Temporal hierarchies in R

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Contents

1 Data and packages

We will model temporal hierarchies in two ways. First, we will use a package that gives us a somewhat limited control in the generation of the hierarchical forecasts. Second, we will use step by step code to do the combination manually, as it is a fairly simple process and gives us full control in generating any base forecasts we may want.

The relevant package is *thief*. For manual implementation we will make use of some supporting packages, namely: *MAPA* (this is in fact the 1st version of temporal hierarchies), *tsutils*, and *abind*.

Let us load the packages.

```
pckg <- c("thief","MAPA","tsutils","abind")
for (i in 1:length(pckg)){
  if(!(pckg[i] %in% rownames(installed.packages()))){
    install.packages(pckg[i])
  }
  library(pckg[i],character.only = TRUE)
}
```

```
## Warning: package 'thief' was built under R version 4.0.5
```

```
## Loading required package: forecast
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
## Loading required package: parallel
```

```
## Loading required package: RColorBrewer
```

```
## Loading required package: smooth
```

```
## Loading required package: greybox
```

```
## Package "greybox", v0.5.9 loaded.
```

```
## This is package "smooth", v2.5.6
```

```
## Warning: package 'tsutils' was built under R version 4.0.5
```

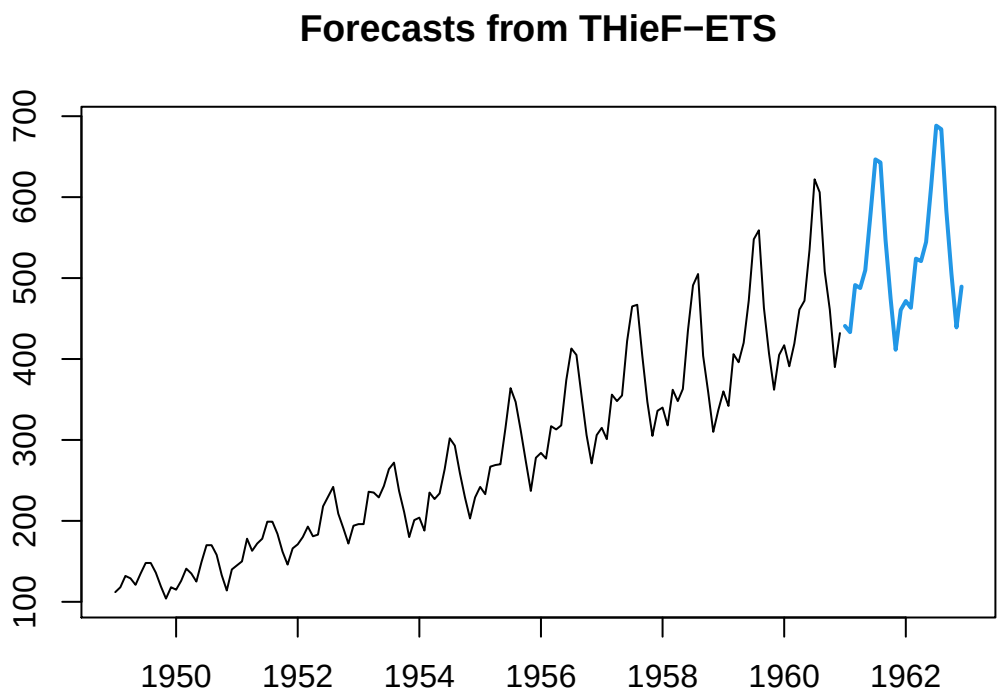
We will use the usual `AirPassenger` time series.

```
y <- AirPassengers
```

2 Temporal hierarchies using the thief package

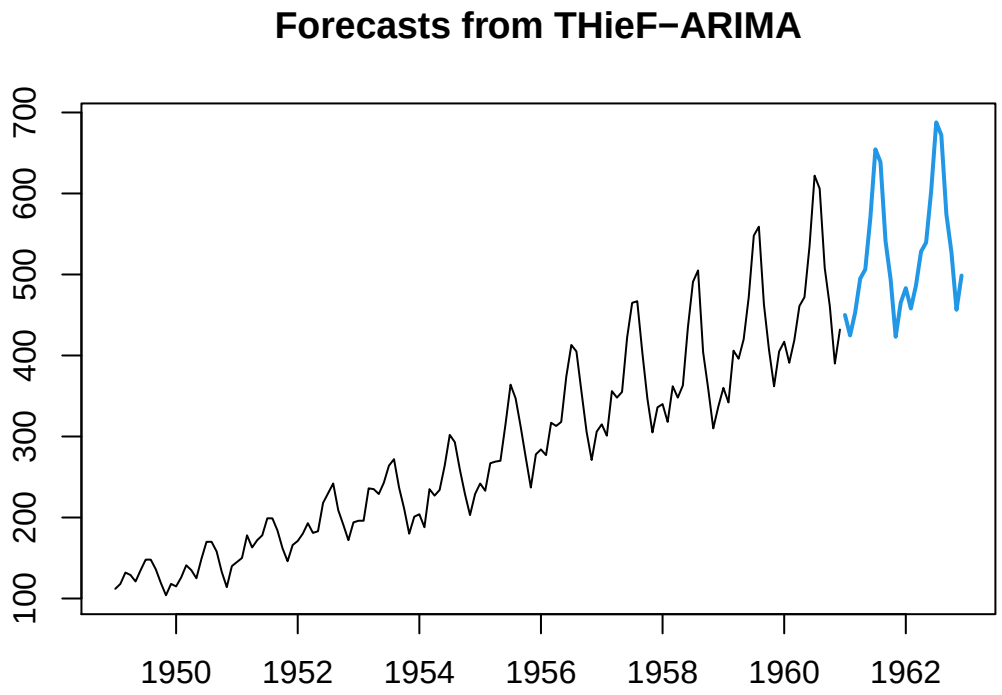
Using the function `thief()` the whole process is automated:

```
frc1 <- thief(y)
plot(frc1)
```



By default it uses ETS as the base forecast, we can change that using the argument `usemodel`. For instance:

```
frc2 <- thief(y,usemodel="arima")
plot(frc2)
```



Similarly, we can control the approximation of the covariance of the forecast errors that contribute to the estimation of the combination weights for the matrix G. This can be done with the argument `comb`. Look at the documentation of the function for more details `?thief`.

3 Manual implementation of THieF

First, we construct the summing matrix S, used to map the hierarchy and pick up from that the useful aggregation levels (we use only those that would not result in a fractional seasonality).

```
S <- tsutils::Sthief(y)    # Get the S matrix
ff <- frequency(y)         # Get sampling frequency of target series
AL <- ff/(1:ff)            # Calculate frequencies of various aggregation levels
AL <- AL[AL %% 1 == 0]     # And exclude those that would not be integer
k <- length(AL)            # Find how many are left
```

Next we create the temporally aggregated time series

```
Y <- MAPA::tsaggr(y,AL)[[1]]
```

We set the target forecast horizon, and calculate the corresponding forecast horizons for all temporal aggregation levels

```
hrz <- 16 # Target horizon
hAggr <- (ceiling(hrz/ff)*ff)/AL
hAggr
```

```
## [1]  2  4  6  8 12 24
```

Now we produce all the base forecasts, i.e., the independent forecasts for each level of the temporal hierarchy. For this example I am using ETS, but any forecasting method could be used. Note that this does not need to be the same across the various levels of the tmeporal hierarchy.

```
frc <- mse <- list()
for (i in 1:k){
  yTemp <- Y[[i]]
  fit <- ets(yTemp)
  mse[[i]] <- fit$mse
  frcTemp <- forecast(fit,h=hAggr[i])$mean
  # Re-structure forecasts
  frc[[i]] <- matrix(frcTemp,ncol=hAggr[1]) # Organised as column per year
}
```

Next, we re/arrange the forecasts into a matrix. First comes the most aggregate, and the rest follow.

```
frcAll <- abind(frc,along=1)
frcAll
```

```
##             [,1]      [,2]
##  [1,] 6117.6361 6596.1292
##  [2,] 3007.6633 3250.5458
##  [3,] 3129.1045 3371.9871
##  [4,] 1833.0574 1941.5106
##  [5,] 2393.3701 2532.2736
##  [6,] 1898.4091 2006.5253
##  [7,] 1355.3595 1459.7158
##  [8,] 1593.3745 1713.7677
##  [9,] 1887.3399 2027.3332
## [10,] 1383.1478 1483.8981
## [11,]  867.1471  929.9355
## [12,]  963.5666 1032.5139
## [13,] 1094.7423 1172.1632
## [14,] 1332.7008 1425.8650
## [15,] 1037.7271 1109.4450
## [16,]  881.9057  942.1689
## [17,]  441.8018  459.0139
## [18,]  434.1186  450.6333
## [19,]  496.6300  515.0797
## [20,]  483.2375  500.7700
## [21,]  483.9914  501.1423
## [22,]  551.0244  570.0974
## [23,]  613.1797  633.9130
## [24,]  609.3648  629.4938
## [25,]  530.5408  547.6630
## [26,]  463.0332  477.6340
## [27,]  402.7478  415.1573
## [28,]  451.9694  465.5780
```

Each column of frcAll corresponds to a period in the most temporally aggregate level, here the annual level. Rows contain the values for the forecasts at the various nodes of the hierarchy.

Now we estimate the G matrix that contains the combination weighs. We wil use here two easy approximations, the structural and the variance scaling.

```
# Structural:
W <- diag(1/rowSums(S))
Gstr <- solve(t(S)%*%W%*%S)%*%t(S)%*%W
# Variance:
mse <- unlist(mse)
W <- diag(1/mse[rep((1:k),rev(AL))])
Gvar <- solve(t(S)%*%W%*%S)%*%t(S)%*%W
```

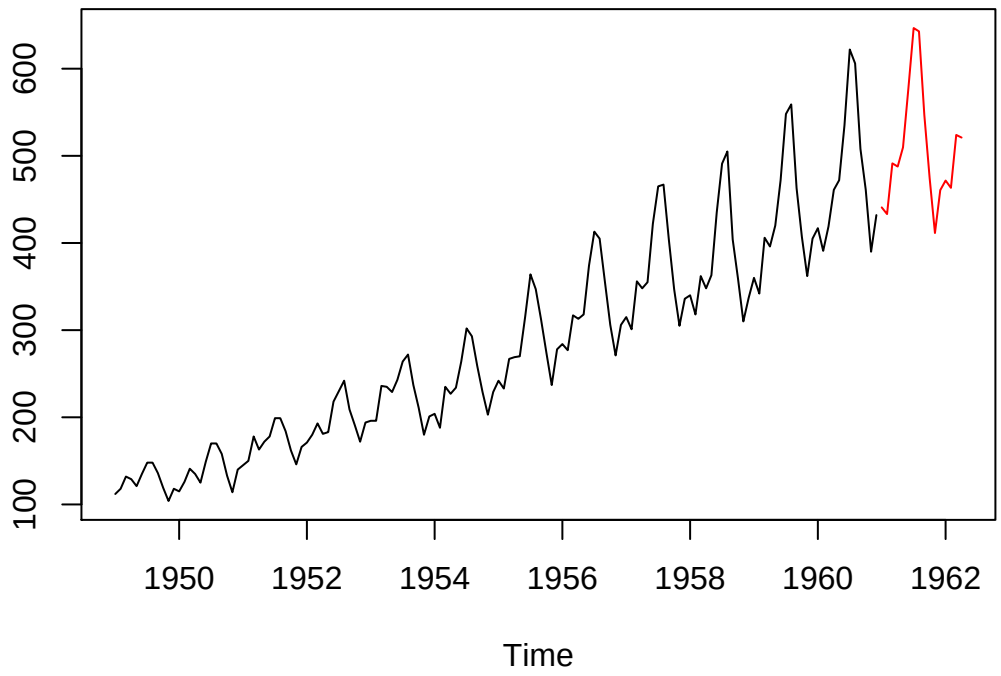
Note that the caclulation of G is the same in both cases. It is only the W that is different.

Now we reconclile (combine) the forecasts. For this example we use the Structural scaling, but we only need to replace G to obtain any other result.

```
# Create the bottom level forecasts
frcBRec <- Gstr %*% frcAll
frcFinal <- as.numeric(frcBRec)[1:hrz]
# We can also translate this into a time series object
frcFinal <- ts(frcFinal,frequency=frequency(y),start=end(y)[1] + deltat(y)*end(y)[2])
frcFinal
```

```
##           Jan      Feb      Mar      Apr      May      Jun      Jul      Aug
## 1961 440.8650 433.1818 491.3771 487.7213 509.7371 576.7701 646.6260 642.8111
## 1962 471.6625 463.2819 524.0035 520.9690
##           Sep      Oct      Nov      Dec
## 1961 547.0800 474.7431 411.4239 460.6455
## 1962
```

```
ts.plot(y,frcFinal,col=c("black","red"))
```



Or we can use the S-matrix to generate the forecasts for the complete hierarchy

```
frcARec <- S %*% frcBRec # Which has the same structure as frcAll
```

Happy forecasting!