Wednesday, Nov 3

Independent Versus Dependent Samples

Example: Consider a study like the twin study for investigating the relationship between schizophrenia and the volume of the left hippocampus.

	Schizophrenia		
Pair	Unaffected	Affected	Difference
1	1.94	1.27	0.67
2	1.44	1.63	-0.19
3	1.56	1.47	0.09
4	1.58	1.39	0.19
5	2.06	1.93	0.13
6	1.66	1.26	0.4
7	1.75	1.71	0.04
8	1.77	1.67	0.1
9	1.78	1.28	0.5
10	1.92	1.85	0.07
11	1.25	1.02	0.23
12	1.93	1.34	0.59
13	2.04	2.02	0.02
14	1.62	1.59	0.03
15	2.08	1.97	0.11
Size:	15	15	15
Mean:	1.759	1.56	0.199
SD:	0.242	0.301	0.238

How would we make inferences if the samples are *dependent*, and how would we make inferences if the samples are *independent*?

Matching in Observational Studies

Example: Consider a study to compare the foot hair density of Hobbits who smoke pipe-weed with that of Hobbits that do not smoke. But we use an *observational* study and are concerned that smoking and foot hair density are also related to the confounding variables *age* and *Farthing*. We can control for age and Farthing by matching Hobbits based on those two variables.

	Smoker			Non-Smoker		
Pair	Age	Farthing	FHDI	FHDI	Farthing	Age
1	60	W	74.8	70.1	W	59
2	108	W	96.2	45.1	W	109
3	80	E	69.3	60.2	E	80
4	43	N	96.9	76.3	N	43
5	96	S	77.6	69.2	S	96
⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	98	W	70.5	49	W	95

This creates a matched-pairs design with dependent samples.

	FHDI		
Pair	Smoker	Non-Smoker	Difference
1	74.8	70.1	4.7
2	96.2	45.1	51.1
3	69.3	60.2	9.1
4	96.9	76.3	20.6
5	77.6	69.2	8.4
⋮	⋮	⋮	⋮
100	70.5	49	21.5

Standard Errors for Dependent Versus Independent Samples

Suppose we have two samples, both of equal size n (so we'll leave off the subscript). The standard error of $\bar{x}_1 - \bar{x}_2$ is

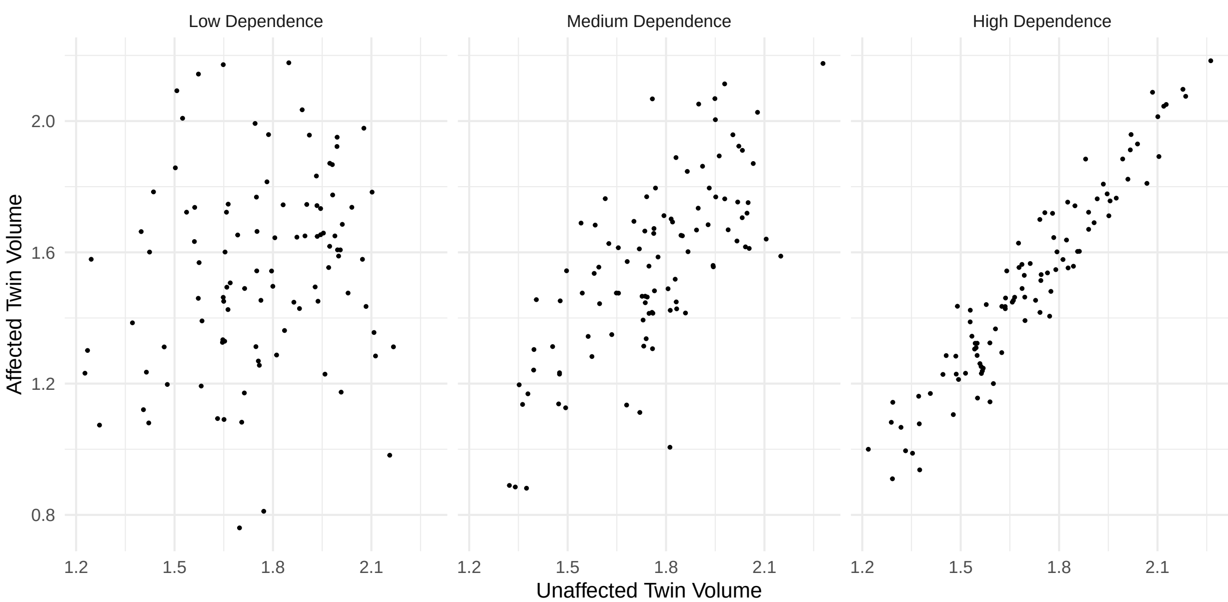
$$\underbrace{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n}}}_{\text{independent}} \geq \underbrace{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n} - \rho\frac{2\sigma_1\sigma_2}{n}}}_{\text{dependent}} = \frac{\sigma_d}{\sqrt{n}}.$$

Here σ_d is the standard deviation of the *differences* of matched observations, and ρ is the *correlation coefficient*. The correlation coefficient can be between -1 and 1.

1. If the samples are *independent* then $\rho = 0$.
2. If the samples are *dependent* then (usually) $0 < \rho < 1$.

What does this imply about the standard errors for dependent versus independent samples?

Example: Suppose we had a matched-pairs design using genetically-related individuals (e.g., cousins, siblings, or identical twins) for a study like that that investigated the relationship between schizophrenia and left hippocampus volume.



The Other Standard Error for $\bar{x}_1 - \bar{x}_2$

If two samples are independent, the standard error we have been using is

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

An alternative is to use

$$s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad \text{where} \quad s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}},$$

and the degrees of freedom becomes $n_1 + n_2 - 2$.

Why would we use this alternative standard error?