Wednesday, Feb 15

The "Anatomy" of Confidence Intervals

Many confidence intervals (and all that will be discussed in this class) have the form

$$\overbrace{\text{point estimate} \pm \underbrace{\text{standard score} \times \text{standard error}}_{\text{margin of error}}}^{\text{confidence interval}}.$$

Confidence Level

The **confidence level** of a confidence interval formula is the probability an interval produced by the formula will contain the parameter *before the data are collected* (after the data are collected the interval either does or does not contain the parameter). It is controlled through the *standard score*.

Example: Recall the study with the platies. Out of a sample of 84 observations, the yellow-tailed male was preferred on 67 observations. Let p be the *probability* of a preference for the yellow-tailed male. Let p be the probability that a female will prefer to the yellow-tailed male. What is our estimate of p using the confidence interval

$$\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/n},$$

with a *confidence level* of 95%?

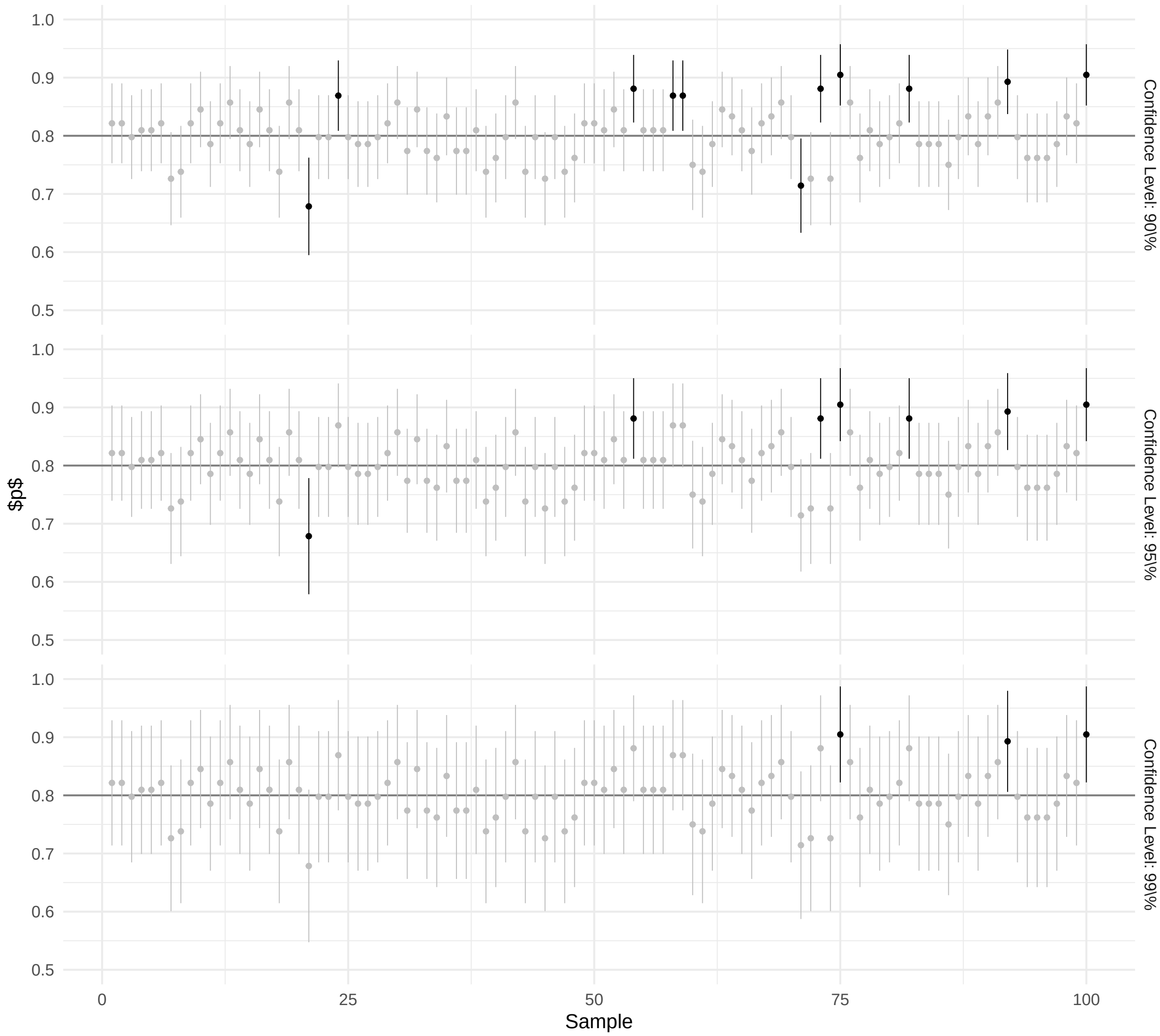
How about a confidence level of 90% or 99%? Note that we can look up z for *any* desired confidence level using statdistributions.com.

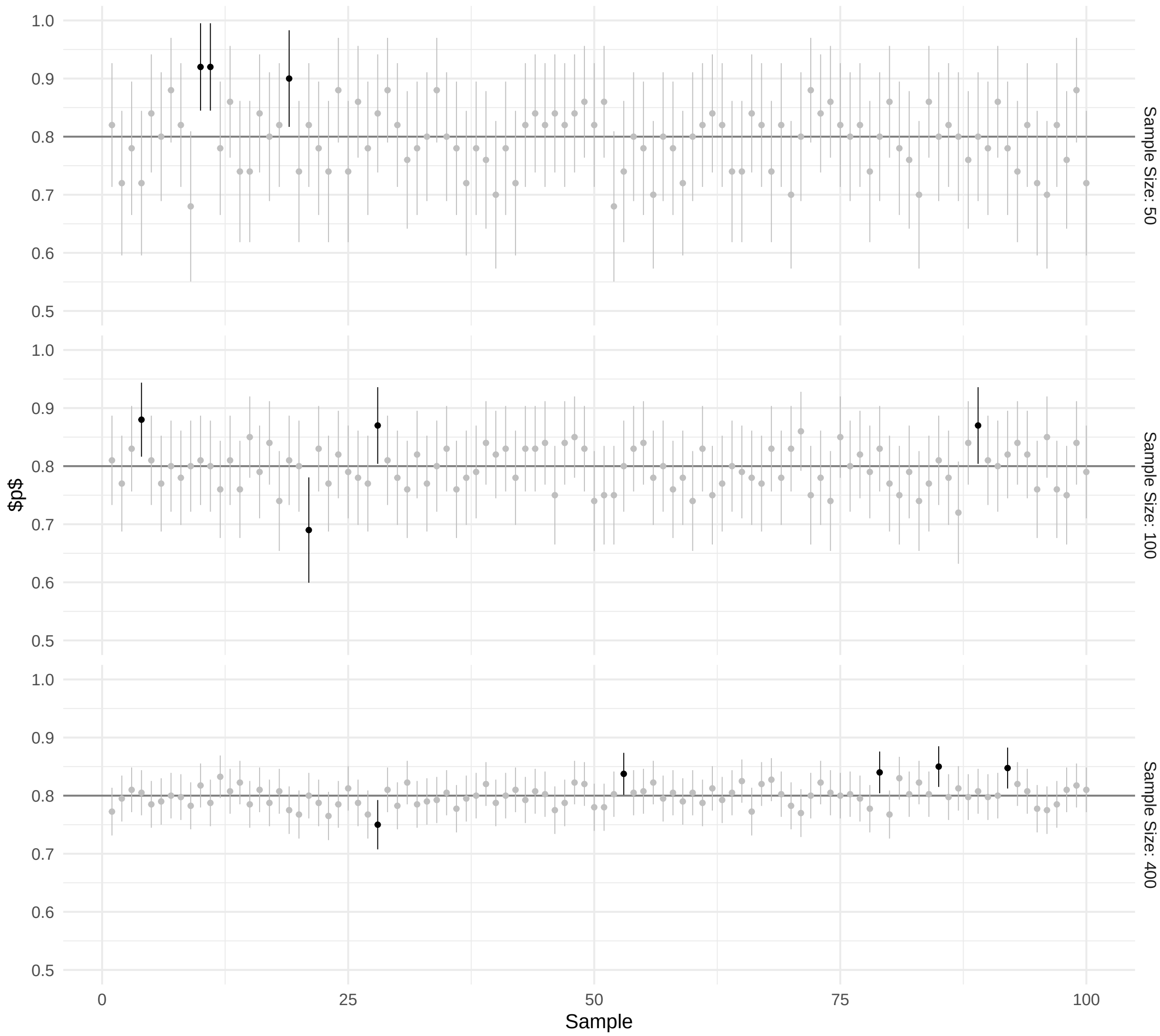
Level	z
68%	0.994
90%	1.645
95%	1.960
99%	2.576

Suppose that $p = 0.8$. What would happen if we repeated the study many times over, each time computing a confidence interval

$$\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/n}.$$

to estimate p? Each panel below shows 100 confidence intervals.





1.0
0.9
0.8
0.7
0.6
0.5
p
Sample Size: 50
Sample Size: 100
Sample Size: 400
0
25
50
75
100
Sample

How does increasing the *confidence level* affect the margin of error and confidence interval?

How does increasing the *sample size* affect the margin of error and confidence interval?

Suppose I obtain 1000 samples, and from each sample I computed a confidence interval to estimate p using the formula

$$\hat{p} \pm 1.96\sqrt{\hat{p}(1-\hat{p})/n}.$$

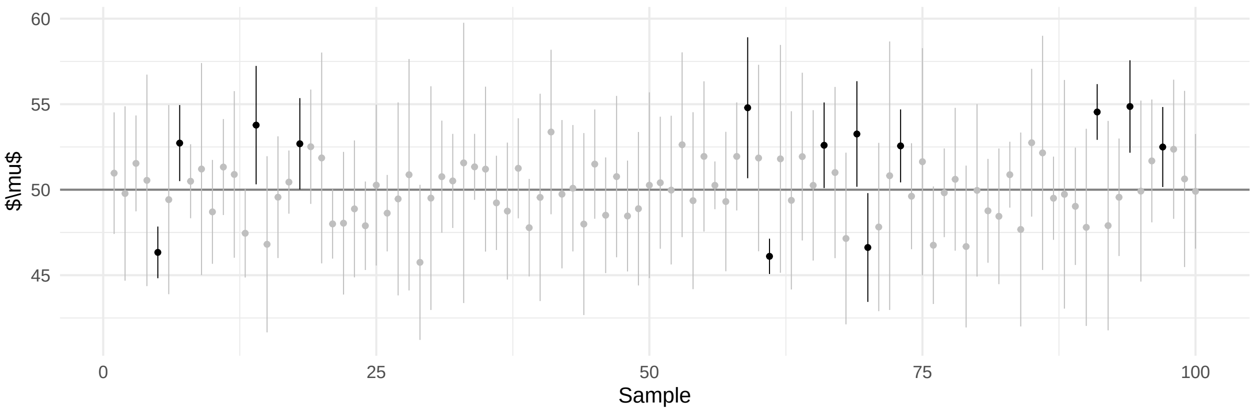
Approximately how many confidence intervals would contain p? What if we replaced 1.96 with 2.576?

Confidence Intervals for μ

The *actual* confidence level of the confidence interval

$$\bar{x} \pm zs/\sqrt{n}$$

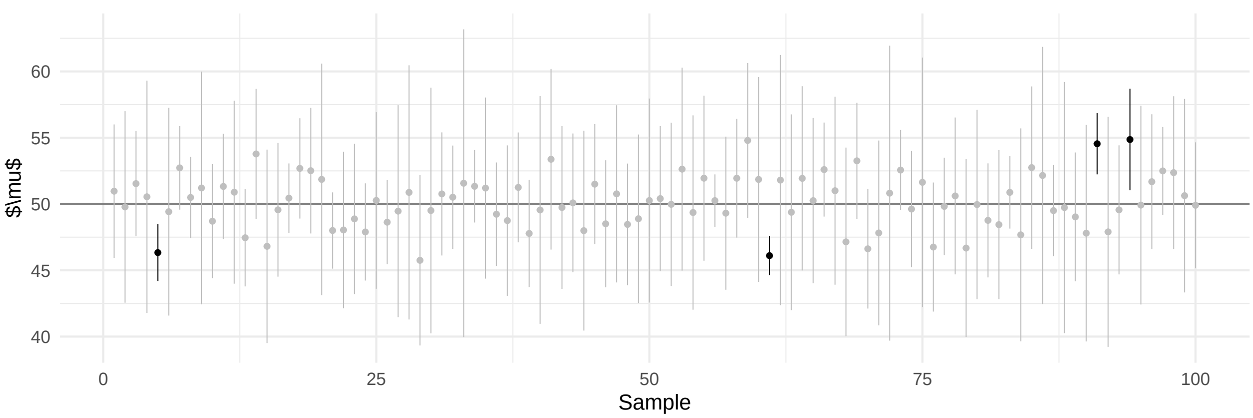
is less than the *specified* confidence level, particularly if n is small.



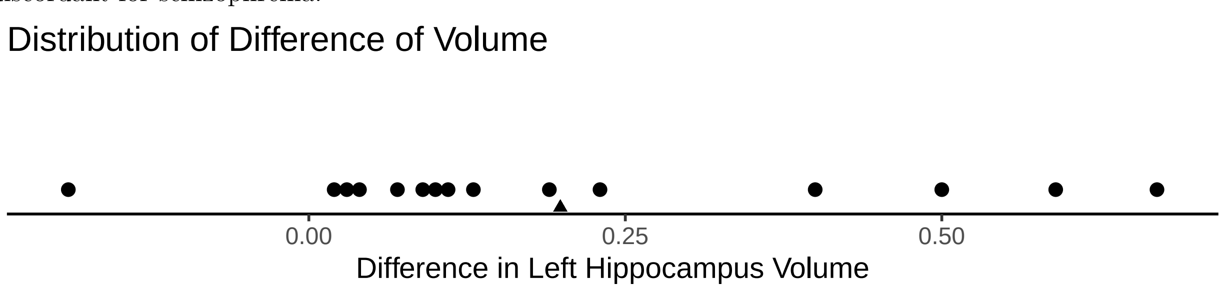
A solution is to modify the confidence interval as

$$\bar{x} \pm ts/\sqrt{n},$$

where t is a "t-score" from the t-distribution with degrees of freedom $n-1$.



Example: Consider the following data from a study of the volume of the left hippocampus for twin pairs discordant for schizophrenia.[1]

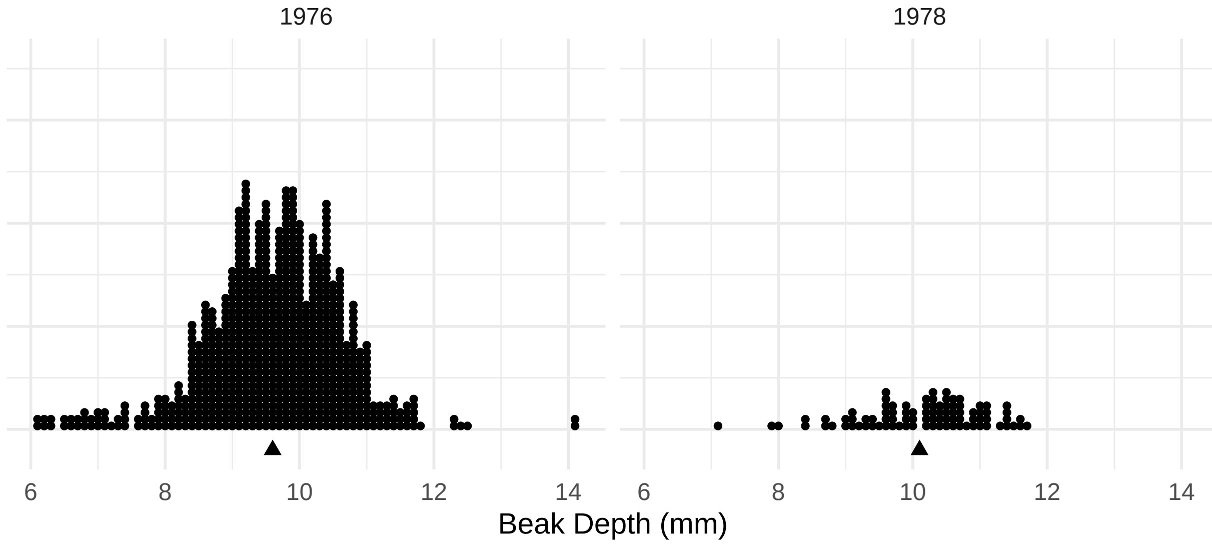


[1]Suddath, R. L., Christison, G. W., Torrey, E. F., Casanova, M. F., & Weinberger, D. R. (1990). Anatomical abnormalities in the brains of monozygotic twins discordant for schizophrenia. *New England Journal of Medicine*, *322*, 789–794.

	Twin		
Pair	Unaffected	Affected	Difference
1	1.94	1.27	0.67
2	1.44	1.63	-0.19
3	1.56	1.47	0.09
4	1.58	1.39	0.19
5	2.06	1.93	0.13
$\vdots$	$\vdots$	$\vdots$	$\vdots$
15	2.08	1.97	0.11

The mean difference from the sample is $\bar{x} = 0.2$ cubic cm, and the standard deviation from the sample is $s = 0.24$ cubic cm. Let μ be the mean difference in volume for the probability distribution of one observation of the difference in mean volume. What are the point estimate, margin of error, and confidence interval (with a confidence level of 95%) for estimating μ?

Example: Recall the study of beak length of finches on Daphne Major in 1976 and 1978.[2]



Let μ_{76} and μ_{78} be the means of the distributions of beak length in 1976 and 1978, respectively (i.e., the mean beak length of *all* finches on the island those years). What are the point estimates, margins of error, and confidence intervals for μ_{76} and μ_{78}?

Year	$\bar{x}$	s	n
1976	9.6	1.0	751
1978	10.1	0.9	89

Important: From now on we will not necessarily being using 2 as our standard score in confidence intervals. For confidence intervals for p, look up the value of z corresponding to the desired confidence level. For confidence intervals for μ, look up the value of t corresponding to the desired confidence level *and* degrees of freedom $(n-1)$.

[2]Grant, P. (1986). *Ecology and evolution of Darwin's finches.* Princeton, N.J.: Princeton University Press.