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TECHNICAL CONTRIBUTIONS

EEG ANALYSIS BASED ON TIME DOMAIN PROPERTIES

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The need for quantitative methods in the description of an EEG trace has initiated many attempts to define suitable parameters. In a first approach to a system of such parameters it is necessary to recognize the qualitative difference between two main types of problems: (a) to define descriptive qualities for the general characterization of an amplitude/time pattern, *e.g.*, by the application of mathematical-statistical methods which also include frequency considerations; (b) to describe individual patterns or to detect specific patterns which can be defined in advance.

The following report will concern the first type only, in which the traditional way has been to convert the amplitude/time pattern into a frequency distribution. Since the phase information is then normally omitted, this conversion leads to a pattern of reduced complexity, although the basic problems of finding adequate descriptive qualities for a pattern remain. Since the generation of the EEG cannot be associated with the sine function concept on which the time–frequency conversion is based, the amplitude/frequency pattern must be considered as a purely descriptive system, not necessarily having any direct connection with the generating physical system. From this point of view a descriptive system based on time may be regarded as more relevant.

Measurements and evaluation of time intervals between zero line crossings in an EEG trace may look like a time approach but normally result in more or less clearly expressed frequency considerations. One example of a system which is really based on time was presented by Byford (1969), in which he suggested a characterization of the EEG trace by means of a histogram of occurring values of the first time derivative during the actual epoch. Although a high degree of data reduction is obtained by this method, we still end up with a set of patterns, the histograms, which do not directly lend themselves to quantification. The method to be described here uses a similar approach, taking into account not the detailed histogram, but one of its basic parameters, the standard deviation, and it is further generalized to include also the standard deviations of the amplitude and the second derivative. From these standard deviations has been derived a set of parameters intended as a clinically useful tool for the quantitative description of an EEG trace within each epoch.

The following parameters have been derived:

1. *Activity*, giving a measure of the squared standard deviation of the amplitude, sometimes referred to as the variance or mean power.
2. *Mobility*, giving a measure of the standard deviation of the slope with reference to the standard deviation of the amplitude. It is expressed as a ratio per time unit and may be conceived also as a mean frequency.
3. *Complexity*, giving a measure of excessive details with reference to the "softest" possible curve shape, the sine wave, this corresponding to unity. It is expressed as the number of standard slopes actually generated during the average time required for generation of one standard amplitude as given by the mobility. Due to the non-linear calculation of standard deviation this parameter will quantify any deviation from the sine shape as an increase from unity.

These three parameters will together characterize the EEG pattern in terms of amplitude, time scale and complexity. The three parameter values referring to each epoch can be either printed on-line at the end of the epoch or transferred to automatic calculation of averages, variabilities etc.

The statistically sound nature of the parameters makes them "survive" a Fourier transformation, signifying that they also have a meaning in the description of the power spectrum associated with the time domain pattern. Since the frequency aspect is more familiar to most EEG interpreters, the following presentation of the method will start from the frequency domain, where the shape of the power spectrum can be expressed in terms of statistical moments.

METHOD

The EEG trace within a certain epoch can be expressed as a function of time $f(t)$ and, by means of the Fourier transform, translated into a function of frequency $F(\omega)$. The data reduction obtained by neglecting the individual detail when extracting the general characteristics of the EEG corresponds to a neglect of the phase information in the frequency description. The phase is excluded by the multiplication of $F(\omega)$ by its conjugate $F^*(\omega)$, giving the power spectrum $S(\omega)$ as a result;

$$F(\omega)\cdot F^*(\omega) = S(\omega)$$

The complete frequency description as derived by means of the Fourier transform is always symmetrical with respect to zero frequency, *i.e.*, it has identical branches stretching into both positive and negative frequency (the latter being in fact no more mysterious than the analytical concept of frequency itself). A consequence of this symmetry is that, in a statistical approach to the shape of the frequency distribution, all odd moments will become zero. Thus there will be no information in the linear average or in the skewness, since these qualities constitute the first and the third moments, respectively. The general definition of a moment m of order n is

$$m_n = \int_{-\infty}^{+\infty} \omega^n \cdot S(\omega)\mathrm{d}\omega$$

Table I shows the systematic way in which the proposed parameters are derived with respect to spectral moments. The corresponding time operations are indicated in the right column of the Table. The transformation of the parameters between the frequency and the time domains is based on the energy equality within the actual epoch, *i.e.*, the total power in the frequency domain is identical to the mean power in the time domain. Thus

$$m_0 = \int_{-\infty}^{+\infty} S(\omega)\mathrm{d}\omega = \frac{1}{T}\int_{t-T}^{t} f^2(t)\mathrm{d}t$$

TABLE I

Derivation of parameters

Frequency		Time
Spectral moment	Parameter definition	Time operation
m_0	m_0 = activity = σ_a^2	$f(t)$
$m_1 = 0$	—	—
m_2	$(m_2/m_0)^{\frac{1}{2}}$ = mobility = σ_d/σ_a	$\mathrm{d}(f)/\mathrm{d}t$
$m_3 = 0$	—	—
m_4	$\dfrac{(m_4/m_2)^{\frac{1}{2}}}{(m_2/m_0)^{\frac{1}{2}}}$ = complexity = $\dfrac{\sigma_{dd}/\sigma_d}{\sigma_d/\sigma_a}$	$\dfrac{\mathrm{d}^2(f)}{\mathrm{d}t^2}$
$m_5 = 0$	—	—
$m_6, m_8, \ldots$	....	....

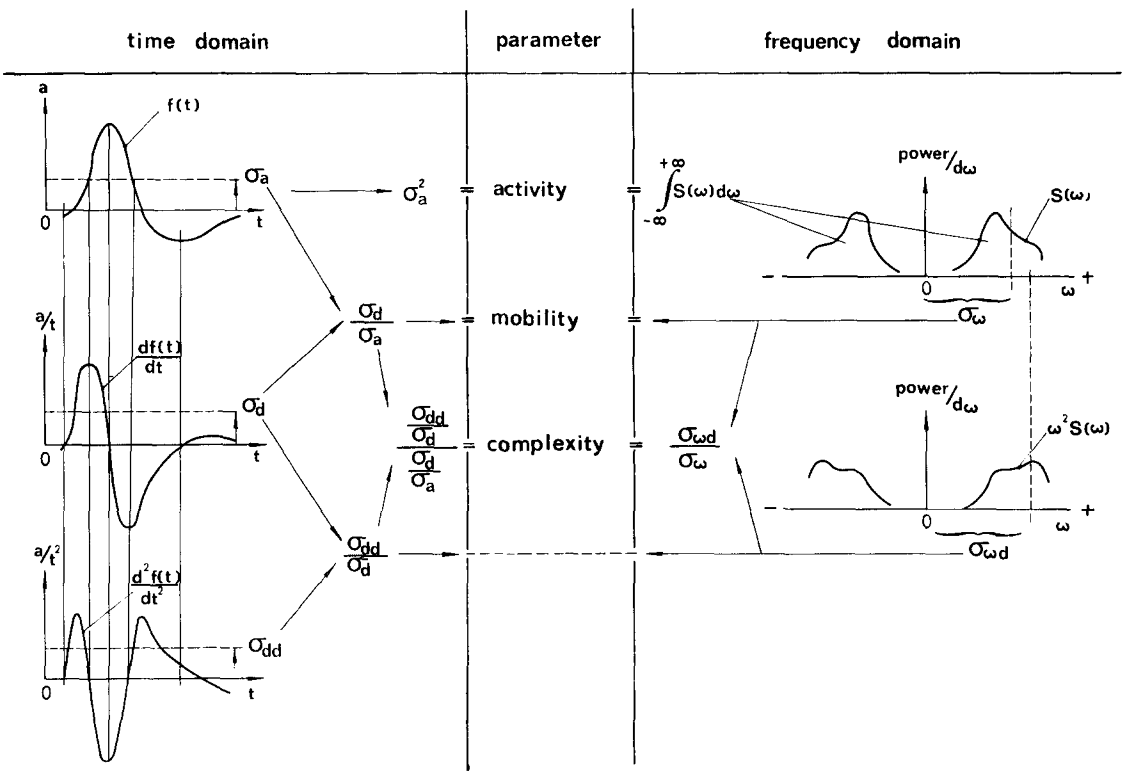

Fig. 1

Parametric connections between time domain properties and spectral characteristics of an arbitrary signal.

The mean power of the time function is recognized by the statistician as its variance σ^2.

Also, the second moment m_2 can be considered as a power, but then of another spectrum $\omega^2 \cdot S(\omega)$, corresponding to a frequency function $\omega \cdot F(\omega)$. The time equivalent of an ω-multiplied frequency function is the first derivative of the time function. Thus

$$m_2 = \int \omega^2 \cdot S(\omega)\mathrm{d}\omega = \frac{1}{T}\int_{t-T}^{t} \left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)^2 \mathrm{d}t$$

A repetition of this procedure gives the moment

$$m_4 = \int \omega^4 \cdot S(\omega)\mathrm{d}\omega = \frac{1}{T}\int_{t-T}^{t} \left(\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}\right)^2 \mathrm{d}t$$

Obviously the even moments of the power spectrum correspond to variances in the time domain. Since linear measures are more easily conceived in the frequency as well as in the time domains, the parameters (except activity) are preferably defined by means of the standard deviations, *i.e.*, the square roots of the variances. These quantities are indicated in Fig. 1. The following statements can be made concerning the parameters.

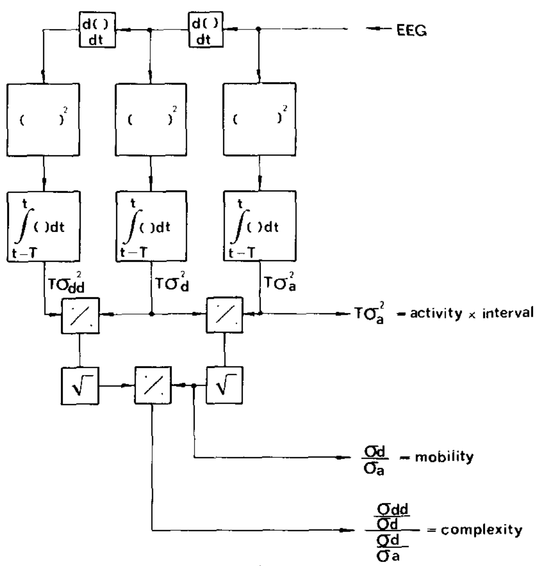

Fig. 2
Flow graph illustrating the calculation of parameter values.

1. *Activity*

The activity is quantified by means of the amplitude variance, which has the necessary additive property to allow integration of different observations during the epoch into one representative figure. In the frequency domain the variance of the time function can be conceived as the surface of the power spectrum.

2. *Mobility*

The mobility is defined as the square root of the ratio between the variances of the first derivative and the amplitude. Since these quantities are equally dependent of the mean amplitude, the ratio will be dependent on the curve shape only and in such a way that it measures the relative average slope. Its frequency domain interpretation becomes obvious if the ratio is considered as a power ratio:

$$\frac{\sigma_d^2}{\sigma_a^2} = \frac{\int \omega^2 S(\omega)\mathrm{d}\omega}{\int S(\omega)\mathrm{d}\omega} = \int \omega^2 \frac{S(\omega)}{\int S(\omega)\mathrm{d}\omega}\mathrm{d}\omega = \int \omega^2 p(\omega)\mathrm{d}\omega$$

$$\left(\int p(\omega)\mathrm{d}\omega = \int \frac{S(\omega)}{\int S(\omega)\mathrm{d}\omega}\mathrm{d}\omega = \frac{\int S(\omega)\mathrm{d}\omega}{\int S(\omega)\mathrm{d}\omega} = 1\right)$$

Since $p(\omega)$ is a density function whose integral is unity, the ratio can be recognized as the frequency variance of the power spectrum, *i.e.*,

$$\frac{\sigma_d^2}{\sigma_a^2} = \int \omega^2 p(\omega)\mathrm{d}\omega = \sigma_\omega^2$$

By considering the square root of the ratio it can be illustrated as the standard deviation of the power spectrum along the frequency axis (Fig. 1, upper spectrum).

3. *Complexity*

This parameter is dimensionless and derived as the ratio between the mobility of the first derivative of the EEG and the mobility of the EEG itself. The mobility of the first derivative is obtained in a way analogous to the mobility of the original signal (Fig. 1, lower spectrum), using the variance of the second derivative. The minimum value of the complexity is unity, corresponding to $\sigma_{wd} = \sigma_w$ which can occur only in the case of a discrete frequency in the spectrum, meaning a pure sine function in the time domain.

Although this descriptive system makes systematic use of the spectral moments, all parameters are instantaneously accessible in the time domain. The operations required

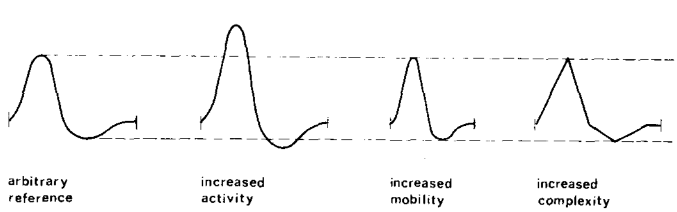

Fig. 3
Characteristic changes of a curve shape, illustrating the dependence of the individual parameters.

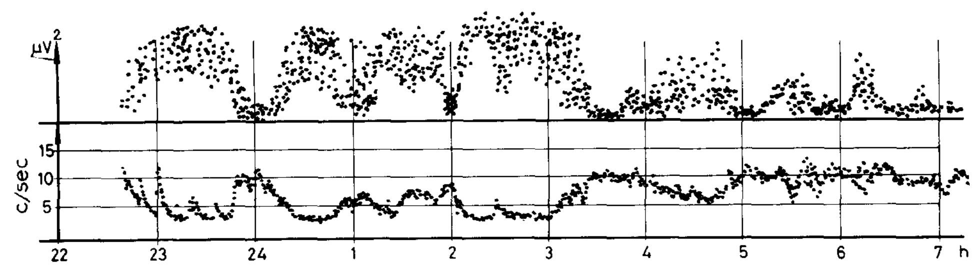

Fig. 4
Recording of amplitude variance (activity) and mean frequency during one night's sleep.

for the time domain calculation are shown in the flow graph in Fig. 2. A few basic operations occur in a regular structure and, when the analysis is applied simultaneously to several EEG channels, a sequential calculation by means of digital technique is probably the most economic solution.

RESULTS

Fig. 3 illustrates how the individual parameters vary with specific changes in the signal. Preliminary recordings of activity and another parameter (the mean frequency of a half-spectrum), very similar to mobility, have been made from sleep EEGs. A chart recorder was used to plot the averages from each epoch of 10 sec. The result from one night's sleep is shown in Fig. 4, in which the lower curve follows the mean frequency. There is undoubtedly a strong correlation between metabolic activity and mobility (average rapidity of the amplitude changes) (Sulg 1949). The results from a correlation analysis concerning regional cerebral blood flow and the proposed parameters, based on an extensive material, will be available later.

DISCUSSION

The method described here offers a way of quantifying the general characteristics of an EEG trace. It is shown that the proposed parameters can be expressed by the first 5 moments in a statistical description of the power spectrum and the question may be raised whether there is any more relevant information available in the power spectrum. Of course its detailed shape may be caused by special organizations or patterns, but the analysis of such phenomena will require phase information, which is deliberately avoided in the power spectrum. Probably no equipment for analysis of general characteristics of an EEG will be able to compete with the eye of a trained interpreter in the detection of specific patterns. On the other hand there is no possibility of making quantitative statistical estimates with any reasonable accuracy by means of subjective inspection. Quantification of very subtle changes in the general characteristics is sometimes desirable in order to determine correlations with physiological variables, drugs, etc. and then usually requires frequency analysis to give the power spectrum. By means of the parameters proposed here the basic properties of this spectrum can be derived without any frequency analysis, thus requiring less complex equipment.

As the parameters are based on the concept of variance they will inherit some of the characteristic additive properties of variance. This means that if a complex curve is composed of a large number of superimposed elements, the parameter values measured in the complex curve will pertain to the average basic element. The implications of these parameter properties with reference to EEG analysis are not yet fully recognized.

SUMMARY

A method to describe the general characteristics of an EEG trace in a few quantitative terms is introduced. Its descriptive parameters are entirely based on time, but they can be derived also from the statistical moments of the power spectrum. Thus the method provides a bridge between a physical time domain interpretation and the conventional frequency domain description. Further, the parameters are based on the concept of variance, giving them an additive property so that the measured values pertain also to any basic elements from which a complex curve may be composed by superposition.

The proposed method offers a way to on-line measurement of basic signal properties by means of a time-based calculation, requiring less complex equipment compared to conventional frequency analysis. The data-reducing capability of the parameters has been experimentally stated in the recording of "sleep profiles".

RESUME

ANALYSE EEG BASEE SUR LES SERIES TEMPORELLES

L'auteur introduit une méthode de description des caractéristiques générales d'un tracé EEG en un nombre limité de termes quantitatifs. Ses paramètres descriptifs sont entièrement basés sur le temps, mais peuvent être dérivés également des moments statistiques du spectre de puissance. Ainsi, cette méthode fait la jonction entre une interprétation du domaine des séries temporelles physiques et la description du domaine fréquentiel conventionnel. De plus les paramètres sont basés sur le concept de variance, leur donnant une propriété supplémentaire de telle sorte que les valeurs mesurées se

rapportent également à chaque élément de base à partir duquel une courbe complexe peut être composée par superposition.

La méthode proposée offre un moyen de mesurer "on-line" des propriétés de base du signal au moyen d'un calcul basé sur le temps, nécessitant un équipement moins complexe que l'analyse de fréquence conventionnelle. La capacité de réduction des données des paramètries a été spécifiée expérimentalement dans l'enregistrement des "profils de sommeil".

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A DIGITALLY CONTROLLED CONSTANT CURRENT STIMULATOR[1]

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In electrophysiological experiments it is often convenient to be able to vary stimulus intensity automatically during the course of an experiment. This is particularly true if the experimental parameters are controlled by a digital computer or computer-like device. However, the control lines emanating from many laboratory computers are referenced to ground, whereas the physiological situation requires that the entire stimulator should be totally isolated in order to minimize stimulus escape. The stimulator described here produces digitally controlled, ground free constant current pulses of from 0.25 to 15.75 mA in 0.25 mA steps. In addition, a precision potentiometer provides for continuously variable pulses of from 0.5 to 10 mA which may be added to the digitally derived pulses. The output current is voltage limited at approximately 200 V; consequently, the maximal output current into a 10 kΩ load is 20 mA. Pulse width under internal control is preset at 100 μsec; however, provision is made for external duration control when pulse widths of greater than 100 μsec are required.

Refer to schematic (Fig. 1). The stimulator is divided into two sections: a control section and an isolated current generator section. Power for the control section is derived from an external 4.75 V power supply; power for the isolated current generator is derived from three 67.5 V batteries (Burgess XX45 or equivalent). The output section is battery powered to minimize capacitive coupling and thus to reduce stimulus artifact. As the original unit was designed for use with a solid state programmer, all control inputs are compatible with TTL[2] logic. Stimulus levels may be set manually by disconnecting the control line inputs and setting the appropriate intensity control switches on the stimulator. (If automatic control is used, all manual intensity control switches must be set to the "on" position.)

Operation is as follows. A positive pulse on "Trig In" triggers the one-shot (03A and Q2). The one-shot keys the 5 Mc/sec multivibrator (03C and 03D). The multivibrator output is ended with each of the control input lines. Assuming line 8 only is up, the 5 Mc/sec signal appears on pin 3 of 02A and is coupled through C6 and T7 to D2, C13 and R11 where it is rectified and smoothed. The output of the filter network drives Q4 into saturation, thus switching the lower end of TP2 to the common line of the isolated output driver section. As the base of Q3 is held at a fixed voltage with respect to the common line, the collector current can be set by the series resistance R21 and TP2 since the emitter to base voltage and the current gain of Q12 are essentially constant. All other digital control inputs operate in a similar fashion. The continuously variable current control is implemented by switching the emitter of Q10 to some voltage above the common line voltage as determined by the 50 kΩ knobpot.

If stimulus pulse durations of greater than 100 μsec are required, the 5 Mc/sec multivibrator may be keyed by applying a pulse on the "Ext Dur" input line. The output pulse duration will then be that of the driving pulse. Note that the appropriate control inputs must be up while the multivibrator is being keyed.

To minimize capacitive coupling to ground and to the

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[2] Transistor–Transistor Logic, a mode of integrated circuit logic.