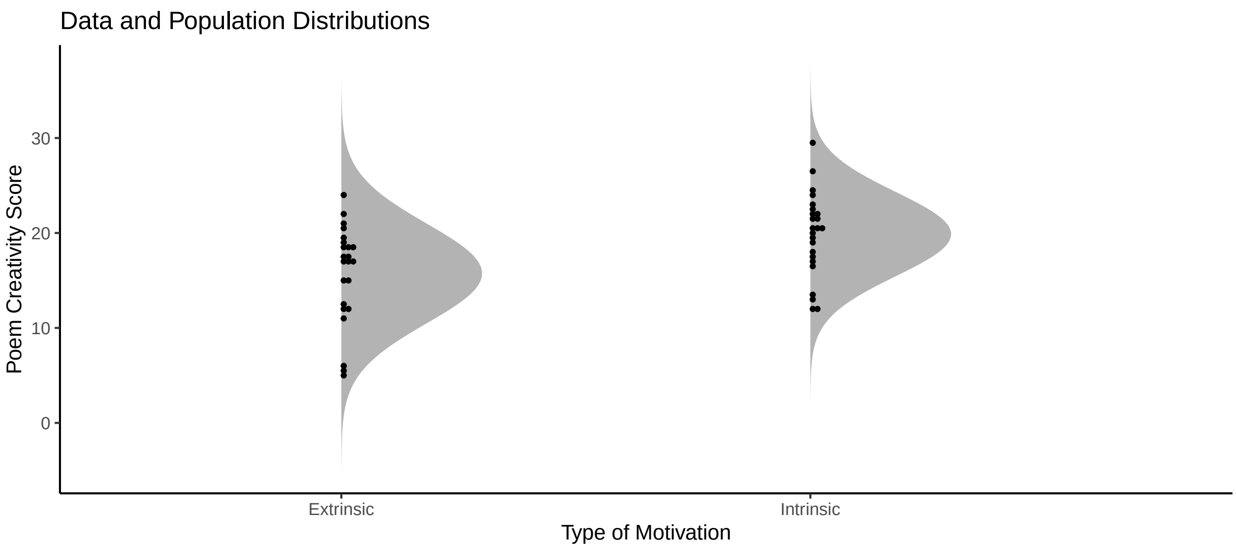
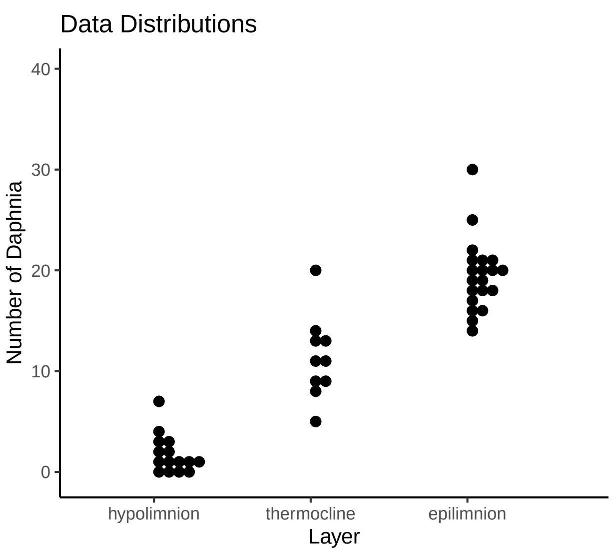
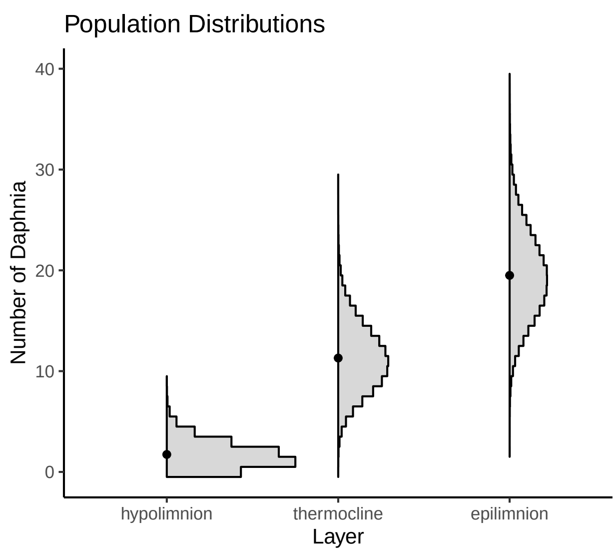
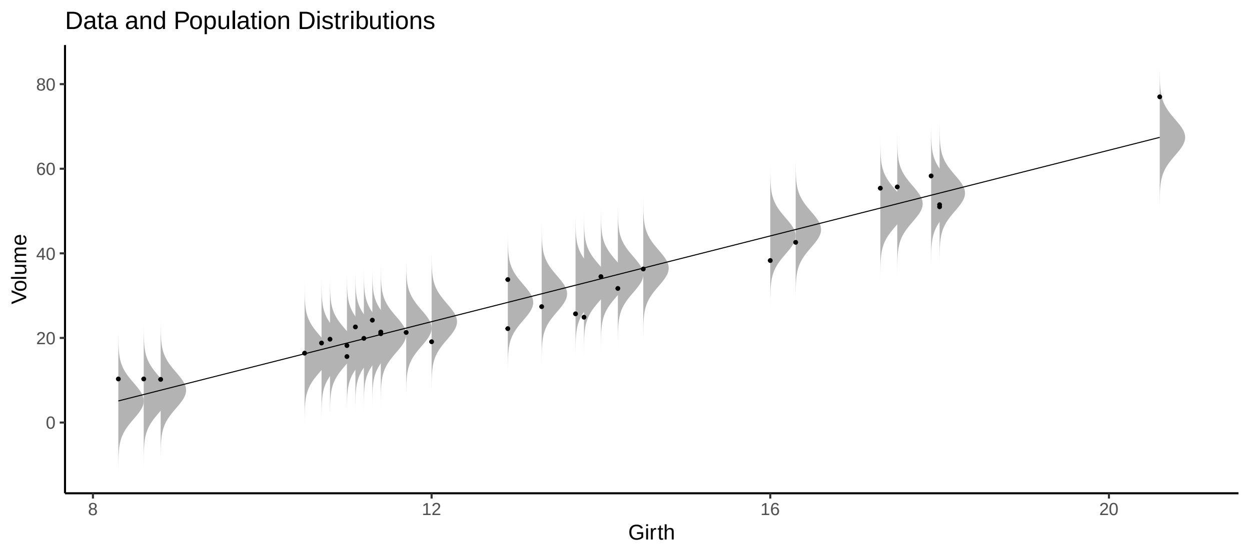
Wednesday, May 4









The Linear Regression Model

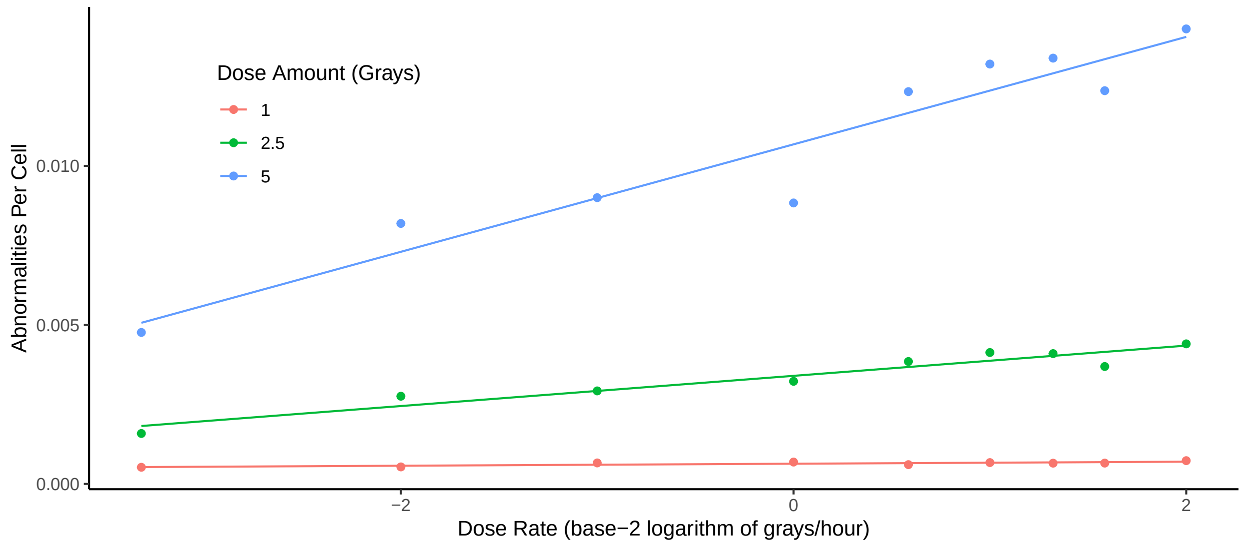
The linear regression **model** has the form

$$\mu_y = \alpha + \beta x,$$

where μ_y is the *mean of the population distribution of the response variable* y (e.g., mean tree volume), and x is the *value of the explanatory variable* (e.g., tree girth). The quantities α and β are the intercept and slope *parameters*, respectively.

Study Question: What do the four symbols in $\mu_y = \alpha + \beta x$ represent?

Example: The plot below shows the data from a study of the relationship between the number of chromosomal abnormalities per cell (μ_y) and the rate of exposure to gamma radiation (x). But this relationship was studied at three different total dose amounts. Three linear regression models are used here.



Multiple Linear Regression

The **multiple linear regression model** has the form

$$\mu_y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$$

where $x_1, x_2, \ldots x_k$ are the values of k explanatory variables. For example, we might have

$$\mu_y = \alpha + \beta_1 x_1 + \beta_2 x_2$$

where μ_y is the *mean of the population distribution of the response variable* y (e.g., mean tree volume), x_1 is the *value of one explanatory variable* (e.g., tree girth), and x_2 is the *value of a second explanatory variable* (e.g., tree height).

The generic term **linear regression** is usually used to refer to the case where there is one or more explanatory variables. The case where there is only one explanatory variable is sometimes referred to as **simple linear regression**.

Study Question: How is *multiple linear regression* different from *simple linear regression*?

Nonlinear Regression

A **nonlinear regression model** is any regression model that cannot be written as

$$\mu_y = \alpha + \beta x$$

or

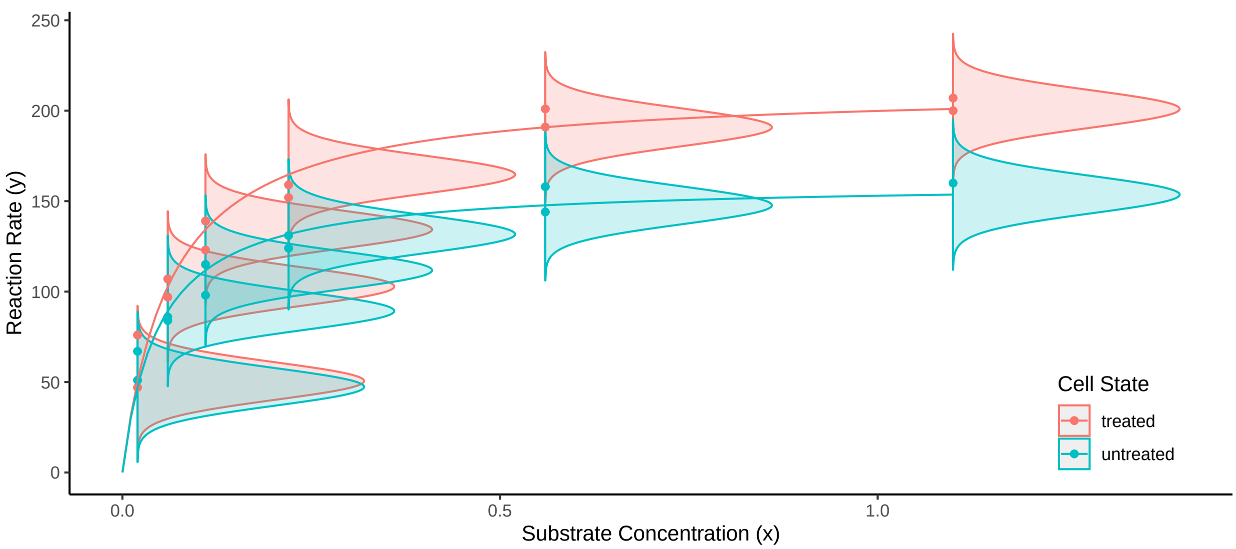
$$\mu_y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k.$$

Study Question: What is *nonlinear regression*?

Example: In biochemistry, the relationship between the mean reaction rate (μ_y) and the concentration of a substrate (x) is often modeled as

$$\mu_y = \frac{\delta x}{\gamma + x}.$$

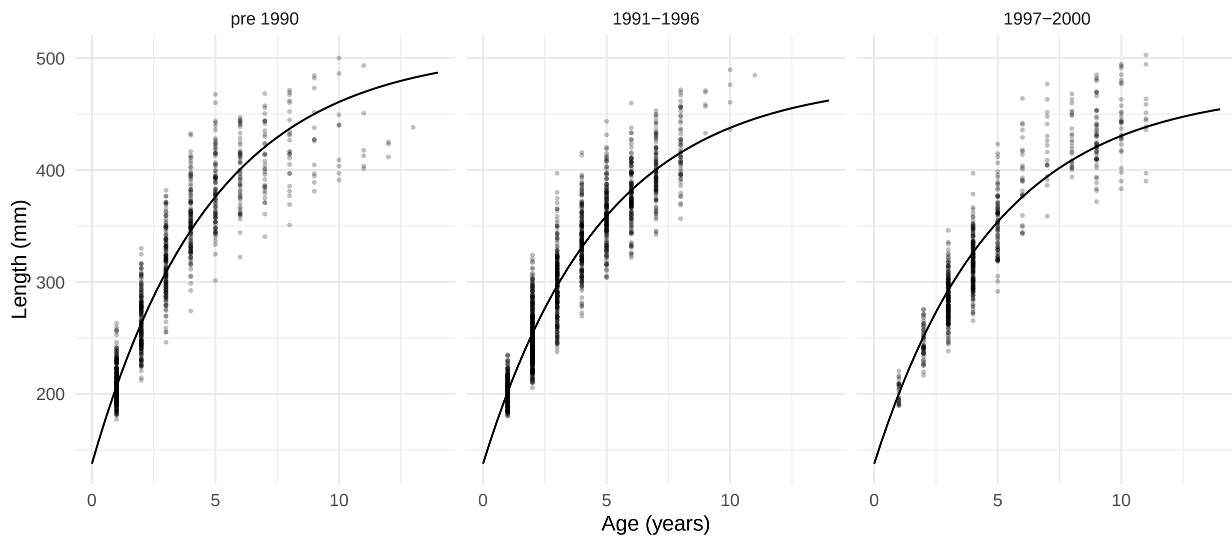
Here δ is the maximum achievable mean reaction rate, and γ is the substrate concentration that yields a mean reaction rate half way between 0 and δ.



Example: In fisheries science, a nonlinear regression model (the *von Bertalanffy model*) is used to model the relationship between mean length (μ_y) and age (x) of fish. This model can be written as

$$\mu_y = \alpha + (\delta - \alpha)e^{-x \log(2)/\gamma}.$$

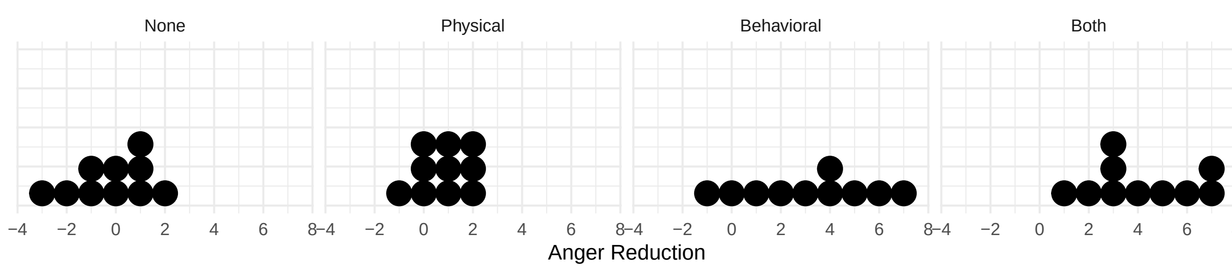
Here α is the maximum value of μ_y that we approach as fish age, δ is the value of μ_y before they reach one year of age, and γ is how many years it takes for μ_y to be half way between δ and α.



Categorical Explanatory Variables

What if we have one or more *categorical* explanatory variables? Regression can accommodate categorical explanatory variables using some tricks. But often the statistical methodology is described as the *analysis of variance* (ANOVA).

Example: The dot plots below show four samples of observations of the variable *anger reduction*. The four samples correspond to four levels of a categorical treatment variable of *anger management exercises* (none, physical, behavioral, and both physical and behavioral).



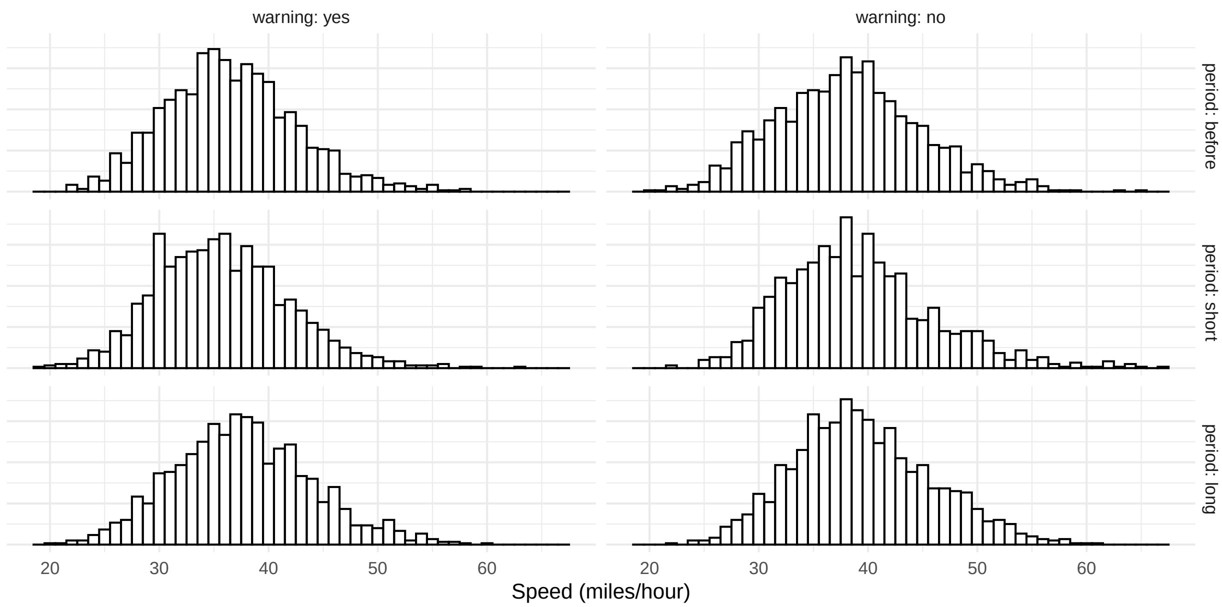
Here are some descriptive statistics for each group.

Group	n	mean	sd
None	10	-0.2	1.5
Physical	10	0.8	1.0
Behavioral	10	3.1	2.6
Both	10	4.1	2.1

Possible research questions to address using statistical inference:

1. Are physical exercises effective for anger reduction?
2. Are behavioral exercises effective for anger reduction?
3. Is one type of exercise more effective than the other?
4. Is it useful to use both types of exercises rather than just one?

Example: The histograms below show data from an observational study of the effect of warning signs on car speed. Here there are *two* categorical explanatory variables: warning (yes or no) and period (before, short, and long).



Here are some descriptive statistics for each group.

warning	period	n	mean	sd
yes	before	1400	36.5	6.0
yes	short	1400	35.8	6.1
yes	long	1362	37.7	6.4
no	before	1400	38.2	6.6
no	short	1400	39.2	6.8
no	long	1475	39.5	6.4

Possible research questions to address using statistical inference:

1. How much (if at all) does a warning sign decrease average speed?
2. How does time affect the effectiveness of a warning sign on reducing speed?

Study Question: When would a researcher use an *analysis of variance*?