Homework Problem Set 5: Deriving Sampling Distributions

Solutions are given at the end.

Sampling Distribution of a Mean

Consider again the probability distribution for a single observation for the tree volume survey from lecture (i.e., a *population distribution*), but with different probabilities. Here x denotes the volume of a randomly selected tree.

x	$P(x)$
20	0.7
30	0.3

Use the five-step method to derive *two* sampling distributions of the mean from a sample of observations ($\bar{x}$): one based on a sample size of $n = 2$, and a second based on a sample size of $n = 3$. (Hint: When $n = 2$ there are four possible samples in the sample space, and one of these samples is 20, 30. But when $n = 3$ there are eight possible samples, and one of these samples is 20, 20, 30.)

Sampling Distribution of a Median

In the previous problem you derived the sampling distribution of the *mean* volume of a random sample of $n = 3$ observations of the volumes of trees. Now use the five-step method to derive the sampling distribution of the *median* volume of a random sample of $n = 3$ observations of the volumes of trees. Recall that the median is defined as the middle observation when the observations are arranged in increasing order. (Hint: You can use the same sample space that you obtained when you were deriving the sampling distribution of the mean.)

Sampling Distribution of a Proportion

1. Consider again the population distribution from lecture for the preference of one female platy fish, but with different probabilities.

x	$P(x)$
C	0.4
Y	0.6

   Here C and Y represent a preference for the clear-tailed and the yellow-tailed male, respectively. Use the five-step method to derive the sampling distribution of the proportion ($\hat{p}$) of platy fish in a sample of $n = 2$ observations that show a preference for the yellow-tailed male. Then use the five-step method to derive the sampling distribution when the sample size is $n = 3$.

2. The *number* of observations on which the female platy prefers the yellow-tailed male has a *binomial distribution*. Note that we define a "success" as a female preferring the yellow-tailed male, and so the probability of a success is as given by the population distribution. Derive the sampling distribution for the proportion of observations out of $n = 2$ on which the female platy prefers the yellow-tailed male using the formula for the binomial distribution. Then use the binomial distribution formula to derive the sampling distribution for a sample size of $n = 3$. Note that you should get the same sampling

distributions using the formula for the binomial distribution as you did when you used the five-step method in the previous problem.

Sampling Distribution of a Mean (Solution)

The sampling distribution of $\bar{x}$ when $n = 2$ is shown in the following table.

$\bar{x}$	$P(\bar{x})$
20	0.49
25	0.42
30	0.09

The sampling distribution of $\bar{x}$ when $n = 3$ is shown in the following table.

$\bar{x}$	$P(\bar{x})$
20.00	0.343
23.33	0.441
26.67	0.189
30.00	0.027

Note that the sample mean has been rounded in the case when $n = 3$.

Sampling Distribution of a Median (Solution)

The following table shows the sampling distribution of the median. Here I am using m to represent the median.

m	$P(m)$
20	0.784
30	0.216

Sampling Distribution of a Proportion (Solution)

The following table shows the sampling distribution of the proportion when $n = 2$.

$\hat{p}$	$P(\hat{p})$
0.0	0.16
0.5	0.48
1.0	0.36

The following table shows the sampling distribution of the proportion when $n = 3$.

$\hat{p}$	$P(\hat{p})$
0	0.064
1/3	0.288
2/3	0.432
1	0.216