Friday, Oct 15

The p-Value (Probability Value)

The **p-value** of a significance test is *the probability of a value of the test statistic as or more extreme than the observed value of the test statistic when H_0 is true.*

Definition and Calculation of the p-Value

The definition of the p-value depends on the *alternative hypothesis*. Let p_0 denote the value of p hypothesized by the null and alternative hypotheses, so that the test statistic is

$$z_{\text{obs}} = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}.$$

Here is how we define and compute the p-value.

1. $H_a: p > p_0$. The p-value is $P(\, z \geq z_{\text{obs}} |\, H_0)$ (i.e., the probability of a value of z *greater than or equal to* the value we observed *given that H_0 is true*).
2. $H_a: p < p_0$. The p-value is $P(\, z \leq z_{\text{obs}} |\, H_0)$ (i.e., the probability of a value of z *less than or equal to* the value we observed *given that H_0 is true*).
3. $H_a: p \neq p_0$. The p-value is $P(\, |z| \geq |z_{\text{obs}}| \mid H_0)$ (i.e., the probability of a value of z that is *at least as large in absolute value* than the value we observed *given that H_0 is true*).

We need a resource like statdistributions.com to compute the p-value.

Note: The "p" in "p-value" is *not* the same as the p in the hypotheses and test statistic.

Example: How would we compute the p-values for the coin flip, pounce game, and platy examples?

The Decision Rule

Let α be the **significance level**. The decision rule is then as follows.

1. If p-value $\leq \alpha$ then *reject* H_0 (results *are* statistically significant).
2. If p-value $> \alpha$ then *do not reject* H_0 (results *are not* statistically significant).

Example: What would our decisions be for the coin flip, pounce game, and platy examples if $\alpha = 0.05$?

Steps for a Statistical Test for p

1. State the null and alternative hypotheses concerning p.
2. Check the sample size. For the "sufficiently large sample size" we need

$$np_0 \geq 15 \quad \text{and} \quad n(1 - p_0) \geq 15,$$

   where p_0 is the value of p hypothesized by the null hypothesis. If these conditions are not met, then the calculation of the p-value in the fourth step below may not be accurate.
3. Compute the test statistic $z = (\hat{p} - p)/\sqrt{p(1-p)/n}$.
4. Compute the p-value using statdistributions.com.
5. Make a decision by using the decision rule.

Example: Consider a study of just noticeable differences for pitch.

Reference	Stimulus	Correct	Total
100 Hz	101 Hz	48	100
100 Hz	102 Hz	54	100
100 Hz	103 Hz	69	100
100 Hz	104 Hz	72	100

Can the subject discriminate between, say, 100 Hz and 103 Hz?

Example: Does taking garlic supplements repel ticks? A study published in the *Journal of the American Medical Association* used a cross-over design to determine if daily consumption of garlic would reduce tick bites. A total of 66 Swedish military conscripts took 1200 mg of garlic daily during one period, and a placebo during the other period. 37 subjects reported fewer tick bites during the period they took garlic supplements. Would we conclude that garlic supplements repel (some) ticks?