Friday, Oct 22

Statistical Test Errors

The *decision* to reject or not reject H_0 may be a correct or incorrect decision.

	Decision	
Reality	Do Not Reject H_0	Reject H_0
H_0 true	correct decision	type I error
H_0 false	type II error	correct decision

We have two types of errors:

1. A **type I error** occurs when the null hypothesis is *true* but it is *rejected* — i.e., *rejecting* a *true* null hypothesis.
2. A **type II error** occurs when the null hypothesis is *false* but it is *not rejected* — i.e., *failing to reject* a *false* null hypothesis.

Example: Recall the twin study that examined the relationship between schizophrenia and left hippocampus volume. Suppose the hypotheses are $H_0 : \mu = 0$ (there is no relationship) and $H_a : \mu > 0$ (there is a relationship).

	Decision	
Reality	Do Not Reject H_0	Reject H_0
there is no relationship	correctly conclude there is no relationship	incorrectly conclude there is a relationship
there is a relationship	incorrectly conclude there is no relationship	correctly conclude there is a relationship

We rejected H_0. What kind of error might we have made?

Example: Recall the study with the cross-over design that investigated if garlic repels ticks. Suppose the hypotheses are $H_0 : p = 0.5$ (garlic is not effective) versus $H_a : p > 0.5$ (garlic is effective).

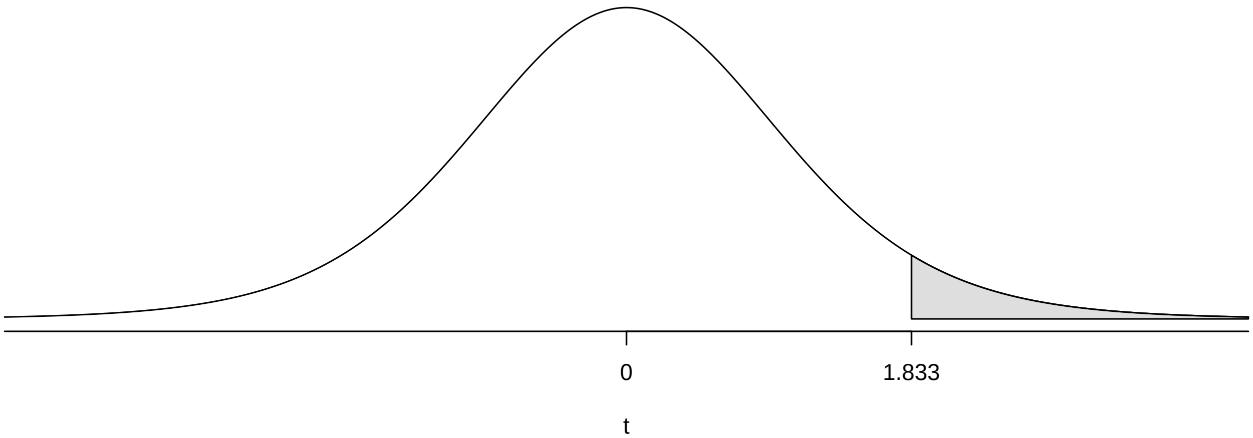
	Decision	
Reality	Do Not Reject H_0	Reject H_0
garlic is not effective	correctly conclude that garlic is ineffective	incorrectly conclude that garlic is effective
garlic is effective	incorrectly conclude that garlic is ineffective	correctly conclude that garlic is effectve

We did not reject H_0. What kind of error might we have made?

Probability of a Type I Error

The probability of a type I error is the probability of *rejecting* H_0 when it is *true*.

Example: Suppose we have the hypotheses $H_0 : \mu = 0$ versus $H_a : \mu > 0$ and plan to use a significance level of $\alpha = 0.05$. The *critical value* of t is the value of the test statistic with a p-value *equal* to the significance level. Assume a sample size of $n = 10$.



So we can state the decision rule as follows.

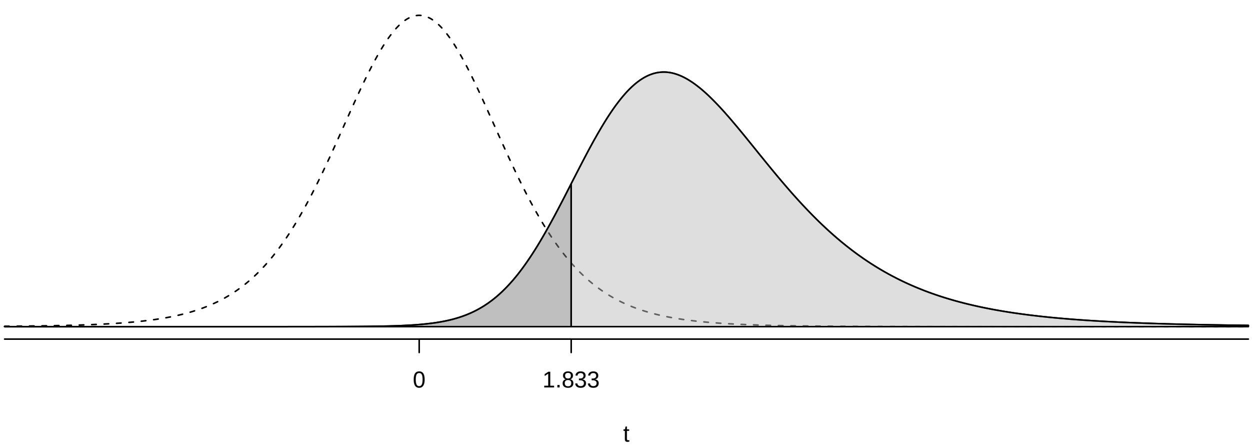
1. If $t \geq 1.833$ then p-value $\leq \alpha$ so *reject* H_0.
2. If $t < 1.833$ then p-value $> \alpha$ so *do not reject* H_0.

Thus the probability of a type I error is the probability of rejecting H_0 when H_0 is true, which is $P(t \geq 1.833|H_0) = \alpha$. Thus, *the probability of rejecting the null hypothesis when it is true (i.e., a type I error) equals* α.

Probability of a Type II Error

The probability of a type II error is the probability of *not rejecting* H_0 when it is *false*.

Example: Suppose again that we have the hypotheses $H_0 : \mu = 0$ versus $H_a : \mu > 0$ and plan to use a significance level of $\alpha = 0.05$. The *critical value* of t is the value of the test statistic with a p-value *equal* to the significance level. Assume a sample size of n=10. But now suppose that *in reality* $\mu > 0$ (e.g., $\mu = 1$). Note that the sampling distribution of the test statistic when H_0 is true is shown by the dotted line, while the sampling distribution of the test statistic when H_0 is false is shown by the solid line.

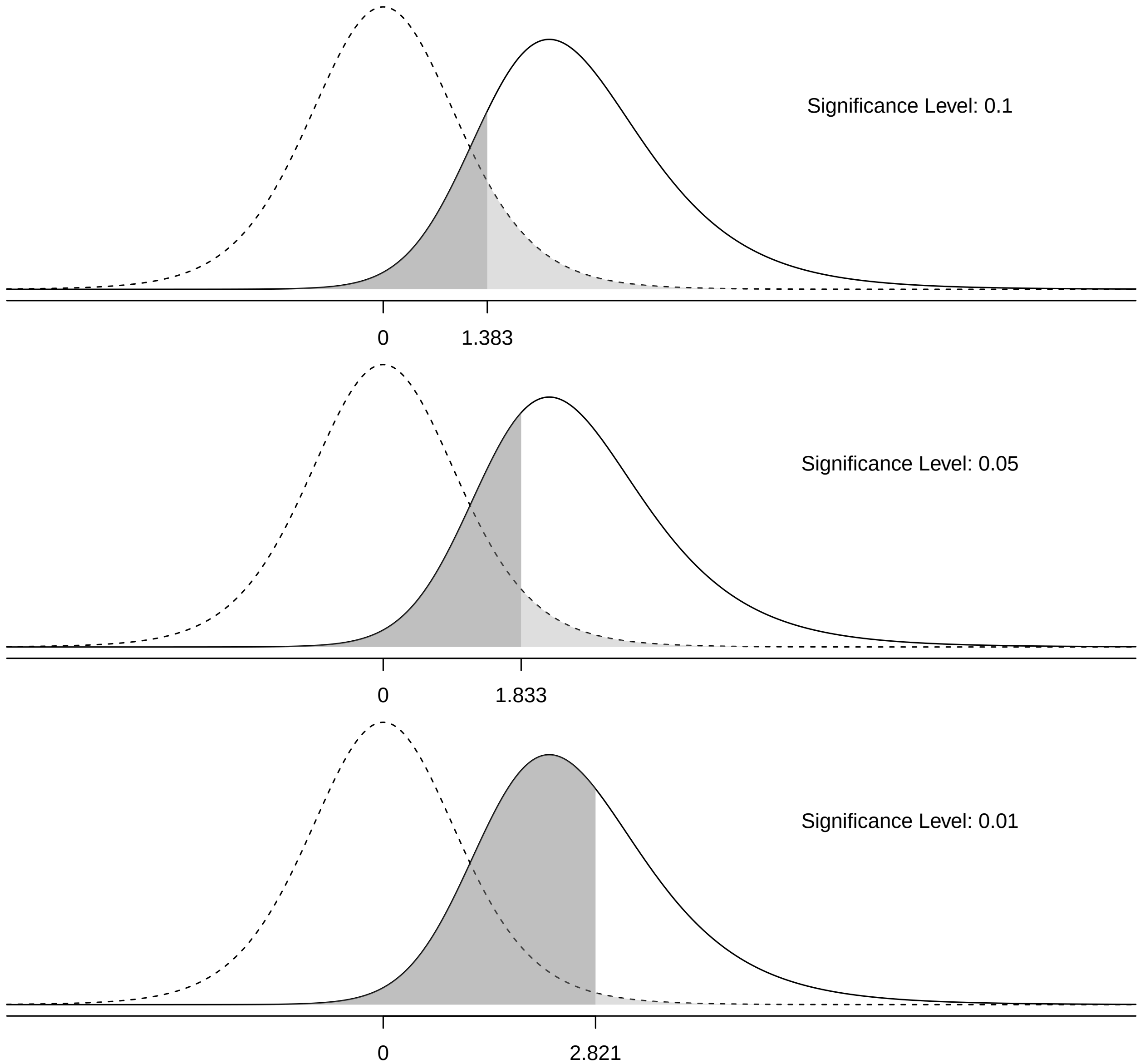


So the probability of a type II error (i.e., the probability of *not rejecting* H_0 when it is *false*) here is $P(t < 1.833|H_a)$.

It is not as simple to compute the probability of a type II error because it depends on several factors.

Effect of α on Error Probabilities

The probability of a type I error is the *light* grey area, and the probability of a type II error is the *dark* grey area.



If we decrease α we will (a) decrease the probability of a type I error and (b) increase the probability of a type II error.

If we increase α we will (a) increase the probability of a type I error and (b) decrease the probability of a type II error.