



 POLITECNICO DI MILANO



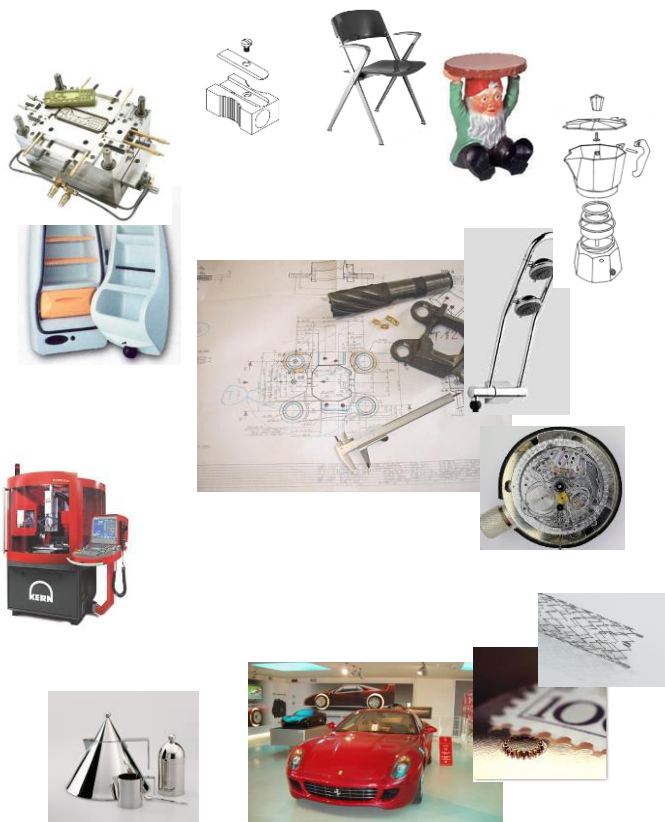
CMM: Uncertainty



The problem

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How to choose the correct CMM for the measurement of a part?



- Is the machine able to perform the measurement? → CAIP
- Is the measurement result accurate enough? → ...?



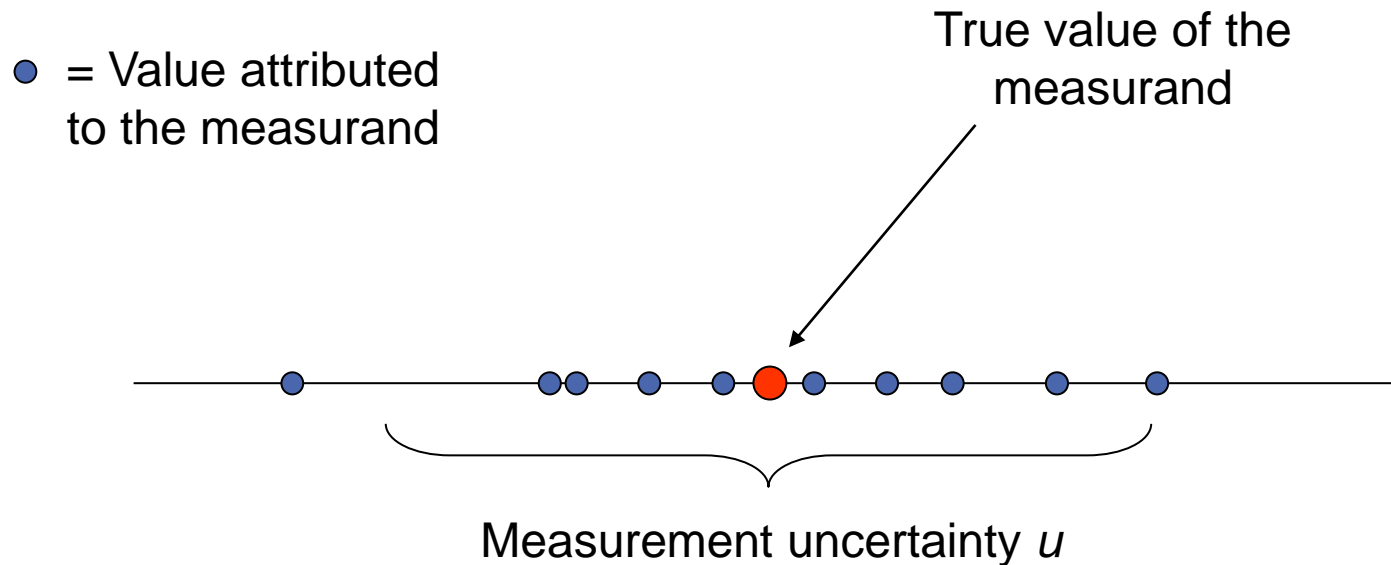
Measurement uncertainty

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Definition of measurement uncertainty (UNI CEI ENV 13005):

“non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used”

It's a measurement of the quality of the measurements!

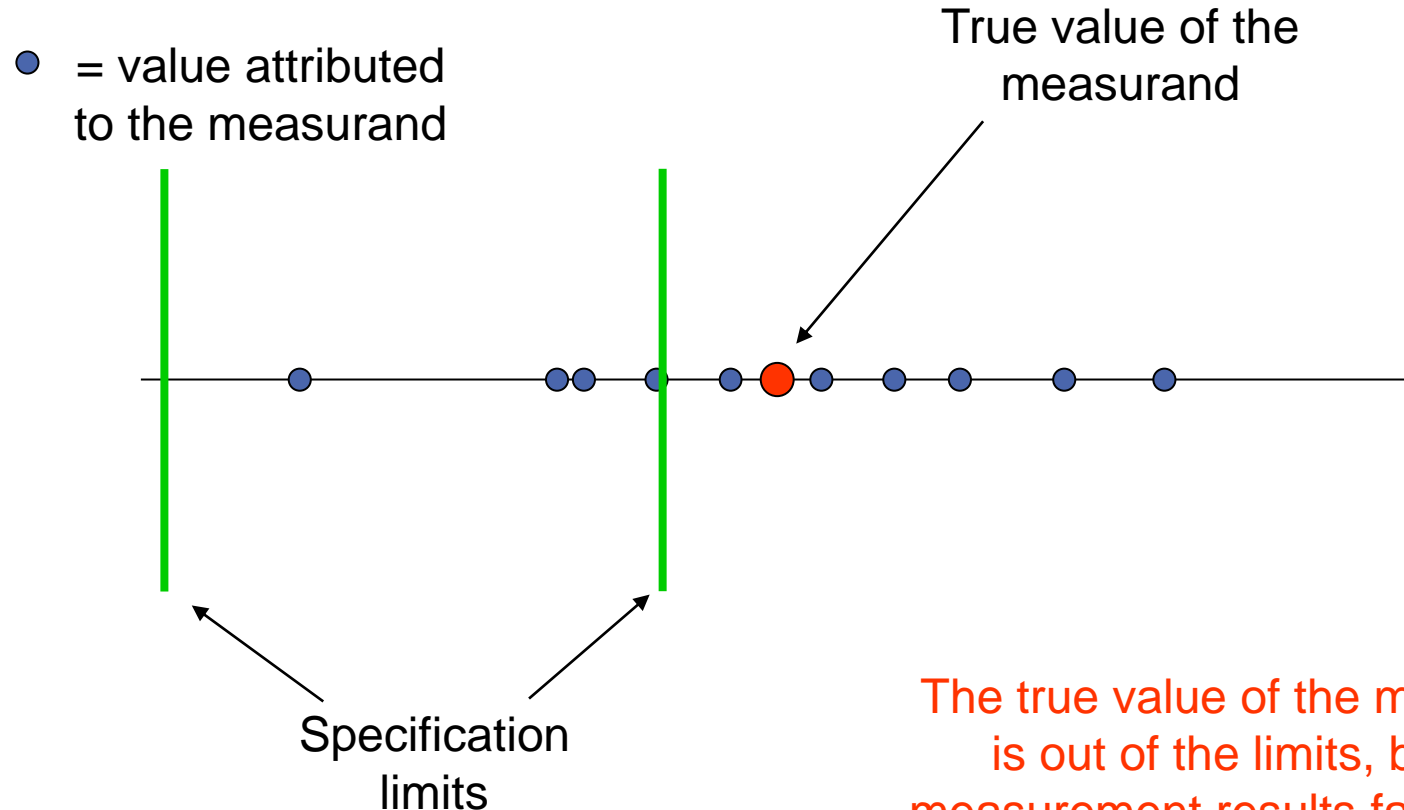




Impact of the uncertainty

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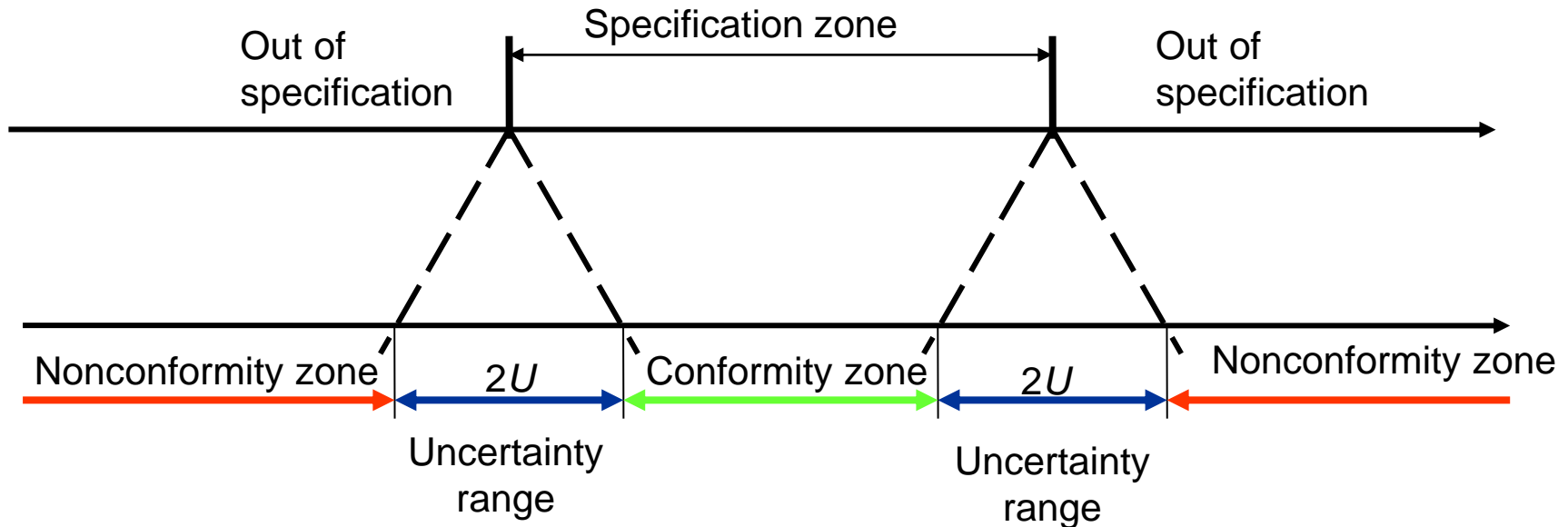
Problem: let's suppose there exist any specification limit



The true value of the measurand is out of the limits, but few measurement results fall between them. How to manage this situation?



ISO 14253-1 Standard

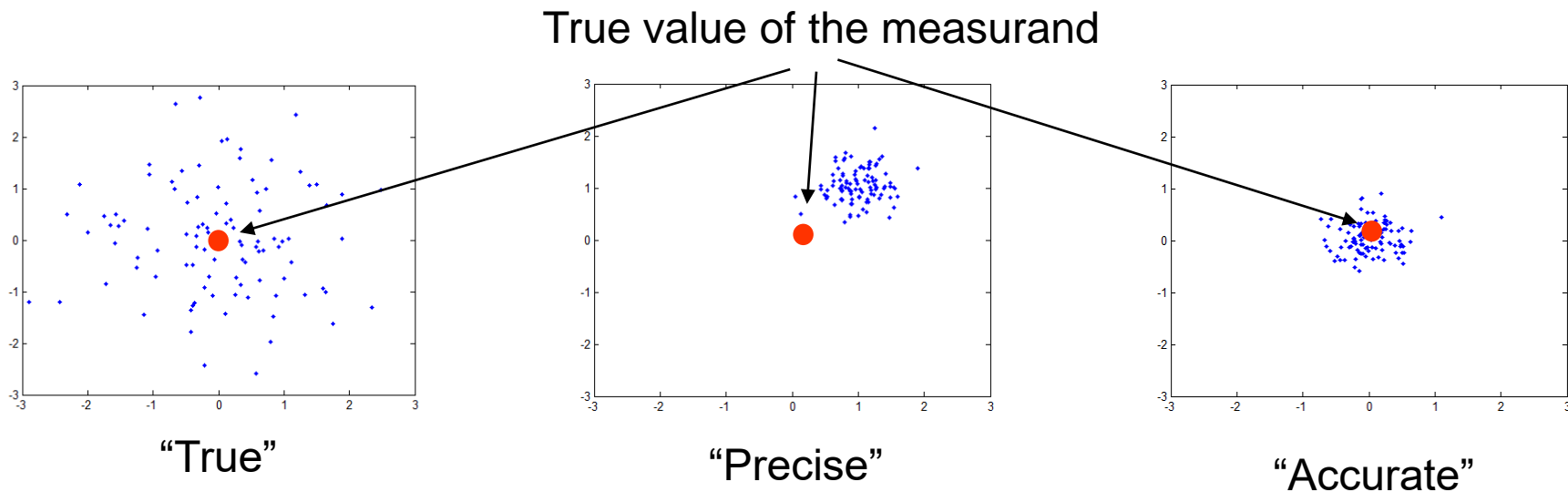


- A measurement result either in the conformity or non conformity zone always denotes a conformity or a nonconformity to some specification statement.
- A measurement result in the uncertainty range denotes a nonconformity if the aim is to prove conformity, or a conformity if the aim is to prove nonconformity.



Systematic and Random errors

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General model of a measurement:

$$y = x + \varepsilon$$

Measurement error $\rightarrow \varepsilon \sim D(\mu, \sigma)$

Measurement result $\rightarrow y \sim D(\mu + x, \sigma)$

Bias or systematic error $\rightarrow \mu$

Random Error $\rightarrow \sigma$

True value of the measurand $\rightarrow x$



Measurement uncertainty

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$$y = X + \varepsilon$$

$$\varepsilon \sim D(\mu, \sigma)$$

$$y \sim D(\mu + X, \sigma)$$

The bias is corrected:
calibration



$$y' = X - \mu + \varepsilon$$

$$\varepsilon \sim D(\mu, \sigma)$$

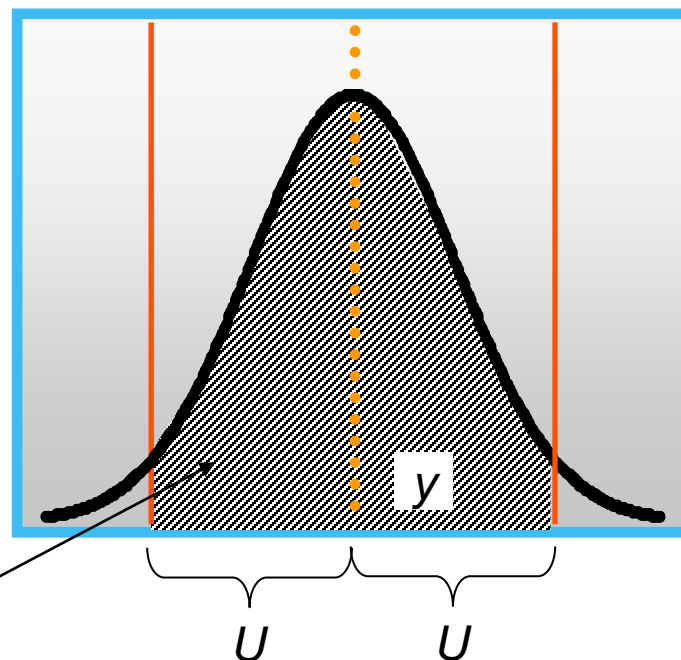
$$y' \sim D(X, \sigma)$$

In general, the random error cannot be corrected: it generates the measurement uncertainty (cfr ISO GUIDE 98-3 “GUM” and ISO GUIDE 99 “VIM”)

$$U = \sigma \text{ “standard uncertainty”}$$

$$U = k * u = k * \sigma$$

“expanded uncertainty”



“Confidence level”



Measurement uncertainty must be estimated:

- Category A estimation: based on the variance of multiple observations

$$y = \frac{1}{n} \sum_{j=1}^n y_j \quad u^2(y) = \frac{1}{n(n-1)} \sum_{j=1}^n (y_j - y)^2$$

- Category B estimation: based on other information
 - Former measurements
 - Operator experience
 - Instrument manufacturer information
 - Instrument calibration certificates
 - Reference uncertainty from manuals
 - Simulation



Let's suppose the measurement result comes from the combination of several different measurement results:

$$y = f(y_1, y_2, \dots, y_n) \quad Y = f(Y_1, Y_2, \dots, Y_n)$$

If the n measurements are independent, then

$$u_c^2(y) = \sum_{i=1}^n \left[\left. \frac{\partial f}{\partial Y_i} \right|_{y_i} u(y_i) \right]^2$$

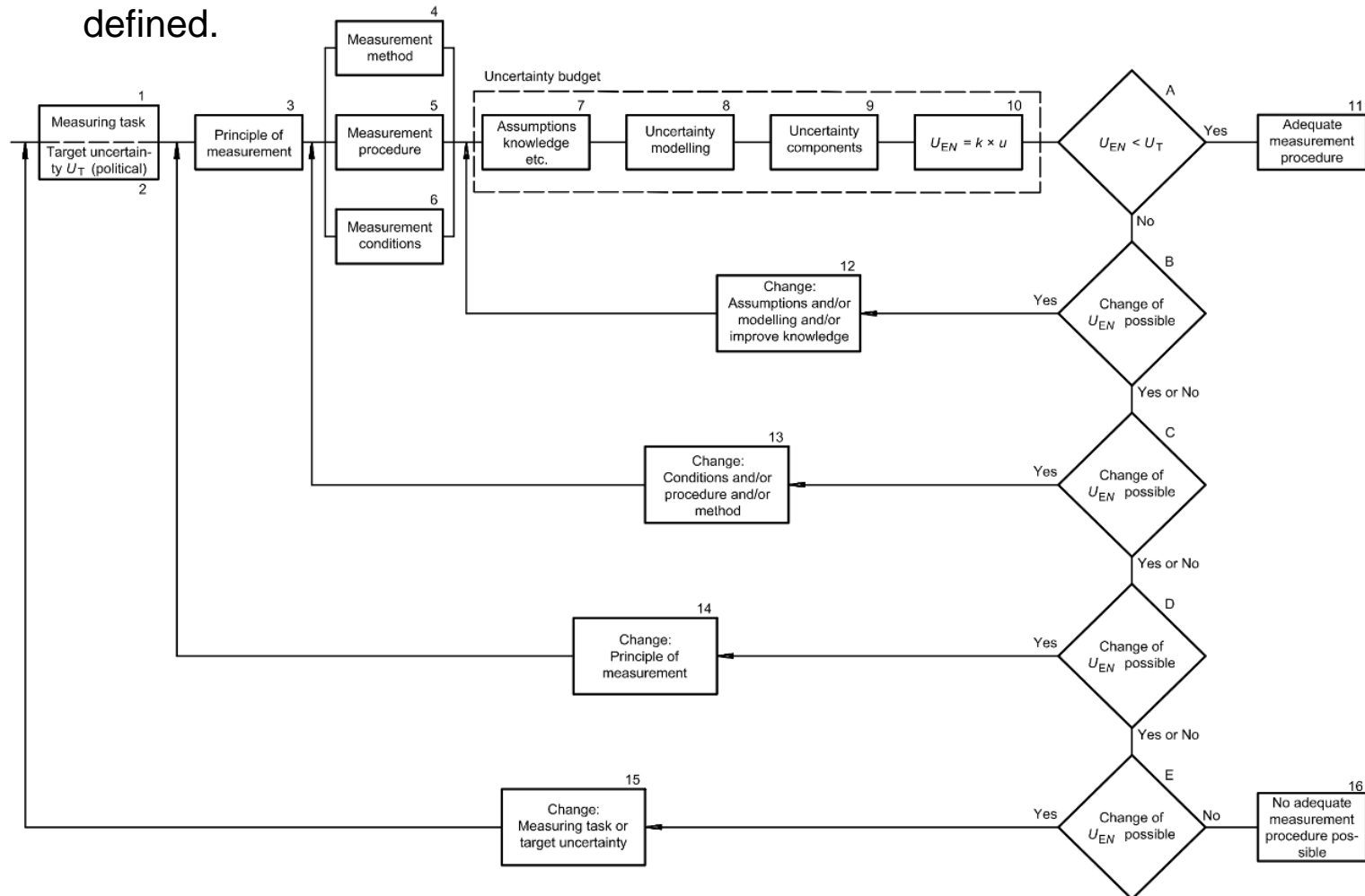


Measurement uncertainty - GPS

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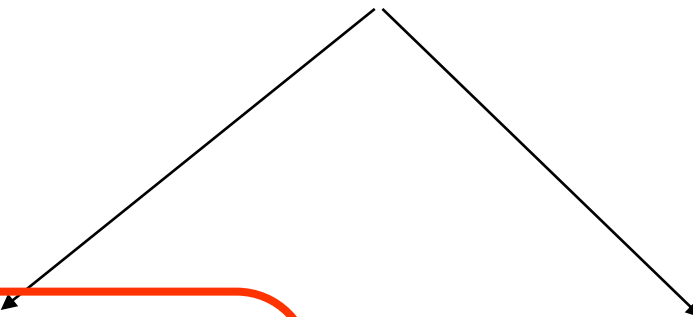
In the field of GPS, the reference standard is the ISO/TS 14253-2

- PUMA (Procedure for Uncertainty Management) method: the measurement uncertainty is cyclically estimated, until an adequate measurement procedure is defined.





However, the PUMA method does not describe how to estimate the uncertainty, but just how to manage it. The problem of the evaluation of the uncertainty and the performance of a CMM has been divided in two series of standards:



ISO 10360

“Acceptance and reverification tests for coordinate measuring machines”

They define the performance of a CMM through a series of tests and related performance indicators (MPE).

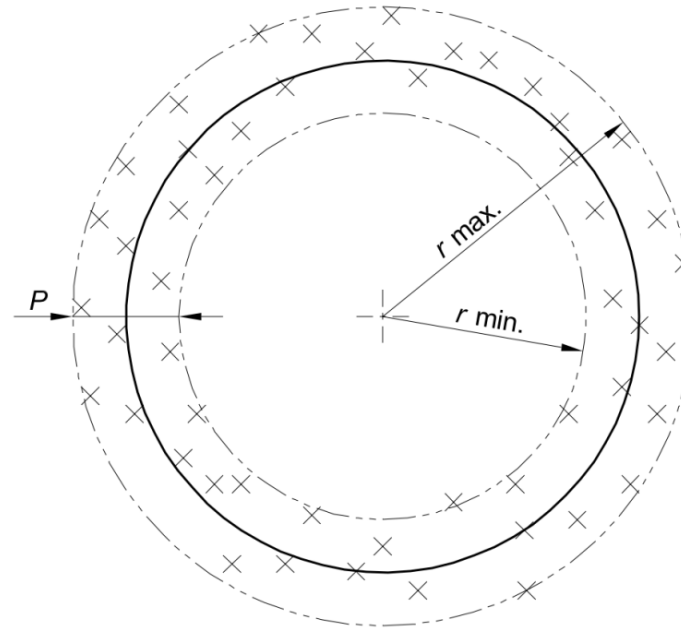
ISO 15530

“Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement”

Simplified methods for the evaluation of the uncertainty of CMMs



- $P_{FTU,MPE}$ maximum permissible single-stylus form error
- extreme value of the single-stylus form error, PFTU, permitted by **specifications, regulations**, etc. for a CMM.
- $P_{FTj,MPE}$ maximum permissible multi-stylus form error
- extreme value of the multi-stylus form error, PFTU, permitted by **specifications, regulations**, etc. for a CMM.
- MPE_{Tij} maximum permissible scanning probing error
- extreme value of the multi-stylus form error, PFTU, permitted by **specifications, regulations**, etc. for a CMM.





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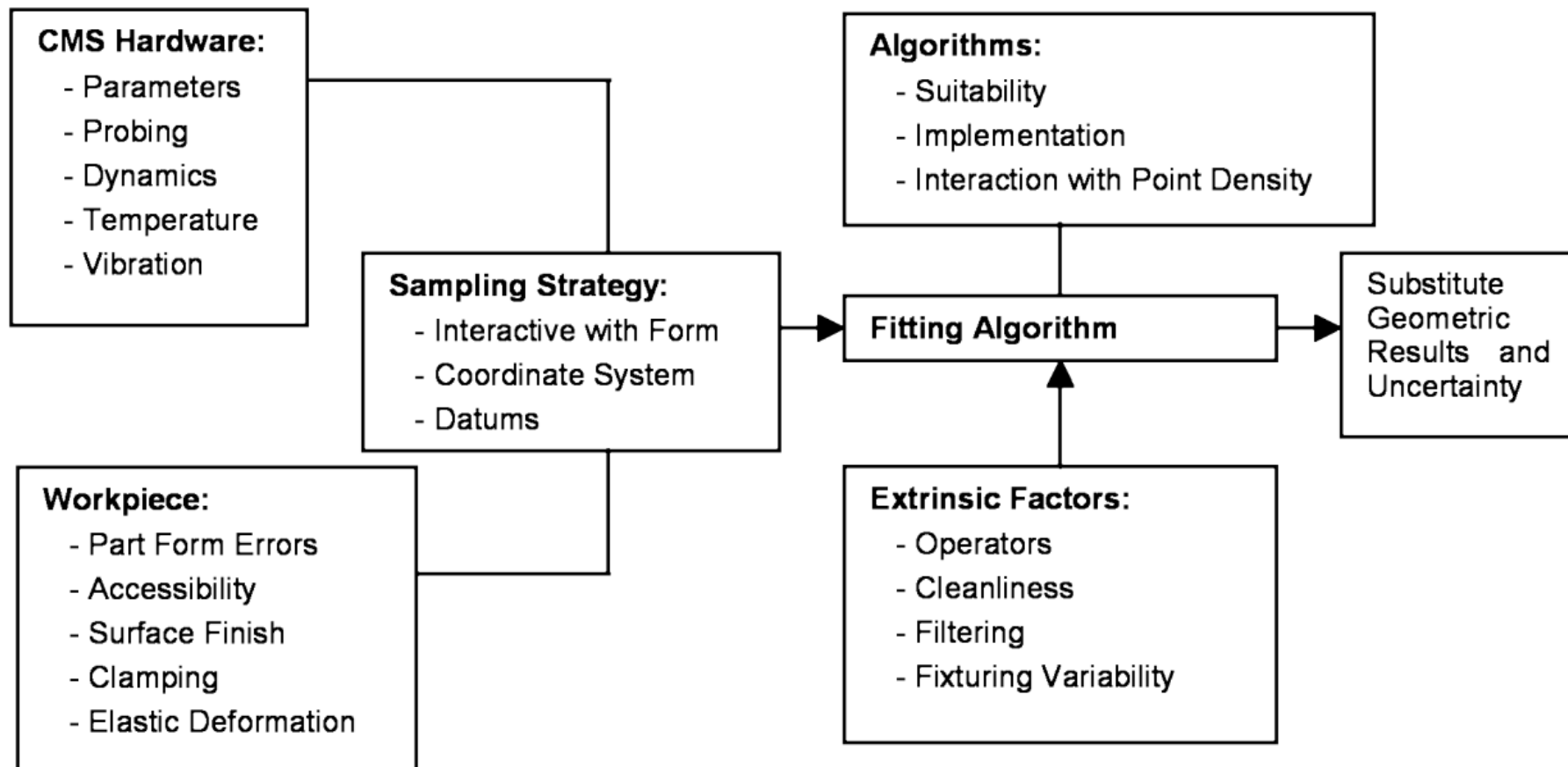
ISO 15530

“Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement”

Simplified methods for the evaluation of the uncertainty of CMMs



Task specific uncertainty





The limits of the proposed procedure lead to the search for a procedure which is

- fast
- easy to apply
- flexible

I.e. able to evaluate the task specific measurement uncertainty.

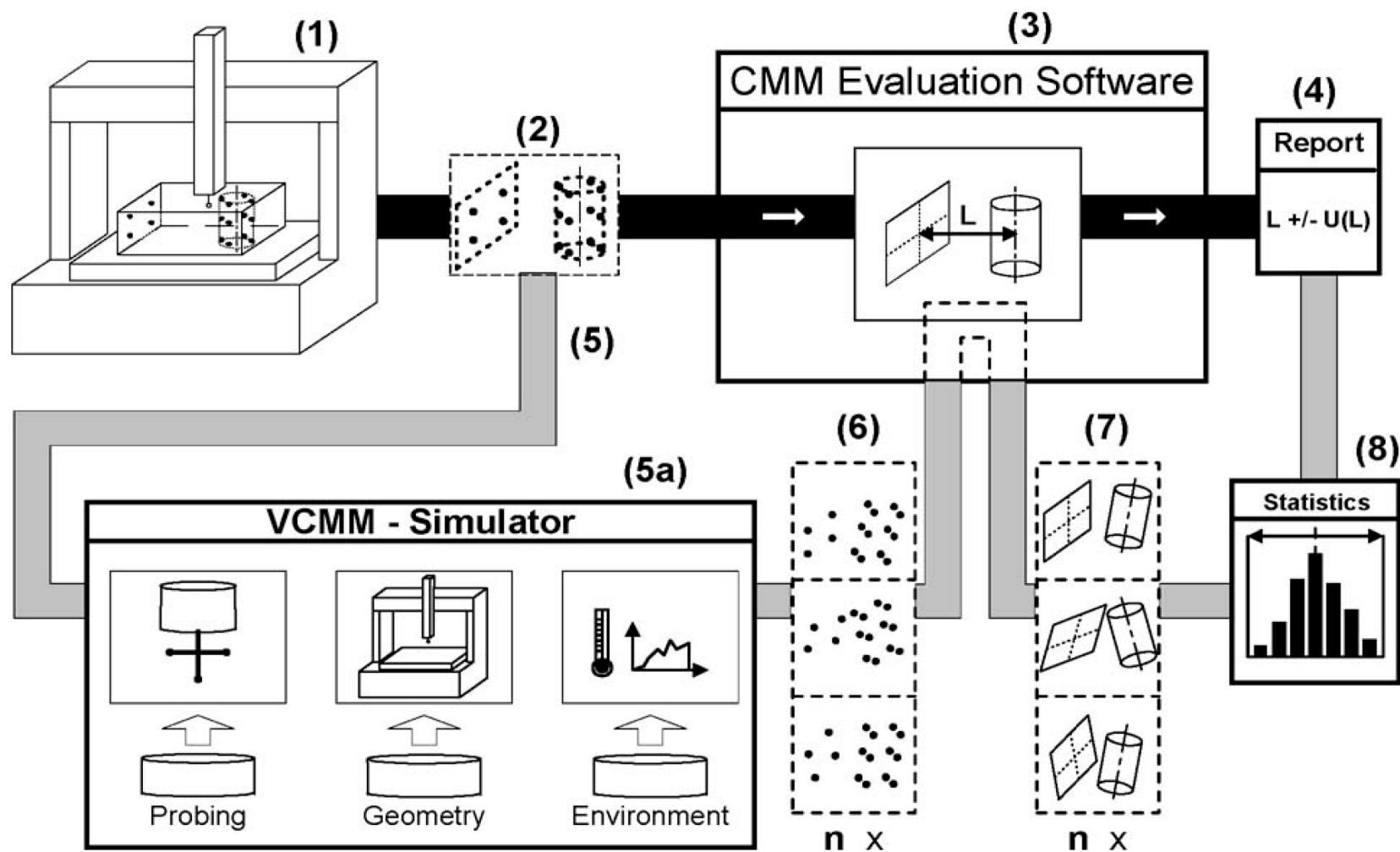
A methodology characterized by these features is the one based on a “Virtual CMM” (VCMM)

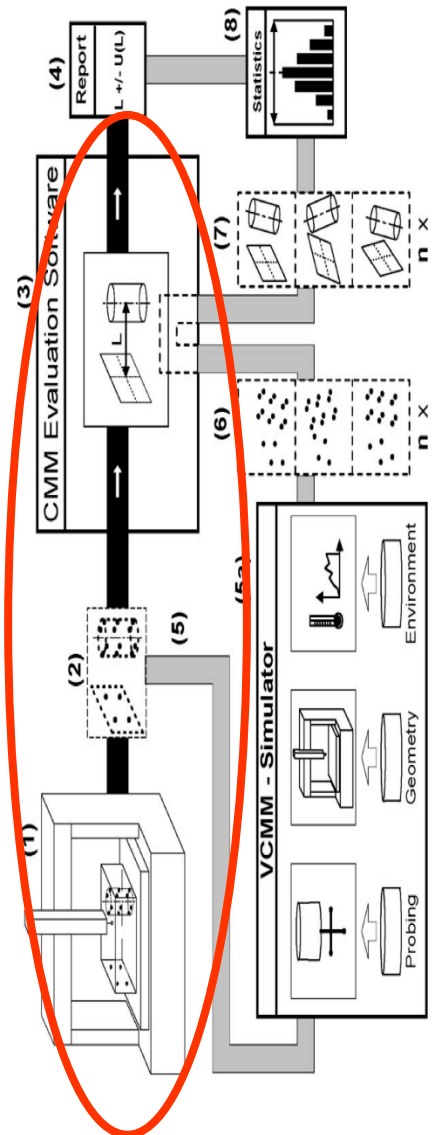
“The virtual coordinate measuring machine (VCMM) approach estimates an uncertainty statement for a particular measurement task on a particular CMM according to **Monte Carlo simulation** results” (Wilhelm, R. G.; Hocken, R. & Schwenke, H. **Task Specific Uncertainty in Coordinate Measurement**. *CIRP Ann-Manuf. Technol.*, Elsevier, 2001, 50, 553-563



VCMM – scheme of the procedure

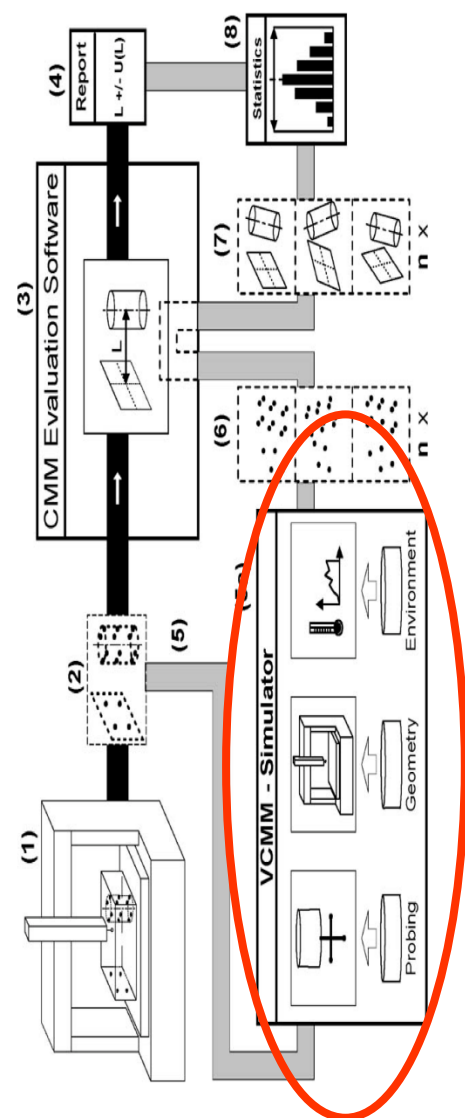
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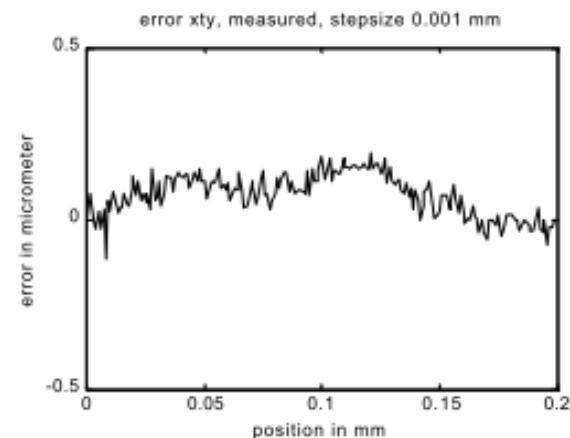
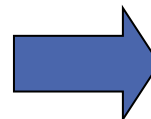
The VCMM needs some input. These can e.g. include (ISO/TS 15530-4):

- The CMM performance according to the 10360 standard
- The measurement of one or more calibrated artifacts in some specified position within the measuring volume of the CMM
- A “Gauge R&R” study
- Some expert evaluation



The core of a VCMM is the measurement simulator. Simulations can be operate in several different ways:

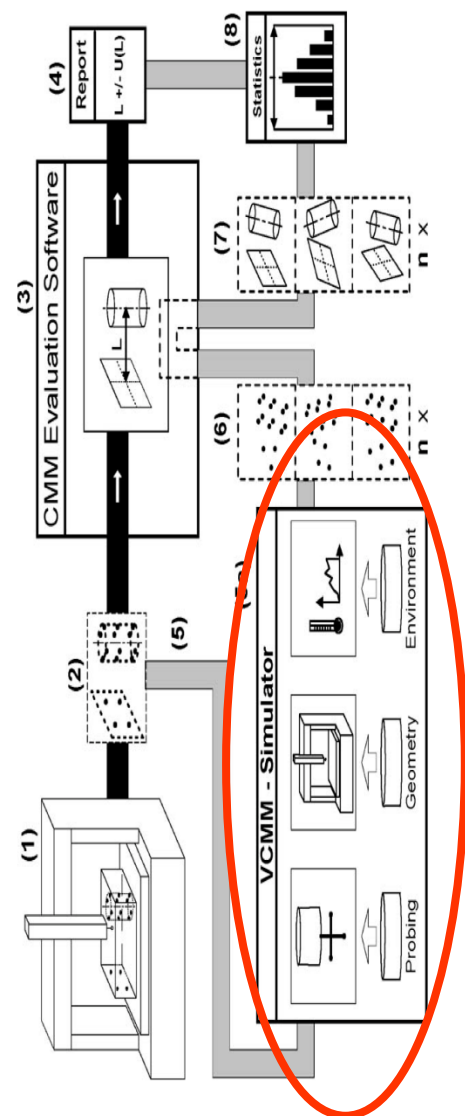
- By applying real measurement errors on single points to the actual measurement results



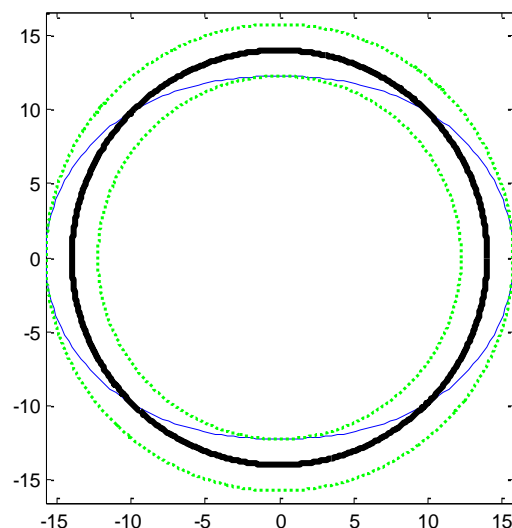
- Based on some spatial statistical model of the measurement error at a specific point

$$\mathbf{e} = N(\mathbf{0}, \mathbf{\Sigma})$$

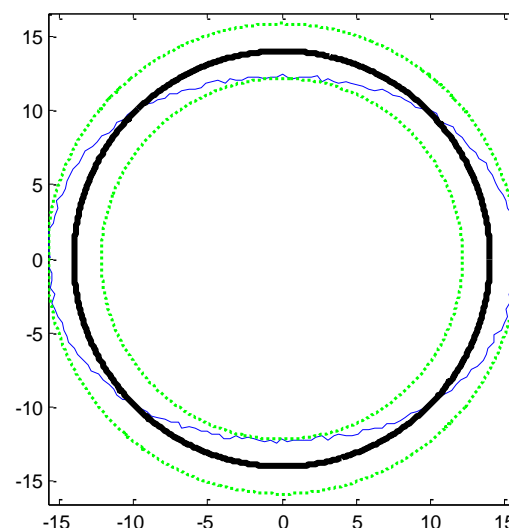
$$\sigma_{i,j} = \sigma_{i,j} (x_i, y_i, z_i, x_j, y_j, z_j)$$



A series of «nominal» profile are simulated for which the expected measurement result is known. The nominal profiles are perturbed according the VCMM model, and a perturbed measurement result is obtained.

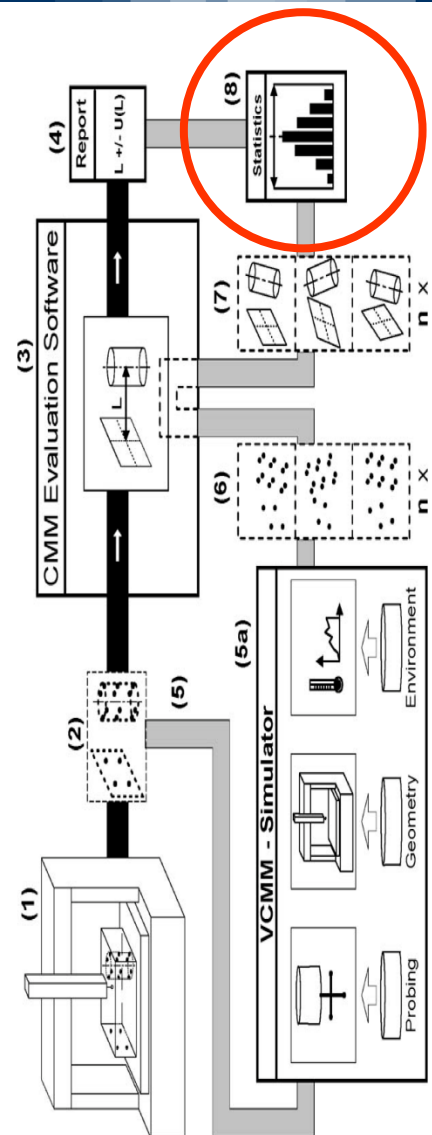


Nominal profile



Perturbed profile

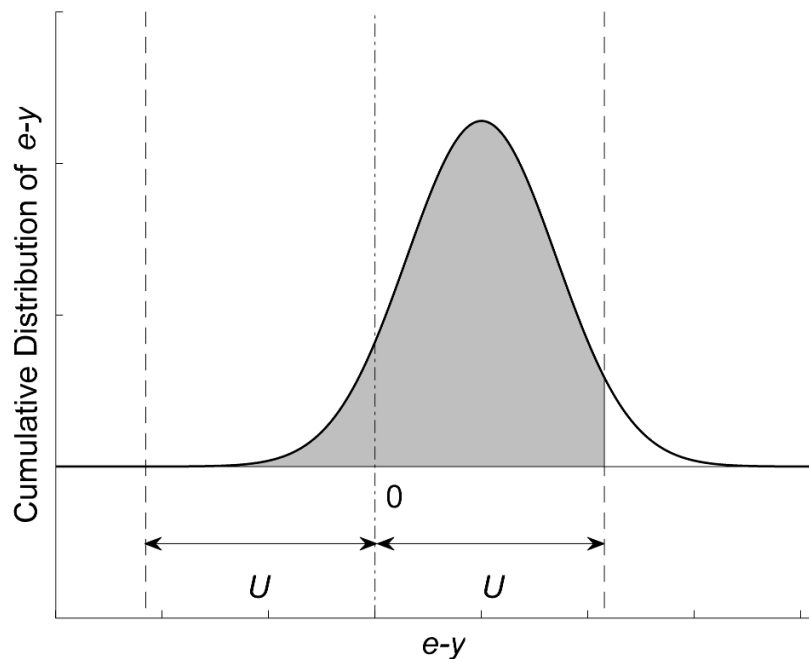
Simulated measurement error



The uncertainty can be extrapolated in several ways. For instance* one can calculate U so that

$$G(U) - G(-U) = 0.95$$

Where G is the empirical cumulative function of the measurement errors.



*[https://doi.org/10.1016/S0007-8506\(07\)62973-4](https://doi.org/10.1016/S0007-8506(07)62973-4)

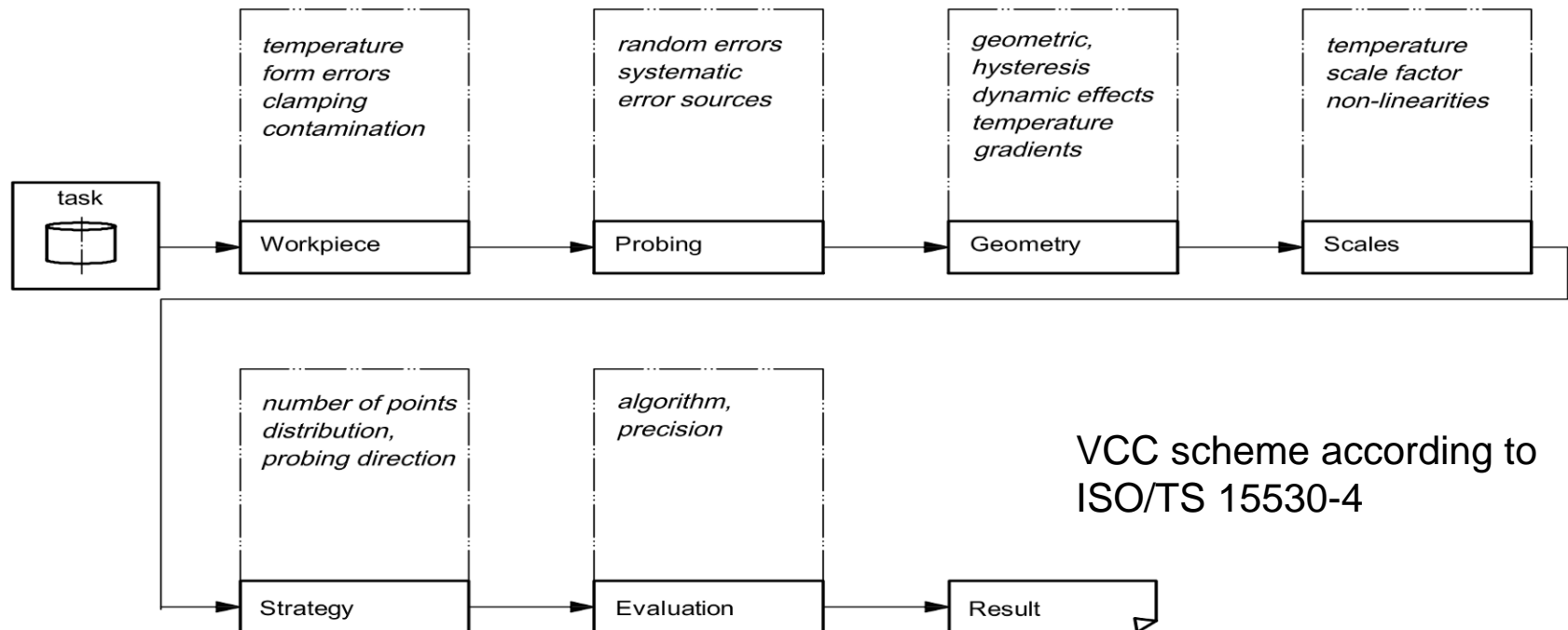


Measurement uncertainty – ISO/TS 15530-4

VCMMs are regulated by the ISO/TS 15530-4 standard: “Evaluating task-specific measurement uncertainty using simulation”

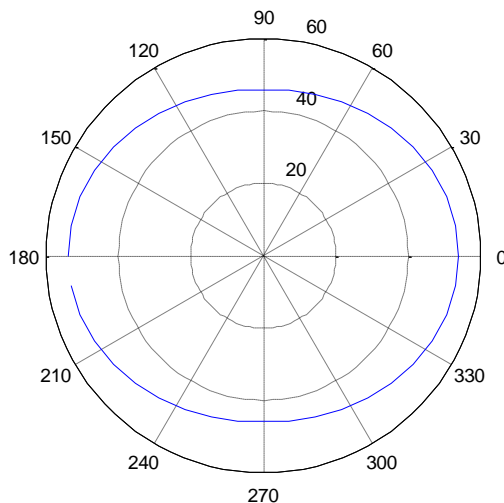
The standard requires:

- The fundamental requirements which define the VCMM
- The fundamental methods for the VCMM validation

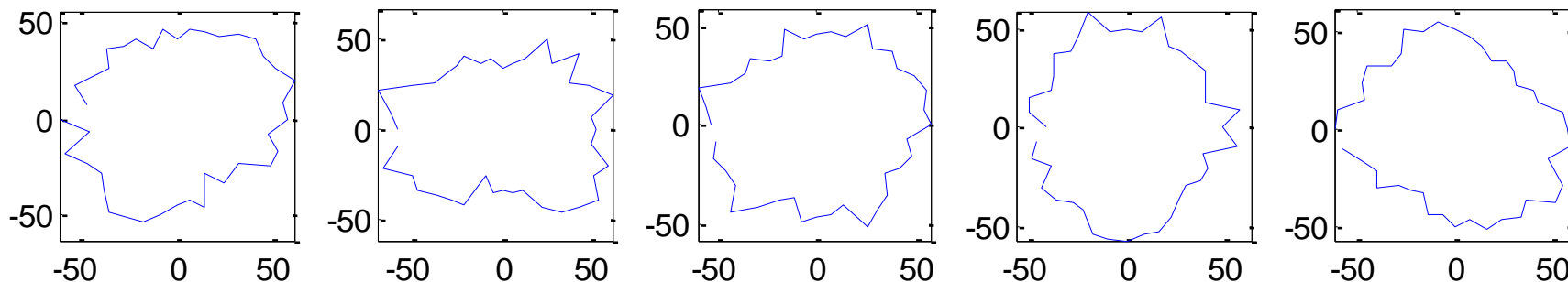




Let's suppose we aim at estimating the measurement uncertainty for the verification of roundness on a calibrated plug gauge measured on 40 points.



Nominal profile
(roundness deviation 0.32 μm)

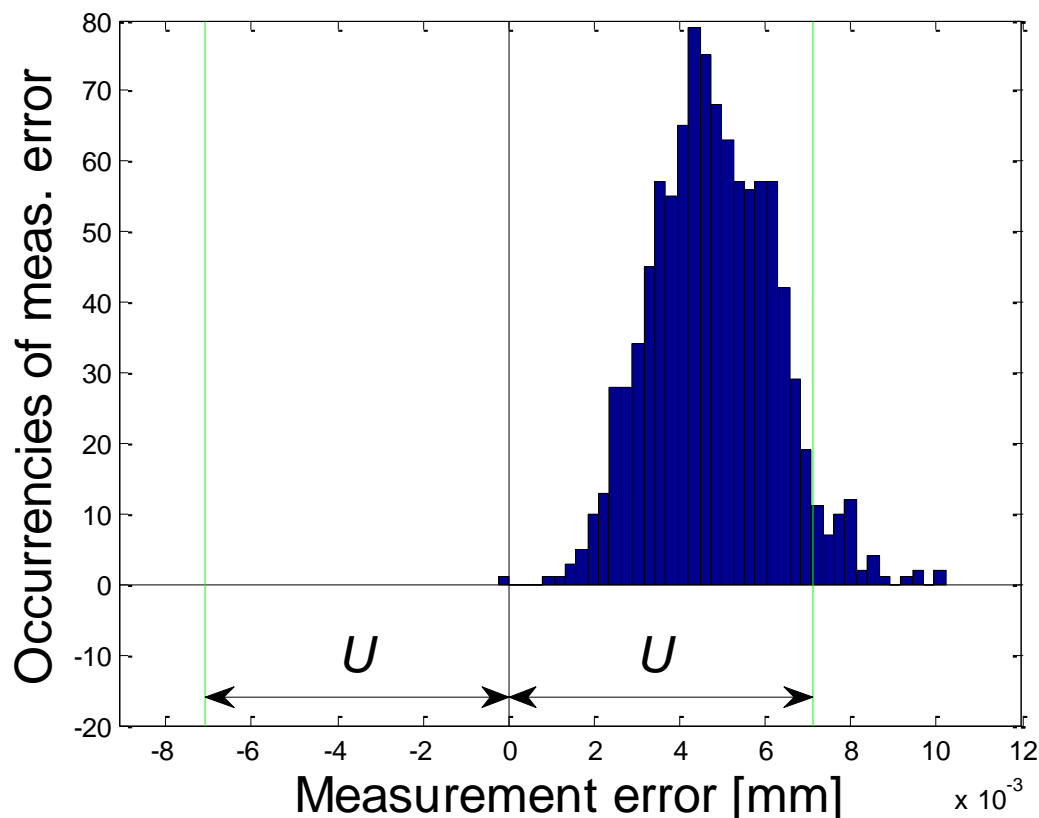


... 1000 replicas → 1000 simulated measurement errors



Let's put everything on an histogram...
And let's extract the uncertainty.

$$G(U) - G(-U) = p$$





Let's apply the ISO/TS 15530-4 to validate the result.

- Let's take a calibrated plug gauge (calibrated roundness deviation $0.32 \pm 0.15 \mu\text{m}$).
- Let's perform 100 measurements.
- Let's estimate the uncertainty: $U = 1.0 \mu\text{m}$

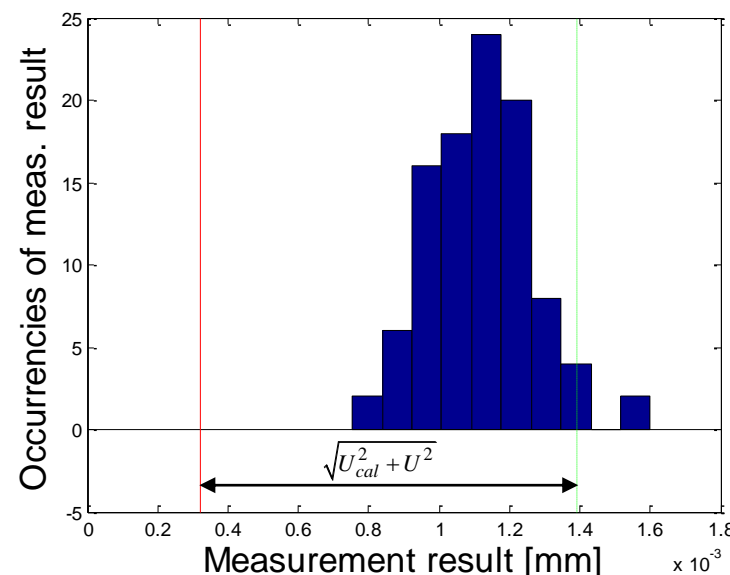
Results are compared by means of the formula:

$$|y_{cal} - y| \leq \sqrt{U_{cal}^2 + U^2}$$

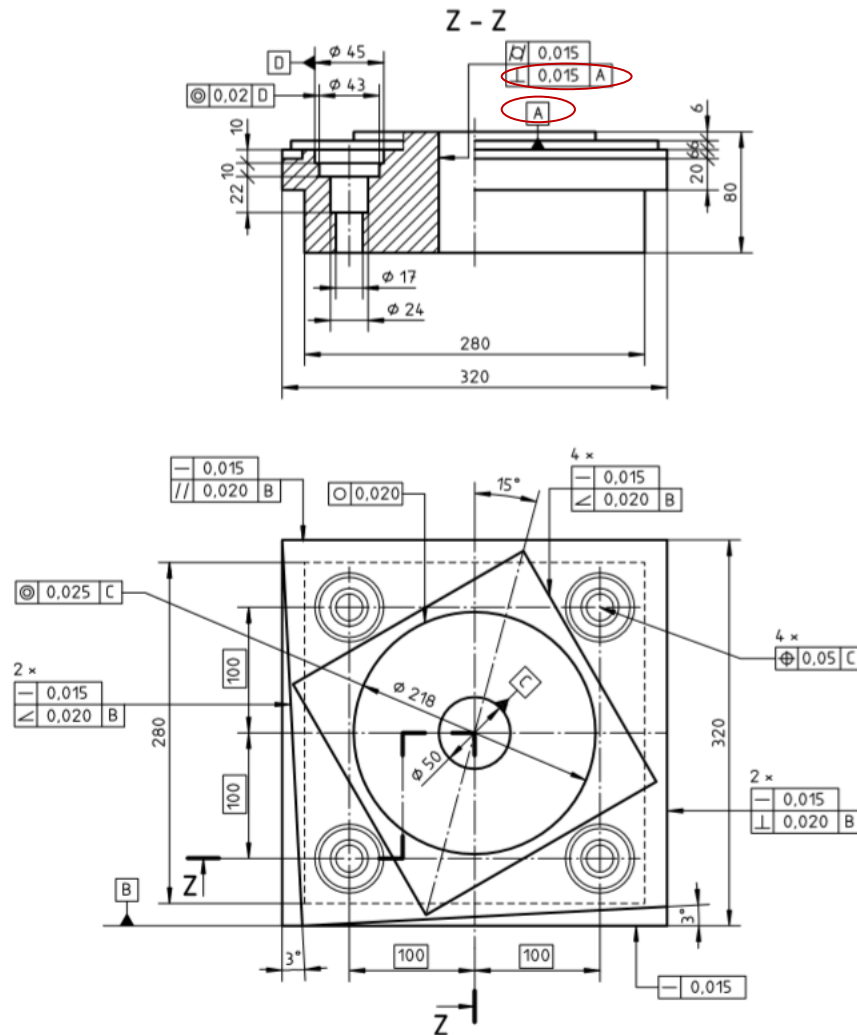
Diagram illustrating the VCMM formula components:

- y_{cal} : Reference value (green box)
- y : Measured value (red box)
- U_{cal} : Calibration uncertainty (yellow box)
- U : VCMM uncertainty (blue box)

Which is satisfied 97 times,
coherent with what was expected (95).



Focus on the perpendicularity tolerance of the center hole



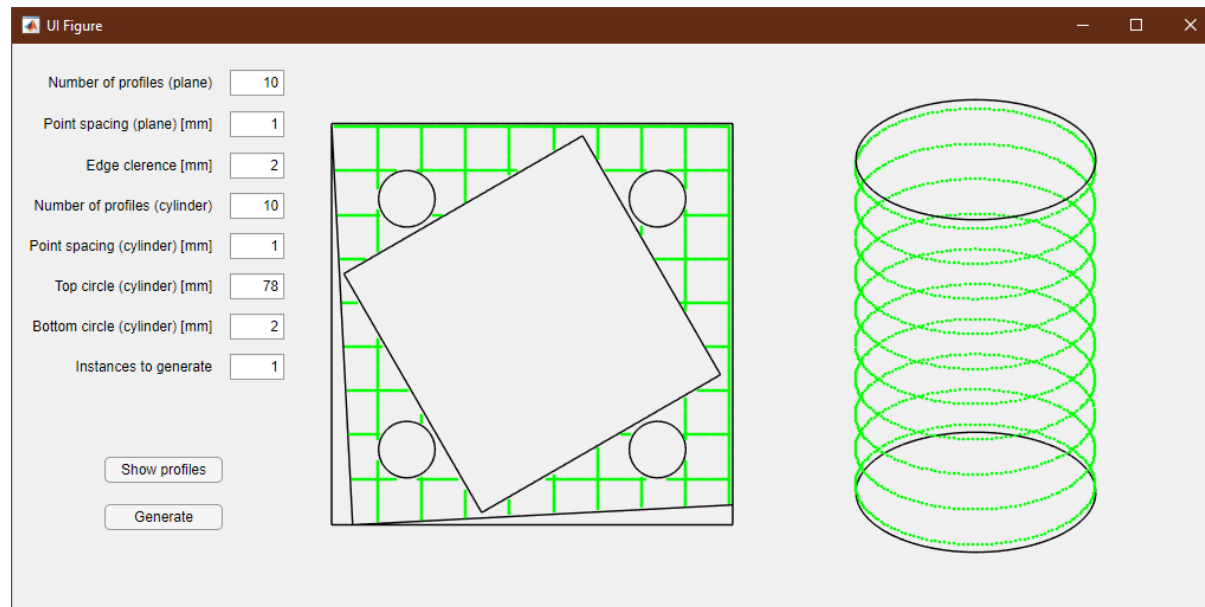


What to do now...

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Step 1: generate reference values and cloud of points for the central hole and datum A

- Define the sampling strategy
 - The strategy must be coherent with the probe configuration and part fixturing!
- Choose how many instances to generate
- Use the provided matlab software





Step 3: Perturb the generated cloud of points.

- Consider the maximum probing error as defined in the brochure as reference for the generation probing error. Justify why you generated the error this way.

Step 4: fit the datum and the toleranced feature and calculate the squareness deviation.

Step 5: from the calculated squareness deviations estimate the uncertainty

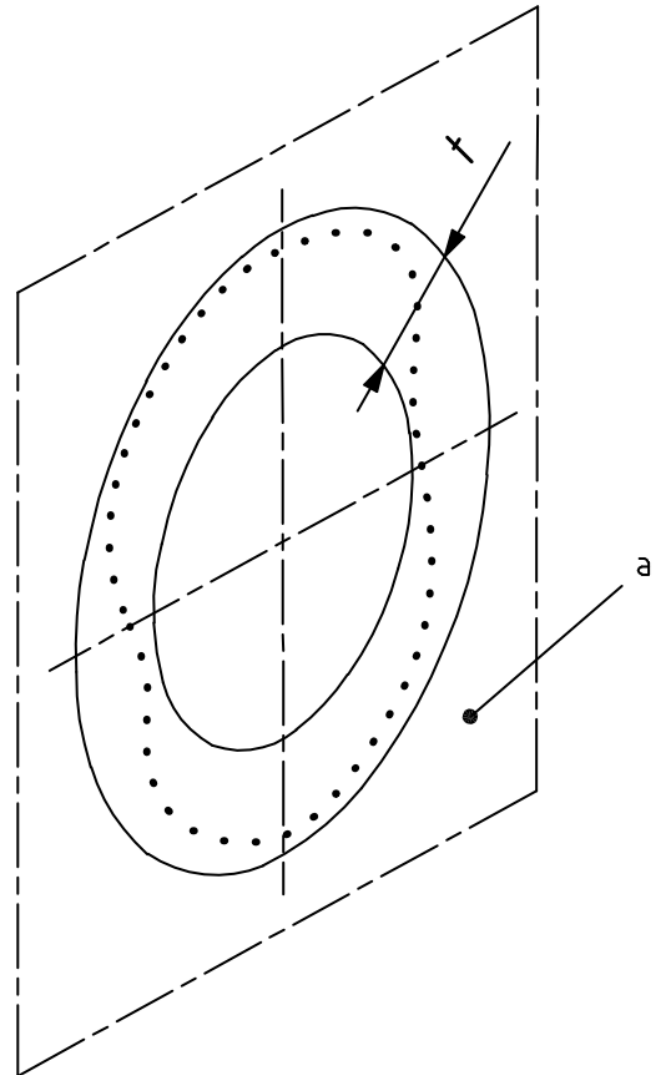
Step 6: evaluate whether the uncertainty is adequate or not.



Es. 1: roundness

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- Generate a round $\varnothing 10$ mm profile characterized by a 0.02 mm global roundness deviation in np points.
- Simulate n profiles in which a perturbation is added, the perturbation being generated from a $N(0, 0.001^2)$ gaussian distribution.
- Calculate the roundness deviation (use minimum zone fitting) per each profile.





Es. 2: roundness - correlation

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- Generate a round $\varnothing 10$ mm profile characterized by a 0.02 mm global roundness deviation in np points.
- Generate a perturbation being as*

$$p_i = \sum_{j=1}^{np} \frac{\rho_{i,j}}{n_i} p_j + \varepsilon_i$$

$$(\mathbf{I} - \rho \mathbf{N}) \mathbf{p} = \boldsymbol{\varepsilon}$$

$$\rho_{i,j} = \begin{cases} \rho, & \text{points } i \text{ and } j \text{ are adjacent} \\ 0, & \text{else} \end{cases}$$

n_i is the number of points adjacent to point i

$$\rho = 0.9$$

$$\varepsilon_i \sim N(0, 0.001^2)$$

- Rescale the perturbation so that its variance equals the original variance of the generated ε_i . Apply it to the nominal profile to generate the perturbed profile.
- Calculate the roundness deviation (use minimum zone fitting) per each profile.

*Spatial autocorrelation model

