

Friday, Mar 25

## Statistical Test Errors

The *decision* to reject or not reject  $H_0$  may be a correct or incorrect decision.

Reality	Decision	
	Do Not Reject $H_0$	Reject $H_0$
$H_0$ true	correct decision	type I error
$H_0$ false	type II error	correct decision

We have two types of errors:

1. A **type I error** occurs when the null hypothesis is *true* but it is *rejected* — i.e., *rejecting a true* null hypothesis.
2. A **type II error** occurs when the null hypothesis is *false* but it is *not rejected* — i.e., *failing to reject a false* null hypothesis.

**Example:** Recall the twin study that examined the relationship between schizophrenia and left hippocampus volume. Suppose the hypotheses are  $H_0 : \mu = 0$  (there is no relationship) and  $H_a : \mu > 0$  (there is a relationship).

Reality	Decision	
	Do Not Reject $H_0$	Reject $H_0$
<b>there is no relationship</b>	correctly conclude there is no relationship	incorrectly conclude there is a relationship
<b>there is a relationship</b>	incorrectly conclude there is no relationship	correctly conclude there is a relationship

We rejected  $H_0$ . What kind of error might we have made?

**Example:** Recall the study with the cross-over design that investigated if garlic repels ticks. Suppose the hypotheses are  $H_0 : p = 0.5$  (garlic is not effective) versus  $H_a : p > 0.5$  (garlic is effective).

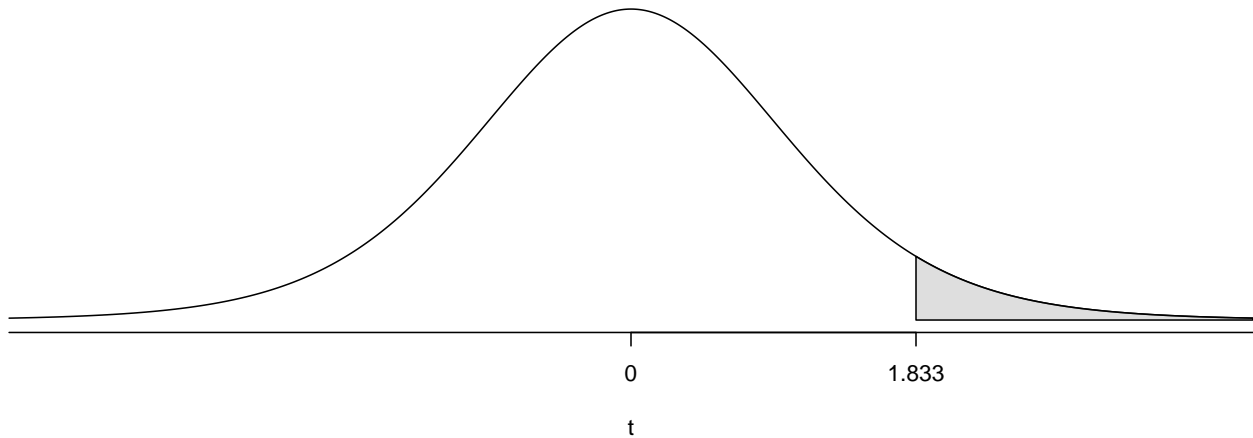
Reality	Decision	
	Do Not Reject $H_0$	Reject $H_0$
<b>garlic is not effective</b>	correctly conclude that garlic is ineffective	incorrectly conclude that garlic is effective
<b>garlic is effective</b>	incorrectly conclude that garlic is ineffective	correctly conclude that garlic is effective

We did not reject  $H_0$ . What kind of error might we have made?

## Probability of a Type I Error

The probability of a type I error is the probability of *rejecting*  $H_0$  when it is *true*.

**Example:** Suppose we have the hypotheses  $H_0 : \mu = 0$  versus  $H_a : \mu > 0$  and plan to use a significance level of  $\alpha = 0.05$ . The *critical value* of  $t$  is the value of the test statistic with a p-value *equal* to the significance level. Assume a sample size of  $n = 10$ .



So we can state the decision rule as follows.

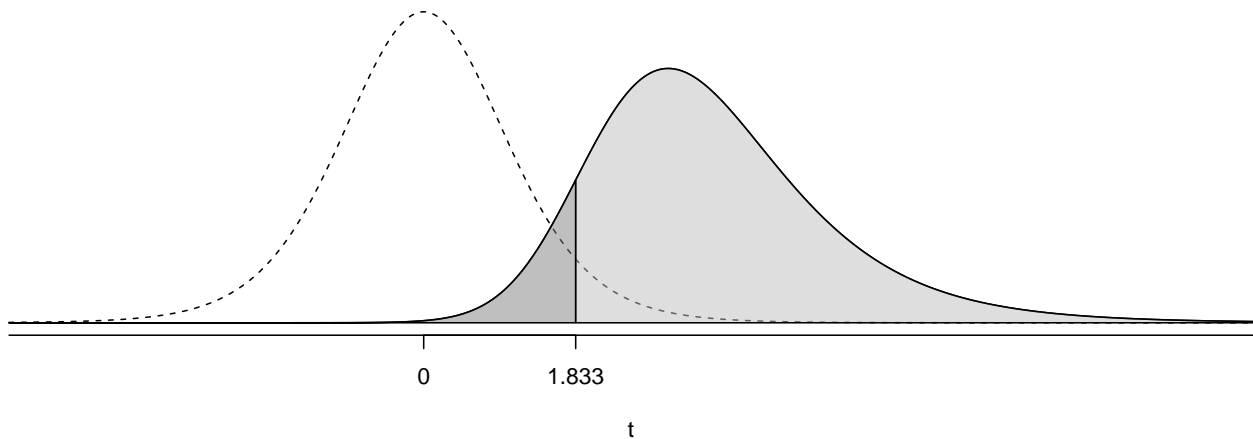
1. If  $t \geq 1.833$  then  $p\text{-value} \leq \alpha$  so *reject*  $H_0$ .
2. If  $t < 1.833$  then  $p\text{-value} > \alpha$  so *do not reject*  $H_0$ .

Thus the probability of a type I error is the probability of rejecting  $H_0$  when  $H_0$  is true, which is  $P(t \geq 1.833|H_0) = \alpha$ . Thus, *the probability of rejecting the null hypothesis when it is true (i.e., a type I error) equals  $\alpha$ .*

## Probability of a Type II Error

The probability of a type II error is the probability of *not rejecting*  $H_0$  when it is *false*.

**Example:** Suppose again that we have the hypotheses  $H_0 : \mu = 0$  versus  $H_a : \mu > 0$  and plan to use a significance level of  $\alpha = 0.05$ . The *critical value* of  $t$  is the value of the test statistic with a p-value *equal* to the significance level. Assume a sample size of  $n=10$ . But now suppose that *in reality*  $\mu > 0$  (e.g.,  $\mu = 1$ ). Note that the sampling distribution of the test statistic when  $H_0$  is true is shown by the dotted line, while the sampling distribution of the test statistic when  $H_0$  is false is shown by the solid line.

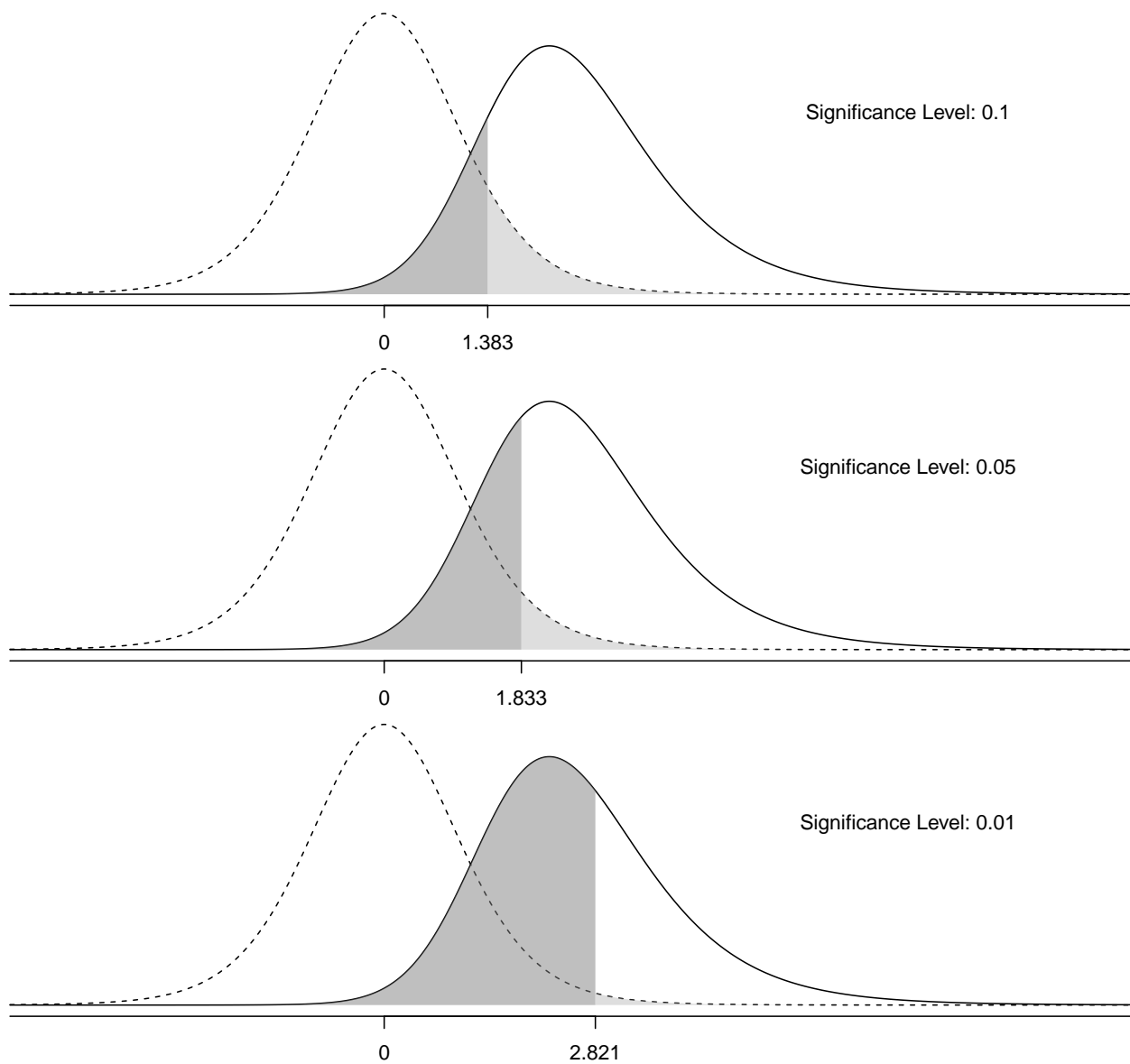


So the probability of a type II error (i.e., the probability of *not rejecting*  $H_0$  when it is *false*) here is  $P(t < 1.833|H_a)$ .

It is not as simple to compute the probability of a type II error because it depends on several factors.

## Effect of $\alpha$ on Error Probabilities

The probability of a type I error is the *light* grey area, and the probability of a type II error is the *dark* grey area.



If we decrease  $\alpha$  we will (a) decrease the probability of a type I error and (b) increase the probability of a type II error.

If we increase  $\alpha$  we will (a) increase the probability of a type I error and (b) decrease the probability of a type II error.