34011.43[1]	62	0.03

5 Conclusion

In this paper, we study the single-depot vehicle scheduling problem (SDVSP) applied to electric buses. The main question that we want to answer is whether it is beneficial for public transport companies to invest in an electric bus fleet instead of regular buses. Therefore, we study the SDVSP with regular buses and compare the results to the SD-VSP with electric buses. Besides that, in order to determine how sensitive the results are to different driving ranges of electric buses, we conduct a sensitivity analysis of the SDVSP with electric buses and different battery lives. Finally, we have studied two other extensions of the SDVSP: multiple vehicle types vehicle scheduling problem (MVT-VSP) and the optimal placement of an additional charger. The SDVSP with regular buses was studied using the single-commodity flow and set covering formation, and all the other problems were solved using the set covering formulation.

Our first main result is that a bus fleet with only electric buses has a higher total cost per working day compared to a bus fleet with only regular buses. Moreover, more electric buses are needed to cover all trips than if we would use regular buses, which indicates that it is not optimal for a public transport company to only invest in electric buses.

However, the above result assume that the capital and operational costs of electric and regular buses are the same. In section 2.2 we motivated that this need not be the case and, therefore, studied the MVT-VSP. In this problem both electric and regular buses can be used. However, electric buses have a higher capital cost, lower operational cost and speed than regular buses. We found that it is optimal to invest in a mixed bus fleet with both electric and regular buses.

Furthermore, we found that a reduction of the battery life to six or seven hours makes it impossible to cover all the trips. Furthermore, if the battery life were to increase to ten hours, the total costs will decline by approximately 20 percent compared to the – original – eight hour case. These results imply that the technological advancements regarding the battery life are important for a feasible and cost-efficient bus schedule.

Given the above result, a public transport company may consider to invest in an extra recharging station for the buses at a depot in order to make the vehicle schedule feasible. We found, using limited results, that it is optimal to put an additional recharging station at location D.

For further research we suggest research on the placement of an additional charging

[1]After reviewing our paper, we came to realize that the higher total cost with an additional charging station than without does not make sense. Therefore, we rerun the *AIMMS* code and found that we copied the wrong results to the table. Total Costs and Buses should be 27203.31 and 65, respectively.

station using another method such as the single-commodity flow formulation. Furthermore, a SDVSP with partial recharging could be researched to investigate its potential benefits.

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