



Wavelet Based Filters for Artifact Elimination in Electroencephalography Signal: A Review

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Abstract—Electroencephalography (EEG) is a diagnostic test that records and measures the electrical activity of the human brain. Research investigating human behaviors and conditions using EEG has increased from year to year. Therefore, an efficient approach is vital to process the EEG dataset to improve the output signal quality. The wavelet is one of the well-known approaches for processing the EEG signal in time–frequency domain analysis. The wavelet is better than the traditional Fourier Transform because it has good time–frequency localized properties and multi-resolution analysis where the transient information of an EEG signal can be extracted efficiently. Thus, this review article aims to comprehensively describe the application of the wavelet method in denoising the EEG signal based on recent research. This review begins with a brief overview of the basic theory and characteristics of EEG and the wavelet transform method. Then, several wavelet-based methods commonly applied in EEG dataset denoising are described and a considerable number of the latest published EEG research works with wavelet applications are reviewed. Besides, the challenges that exist in current EEG-based wavelet method research are discussed. Finally, alternative solutions to mitigate the issues are recommended.

Keywords—Electroencephalography, Data processing, Wavelet transform, Denoising.

ABBREVIATIONS

AC	Alternating current
AMUSE	Algorithms for multiple unknown signals extraction

AWICA	Automatic wavelet independent component analysis
BIT	Brain imaging tools
BSS	Blind source separation
EEG	Electroencephalography
CCA	Canonical correlation analysis
CWT	Continuous wavelet transform
DC	Direct current
DFA	Detrended fluctuation analysis
DOST	Discrete orthonormal S-Transform
DWT	Discrete wavelet transform
ECG	Electrocardiography
EMD	Empirical mode decomposition
EMG	Electromyography
EOG	Electrooculography
ERP	Event-related potentials
fMRI	Functional magnetic resonance imaging
FRWT	Fractional wavelet transform
FT	Fourier transform
FWT	Fast wavelet transform
GSR	Galvanic skin response
GUI	Graphical user interface
Hz	Hertz
IDWT	Inverse discrete wavelet transform
JADE	Joint approximate diagonalization
MEG	Magnetoencephalography
MRI	Magnetic resonance imaging
MSE	Mean square error
mV	Millivolt
NIRS	Near-infrared spectroscopy
NREM	Non-rapid eye movement sleep
PCA	Principal component analysis
PET	Positron emission tomography
PSNR	Peak-to-noise signal ratio

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SOBI	Second order blind identification
STFT	Short-time Fourier transform
SWT	Stationary wavelet transform
VMD	Variational mode decomposition
WPD	Wavelet packet decomposition

INTRODUCTION

Research on the brain has been conducted for thousands of years ago, and in the last 30 years, rapid development has been found.^{85,88} This contributes to the advancement of brain imaging tools (BIT) which facilitated researchers to directly examine brain activities while subjects perform various cognitive, motor, and perceptual assessments.^{54,92} Up to now, there are different brain imaging tools (BIT) available such as EEG, magnetoencephalography (MEG), positron emission tomography (PET), functional magnetic resonance imaging (fMRI), and near-infrared spectroscopy (NIRS). Through a combination of BIT with sophisticated experimental designs and data analysis methods, the roles of brain regions and their interactions can be investigated. The EEG is one of the most BIT used for brain research because it is non-invasive, non-painful, and non-harmful to the subject, portable, accessible, and cheaper than other BIT.^{13,57} It was invented a century ago by the German psychiatrist Hans Berger. The EEG obtains electrical activities from the brain through metal disc electrodes placed on the scalp. This is because the brain cells communicate *via* electrical impulses, and when the media is attached to the head, the signal can be recorded. The EEG system usually comes with 10 to 256 electrodes placement to detect the signal at a specific location.

The EEG signals usually reflect two types of brain activities which are spontaneous and event-related. Spontaneous activities refer to the neuronal response that causes no identifiable stimulus or behavioral manifestations. This type of activity is essential for detecting and evaluating brain disorders. Meanwhile, the event-related potentials (ERP) reflect the signals because of specific thoughts or stimuli. The ERP signals are low in amplitude, about less than a microvolt to several microvolts, whereas the spontaneous EEG had tens of microvolt.^{41,91} Therefore, the signal processing technique is applied to identify and process the low-amplitude signals that are always contaminated with artifacts/noises. The basic signal processing stages are amplification, filtering, digitization, analysis, and storage. The processing is accomplished with a simple electronic circuit or with digital computers.

The noise can be defined as any unwanted interference which alters the accuracy of the signal reading.

Most of the noise had quite a similar frequency to the EEG signal, which makes overlapping to occur, such as the unwanted DC components (amplitude: 300 mV, frequency: DC), power line noise (fundamental frequency: 600 mV of amplitude, and 50 Hz/60 Hz of frequency, and harmonic frequency: 600 mV amplitude and 100 Hz/120 Hz of frequency), and noise generated from chemical processes (amplitude: 100 mV, frequency 0.05–1 Hz). Generally, two main noise sources always exist in EEG: internal sources or external sources.^{38,58} The external sources of EEG noise are AC interference, reversed leads, and poor contact between electrode gel and head scalp, which disrupt signals. Meanwhile, the internal sources of noise are generated from muscle and other organs' movements and emanate from the surface of the epidermis. Figure 1 shows the EEG signal that is contaminated with artifacts.

The main aim of processing the raw EEG is to remove the noise that may disturb the signal's actual information and extract the crucial features of spontaneous or ERP information.^{74,96} There are two major techniques to identify that information: time-locked averaging techniques and spectral analysis techniques. The time-locked averaging methods are employed to determine the evoked activities. It refers to time-locked presentation stimuli that can be either stimulus-locked or response-locked. Furthermore, the spectral analysis techniques detect the specific frequency of EEG signals associated with human activities and behaviors. Among the well-known methods for processing EEG signals' time-domain and frequency-domain features is Fourier transform (FT) and wavelet transforms methods.^{72,17} The FT method is traditionally selected because it is time-shift invariant in both the time and frequency domains. But, the limitation of the FT method is it ignores any time-varying spectral content because it considers the signal is stationary over time. Therefore, the FT method is not efficient for processing EEG signals present in non-stationary properties. Thus, an alternative method is required where the wavelet has been employed for EEG signal processing. Figure 2 illustrates the EEG processing for acquired related signal information.

The wavelet is a time–frequency analysis based on the Wigner-Ville distribution that considers as one of the most efficient and powerful processing approaches for most biosignals.^{72,18} The wavelet can be applied to many tasks in signal processing, where it can process both the continuous and discrete-time data. For example, a wavelet was used to process EEG, electromyography (EMG), electrooculography (EOG), and galvanic skin response (GSR) signals. It is widely preferred for processing the non-stationary signals because the wavelet provides an alternative to the

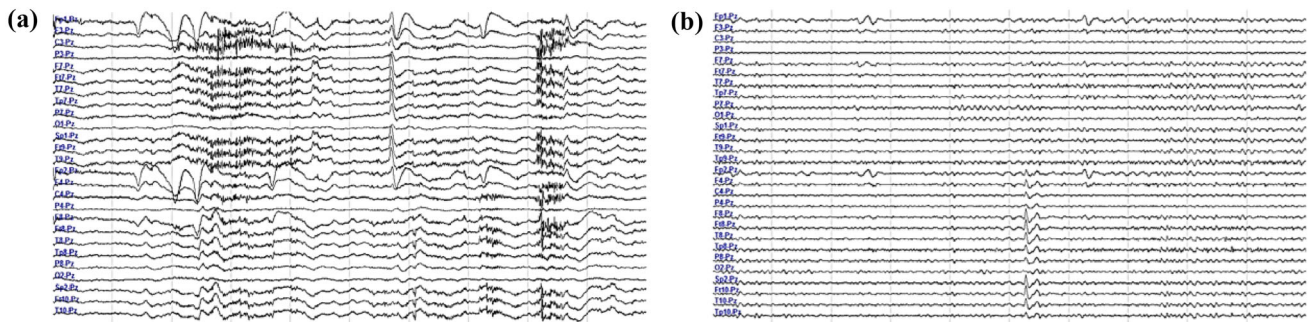


FIGURE 1. A screen of EEG signals (a) contaminated with eye blinks, muscle artifacts, and high frequencies, and (b) after the denoising process.⁷³

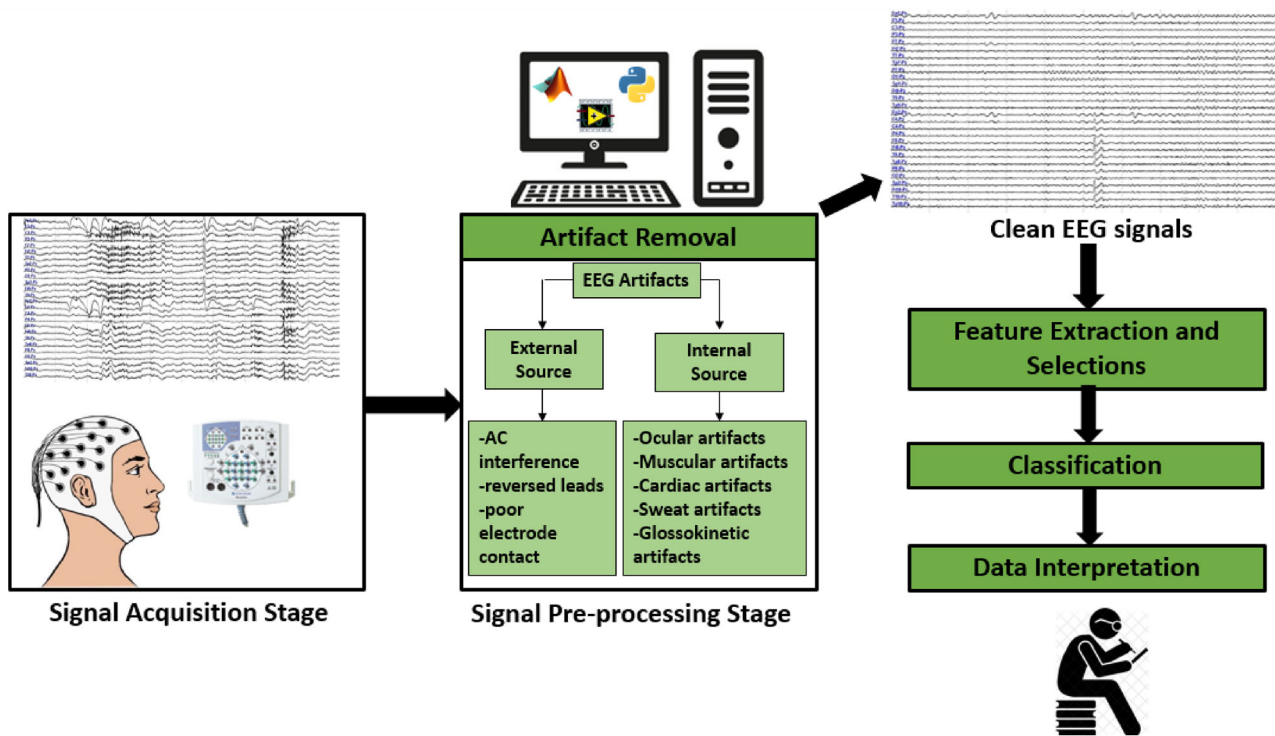


FIGURE 2. The flow of EEG signal processing.

Short-Time Fourier Transform (STFT) or Gabor transform. The STFT uses a single analysis window, whereas the wavelet uses short windows at high frequencies and long windows at low frequencies. This process is known as “constant-Q” or constant relative bandwidth frequency analysis. The wavelet analysis is generated from a single prototype wavelet *via* contractions and dilations (scaling) and shift.^{98,4} This prototype can be imagined as a band-pass filter, and the constant-Q property of the other band-pass filters (wavelets) follows because they are scaled versions of the prototype. Alternatively, the notion of scale is introduced to replace the frequency in the wavelet method, where the signal represents in the time-scale plane. The wavelet approach can be divided into sev-

eral types, and selection depends on application goals. The main basic kinds of WT are continuous wavelet transform (CWT), discrete wavelet transform (DWT), and stationary wavelet transform (SWT). Each of the WT types has a different influence on the accuracy of the process biosignal. Therefore, many studies investigated the best wavelet types with their properties to determine the most efficient and highest accuracy.^{37,8}

According to the Web of Science database using key search “electroencephalography/EEG and wavelet,” there are about 1377 publications from 2010 to 2020. The number of publications had increased from year to year, as shown in Fig. 3. The increment of published work is about 330% for ten years. The significant influence of publication increment is due to the ease of

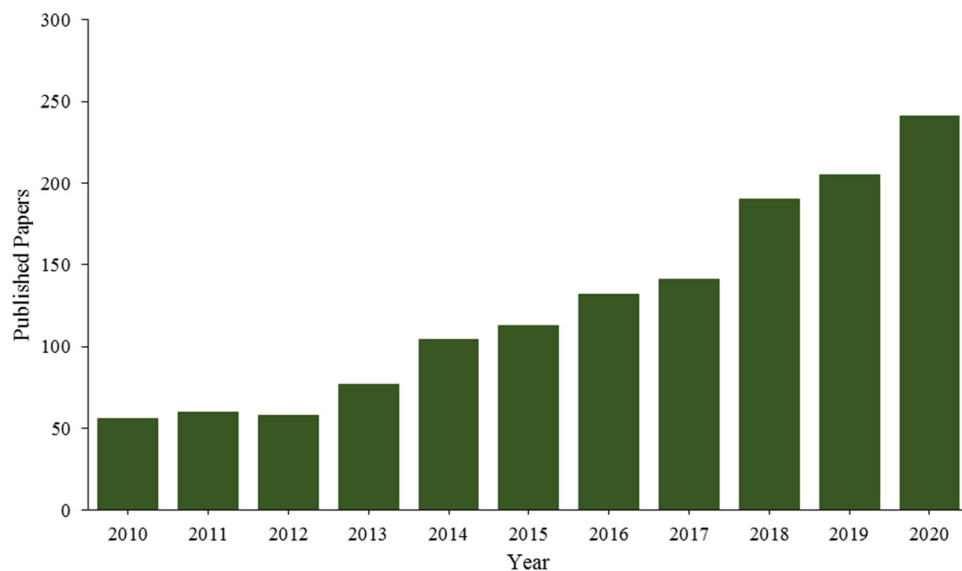


FIGURE 3. Number of publications per year from 2010 to 2020 using key search “electroencephalography/EEG and wavelet” as derived from Web of Science database (accessed on March 3, 2022).

processing using the wavelet method. It can be done either using the WT toolbox or simple coding in MATLAB software. Besides, the researcher also found that the WT method is more suitable and efficient for processing the non-stationary signal such as EEG, where more useful information can be preserved than the traditional FT method.^{98,4,71} According to Daud *et al.*,²³ high EEG data loss is found using the 4th order Butterworth band-pass filter compared to the SWT method based on the mean square error (MSE) and peak-to-noise ratio (PSNR) result during filtering (remove noises) process. Therefore, it showed that the SWT method is more efficient in eliminating noises than the Butterworth band-pass filter. In the current literature, some research has reviewed and described the application of the wavelet technique in denoising EEG signals. For instance, Ranjan *et al.*,⁷⁰ had overview the wavelet method in eliminating the ocular elimination from EEG signal. However, this work not only focused on wavelet but also included the other method. Besides, Islam *et al.*,³⁴ also shortly review the wavelet technique for artifact detection and removal from scalp EEG. Therefore, limited discussion on wavelet technique on EEG denoising was found. Meanwhile, the other review works showed the application of wavelet in various signals and images denoising, such as electrocardiography (ECG) signals,⁸⁰ magnetic resonance imaging (MRI) and ultrasound brain images,^{65,69} PET dataset,⁸⁴ and tomography images.⁶⁸ Therefore, minimal review works on wavelet-based techniques for EEG denoising were found that motivate this current work to provide a comprehensive discussion on these techniques.

This review article comprehensively describes the application of the wavelet technique in denoising EEG signals. The recent published articles are used as the study case. The first part of article briefly overviews the basic theory, properties, and characteristics of EEG and wavelet approaches. Then, a considerable number of latest published EEG research works with wavelet applications are reviewed. The most important part, this review briefly describes two main standard methods used in denoising EEG signals: (1) basic wavelet and (2) advanced wavelet. Finally, current EEG-based wavelet approach research challenges are discussed, and alternative solutions to mitigate the issues are recommended. To the best of the author's knowledge, there are very limited studies on review articles about applying the wavelet method in EEG denoising. Therefore, it is deemed essential to discuss and overview this intensely.

SCOPE OF FOURIER TRANSFORM AND WAVELET METHODS

The wavelet method is often distinguished with the Fourier transform (FT). The FT is a powerful method for processing and analyzing the components of the stationary signal.^{72,17} The FT is a continuous function that converts a mathematical function from the time domain to the frequency domain. It is generated by a set $W_n(t) = e^{int}$, $n = 0, 1, \dots$ of orthogonal functions, of period $2n$. It provides two-dimensional information about any signal, which are frequency components present in a signal and their respective amplitudes.

TABLE 1. Comparison between stationary and non-stationary signals.

Parameter	Stationary signals	Non-stationary signals
Definition	The signal that its frequency or spectral contents are not changing with respect to time	The signal that the frequency was changing with respect to time
Time period	Constant	Varies and not constant
Frequency	Constant	Change constantly
Spectral contents	Constant	Dynamic and keep changing
Fourier equation	Good at representing stationary signals	Non-good at representing non-stationary signals
Type of signal	Single-tone or multitone sine wave of constant frequency, white noise, temperature	Speech signals, biosignal

Equation (1) represents the mathematical equation of FT.

$$f(w) = \int_{-\infty}^{\infty} f(t) \cdot e^{-iwt} - dt, w = 2\pi f \text{ and } |f(w)|$$

= amplitude of each component w of the signal

(1)

Besides, the FT can also transform the signal from the frequency domain to the time domain based on the reconstruction of the original function called inverse Fourier transform (Eq. (2)).

$$f(t) = \int_{-\infty}^{\infty} f(w) \cdot e^{-iwt} - dw \quad (2)$$

Respective to FT properties; thus, it is more suitable for processing the stationary signal than non-stationary signal. Examples of stationary signals are white noise, temperature, and a combination of cosine and sine signals (sinusoids). The FT is less useful and inappropriate in processing and analyzing non-stationary signals.⁷² The comparison between stationary and non-stationary signals is described in Table 1.

Therefore, the wavelet is proposed as an alternative to the FT method. The wavelet method allows the components of a non-stationary signal to be processed. Besides, the wavelet also can be applied for stationary signals. It represents the signal in both the time and frequency domain, which aids the collection of localized information about the signal. The wavelet method has numerous modes of operation and other options than the FT method. The wavelet is a simple wave of duration adjusted with energy concentrated in variable intervals.^{72,18} This makes the wavelet an excellent and valuable method for time series analysis that obtains properties of the ability to change the time and frequency components. The wavelet method can be defined as the convolution of $f(t)$ with a scaled and translated version of ψ , known as wavelet mother.^{72,18} The mathematical equation of the wavelet is represented in Eq. (3).

$$\langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t) \psi\left(\frac{t-b}{a}\right) dt \quad (3)$$

where, $f(t)$ refers to the input signal with a scale parameter and b translation value, $\psi_{a,b}(t)$ refers to the mother function of wavelets, and $\langle f, \psi_{a,b} \rangle$ refers to the spectrum wavelets.

The scale is associated with the frequency, where the high scales refer to low frequencies and low scales refer to high frequencies, whereas the translation is the displacement of the mother function of the input signal. The restoration of the signal from the frequency domain to the time domain, inverse wavelet transform, permits the observation of signal behavior in specific frequency bands and the reconstruction of the original function $f(t)$. The inverse wavelet transform can be determined by the sum of the real part of the wavelet spectrum on all scales (Eq. 4).

$$x_n = \frac{\delta_j \delta_t^{0.50}}{C_\delta \psi_0(0)} \times \sum_{j=0}^J \frac{R\{f, \psi_{a,b}\}}{s_j^{0.50}} \quad (4)$$

where, $s_j^{0.50}$ refers to the factor that transforms the wavelets into energy density and $\delta_j, \delta_t^{0.50}, C_\delta, \psi_0(0)$ refers to specific constants of the base function used.

The difference in properties between Fourier transform and wavelet methods is described in Table 2.

ARTIFACTS IN ELECTROENCEPHALOGRAPHY

The main concern when dealing with EEG signals is assuring clean data with a high signal-to-noise ratio. The EEG signal is easily contaminated with noise or artifacts due to its small amplitude in the microvolts range. These artifacts need to be eliminated through the denoising process to preserve the valuable information required for applications. Generally, artifacts refer to unwanted components or signals that the brain does not generate.^{57,38} The first step requires in

TABLE 2. Comparison of properties between Fourier transform and wavelet methods.

Fourier transform	Wavelet
This method is appropriate for stationary signals	This method is appropriate for stationary and non-stationary signals
The input signal is transformed from the time domain to the frequency domain and <i>vice versa</i> . It provides two-dimensional information about signal frequency component and their respective amplitudes	The input signal is transformed from the time domain to the frequency domain and <i>vice versa</i> . It provides three-dimensional information about signal frequency components, respective amplitudes, and time axes at different frequency components
This method exhibits zero time resolution and very high-frequency resolution	This method obtains high time resolution and high-frequency resolution. Besides, the time and frequency resolution can be changed
The input signal is transformed into sine and cosine waves of different frequencies and amplitudes	The input signal is transformed into a scaled and translated version of selected mother wavelets
Not suitable for examining the local behavior of the signal	Suitable for examining the local behavior of the signal
This method uses a real or complex function, whereas the output is often complex	This method used real or complex function, whereas the output could be real or complex
The output signals are smooth, regular, well defined, and can be predicted	The output signal is very irregular and cannot be predicted. Therefore, the mother wavelets are employed to assume the local behavior of the signal, such as spikes and irregularities

removing them is by identifying the types and characteristics of that artifacts. There are two standard artifacts in EEG signals: physiological/biological and non-physiological/technical artifacts. The physiological artifacts originate from ocular, muscle, cardiac, perspiration, and respiration.^{57,38} Meanwhile, the non-physiological artifacts occur due to electrode pop, cable movement, incorrect reference placement, body movement, AC electrical, and electromagnetic interferences.^{57,38}

The physiological artifacts are the distortions in the signal of interest because of physiological processes in the human body that are classified as intrinsic artifacts. EEG signals' most common physiological artifacts are eye movements and blinks.^{21,67} The changes in the resting potential of the retina during eye movements, eye blinks, and muscle activities of the eyelid generates disturbances in EEG recordings. The artifacts-based eye activities can be measured through EOG and exist in low-frequency components of the EEG, which is up to 10 Hz. The amplitudes of EOG artifacts are up to 1 mV, which attenuated the distance toward the occipital brain regions. Besides, the movements of facial, jaw, and neck muscles also generate electrical activities that EEG tools can record. This artifact is known as EMG and is mainly found over the temporal lobes.^{99,51} The most observed in EEG signals is when the teeth chewing and clenching and neck muscle movements.

The EMG artifact usually exists from 20 Hz to a few hundred frequencies hertz and has amplitude in the millivolt range across the EEG signal. Therefore, a solution is required to minimize the EMG and EOG artifacts inherent in human behavior. The other physiological artifact originates from the heart, called an ECG. Commonly, the ECG artifacts are found in the frequency range of 0.5–40 Hz.^{34, 48} In addition, the

heart beating also generates artifacts that result from voltage changes in the area near the blood vessels that expand and contract. These artifacts are dominantly present over the temporal arteries and veins. The heartbeat artifact has a low amplitude and low-frequency range of about 0.5–3 Hz. Therefore, it usually influences the EEG recording that involves deep or slow-wave sleep and NREM sleep studies.

The non-physiological artifacts come from external factors such as environment and experiment errors classified as extrinsic artifacts. This artifact usually occurs due to the movement of EEG recording electrodes, which are known as “electrode-pop” artifacts, electrode misplacement, and cable movements.³⁸ In particular, the movement of the electrode causes changes in the DC contact potential at the electrode–skin interface. As a result, it leads to inconsistent changes in the baseline level of EEG signals, such as fast, slow, and gradual return to the original baseline level. This artifact can be observed in EEG signals by sharp epileptic waves and deceptive spikes. Meanwhile, the loose electrodes led to substantial 50/60 Hz powerline interference. This artifact contributed to the flowing electromagnetic field of currents in nearby electrical devices or powerlines. The non-physiological artifacts are much easier to be eliminated from EEG signals through the simple filter and proper procedure/planning because their frequency is inconsistent with the desired signals. However, the physiological artifacts are difficult to eliminate, requiring particular algorithms and advanced filtering techniques.^{60,32} Table 3 summarizes the contaminated artifacts in the EEG signal.

Numerous artifact handling techniques were introduced to process the raw EEG signals to obtain real and concise data.^{58,32} The clinicians and researchers manually identified the artifacts at the early stage,

TABLE 3. Types of common artifacts contaminated in EEG signal.

Non-physiological/technical artifacts (External sources)				Physiological/biological artifacts (Internal sources)		
Originate from physiological processes in the human body				Originate from external factors such as environment artifacts and experiment errors		
Ocular	Cardiac	Muscle	Others	Instrumental	Interference	Movement
Eye movement	ECG pulse	Scalp contraction	Respiration	Poor ground	Optical	Body movement
Eye blink		Swallowing	Gloss kinetic	Cable movement	Sound	Head movement
Eye flutter		Chewing	Skin	Electrode	Electromagnetic waves	Limbs movement
REM sleep		Talking		Displacement and pop-up	Electrical	Tremor
		Clenching			Magnetic	Other movements
		Sniffing				

resulting in a tedious and long-time process. Then, the algorithms with an automated way are introduced to identify the clean or artifact composited EEG segment. The artifacts handling can be divided into an artifact rejection and artifact reduction methods.³⁸ The artifact rejection process identifies the artifacts and rejects the contaminated EEG segments (i.e., epoch). This approach is suitable for long acquisition duration but inefficient for short acquisition duration. The drawback of using this approach for a short acquisition duration is substantial data loss, leading to insufficient data analysis information.³⁸

Hitherto, the real-time automatic artifact reduction is preferred. The artifact reduction or removal approach detects artifact components in the EEG signal and separates them from the neuronal sources. In this approach, the EEG signal might be used only or included with other physiological signals such as EMG, ECG, EOG, or movement (gyroscope and accelerometer). Most artifact reduction approaches assume that the measured signal is composed of the interest signal and artifacts. Thus, the methods that are used for artifact removal consist of regression and combination,^{94,76,24} empirical mode decomposition,^{78,59,95,63} wavelet transforms,^{89,1,55,87} and blind source separation (BSS) approaches (i.e., principal component analysis and independent component analysis).^{64,2,20} Commonly, these approaches are combined, known as the hybrid method. Besides, other techniques such as Bayesian filtering, Wiener filtering, and adaptive filtering are also used. For this recent review, the application of the wavelet transform approach in artifacts elimination is discussed.

WAVELET FILTERS

Denoising means removing noise or artifacts and unwanted information in the desired signal.⁴² The application of wavelets methods for artifact elimination was first introduced by Donoho and Johnstone.⁹³

The elimination of artifacts in EEG signals is one of the crucial parts that require great attention because artifacts in any physiological signal can lead to misinterpretation of data information. However, the inconvenient selection of the denoising approaches could lead to the loss of important data information, and the required artifacts are not efficiently removed from EEG signals.^{57,39} The application of wavelet-based approaches in signal denoising has opened up a new dimension in biosignal processing. It has been proved that efficient in removing noise from non-stationary and time-frequency signals.

The wavelet method refers to the linear transformation with the mathematical technique that can decompose the input signal into detail coefficients ($cD[n]$) and approximation coefficients ($cA[n]$).^{4,26} The $cD[n]$ can be defined as a high-frequency coefficients and $cA[n]$ is a low-frequency coefficients. Besides, the output of wavelet also control by scaling and shifting/translation factors of a single wavelet function (mother wavelet). The desired frequency range of signal can assign to each scale component and resolution that match its scale. The decomposition of input signal in wavelet can be mathematically expressed based on Eqs. (5) and (6).^{4,26}

$$cD[n] = \sum_{i=-\infty}^{\infty} s[i]h[2n-i], \quad (5)$$

$$cA[n] = \sum_{i=-\infty}^{\infty} s[i]g[2n-i], \quad (6)$$

where i refers to the sampling data point, n refers to the number of the sampling data, $s[i]$ refer to the discrete radar signal with noise, $h[2n-i]$ refer to high pass filters and $g[2n-i]$ refer to low pass filters that vary based on the mother wavelet function. The presence of frequency filters facilitates the wavelet to extract the required frequency band from the input signals. Figure 4 represents the y level of the decomposition process the input signal into the $cD[n]$ and $cA[n]$.

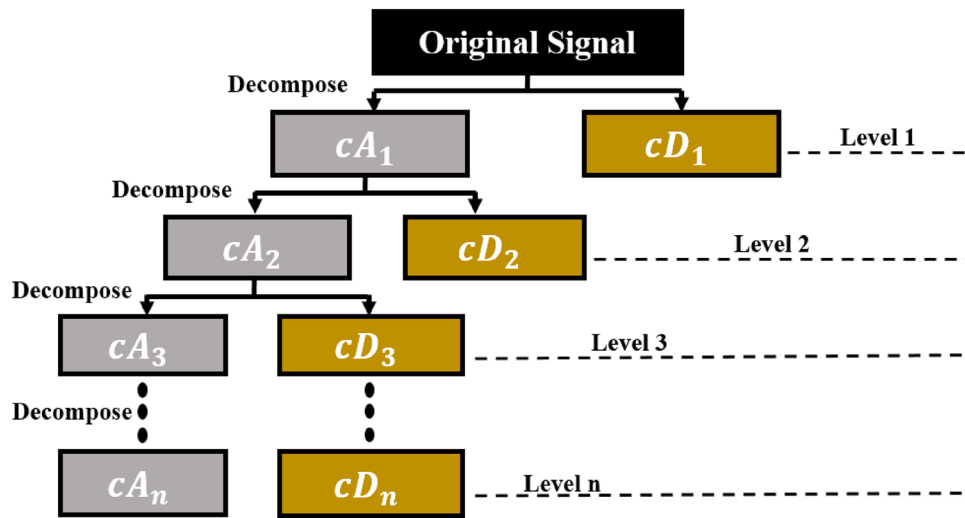


FIGURE 4. The decomposition process of the input signal using the wavelet method.

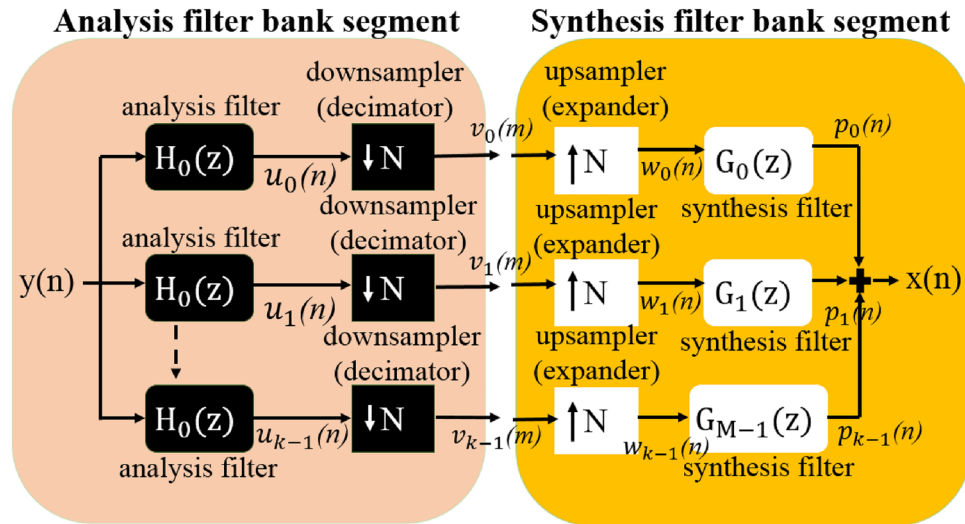


FIGURE 5. Wavelet filter bank: (a) analysis filters and downsamplers and (b) synthesis filters and upsamplers.

The cascaded arrangement of filters such as low-pass, high-pass, and band-pass filters connected by sampling operators to obtain the required decomposition and recombination of a signal from a spectrum perspective is named wavelet filter.⁹ In this case, the sampling operators could either be up-samplers or down-samplers. The up-samplers are known as expanders, and the down-samplers are called decimators. Through this filtration, the desired components can be reliably extracted and analyzed. Figure 5 shows the wavelet filter bank that is executed in the typical denoising process. From Fig. 5, the filter bank consists of two sections: (1) the analysis filter bank section that includes the analysis filters and downsamplers. The synthesis filter bank section consists of upsamplers and synthesis filters. These sections work together to ex-

ecute the opposite operation in filtering the input signal frequencies.

The analysis filter bank is composed of either low-pass filters or high-pass filters. The filter will only allow the loaded signal $y(n)$ specific frequency components. Therefore, the desired features of the input signal can be obtained and investigated *via* the analysis filter bank. Like the analysis filter bank, the synthesis filter bank is also composed of low-pass and high-pass filters. The output from these filters is summed to a common output, and the frequency responses are matched with the analysis filters.^{26,9} Meanwhile, the down-sampler refers to the frequency downsampling of the input signal by a factor of N . Thus, it indicates that the N th samples are retained in a provided sequence. In contrast, the up-sampler expands or upsampling the

input signal by a factor N . It executes by adding zeros at every n th position in the input signal sequence. The wavelet filter banks should meet specific properties to guarantee that the input signal is processed correctly and obtain the desired outcomes. Among the properties are perfect reconstruction, orthogonality, and paraunitary condition.

The main factors influencing wavelet performance are the mother wavelet, coefficient feature, frequency band, and decomposition level.^{79,10,3,45} Mainly, the wavelet transform is classified into continuous wavelet transform (CWT), discrete wavelet transform (DWT), and stationary wavelet transform (SWT). All wavelet classes are considered the time–frequency properties for processing the continuous-time (analog signals) related to harmonic analysis. The CWT executes every scale and shift, whereas the DWT operates a specific subset of scale and shift values. Meanwhile, the SWT is almost similar to DWT, but it has an invariance shift value where the output remains unaffected. Besides those three main wavelet classes, several other types of wavelets are suitable for various applications, such as Fast wavelet transform (FWT),^{31,40} Lifting scheme, Generalized Lifting Scheme,^{97,12} wavelet packet decomposition (WPD),^{77,101} and Fractional wavelet transform (FRWT).^{29,28} The wavelet classes or families are defined based on wavelet function. The wavelet

function consists of the scale and translation copies of the mother wavelet function $\varphi(x)$ and the scaling function $\phi(x)$ that continuously differentiable function with compact support. There are various mother wavelet functions: Morlet, Meyer, Coiflet, Mexican Hat, Haar, Symlet, Daubechies, Gaussian wave, and others. The types of mother wavelets are summarized in Table 4. Each has different characteristics that require an adequately chosen mother wavelet to fulfill the application requirements.^{3,45} The coefficients of WT designate the projection of the signal over a set basis function executed as the dilation and translation of the scaling function and the mother wavelet function. Specifically, the low-pass coefficients are related to the scaling function, and the high-pass coefficients are associated with the mother wavelet functions, as represented in Eqs. (7)–(9).

$$g(h) = -1^n h(1 - n), \quad (7)$$

$$\phi(x) = \sum_n h(n) \sqrt{2} \phi(2x - n), \quad (8)$$

$$\varphi(x) = \sum_n g(n) \sqrt{2} \phi(2x - n), \quad (9)$$

TABLE 4. Types of mother wavelets.

Wavelet functions	Descriptions	Members
Haar	The Haar is the simplest and first wavelet. It has discontinuous and resembles a step function with a square wave	haar
Meyer	The wavelet and scaling function of Meyer is determined in the frequency domain	–
Discrete Meyer	The discrete approximation of the Meyer wavelets is symmetric and continuous with compact support	dmey
Morlet	The Morlet does not have a scaling function, but it is explicit	–
Mexican Hat	The Mexican Hat does not have a scaling function and obtains from a function proportional to the 2 nd derivative function of the Gaussian probability density function. It is also defined as the Ricker wavelet	–
Coiflets	The Coiflet is a symmetric wavelet with vanishing moments of additional properties	coif1, coif2, coif3, coif4, coif5
Daubechies	The Daubechies are continuous wavelets with compact support and is quite asymmetric	db1, db2, db3, db4, db5, db6, db7, db8, db9, db10
Symlets	The Symlets is a modified version of Daubechies function. It has symmetrical properties	Sym2, sym3, sym4, sym5, sym6, sym7, sym8
Biorthogonal	The Biorthogonal function has a linear phase property and uses two wavelets. One is used for decomposition and the other for reconstruction instead of the same single 1	bior1.1, bior1.3, bior1.5, bior2.2, bior2.4, bior2.6, bior2.8, bior3.1, bior3.3, bior3.5, bior3.7, bior3.9, bior4.4, bior5.5, bior6.8
Reverse Biorthogonal	The Biorthogonal function has a linear phase property and uses two wavelets. One is used for decomposition and the other for reconstruction instead of the same single 1	rbior1.1, rbior1.3, rbior1.5, rbior2.2, rbior2.4, rbior2.6, rbior2.8, rbior3.1, rbior3.3, rbior3.5, rbior3.7, rbior3.9, rbior4.4, rbior5.5, rbior6.8

TABLE 5. Metrics for evaluating EEG denoising performance.

Metrics	Description
Signal power-related evaluation	
Signal to artifact ratio (SAR)	The SNR is a time-domain metric that refers to the ratio between corrected and noisy EEG signals. It determines before and after artifact removal. The effective denoising technique will produce high values of SAR after (SAR_A) denoising than SAR before (SAR_B) denoising value
Peak signal to noise ratio (PSNR)	The PSNR is a time-domain metric that refers to the ratio between the maximum possible input signal power and artifacts components in the EEG signal. The PSNR is expressed in decibels
Power spectral density distortion (PSD)	The PSD calculates distortion in EEG signal through the percentage ratio between PSD in denoised EEG signal and the PSD raw (reference) signal
Error related evaluation	
Mean square error (MSE)	The MSE determines the difference between the raw EEG signal and the denoised EEG
Root mean square error (RMSE)	The RMSE determines the root of the difference between the raw EEG signal and the denoised EEG
Normalized mean square error (NMSE)	The NMSE refers to normalized MSE. It calculates the difference between the raw and denoised EEG to the raw EEG signal
Mean absolute error (MAE)	The MAE measures the difference between the power density of input and output signal
Relative error (RE)	The RE determines the ratio between signal error and the raw/input EEG signal
Signal information-based evaluation	
Mutual information (MI)	The MI measures the statistical dependency between artifacts and denoise EEG signals
Correlation coefficient (CC)	The CC can be estimated from the ratio of covariance of input and output signal to the product of variance of input and output signal

DENOISING MEASUREMENT METRICS

The denoising performance criteria of EEG signal can be evaluated and compared based on several metrics as represented in Table 5.

Wavelet-Based Methods for EEG Denoising

This section briefly described the application of wavelet-based methods in denoising the EEG signals. Based on literature studies, numerous wavelet-based approaches have been used, and it had been executed either through a Simulink or an algorithm. Each method had different influences on EEG processing, where their effects are compared with the existing traditional methods. The effect of developing wavelet-based approaches has been investigated on open sources EEG datasets or experimental EEG datasets. Therefore, specific well-known wavelet-based methods are described and systematically reviewed in the following part to examine their influence on EEG processing performance deeply. To aid the reader, the wavelet-based methods are divided into two main types: basic and advanced. The section initially starts with the basic wavelet-based method: stationary wavelet transform (SWT) and discrete wavelet transform (DWT). These basic wavelet methods are commonly used in EEG processing as they are simple and quite effective methods for EEG processing. However, in recent years, more advanced wavelet-based methods have been used to improve the quality of output signals. In this paper, a few types of advanced wavelet-

based methods are selected. A brief discussion on each selected wavelet-based method is found in the following section.

Basic Wavelet-Based Methods

Stationary Wavelet Transform

In wavelet transform algorithms, the SWT is more effective than DWT for EEG processing because its properties that time-invariant and obtain a better time resolution for the artifact characterization, change detection, pattern recognition, and feature extraction.^{23,93,82} The SWT differs from basic DWT in terms of shift-invariance and decimation. The DWT is decimated by two when convolving with a low pass filter and high pass filter for getting wavelet transform coefficients. On the other hand, the DWT has a similar resolution to the input signal. Meanwhile, the SWT operation is not decimated when convolve with low pass and high pass filters. This resulted the output of SWT double the number of coefficients of the samples in the input signal. Therefore, a smoother EEG is achieved after the threshold process in the wavelet domain. The EEG signal is processed in SWT by decomposing the input signal into low and high-frequency bands through high-pass and low-pass filters, as illustrated in Fig. 6.^{22,49,81} The yield decomposes signals are designated as the detail (low-frequency band) and approximation (high-frequency band) components. Like other basic wavelet transforms, the SWT process must be specified based on two param-

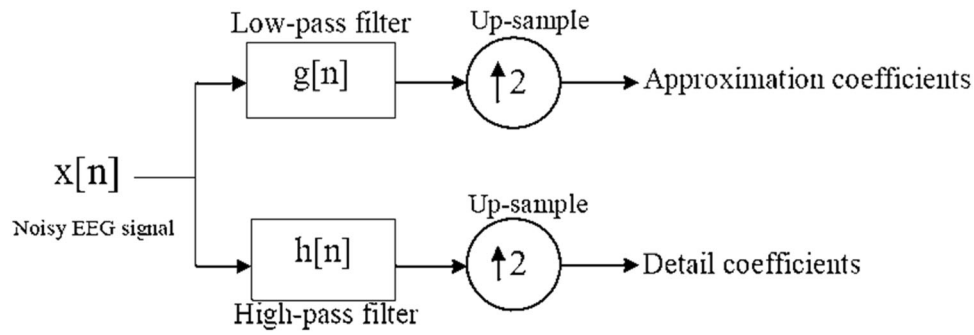


FIGURE 6. Structure of stationary wavelet transform decomposition process at level 1.

TABLE 6. Comparative analysis of SWT filters for EEG denoising from past works.

SWT filter	EEG dataset	Optimal wavelet filter	Ref
db3 of wavelet function with seven decomposition level	Subjects perform audiovisual assessment task	db3 with decomposition level 5	Daud <i>etal.</i> ²²
sym3, haar, coif3, bior4.4	Subjects in rest and blinking state	Coif3	Khatun <i>etal.</i> ⁴⁶
Haar, Symlets, Daubechies, Coiflets, Discrete Meyer, Biorthogonal, and reverse Biorthogonal with 51 wavelet members	Presurgical EEG signal from a Pharmacoresistant subject	Sym20	Frikha <i>etal.</i> ²⁵

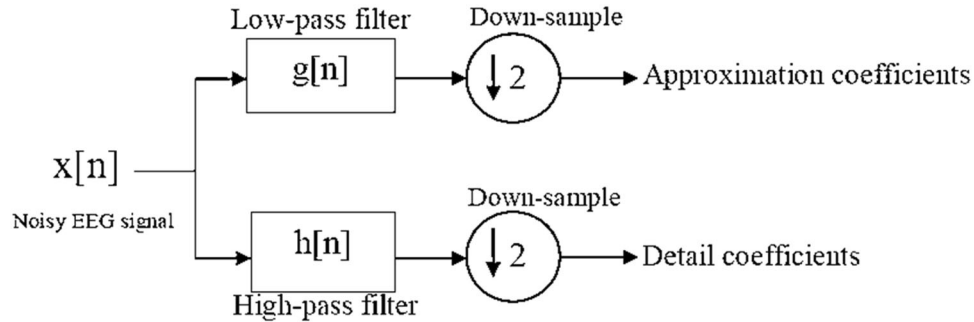


FIGURE 7. Structure of discrete wavelet transform decomposition process at level 1.

eters before execution: wavelet basis function and decomposition level.^{22,49,81,61} The similarity between the wavelet basis function and the desired signal is the basic criterion for selecting the bases wavelet function.

However, the main challenge in the SWT-based algorithm is to choose the optimal level of decomposition.^{61,53} The selection is essential because the EEG signals sample at different rates may need a different number of decomposition levels. The straightforward technique is to employ full tree decomposition of the noisy EEG signal and then filter the artifactual components with various SWT filters. But, this technique led to unnecessary computational burdens. Therefore, limited comparative studies are found to determine the best wavelet properties. Most of the studies randomly

chose either one among them for denoising purposes. Table 6 shows the comparative studies to obtain the optimal SWT filter for denoising artifacts in EEG signals.

Discrete Wavelet Transform

The DWT is classified as a linear transformation method that performs on a coefficient vector. The length is the integer power of two, where it transforms into a numerically different vector of the same size.^{39,19,11,43} The execution process of DWT for one-dimensional (1-D) signals splits the input signal into two sections: a high-frequency component and a low-frequency component. This splitting process is known as signal decomposition.^{39,19,11,43} The edge compo-

TABLE 7. Comparative analysis of DWT filters for EEG denoising from past works.

DWT filter	EEG dataset	Optimal wavelet filter	Ref
Haar, db4, sym6, coif1, dmey	Subject performs a various mental task	Sym6, coif1, dmey	Patil <i>et al.</i> ⁶⁶
db2,db6, db8, and dmey	Epileptic patient and healthy subject	db8, dmey	Al-Kadi <i>et al.</i> ⁶
Db, sym, coif, bior, rbio, and dmey	Motor imagery	db5, db7, db9, sym7, coif5, rbio1.5, dmey	Gorji <i>et al.</i> ²⁷
Daubechies (db1-db20), Symlets (sym1-sym20), Coiflets (coif1-coif5)	Subject perform working memory task	sym9	Al-qazzaz <i>et al.</i> ⁷
db2, db6, db8, and dmey	Subject in various sleep stages	db8	Thamarai <i>et al.</i> ⁸⁶
db4, db10, sym8, and Haar with decomposition level 4	Physionet database	db4	Kumar. ⁴⁷
Symlet, haar, coif, bior4.4	Physionet database	haar	Harender <i>et al.</i> ³⁰

nents of the signal are primarily confined to the high-frequency part. The signal will pass through a series of high-pass filters to determine the high-frequency components and low-pass filters to determine the low-frequency components. These filters come with different cutoff frequencies to process the input signal at different resolutions. As a result, the wavelet coefficients are reduced to half of those at the next lower scale. Figure 7 illustrates the structure of the DWT decomposition process of raw EEG signals.

The wavelet becomes more localized in time when the scale decreases. The original signal is initially passed through the half-band high-pass and low-pass filters. After filtration, half of the samples can be removed. Based on Nyquist's rule, the recent signal has the highest frequency of $\pi/2$ radians instead of π . Then, the signal is subsampled by two that discard every second sample. This process can be repeated for further decompositions. The produce outputs of the low-pass and high-pass filters are known as DWT coefficients. These DWT coefficients allow the reconstruction of the original signal through the process known as the inverse discrete wavelet transform (IDWT).⁴³ The reconstruction process is done in reverse order to the decomposition process. First, the signals at every level are upsampled by 2, then passed *via* the synthesis filters and low-pass and high-pass, respectively, before adding to each other. The analysis and synthesis filters are identical to each other, except for time reversal. Table 7 shows the comparative studies to obtain the optimal DWT filter for denoising artifacts in EEG signals.

Advanced Wavelet-Based Methods

Stationary Wavelet Transform Based Kurtosis

Improvement of the basic SWT is required to improve the signal process outcome, especially for denoising purposes. As stated earlier, selecting a decomposition level is essential for SWT process design. In this method, kurtosis is automatically used as the criterion for SWT decomposition when it reaches the artifact/noise components.⁸¹ Thus, the kurtosis will detect the presence of artifacts in the EEG signal. The kurtosis value is also crucial because values below three will attribute to either abruption or persisting varying sample values in EEG recording. The EEG signal is quite dynamic; thus, that behavior cannot represent brain activity. During the SWT decomposition process, the kurtosis of approximation components is calculated at every two consecutive levels.⁸¹ Therefore, the absolute difference λ of the computed kurtosis values is the decisive factor for continuing or terminating the SWT decomposition process. The SWT is assumed to achieve the optimal decomposition level of artifact denoising if the $\lambda > T$ and the process stop. The T value is determined based on the lowest error between the artifact-free and denoised EEG signals. Meanwhile, the final approximation component is denoised to remove the artifact of low-frequency components. The obtained detail components were also denoised to eliminate the artifact of high-frequency components. The filtration of detail components is executed according to the thresholding function. Shahbakti *et al.*,⁸¹ had applied SWT-Kurto-

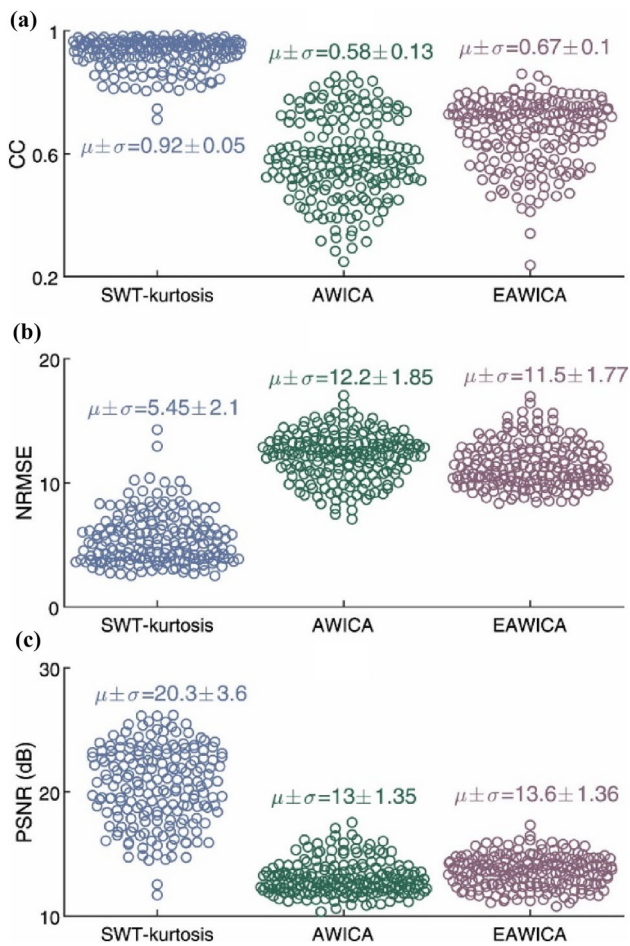


FIGURE 8. Comparison of SWT-kurtosis, AWICA, and EAWICA denoising performance based on Beeswarm plot: (a) CC, (b) NRMSE, and (c) PSNR. The μ and σ represent the mean and standard deviation, respectively.⁸¹

sis based algorithm for removing the electrical shift and linear trend artifacts (ELST) that are present in the EEG signals contribute to the temporary fluctuation of electrode–skin contact. The performance of this SWT-kurtosis technique in denoising ELST was outstanding when compared with the Automatic Wavelet Independent Component Analysis (AWICA) and Enhanced AWICA (EAWICA) algorithms (Fig. 8).

Discrete Wavelet Transform Based Scalar Quantization

The scalar quantization is usually used to approximate and compress the signals, where the lossy compression leads to inevitable information loss. However, Balamareeswaran *et al.*,¹¹ proposed the quantization-based DWT method for effectively filtering and compressing the signal. Besides, this method also minimizes the complexity of the circuit. Scalar quantization is

classified into uniform quantization, non-uniform quantization, and adaptive uniform quantization. Balamareeswaran and teamwork selected the adaptive uniform quantization in their research to be combined with the DWT method. Uniform quantization is a backward adaptive quantization that employs the feedback from the output of the quantizer output or equally from the coded sequence of the process. The step quantizer boundary (step size) varies with the adjustable quantization scheme. The output will be averaged within the input of the sequence to the quantizer. In this method, the EEG signal was initially decomposed into coefficients and then thresholding to remove the unwanted signal. Lastly, the signal is reconstructed and undergoes the transceiver's scalar quantization process for signal compression before determining the denoising performance parameter. The reconstructed signal's main aim in compressing or quantizing after filtration is to reduce the number of bits required to store the transformed coefficients by minimizing the precision of those values.¹¹ This compresses the signal to a greater extent. The flows of the Discrete Wavelet Transform based Scalar Quantization method in denoising EEG signal are indicated in Fig. 9.

Wavelet-Based Semblance

The wavelet-based semblance methods motivate by Fourier transform semblance method that suffers from problems associated with the changes in frequency content or location of the dataset. Through wavelet, the semblance analysis allows the local phase relationships between the two datasets to be studied as a function of both scale (or wavelength) and time. The semblance analysis will compare two signals based on CWT and DWT *via* the phase correlations between its wavelet decomposition. Saavedra *et al.*,⁷⁵ proposed this method for denoising and time-window selection of EEG signals to improve event-related potential (ERP) detection. Table 8 shows that this method provides better letter-task accuracy and stability than other methods. However, this method is rarely used in EEG denoising may be due to the complicated execution process.

Discrete Orthonormal Stockwell Transform

The S-transform is also known as Stockwell transform, a hybrid method between short-time Fourier transform (STFT) and wavelet transform. It is classified as a frequency-dependent STFT or a wavelet transform with a corrected phase. The S-transform extends CWT that moves and localizes scalable

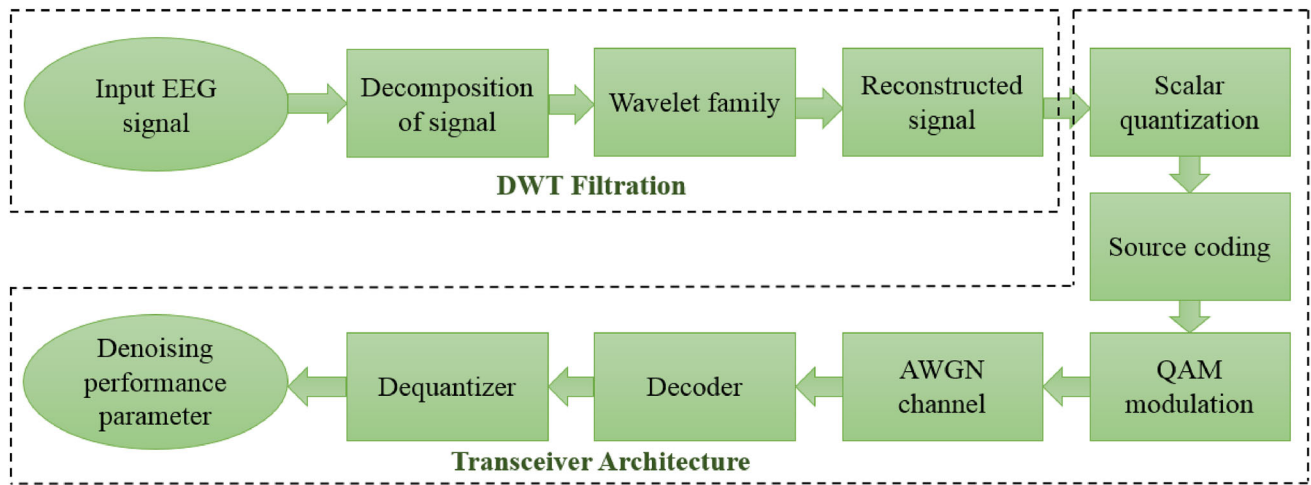


FIGURE 9. Discrete Wavelet Transform based Scalar Quantization method.

TABLE 8. The percentage accuracy of letter-task for selected methods using Coiflet level 3.

Method	Mean	Standard deviation	Minimum	Maximum
0.1–20 Hz filter	53.6	14.1	28.3	79.5
XDAWN	51.0	15.8	24.4	80.0
VisuShrink	54.8	13.9	33.0	78.6
Wavelet-based semblance	55.8	13.5	34.3	81.0

Gaussian window. The advantages of S-transform are it provides good localization in the frequency domain for low frequencies and good localization in the time domain for higher frequencies. Besides, unlike the CWT method, which only localizes the amplitude of the signal, the S-transform preserves both phase and amplitude information. The S-transform provides promising results for various applications, but its high computational complexity makes it infeasible for real-time applications. Through discrete orthonormal S-Transform (DOST), the computational and memory complexity issues of S-transform can be solved. The DOST employs an orthogonal set of base functions to preserve the local phase properties to detect the spectrum. Upadhyay *et al.*,⁹⁰ proposed DOST for noise suppression and removal from EEG signals. Four main steps are involved in EEG denoising using this method which is: (1) the noise-contaminated in EEG data was detected using blind source separation, (2) automated identification of artifactual independent components, (3) denoising the artifactual independent components *via* DOST method, and (4) reconstruction of clean EEG signal. Their results indicated that the DOST denoising performed better than wICA and IC rejection in removing spiky artifacts. The comparison of EEG signals after denoising using DOST, wICA, and ICA is shown in Fig. 10.

Wavelet Combine Independent Component Analysis

The independent component analysis (ICA) is interrelated with these three assumptions: (i) the experimental data is a spatially stable combination of the activities or events of temporarily independent cerebral and noise sources, (ii) the superposition of potentials obtained from different parts of the body, scalp, and brain is linear at the electrodes, and propagation delays from the sources to the electrodes are negligible, and (iii) the number of sources is no bigger compared to the number of electrodes.¹⁶ The wavelet combined with ICA (wICA) is proposed to reduce the risk of loss of useful cerebral information.^{14,83,62} This is because the artifactual independent components are not entirely rejected in this technique. The wICA method was initially introduced by Castellanos and Makarov,¹⁶ and novel modifications are proposed in more recent studies. Through wICA method, the artifactual independent components are marked after visual or automatic inspection.^{14,83,62} Then, the wavelet denoising of such independent components is executed as an intermediate step. This accommodates the recovery of neural activity that exists in artifactual independent components. The main challenge with this method is the estimation of threshold τ . Up to now, the acceptable threshold $\sigma_n\sqrt{2\log N}$ was proposed by Donoho and Johnstone in VisuShrink algorithm. The σ_n

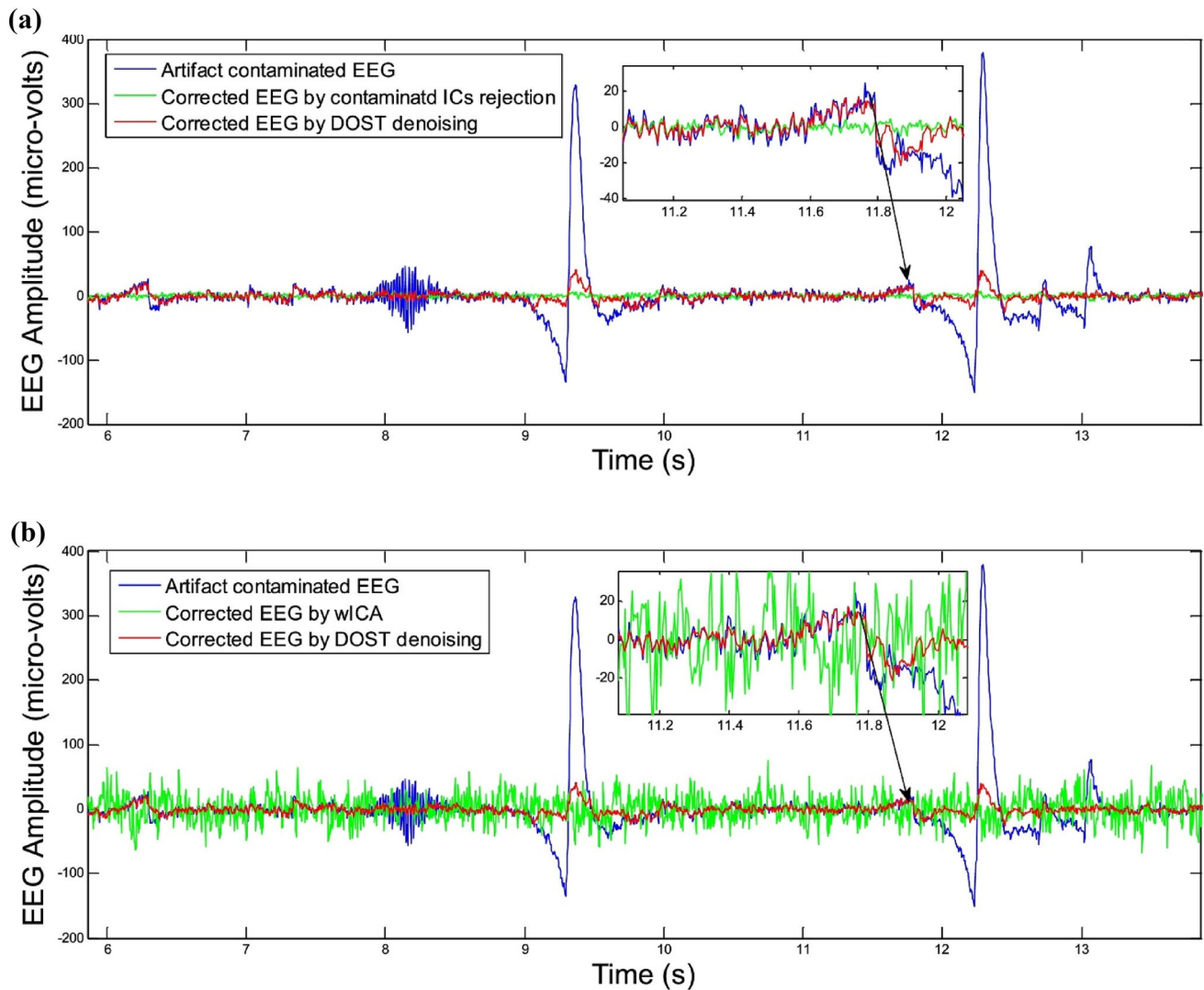


FIGURE 10. EEG signals denoising using different methods: (a) comparison between DOST and IC rejection and (b) comparison between DOST and wICA methods.

TABLE 9. Wavelet-based ICA method for denoising artifacts in EEG signal from past research works.

Type of contaminated artifacts	Type of wavelet combined with ICA	Description	Ref
Electrical shift, linear trend, EOG, and EMG	DWT, db4, decomposition level 4	The wICA had better denoising performance than wavelet and ICA methods	Inuso <i>et al.</i> ³³
Electrical shift, linear trend, EOG, and EMG	SWT, Haar, decomposition level 5	The wICA obtains better artifacts removal than zeroing ICA	Akhtar <i>et al.</i> ⁵
Eye blinks	DWT, Haar	The wICA obtains better eye blinks removal than zeroing ICA	Mahajan <i>et al.</i> ⁵²
50 Hz powerline	SWT, sym, decomposition level 10	The wICA shows better results with higher SNR than a low-pass filter	Sheoran <i>et al.</i> ⁸³
Eye blinks and movements	Haar, decomposition level 6	Eye artifacts can be successfully removed with minimal data loss	Burger <i>et al.</i> ¹⁵
50 Hz powerline	SWT, sym, decomposition level 8	The wICA shows better results with higher SNR than a low-pass filter	Kaushal <i>et al.</i> ⁴⁴
Eye blinks and movements	DWT, sym4, decomposition level 5	The wICA outperformed rejection ICA methods in most performance parameters	Issa and Juhasz. ³⁵

TABLE 10. Accuracy evaluation of VMD-DFA-WPT, VMD-DFA-DWT, and other methods.

Methods	Support vector machine (%)	Random forest (%)
DWT	97.80	94.09
WPT	96.70	94.70
EMD-DFA	98.00	97.21
EMD-DWT	98.01	95.81
EMD-WPT	98.51	98.07
VMD-DFA	99.31	99.10
VMD-DFA-DWT	99.87	99.51
VMD-DWT-WPT	99.97	99.88

stands for estimation of standard deviation of noise. The remaining challenge in wICA method is the selection of the best basis wavelet and threshold value. Table 9 showed the previous research that employed wICA based method to denoise artifacts in EEG signal.

Wavelet Combine Variational Mode Decomposition

Combining the wavelets-based method with variational mode decomposition (VMD) provides more efficient EEG denoising. This hybrid method is proposed to overcome the limitation of partially based variational approaches such as synchrosqueezing and Empirical Wavelet Transform.⁴² The limit of these methods are lack of mathematical modeling, improper handling of noise, no backward error correction permitted by the sifting process, wavelet transform's hard band limits, limited identifying methods for automatic analyzing artifactual independent components, and require to define boundaries of filter banks prior implementation.⁴² The VMD classify as a robust, well-defined time-domain analysis, fully adaptive, and intrinsic variational methodology. The VMD has a high-frequency division where it adaptively calculates the suitable bands and provides a concurrent estimate of corresponding modes. Thus, the VMD can properly equilibrate errors between them. The input signal is breakdowns into band-limited IMFs that are quasi-orthogonal and adaptive. The primary function of VMD is to decompose the real-valued signal 'f' into sub-bands (discrete signal) because these modes obtain exact sparsity property during reconstructing the input.⁴² The methods of EEG denoising using VMD combines wavelet transform method is as follow: (i) the raw contaminated EEG signal is decomposed into N number using VMD, (ii) the denoised signal is reconstructed *via* significant modes defined by detrended fluctuation analysis (DFA), (iii) apply wavelets based

windowing to suppress remaining noise and conserve the neural information exists in these modes, and (iv) the cleaner EEG signals are synthesized by adding windowed IMFs.⁴² Kaur *et al.*,⁴² had proposed VMD combined with DWT and VMD combined with wavelet packet transform (WPT) to denoise EEG signals of depression. In these methods, the detrended fluctuation analysis (DFA) was used to determine the mode selection properties. It began with the decomposition of a signal into several components. Then, the WPT and DWT were used to remove the artifactual components rather than completely rejecting these with DFA as the mode selection basis. The EEG signal based on artificially contaminated and real databases of depression was employed in their research. The results showed that both methods had sufficiently removed the artifacts. The denoising performance of the VMD-DFA-WPT was outperformed by the VMD-DFA-DWT. The accuracy of denoising performance for VMD-DFA-WPT, VMD-DFA-DWT, empirical mode decomposition (EMD) based technique and other methods are shown in Table 10.⁴² Based on accuracy evaluation, the VMD-DFA-WPT and VMD-DFA-DWT showed the highest accuracy than others, which indicates that more artifacts have been removed through these methods.

Wavelet Combine Blind Source Separation

The blind source separation (BSS) and wavelet transform is another method for EEG denoising. The BSS refers to the separation of unknown source signals from mixtures of signals without the aid of information (or with very little information) about the source signals or the mixing process. Among the BSS algorithms are independent component analysis (ICA), canonical correlation analysis (CCA), second-order blind identification (SOBI), joint approximate diagonalization (JADE), algorithms for multiple unknown signals extraction (AMUSE), and principal component analysis (PCA). Mowla *et al.*,⁵⁶ introduced a combination of BSS with the wavelet transform method to denoise the muscle and eyes movements artifacts from EEG signals. Two combinations of BSS and wavelet transform methods were applied in their work: CCA-SWT for denoising muscle artifacts and SOBI-SWT for removing eyes artifact. The advantage of this method is it can localize the artifact to time, frequency, and components. The utilization of multiple BSS methods led to more robust fragmentation and minimized the artifact to a few boundary components. In addition, the wavelet transform method aids the artifact to be removed based on the time and frequency involved. They found that the combination of both

methods is the best for removing the muscle and eyes artifact movements than the single method.

DISCUSSION

Brain signal processing is a critical research area that requires further discovery and studies to improve the quality of output signals and better information interpretation.^{100,50} The EEG is a standard brain imaging tool used to record the electrical activity from the human scalp through electroconductive media. However, the capture signal is always contaminated with the artifact that comes from internal and external sources. The earlier precaution can be considered, such as asking the participant to limit movement, keep away the power line interference sources, and tighten the electrode placement. Nevertheless, this way is not always efficient, especially for long-time signal acquisition and experiments involving movement and physical tasks.^{13,57} The best way is by utilizing a computational approach to filter the artifacts. It is crucial to give enormous attention to the artifact filtration stage because it influences the features extraction and final interpretations. A traditional method such as Fourier transform is one method that can be used to process the EEG signal. But, the researchers found that it is more suitable for processing discrete and stationary signals.

Meanwhile, the EEG is classified as a continuous, non-stationary, and time-frequency transient property, requiring a method to provide three-dimensional information. The wavelet method with a wave-like oscillation is efficient for both non-stationary signal denoising and feature extractor.^{4,26} According to the literature, there are four main types of the dataset used to investigate the effectiveness of the proposed wavelet-based method: open-source EEG dataset, experimentally based EEG dataset, simulated EEG dataset, and semi-simulated EEG dataset. In denoising research, the clean EEG signal will commonly be added with fake or simulated noise to compare the effect before and after filtration. Various types of computational methods are used for artifact denoising from EEG signals. The findings from the literature concluded that the wavelet-based method is among the most effective approaches for EEG processing. Two basic wavelet types are usually utilized in EEG denoising: discrete wavelet transform (DWT) and stationary wavelet transform (SWT). They typically come with low pass and high pass filters to decompose the input signal into detail and approximation coefficients. Therefore, the capability of the wavelet-based method depends on three main criteria: wavelet type, wavelet function, and decomposition level. Besides, various advanced wave-

let-based methods have been proposed to improve the quality and robustness of the processed EEG signal.

This review article has discussed a few methods applied in EEG denoising. It can be said that the advanced method can enhance particular limitations of basic wavelets. For example, the SWT-based kurtosis algorithm and DWT-based scalar quantization can improve the denoising performance than basic SWT and DWT by introducing a kurtosis criterion and quantization that can detect the artifact in EEG signal.⁸¹ Besides, the other methods, such as wavelet-based semblance and DOST methods, also have unique characteristics that enhance EEG denoising, as discussed earlier in Sect. 6. In addition, the advanced method-based combination of wavelet method with other techniques also produces better denoising performance. This review discussed the wavelet combining ICA, wavelet combining VMD, and wavelet combining BSS. These advanced methods have improved the limitation of basic wavelet methods in EEG denoising. However, there is still a limitation with the wavelet-based methods in which there are no standard guidelines and criteria in selecting the mother wavelet and level of decomposition. Finally, we recommended a few suggestions that need to be embedded in future research:

- (a) Standard guidelines of wavelet type and decomposition level need to be investigated.
- (b) Improve the capability of the wavelet-based method in denoising the artifacts. Unfortunately, most of them can only remove certain artifacts without considering eliminating all artifacts contaminated by EEG signals.
- (c) The advanced wavelet-based method has a complex algorithm and process requiring a lengthy processing and understanding time. The new approach should have less computational complexities and execution time.
- (d) Less discovery of continuous wavelet transform (CWT) method that also has the potential to denoise EEG signal.
- (e) Most hybrid wavelet-based methods use the ICA method. Less focus on other methods for combination with the wavelet method.
- (f) Propose a graphical user interface (GUI) using a wavelet-based approach to facilitate people with less knowledge of the computational algorithm.

Conclusion

EEG signals contain valuable information about the neural activity related to human behavior and actions. The EEG signal processing is required because the raw

EEG is always contaminated with several artifacts and the necessary features to be extracted for interpretation. The unsuitable usage of the method will lead to misinterpretation of information and the artifacts being unable to remove altogether. The basic SWT and DWT are among the most common methods preferred for EEG denoising. However, the quality of denoising performance is not too good because not all artifacts can be removed, influencing information interpretation. The usage of advanced approaches can overcome the issue of the basic wavelet method. However, the problem with these methods is a too complex algorithm and process that require a long execution time and a high level of computational and mathematical understanding. Therefore, a further modification in the wavelet-based method needs to be done to accommodate the people. Through this review article, the reader will learn about the common wavelet-based method used in EEG denoising and aids researchers in selecting the best wavelet method for their research purpose. Besides, this review article also can be a guideline for future research related to EEG denoising where more improvement can be suggested.

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CONFLICT OF INTEREST

The authors declare there is no conflict of interest for this article.

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