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TECHNICAL CONTRIBUTION

THE PHYSICAL SIGNIFICANCE OF TIME DOMAIN DESCRIPTORS IN EEG ANALYSIS

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The interpretation of an EEG, constituting a time series of observations of an electrical potential, is complicated by the lack of an adequate model to explain how features of the central nervous system are reflected in the observations. Such a model would be desirable to form the basis for a reduction of the EEG data to minimum redundancy, *i.e.*, to their simplest possible form for quantitative evaluation. Obviously the quantifying parameters should be closely related to basic qualities of the model, since the aim of a quantitative analysis is to assign numerical values to basic features of an observed system, in order to discriminate between different states of the system or to correlate these states with external variables.

The EEG may convey information by the sequential appearance of certain characteristics and by the quantitative relationships between these characteristics without respect to sequential order. The first case implies deterministic components in the material and the methods for their detection will be selective (pattern recognition). In the latter case the irrelevance of the sequential order allows for a more generalized statistical treatment and hence for considerable data reduction.

A basic way of expressing the characteristics of a time series in statistical terms is to refer to its auto-correlation function, which defines statistically the interdependence (cross-transfer ratio) between any two values in the series as a function of their difference in time. The auto-correlation function applies particularly well to physical situations, since the constants which define a physical system and its characteristic response, also determine the auto-correlation function of the response. As an EEG does, in fact, represent a series of measurements of a physical quantity, this suggests that a physical system might be useful as a model for the reduction of EEG data, in which the EEG is considered as a superposition of characteristic responses. A general physical system is obviously valid as a model, as long as there are no restraints on the order (complexity) of the system. The minimum order at which it may still be valid is then a question to be discussed further.

The proposed method concerns the derivation of quantifying parameters and a proof of their efficiency in describing the auto-correlation function. The parameters have been introduced by Hjorth (1970), in a less generalized form, as descriptors of the graphical characteristics of an EEG trace in terms of amplitude, slope and slope spread. The names of these descriptors, "activity", "mobility" and "complexity" have been retained, but the descriptor "complexity" has been redefined as "complexity of the first order", C_1, giving an absolute measure of the slope spread in standard deviations per unit of time.

The following discussion concerns the application of the descriptors to superimposed physical responses, and it is shown that the descriptors convey information which is highly relevant for the description of the generating system.

METHOD

In the attempt to characterize an EEG in terms of interdependence between the values constituting the EEG trace, it is natural to make a further investigation of the auto-correlation function.

According to the Fourier transformation, the auto-correlation function R(t) is related to its frequency domain equivalent, the power spectrum $S(\omega)$, as

$$R(t) = \int_{-\infty}^{+\infty} \cos \omega t \cdot S(\omega) d\omega \Big/ \int_{-\infty}^{+\infty} S(\omega) d\omega$$

An expansion of the cosine factor in the numerator into powers of ωt gives

$$R(t) = \left[\int S(\omega) d\omega - t^2/2! \int \omega^2 S(\omega) d\omega + t^4/4! \int \omega^4 S(\omega) d\omega - \ldots \right] \Big/ \int S(\omega) d\omega$$

Since $\int \omega^n S(\omega) d\omega$ is identical to the spectral moment m_n of order n, the auto-correlation function can be rewritten as

$$R(t) = 1 - t^2/2! \cdot m_2/m_0 + t^4/4! \cdot m_4/m_0 - t^6/6! \cdot m_6/m_0 + \ldots$$

It has been shown, *e.g.* by Hjorth (1970), that the moment of order 2n corresponds to the variance σ_n^2 of the derivative of order n, so that

$$m_0 = \sigma_0^2,\ m_2 = \sigma_1^2,\ m_4 = \sigma_2^2,\ m_6 = \sigma_3^2, \ldots m_{2n} = \sigma_n^2$$

and hence the auto-correlation can be expressed as

$$R(t)=1-\frac{t^2}{2!}\left(\frac{\sigma_1}{\sigma_0}\right)^2+\frac{t^4}{4!}\left(\frac{\sigma_1}{\sigma_0}\right)^2\left(\frac{\sigma_2}{\sigma_1}\right)^2$$
$$-\frac{t^6}{6!}\left(\frac{\sigma_1}{\sigma_0}\right)^2\left(\frac{\sigma_2}{\sigma_1}\right)^2\left(\frac{\sigma_3}{\sigma_2}\right)^2+\ldots$$

By introducing the descriptors

activity $=\sigma_0^2$,

mobility $=M=\sigma_1/\sigma_0\ (M^2=\sigma_1^2/\sigma_0^2)$

and

complexity of order $n=C_n=(\sigma_{n+1}^2/\sigma_n^2-\sigma_n^2/\sigma_{n-1}^2)^{\frac{1}{2}}$ the auto-correlation can be further modified to yield

$$R(t)=1-\frac{t^2}{2!}M^2+\frac{t^4}{4!}M^2(M^2+C_1^2)$$
$$-\frac{t^6}{6!}M^2(M^2+C_1^2)(M^2+C_1^2+C_2^2)+\ldots$$

or, in the form of a general polynomial,

$$R(t)=a_0+a_2\cdot t^2+a_4\cdot t^4+a_6\cdot t^6+\ldots$$

in which

$$a_0=1$$
$$a_2=-a_0\cdot M^2/1\cdot 2$$
$$a_4=-a_2\cdot(M^2+C_1^2)/3\cdot 4$$
$$a_6=-a_4\cdot(M^2+C_1^2+C_2^2)/5\cdot 6$$
$$\vdots$$
$$a_{2n}=-a_{2(n-1)}\cdot\sum_{v=0}^{n-1}C_v^2/((2n-1)(2n))\quad(M=C_0)$$

For non-periodic phenomena with a limited complexity —like that of a physical response—the basic information is essentially contained in the first few polynomial coefficients. This means, with reference to the time domain descriptors, that the number of required (non-redundant) descriptors corresponds to the complexity of the system under observation.

RESULTS

Although no broad experience of the descriptors has been collected as yet, some indications of their usefulness as clinical variables have been reported. The results reported by Binnie *et al.* (1973) show that two descriptors give sufficient information to replace frequency analysis in monitoring the state of the patients in an actual group suffering from minimal hepatic encephalopathy.

Fig. 1 gives an illustration of how the descriptors reflect different states of sleep and waking. The descriptors refer to a fronto-parietal recording from a rat with implanted electrodes[1], and were computed for each epoch of 5 sec. For automatic discrimination between, *e.g.*, paradoxical sleep and waking, by means of the descriptors, it is necessary to insert a threshold for each descriptor between the averages referring to the corresponding states. By defining for each state the criterion that all three descriptors must simultaneously appear on the right side of the corresponding threshold, a sufficient ratio between the numbers of correct and false indications will be obtained, so that the two states can be distinguished in a simple filtering procedure. The purpose of Fig. 1 is, however, merely to illustrate how the characteristics of the different states are preserved by the descriptors, in spite of the extensive data reduction compared with the original EEG trace.

DISCUSSION

The proposed method for time domain analysis of an EEG is basically a means to derive the auto-correlation function of the curve in terms of polynomial coefficients. The auto-correlation function conveys all information which can be obtained from power spectral analysis or from conventional frequency analysis in which the phase relations between the contributing frequencies are normally neglected. A comparison between the methods of frequency domain analysis and time domain analysis reveals two essential advantages of the time domain method; a simple calculation, which can be performed continuously during on-line record-

[1] The recording was made by M. Matějček of Sandoz, Basel.

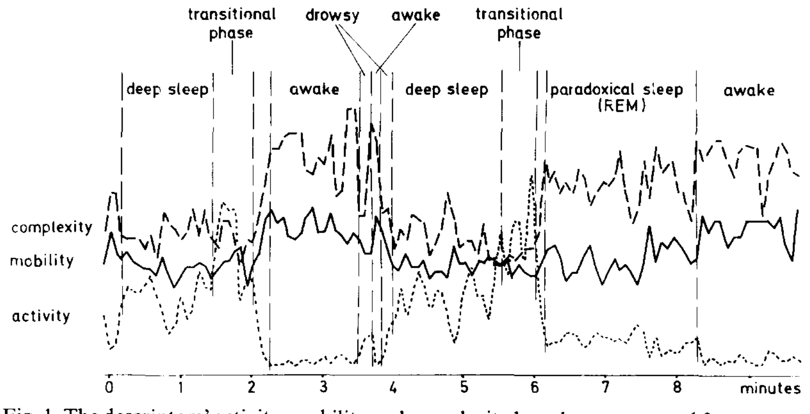

Fig. 1. The descriptors' activity, mobility and complexity have been computed for every epoch of 5 sec, during a frontoparietal recording from a rat with implanted electrodes. The descriptors display characteristic patterns, corresponding to different states of sleep and waking. These states were estimated from the original EEG trace. (By courtesy of M. Matějček)

ing, and the possibility of interpreting the result in terms of the physical system from which the EEG is derived. The advantage of computational simplicity is obvious—especially if the method of Saltzberg and Burch (1971) is applied—but the possibility of relating an observed physical quantity to properties of its source will deserve some further attention.

As an electrical potential or an image of a chemical state, the EEG is subjected to certain basic laws of physical systems. Although the actual system may be complex, some of its basic properties can be expected to characterize or limit the dynamics of quantities occurring in the system. The occurrence of an event in a physical system means that an amount of energy is suddenly made available to or released in the system. This energy is then dissipated by the system in a process which is defined by the determining constants of the system. This makes the process characteristic for the system and it is therefore named the characteristic response of the system.

A study of responses of basic physical systems, in terms of time domain descriptors, gives some interesting results. The system response of a general system of the first order is an exponentially decaying impulse, mathematically expressed as $\exp(-\alpha \cdot \text{time})$, in which expression α is the solely determining constant (the inverse time constant of the system) (Fig. 2). When the descriptors for this response are computed, the mobility turns out to be identical to α and hence describes the system. The corresponding result for a second order system, which has the response $\exp(-\alpha \cdot t) \cdot \sin \beta \cdot t$ (a decaying sinusoid) yields

$$\begin{cases} \text{M (mobility)} = (\alpha^2 + \beta^2)^{\frac{1}{2}} \\ \text{C (complexity)} = 2\alpha \end{cases}$$

After some algebraic manipulations, the two constants can be written as

$$\begin{cases} \alpha = C/2 \\ \beta = M \cdot (1 - C^2/4M^2)^{\frac{1}{2}} \end{cases}$$

Thus the system is fully determined by mobility and first

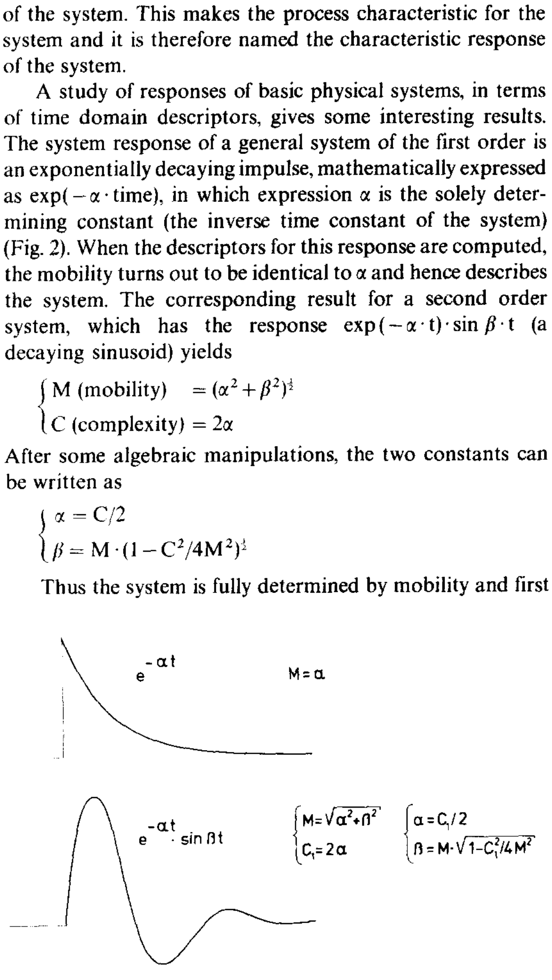

Fig. 2. Characteristic responses of fundamental physical systems to the sudden release of energy. A first order system, defined by the system constant α, gives a response (*upper curve*) which has a mobility (M) identical with the system constant. A second order system requires both mobility and complexity (C_1) of its response (*lower curve*) for determining the two system constants (α and β) as given by the corresponding algebraic expressions.

order complexity, which indicates that the descriptors very efficiently convey basic information on the system, since the number of required descriptors corresponds to the order of the system under observation. Due to practical limitations, there is a fastest change which can occur in the system or be taken care of in the computing process, and this necessitates a correction of the descriptor complexity by a factor $(1 + 1/\alpha \cdot \tau)^{\frac{1}{2}}$, in which τ is a measure of the obtainable time resolution.

The second order physical system may be sufficient as an approximate model for autoregulatory processes in the inter-neuronal medium or in the cell membrane, as suggested by Sokolov (1962) and Sugiyama *et al.* (1970) respectively. Computer simulations of charge–discharge patterns in structures of cells having an excitation threshold and a local mutual dependence on a distributed energy supply, show that the transient response of such a structure is basically that of a second order system. The system constants then correspond to basic properties of the structure—*e.g.*, the compliance of the energy supply—and hence have a direct physiological significance as far as the model gives a good description of the physical (physiological) reality. The computer simulations also show that the energy state in the structure displays a propagating wave pattern during the regulatory process.

The response of a physical system is a definite function of time and it is defined by the constants of the system. The descriptors, however, do not refer to the exact shape of this function, but to ratios between statistical measures of its amplitudes and derivatives. This approach has the great advantage that the descriptor values (except activity) referring to a single response are valid also for a superposition of responses, since the ratios are not affected by the addition of further responses of arbitrary phase and amplitude. The important implication is that basic physical or physiological parameters of a system can be studied from a composite sum of randomly or pseudo-randomly superimposed responses. Fig. 3 shows a single system response and a computer-implemented superposition of 10,000 responses, for three different combinations of the constants of a second order system. When the mobility and the complexity of each superposition are computed and converted mathematically to system constants, these turn out to be identical with the constants of the system from which the superimposed responses were originally derived.

Fig. 3 also illustrates how the system constant β (mainly) is reflected statistically as a predominant rhythm in the superposition. This observation is an interesting complement to the physical chemical approach by Offner (1972), according to which the excitable membrane produces second order responses ("highly damped oscillations") which presumably contribute to the EEG, whereas no source of a sustained oscillation suggestive of the alpha rhythm is found.

The spectral appearance of a typical EEG cannot be satisfactorily explained as emerging from a second order system only (Fig. 3, extreme right). An increase of the order of the system in order to fit a typical EEG spectrum will, on the other hand, lead to a system which is very unlikely in a biological context. One way to extend the possibilities of a second order system to account also for the more detailed features of a typical EEG is to assume that the probability for release of a response is conditioned by the

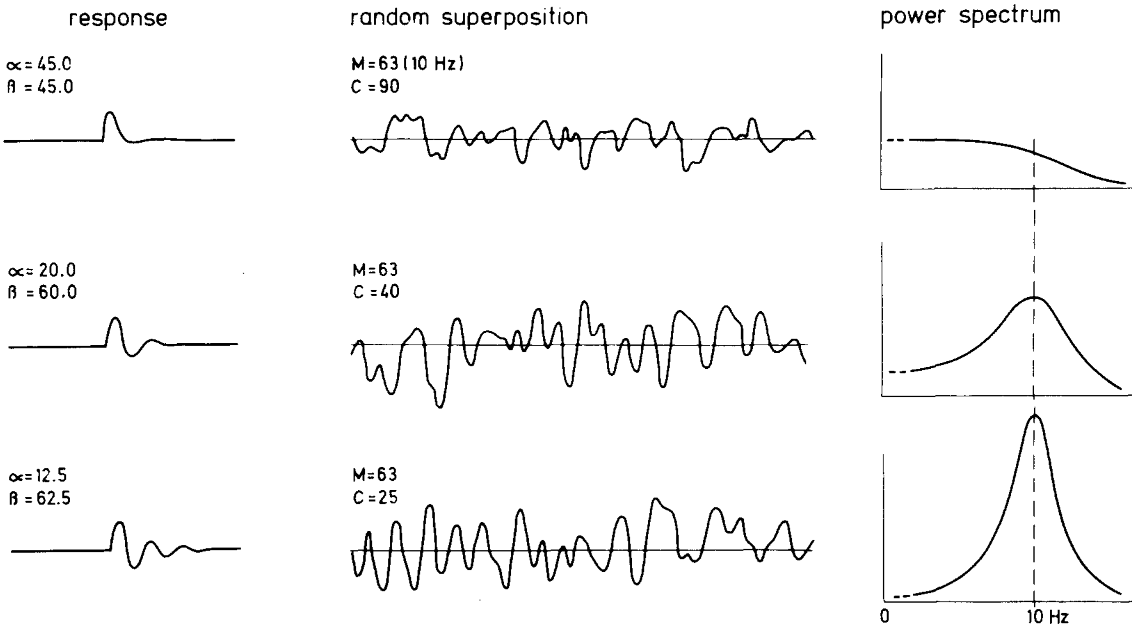

Fig. 3. The single response of a second order physical system, a superposition of 10,000 responses and the corresponding power spectrum, are shown for three different combinations of the system constants α and β. These constants can be restored mathematically from the mobility (M) and the complexity (C) of the superposition.

phase of a preceding response (Fig. 4). Such a deterministic coupling between the responses obviously improves the possibilities of adjusting the physical model to observed EEG data, since the corresponding trend towards periodicity will cause a spectral peak, similar to the spectral representation of the alpha activity. Furthermore, the non-sinusoidal shape of the response will cause harmonics of the periodicity, the second of which is comparable with the beta activity of the EEG. Also the DC component of the response will account for slow potential variations, reflecting variations in the response density, provided the directional distribution of the source structure is not homogenous.

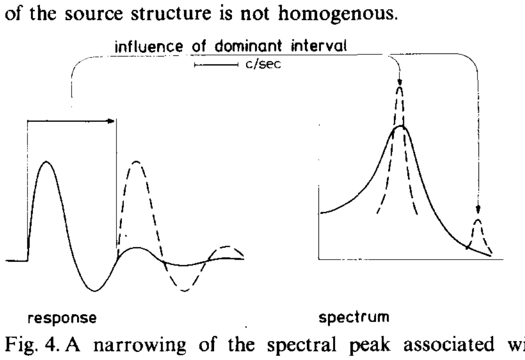

Fig. 4. A narrowing of the spectral peak associated with second order responses, as well as the formation of another peak at twice this frequency; can be explained as the result of the probability for release of a response being conditioned by the phase of a preceding response in a superposition. Such a coupling, giving statistical dominance to a certain interval between responses, will also enhance the occurrence of rhythmic activity in the composite curve.

The characteristic patterns formed by activity, mobility and complexity during different states of sleep, as well as preliminary results from the application of descriptor analysis to clinical situations, indicate that the descriptors may convey basic information concerning physiological states. By correlating the physiological variables with system constants, the generalized physical model can be sustained or rejected as a good description of the physiological reality. The descriptors, however, have a validity of their own as a tool for an objective quantitative description of the EEG curve. The model illustrates, independently of its validity, that the descriptors are very efficient in the handling of data related to a physical situation, and this may be compared with the obscurity of frequency analysis which results in just another (Fourier transformed) pattern. In fact, the concept of frequency is questionable as to its relevance to the description of phenomena which are known not to be periodic or to have a sine shape. This has been further discussed by Levine *et al.* (1972). The physical significance of the time domain descriptors, as discussed in this paper, gives strong support to the time domain approach to EEG analysis.

SUMMARY

Further analysis and development of a method for quantification of EEG characteristics, proposed by Hjorth (1970), show that the proposed time domain descriptors convey all information which is obtainable from conventional frequency analysis. The application of the descriptors to superimposed responses of general physical systems reveals that they convey information which is highly relevant for

describing the generating system. Thus, the descriptors not only constitute a tool for a simplified quantitative analysis of the curve characteristics, but they also offer interpretations in terms of physical models suggesting that the descriptors, or other parameters derived from the descriptors, may have physiological correlates. Such a generalized physical approach to the EEG is supported by clinical results which indicate that the descriptors convey basic information concerning physiological states.

RESUME

LA SIGNIFICATION PHYSIQUE DES DESCRIPTEURS DE SERIES TEMPORELLES EN MATIERE D'ANALYSE EEG

L'analyse plus poussée et le développement d'une méthode de quantification des caractéristiques EEG proposée par Hjorth (1970), montrent que les descripteurs de series temporelles proposés transmettent toute l'information qu'il est possible d'obtenir à partir de l'analyse de fréquences. L'application de ces descripteurs à des réponses surimposées de systèmes physiques généraux révèle qu'ils transmettent des informations hautement adéquates à la description du système générateur. Ainsi, ces descripteurs constituent non seulement un outil d'analyse quantitative simplifiée des caractéristiques des courbes, mais ils offrent également des interprétations en termes de modèles physiques suggérant que les descripteurs, ou d'autres paramètres dérivés des descripteurs, puissent avoir des corrélations physiologiques. Une telle approche physique généralisée à l'EEG est validée par les résultats cliniques qui indiquent que les descripteurs transmettent l'information de base concernant les stades physiologiques.

The valuable contributions by Dr. Colin Binnie and David Lloyd of St. Bartholomew's Hospital to the clinical investigation of the proposed method, as well as the constructive criticism by Dr. Bernhard Watson of the University of London, are gratefully acknowledged.

REFERENCES

BINNIE, C. D., BROWN, R., LLOYD, D. S. L. and SMITH, M. Some electroencephalographic and psychological findings in minimal hepatic encephalopathy. *Electroenceph. clin. Neurophysiol.*, **1973**, *34*: 108P.

HJORTH, B. EEG analysis based on time domain properties. *Electroenceph. clin. Neurophysiol.*, **1970**, *29*: 306–310.

LEVINE, P. A., ELASHOFF, R., CALLAWAY, E., PAYNE, D. and JONES, R. T. Evoked potential analysis by complex demodulation. *Electroenceph. clin. Neurophysiol.*, **1972**, *32*: 513–520.

OFFNER, F. Theory of membrane excitability. *Electroenceph. clin. Neurophysiol.*, **1972**, *33*: 243.

SALTZBERG, B. and BURCH, N. R. Period analytic estimates of moments of the power spectrum: a simplified EEG time domain procedure. *Electroenceph. clin. Neurophysiol.*, **1971**, *30*: 568–570.

SOKOLOV, E. N. Nature of the background rhythm of the cortex of the cerebral hemispheres. In *Basic problems of electrophysiology of the central nervous system*. Publishing House of the Academy of Sciences of the Ukrainian SSR, Kiev, **1962**: 157.

SUGIYAMA H., MOORE, G. P. and PERKEL, D. H. Solutions for a stochastic model of neuronal spike production. *Math. Biosci.*, **1970**, *8/3-4*: 323–341.