Wednesday, Apr 13

Proportional Hazards and the Survival Function

Let $h_0(t)$ and $S_0(t)$ be the "baseline" hazard and survival functions (i.e., the function when all $x_j = 0$). If the proportional hazards assumption hold so that

$$h(t) = h_0(t)e^{\beta_1 x_1}e^{\beta_2 x_2} \cdots e^{\beta_k x_k},$$

then it can be shown that

$$S(t) = S_0(t)^\eta \quad \text{where} \quad \eta = e^{\beta_1 x_1}e^{\beta_2 x_2} \cdots e^{\beta_k x_k}.$$

Thus the effect of increasing x_j in a proportional hazards model can be summarized as follows.

1. If $\beta_j > 0$ then $S(t)$ will be *decreased* as x_j increases, as will $E(T)$.
2. If $\beta_j < 0$ then $S(t)$ will be *increased* as x_j increases, as will $E(T)$.

Note: The signs of the β_j parameters will be *opposite* of what they are in a equivalent accelerated failure time model.

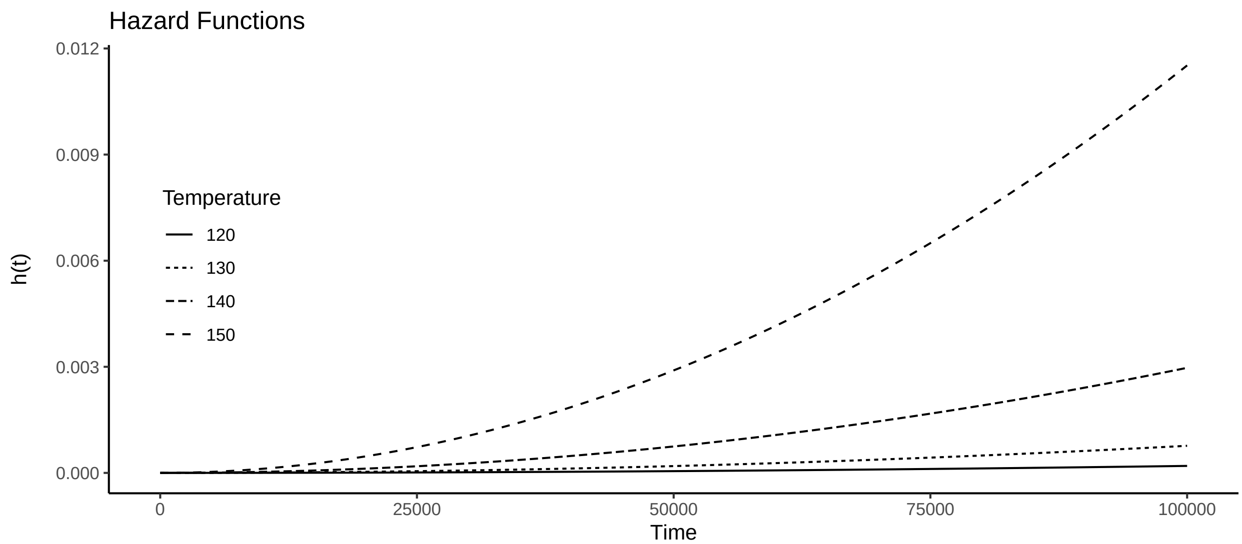
Example: Consider again a proportional hazards model for the `motors` data.

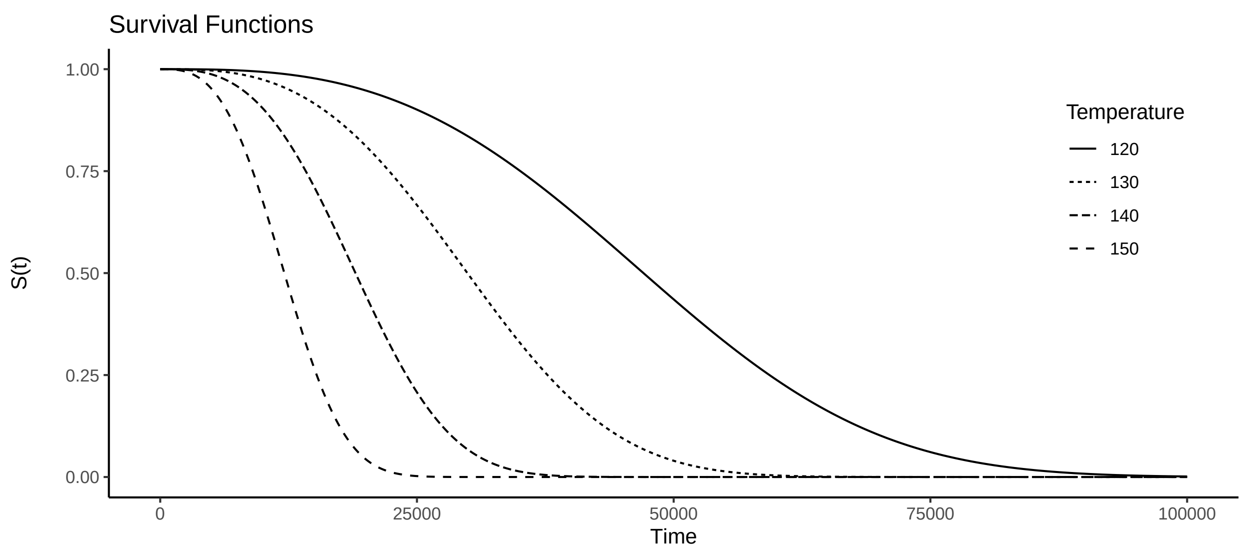
```
library(flexsurv)
m <- flexsurvreg(Surv(time, cens) ~ temp, dist = "weibullPH", data = MASS::motors)
print(m)
```

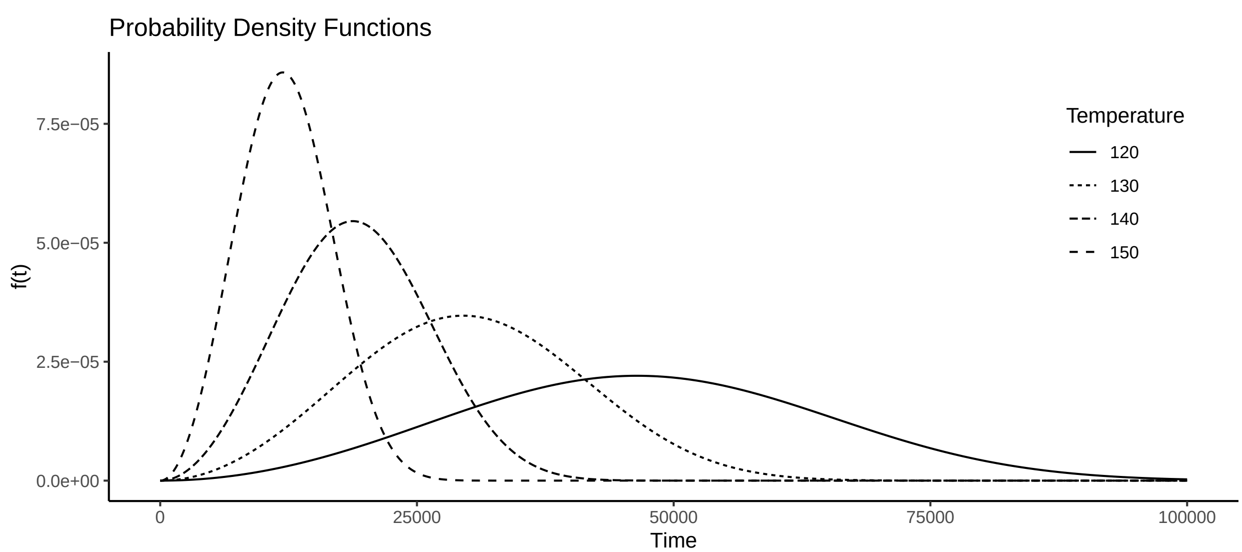
```
Call:
flexsurvreg(formula = Surv(time, cens) ~ temp, data = MASS::motors,
    dist = "weibullPH")

Estimates:
       data mean  est       L95%      U95%      se        exp(est)  L95%      U95%
shape        NA   2.99e+00  1.97e+00  4.55e+00  6.40e-01        NA        NA        NA
scale        NA   6.34e-22  1.79e-30  2.24e-13  6.37e-21        NA        NA        NA
temp   1.82e+02   1.36e-01  8.04e-02  1.91e-01  2.81e-02  1.15e+00  1.08e+00  1.21e+00

N = 40,  Events: 17,  Censored: 23
Total time at risk: 140654
Log-likelihood = -147.4, df = 3
AIC = 300.7
```


Hazard Functions
h(t)
0.012
0.009
0.006
0.003
0.000
Temperature
120
130
140
150
0
25000
50000
75000
100000
Time


Survival Functions
S(t)
1.00
0.75
0.50
0.25
0.00
Temperature
120
130
140
150
0
25000
50000
75000
100000
Time


Probability Density Functions
f(t)
7.5e−05
5.0e−05
2.5e−05
0.0e+00
Temperature
120
130
140
150
0
25000
50000
75000
100000
Time

Semi-Parametric (Cox) Proportional Hazards Model

A proportional hazards model assumes

$$h_i(t) = h_0(t) e^{\beta_1 x_{i1}} e^{\beta_2 x_{i2}} \cdots e^{\beta_k x_{ik}},$$

where again $h_0(t)$ is the "baseline" proportional hazards function. The functional form of $h_0(t)$ and thus $h_i(t)$ depends on the distribution of T_i.

1. A *parametric* proportional hazards model assumes a particular distribution and functional form of $h_0(t)$.
2. The *semi-parametric* proportional hazards model does not assume a particular distribution or functional form for $h_0(t)$.

The *marginal* or *partial* likelihood function permits maximum likelihood estimation of $\beta_1, \beta_2, \ldots, \beta_k$ *without* assuming a particular distribution. It is based only on the *rank order* of the times.

Comments about semi-parametric proportional hazards models.

1. Right-censoring can be easily handled with this model. But other types of censoring require additional assumptions.
2. Estimation of hazard and survival functions relies on a semi-parametric approach.
3. Stratification can be used when hazard functions are proportional within but not between strata.

The function `coxph` from the **survival** package will estimate a Cox proportional hazards model.

Example: Consider a Cox proportional hazards model for the `motors` data.

```r
library(survival) # for coxph function
m <- coxph(Surv(time, cens) ~ temp, data = MASS::motors)
summary(m)
```

```
Call:
coxph(formula = Surv(time, cens) ~ temp, data = MASS::motors)

  n= 40, number of events= 17

       coef exp(coef) se(coef)    z Pr(>|z|)
temp 0.0919    1.0962   0.0274 3.36  0.00079 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     exp(coef) exp(-coef) lower .95 upper .95
temp       1.1      0.912      1.04      1.16

Concordance= 0.84  (se = 0.035 )
Likelihood ratio test= 25.6  on 1 df,   p=4e-07
Wald test            = 11.3  on 1 df,   p=8e-04
Score (logrank) test = 22.7  on 1 df,   p=2e-06
```

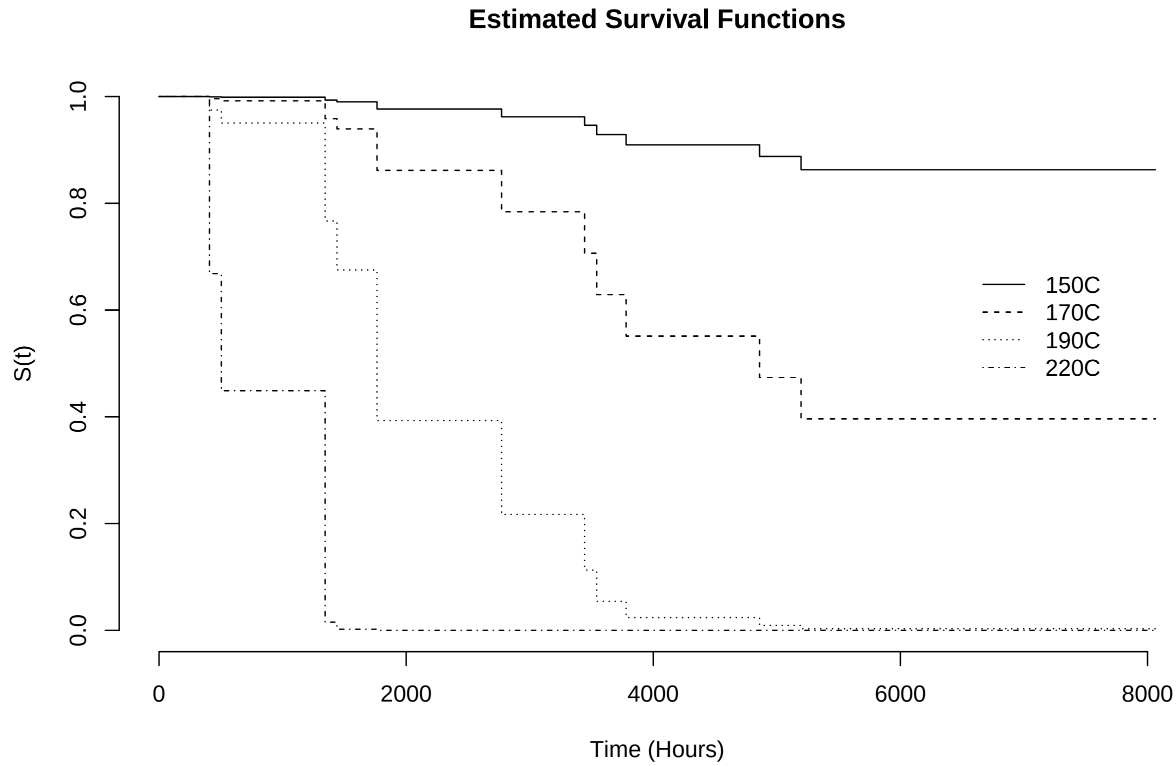
We can plot estimated survival functions from a `coxph` model object.

```r
d <- data.frame(temp = c(150,170,190,220))

# plot estimated survival functions
plot(survfit(m, newdata = d), bty = "n", lty = 1:4, xlab = "Time (Hours)", ylab = "S(t)")

# add a legend
legend(6500, 0.7, legend = c("150C", "170C", "190C", "220C"), lty = 1:4, bty = "n")
```

```r
# add a title
title("Estimated Survival Functions")
```



A common non-parametric estimator of a survival function is the Kaplan-Meier estimator, but it is largely limited to cases where you have a categorical explanatory variable with multiple times observed per category.

Discrete Survival Time Models

Discrete survival time models treat time-to-event as a discrete random variable rather than a continuous random variable. This is done for one of two reasons.

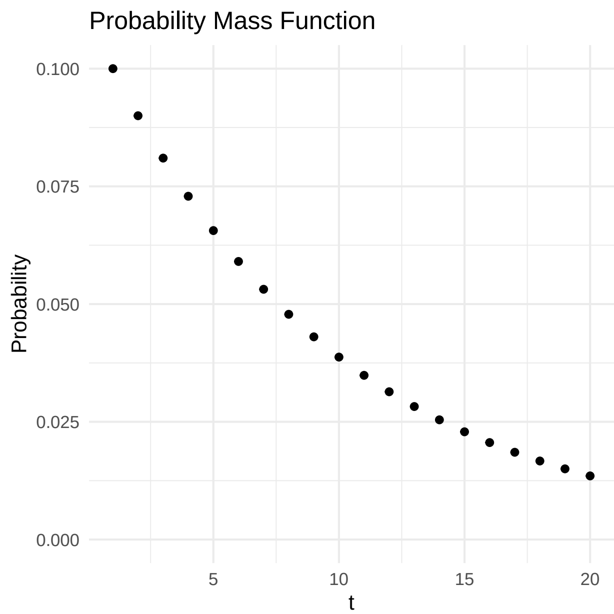
1. Time is actually continuous, but we treat it as discrete for convenience/simplicity, or because the observations are interval-censored (with common intervals, e.g., week, month, year).
2. The "time" is actually a count of "attempts" of an event (e.g., number of cycles until pregnancy, number of times to take a test until it is passed, number of times a machine is run until it fails).

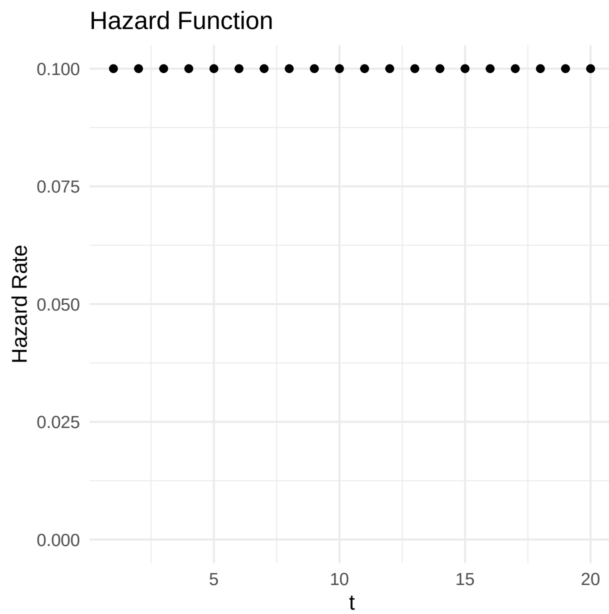
For discrete time, the probability density, survival, and hazard functions are analogous to what they are for continuous time, but simpler because all of them give probabilities.

1. The *probability mass function* is $f(t) = P(T = t)$. This gives the probability that the event will happen at time t.
2. The *survival function* is, as before, $S(t) = P(T \geq t)$. This gives the probability that the event will happen at time t or later.
3. The *hazard function* is $h(t) = P(T = t|T \geq t)$. This gives the probability that the event will happen at time t *given* that it has not yet happened (i.e., the probability that it will happen at time t given that the unit has "survived" to that point).

It is important to not confused the probability mass function which gives the probability that the event will happen at time t, versus the hazard function which gives the probability that the event will happen at time t *given* that it has not yet happened.

Example: Suppose I have a fair ten-sided die. Let t be the number of rolls until I get one. The figures below show the probability mass and hazard function.





Technical Details: Note that $f(t)$, $S(t)$, and $h(t)$ are related because $h(t) = f(t)/S(t)$. Also we can define $f(t)$ entirely in terms of $h(t)$. Consider that if a unit survives to time t, the probability that it *will not* survive past time t is

$$h(t) = P(T = t|T \geq t),$$

and the probability that it *will* survive past time t is

$$1 - h(t) = 1 - P(T = t|T \geq t) = P(T > t|T \geq t).$$

So we can write $f(t)$ in terms of $h(t)$ as follows.

1. For observations that *are not* right-censored at time t,

$$\begin{aligned}
f(1) &= h(1), \\
f(2) &= [1 - h(1)]h(2), \\
f(3) &= [1 - h(1)][1 - h(2)]h(3), \\
f(4) &= [1 - h(1)][1 - h(2)][1 - h(3)]h(4), \\
f(5) &= [1 - h(1)][1 - h(2)][1 - h(3)][1 - h(4)]h(5),
\end{aligned}$$

   and so on. In general for non-censored discrete times

$$f(t) = \begin{cases} h(t), & \text{if } t = 1, \\ h(t)\prod_{j=1}^{t-1}[1 - h(j)], & \text{if } t > 1, \end{cases}$$

   Note that $1 - h(t) = 1 - P(T = t|T \geq t) = P(T > t|T \geq t)$.

2. For observations that *are* right-censored at time t,

$$\begin{aligned}
f(1) &= [1 - h(1)], \\
f(2) &= [1 - h(1)][1 - h(2)], \\
f(3) &= [1 - h(1)][1 - h(2)][1 - h(3)], \\
f(4) &= [1 - h(1)][1 - h(2)][1 - h(3)][1 - h(4)], \\
f(5) &= [1 - h(1)][1 - h(2)][1 - h(3)][1 - h(4)][1 - h(5)],
\end{aligned}$$

and so on. In general for right-censored discrete times

$$f(t) = \prod_{j=1}^{t} [1 - h(j)].$$

Note that $1 - h(t) = 1 - P(T = t | T \geq t) = P(T > t | T \geq t)$.

Discrete Survival Models as Binary Regression Models

Discrete survival time models can be expressed as *binary* regression models. We can model the probability that a unit will not survive past time t *given* that it survived to time t, or we can model the probability that it will survive past time *given* that it survived to time t.

Suppose we code time-till-event with positive integers. For every T we define a set of binary responses such that if $T = t$ then we have t binary responses, $Y_1, Y_2, \ldots, Y_t$, such that

$$Y_t = \begin{cases} 1, & \text{if the event occurs at time } t \text{ (i.e., } T = t), \\ 0, & \text{if the event occurs after time } t \text{ (i.e., } T > t). \end{cases}$$

Note that if T is right-censored then we let $T = t$ where t is the last time we know the event had not failed, but $Y_t = 0$.

Example: The observed event times are $T = t$ where t = 1, 2, 3, 4, or 5. Then we define $Y_1, Y_2, \ldots, Y_5$ as follows.

t	Y_1	Y_2	Y_3	Y_4	Y_5
1	1				
2	0	1			
3	0	0	1		
4	0	0	0	1	
5	0	0	0	0	1

Example: T is censored such that $T > t$ where t = 1, 2, 3, 4, or 5. Then we define $Y_1, Y_2, \ldots, Y_5$ as follows.

t	Y_1	Y_2	Y_3	Y_4	Y_5
1	0				
2	0	0			
3	0	0	0		
4	0	0	0	0	
5	0	0	0	0	0

Not: If time is discrete due to interval-censoring the *maximum possible time* does not need a binary variable.

Technical Details: The distribution of T can be stated in terms of the Y_t. It follows that $h(t) = P(Y_t = 1)$ and $1 - h(t) = 1 - P(Y_t = 1) = P(Y_t = 0)$, so if T is not censored then

$$f(t) = \begin{cases} P(Y_1 = 1), & \text{if } t = 1, \\ P(Y_t = 1) \prod_{j=1}^{t-1} P(Y_t = 0), & \text{if } t > 1, \end{cases}$$

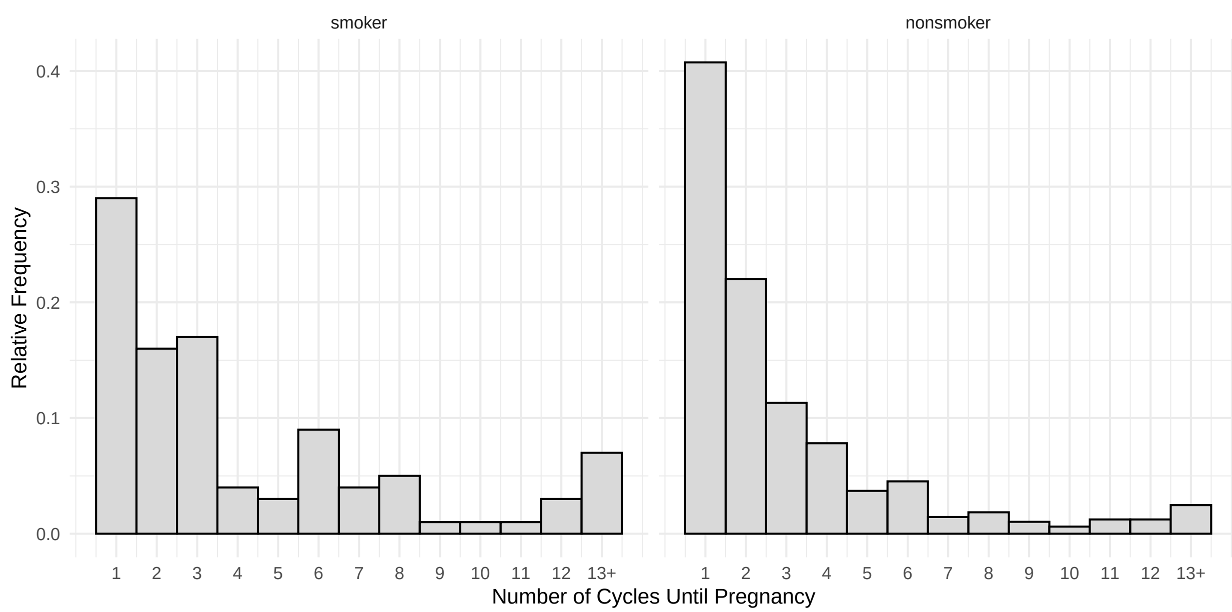
and if T is censored such that $T > t$ then

$$f(t) = \prod_{j=1}^{t} P(Y_t = 0).$$

TLDR: Many discrete time survival models can be estimated as binary regression models (e.g., logistic regression) where the response variable is an indicator variable for if the event happened at a given time.

Example: Consider the following data from a study comparing mothers who smoke to those who do not with respect to the number of menstrual cycles until pregnancy.

```
library(trtools) # for the cycles data
p <- ggplot(cycles, aes(x = cycles, y = 1.0 * ..density..)) +
   facet_wrap(~ mother) +
   geom_histogram(binwidth = 1, center = 1,
      color = "black", fill = grey(0.85)) +
   scale_x_continuous(breaks = 1:13, labels = c(1:12,"13+")) +
   labs(x = "Number of Cycles Until Pregnancy",
      y = "Relative Frequency") + theme_minimal()
plot(p)
```



Note: Using `w * ..density..` computes the relative frequency for the `y` aesthetic, where `w` is the bar width.

It is important to note that all reported values of 13 cycles are actually right-censored and so represent 13 *or more* cycles. The observed censoring times are between 1 and 12 cycles, with all recorded cycles of 13 representing right-censored observations only known to be *more than 12 cycles*. We need to create an indicator variable for *observed* times and to change values of 13 to 12 since that was the last observed time.

```
cycles$status <- ifelse(cycles$cycles == 13, 0, 1)
cycles$cycles <- ifelse(cycles$cycles == 13, 12, cycles$cycles)
```

Here are some mothers of observed (i.e., not censored) times.

```
    cycles    mother status
102      1 nonsmoker      1
```

```
216       1 nonsmoker      1
358       2 nonsmoker      1
437       3 nonsmoker      1
449       3 nonsmoker      1
```

Here are some mothers with censored times.

```
    cycles    mother status
576     12 nonsmoker      0
577     12 nonsmoker      0
581     12 nonsmoker      0
582     12 nonsmoker      0
584     12 nonsmoker      0
```

The function `dsurvbin` from the **trtools** package helps convert a data frame with a discrete time-till-event into a format with binary variables as discussed above (a similar function is available in the **discSurv** package).

```
cycles$subject <- 1:nrow(cycles) # just for clarity
cycles.bin <- dsurvbin(cycles, y = "cycles", event = "status")
```

So depending on the number of cycles up to twelve indicator variable are created *for each observational unit*. For example, here is a mother where pregnancy occurred after *three* cycles.

```
    cycles mother status subject unit t y
541      3 smoker      1      46   46 1 0
542      3 smoker      1      46   46 2 0
543      3 smoker      1      46   46 3 1
```

And here is a mother where pregnancy occurred after *five* cycles.

```
    cycles mother status subject unit t y
793      5 smoker      1      67   67 1 0
794      5 smoker      1      67   67 2 0
795      5 smoker      1      67   67 3 0
796      5 smoker      1      67   67 4 0
797      5 smoker      1      67   67 5 1
```

And here is a mother where pregnancy occurred after *twelve* cycles.

```
     cycles mother status subject unit  t y
1081     12 smoker      1      91   91  1 0
1082     12 smoker      1      91   91  2 0
1083     12 smoker      1      91   91  3 0
1084     12 smoker      1      91   91  4 0
1085     12 smoker      1      91   91  5 0
1086     12 smoker      1      91   91  6 0
1087     12 smoker      1      91   91  7 0
1088     12 smoker      1      91   91  8 0
1089     12 smoker      1      91   91  9 0
1090     12 smoker      1      91   91 10 0
1091     12 smoker      1      91   91 11 0
1092     12 smoker      1      91   91 12 1
```

But for comparison, here is a mother where pregnancy was right-censored and is only known to have occurred (if it occurred) *after twelve cycles*.

```
     cycles mother status subject unit  t y
1117     12 smoker      0      94   94  1 0
1118     12 smoker      0      94   94  2 0
```

```
1119      12 smoker      0       94    94   3 0
1120      12 smoker      0       94    94   4 0
1121      12 smoker      0       94    94   5 0
1122      12 smoker      0       94    94   6 0
1123      12 smoker      0       94    94   7 0
1124      12 smoker      0       94    94   8 0
1125      12 smoker      0       94    94   9 0
1126      12 smoker      0       94    94  10 0
1127      12 smoker      0       94    94  11 0
1128      12 smoker      0       94    94  12 0
```

Let $P(Y_{it} = 1) = \pi_{it}$ be the probability that the i-th mother will become pregnant on the t-th cycle *given* that they did not become pregnant on an earlier cycle. We could consider a logistic regression model such that

$$\pi_{it} = \frac{e^{\eta_i}}{1 + e^{\eta_i}} \quad \text{where} \quad \eta_i = \beta_0 + \beta_1 x_i,$$

where $x_i = 1$ if the mother is a non-smoker, and $x_i = 0$ if the mother is a smoker. We can estimate this model like any other binary regression model.

```
m <- glm(y ~ mother, family = binomial, data = cycles.bin)
cbind(summary(m)$coefficients, confint(m))
```

```
                Estimate Std. Error z value  Pr(>|z|)   2.5 %  97.5 %
(Intercept)      -1.2420     0.1177 -10.550 5.082e-26 -1.4779 -1.0158
mothernonsmoker   0.5414     0.1304   4.151 3.312e-05  0.2894  0.8012
```

```
# odds ratio (odds of pregnancy of a non-smoker versus a smoker)
exp(cbind(coef(m), confint(m)))
```

```
                        2.5 % 97.5 %
(Intercept)     0.2888 0.2281 0.3621
mothernonsmoker 1.7185 1.3356 2.2283
```

```
# estimated probabilities of pregnancy
d <- data.frame(mother = c("nonsmoker","smoker"))
cbind(d, glmint(m, newdata = d))
```

```
     mother    fit    low    upp
1 nonsmoker 0.3317 0.3078 0.3565
2    smoker 0.2241 0.1865 0.2667
```

```
margeff(m,
 a = list(mother = "nonsmoker"),
 b = list(mother = "smoker"))
```

```
 estimate      se   lower  upper tvalue  df    pvalue
   0.1076 0.02396 0.06064 0.1546  4.491 Inf 7.093e-06
```

```
margeff(m, type = "percent",
 a = list(mother = "nonsmoker"),
 b = list(mother = "smoker"))
```

```
 estimate    se lower upper tvalue  df  pvalue
    48.02 14.62 19.37 76.67  3.285 Inf 0.00102
```

Note that with this model the hazard function is "flat" — i.e., the probability of pregnancy each cycle (given pregnancy has not yet happened) is the same.[1] This is reasonable here, but in other cases we might expect

[1] In such cases we say that the number of trials until something happens has a *geometric* distribution.

there to be time-varying effects (e.g., season or temperature in animals), which can be handled easily since we can let an explanatory variable vary over time (recorded as `t` in the data frame). Although over a longer time span we might consider a model where the hazard function decreases due to age.

Example: Consider the following data on the grade when adolescent males first experience sexual intercourse.

```
firstsex <- read.table("https://stats.idre.ucla.edu/stat/examples/alda/firstsex.csv",
    sep = ",", header = TRUE)
head(firstsex)
```

```
  id time censor pt     pas
1  1    9      0  0  1.9789
2  2   12      1  1 -0.5455
3  3   12      1  0 -1.4050
4  5   12      0  1  0.9742
5  6   11      0  0 -0.6356
6  7    9      0  1 -0.2429
```

There is right-censoring (i.e., boys who did not experience sex by the 12th grade). We need a proper status variable for that.

```
firstsex$status <- ifelse(firstsex$censor == 1, 0, 1)
```

One key explanatory variable is whether or not a boy experienced a "parenting transition" prior to the 7th grade. The variable is `pt` but is a binary variable. We'll convert it to a factor with clear level labels.

```
firstsex$transition <- factor(firstsex$pt,
   levels = c(0,1), labels = c("no","yes"))
```

We can verify that these changes were done correctly.

```
head(firstsex)
```

```
  id time censor pt     pas status transition
1  1    9      0  0  1.9789      1         no
2  2   12      1  1 -0.5455      0        yes
3  3   12      1  0 -1.4050      0         no
4  5   12      0  1  0.9742      1        yes
5  6   11      0  0 -0.6356      1         no
6  7    9      0  1 -0.2429      1        yes
```

Now we need to transform the data to create indicator variables for whether or not a boy experienced sex for the first time in a given grade.

```
library(trtools)
firstsex <- dsurvbin(firstsex, "time", "status")
head(firstsex)
```

```
  id time censor pt     pas status transition unit t y
1  1    9      0  0  1.9789      1         no    1 7 0
2  1    9      0  0  1.9789      1         no    1 8 0
3  1    9      0  0  1.9789      1         no    1 9 1
7  2   12      1  1 -0.5455      0        yes    2 7 0
8  2   12      1  1 -0.5455      0        yes    2 8 0
9  2   12      1  1 -0.5455      0        yes    2 9 0
```

Here is a boy who first had sex in the 9th grade.

```
subset(firstsex, id == 1)
```

```
  id time censor pt    pas status transition unit t y
```

```
1  1    9      0  0 1.979      1         no    1 7 0
2  1    9      0  0 1.979      1         no    1 8 0
3  1    9      0  0 1.979      1         no    1 9 1
```

Here is a boy who first had sex in the 12th grade.

```r
subset(firstsex, id == 5)
```

```
   id time censor pt    pas status transition unit  t y
19  5   12      0  1 0.9742      1        yes    4  7 0
20  5   12      0  1 0.9742      1        yes    4  8 0
21  5   12      0  1 0.9742      1        yes    4  9 0
22  5   12      0  1 0.9742      1        yes    4 10 0
23  5   12      0  1 0.9742      1        yes    4 11 0
24  5   12      0  1 0.9742      1        yes    4 12 1
```

Here is a boy who did not first have sex by the 12th grade (but may have first had sex later — i.e., right-censored).

```r
subset(firstsex, id == 3)
```

```
   id time censor pt    pas status transition unit  t y
13  3   12      1  0 -1.405      0         no    3  7 0
14  3   12      1  0 -1.405      0         no    3  8 0
15  3   12      1  0 -1.405      0         no    3  9 0
16  3   12      1  0 -1.405      0         no    3 10 0
17  3   12      1  0 -1.405      0         no    3 11 0
18  3   12      1  0 -1.405      0         no    3 12 0
```

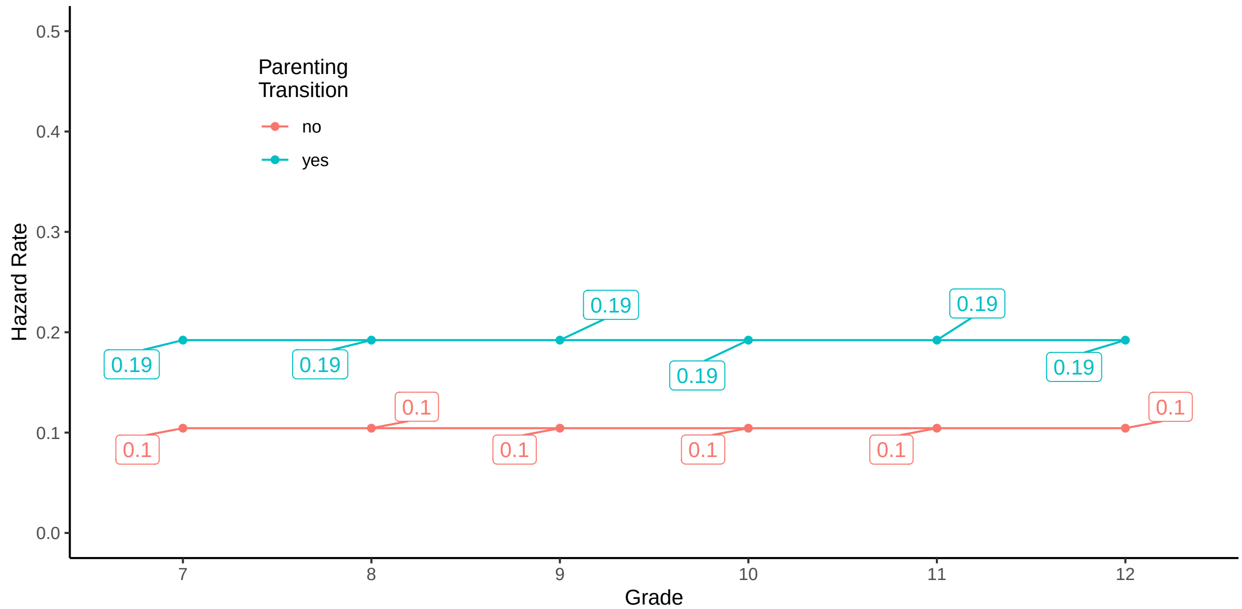
First consider a model for a flat/constant hazard function $h(t) = P(T = t | T \geq t)$, where here T is grade. However we will let the hazard rate depend on whether or not there was a parenting transition.

```r
m <- glm(y ~ transition, family = binomial, data = firstsex)
summary(m)$coefficients
```

```
              Estimate Std. Error z value  Pr(>|z|)
(Intercept)    -2.1493     0.1714 -12.539 4.553e-36
transitionyes   0.7131     0.2084   3.421 6.231e-04
```

```r
d <- expand.grid(t = c("7","8","9","10","11","12"), transition = c("no","yes"))
d$yhat <- predict(m, newdata = d, type = "response")

library(ggrepel) # for geom_label_repel
p <- ggplot(d, aes(x = t, y = yhat, color = transition)) + theme_classic() +
  geom_point() + geom_line(aes(group = transition)) + ylim(0, 0.5) +
  geom_label_repel(aes(label = round(yhat,2)),
   box.padding = 0.75, show.legend = FALSE) +
  labs(x = "Grade", y = "Hazard Rate", color = "Parenting\nTransition") +
  theme(legend.position = c(0.2,0.8))
plot(p)
```




```r
# odds ratio
contrast(m, tf = exp,
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate lower upper
Grade 7      2.04 1.356  3.07
Grade 8      2.04 1.356  3.07
Grade 9      2.04 1.356  3.07
Grade 10     2.04 1.356  3.07
Grade 11     2.04 1.356  3.07
Grade 12     2.04 1.356  3.07
```

```r
# marginal effect (difference)
trtools::margeff(m,
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate      se   lower  upper tvalue  df    pvalue
Grade 7   0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
Grade 8   0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
Grade 9   0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
Grade 10  0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
Grade 11  0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
Grade 12  0.08774 0.02441 0.03991 0.1356  3.595 Inf 0.0003245
```

```r
# marginal effect (factor)
trtools::margeff(m, type = "factor",
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate     se lower upper tvalue  df    pvalue
```

```
Grade 7      1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
Grade 8      1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
Grade 9      1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
Grade 10     1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
Grade 11     1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
Grade 12     1.84 0.3331 1.188 2.493  5.526 Inf 3.274e-08
```

```
# marginal effect (percent)
trtools::margeff(m, type = "percent",
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate    se lower upper tvalue  df  pvalue
Grade 7     84.05 33.31 18.77 149.3  2.524 Inf 0.01162
Grade 8     84.05 33.31 18.77 149.3  2.524 Inf 0.01162
Grade 9     84.05 33.31 18.77 149.3  2.524 Inf 0.01162
Grade 10    84.05 33.31 18.77 149.3  2.524 Inf 0.01162
Grade 11    84.05 33.31 18.77 149.3  2.524 Inf 0.01162
Grade 12    84.05 33.31 18.77 149.3  2.524 Inf 0.01162
```

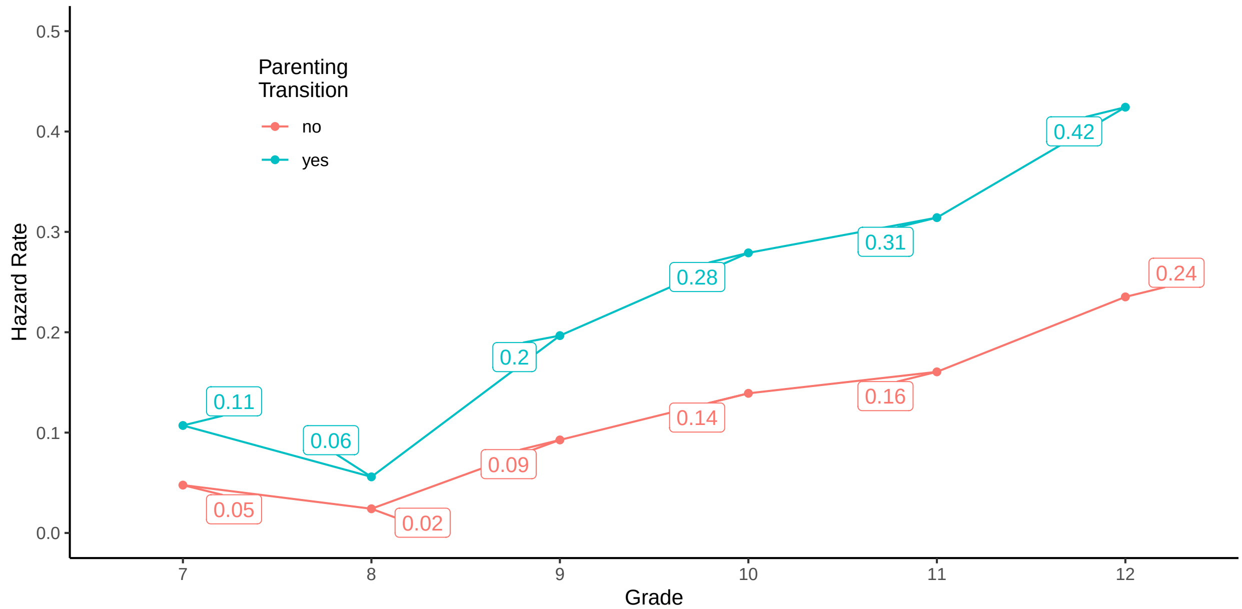
Now consider a model where the hazard rate is not necessarily constant over grades. This can be done by including an "effect" for time (i.e., grade).

```
m <- glm(y ~ transition + t, family = binomial, data = firstsex)
summary(m)$coefficients
```

```
              Estimate Std. Error z value  Pr(>|z|)
(Intercept)    -2.9943     0.3175  -9.431 4.072e-21
transitionyes   0.8736     0.2174   4.018 5.859e-05
t8             -0.7058     0.4728  -1.493 1.355e-01
t9              0.7132     0.3519   2.027 4.267e-02
t10             1.1717     0.3452   3.394 6.887e-04
t11             1.3401     0.3588   3.735 1.877e-04
t12             1.8153     0.3674   4.941 7.780e-07
```

```
d <- expand.grid(t = c("7","8","9","10","11","12"), transition = c("no","yes"))
d$yhat <- predict(m, newdata = d, type = "response")

p <- ggplot(d, aes(x = t, y = yhat, color = transition)) + theme_classic() +
   geom_point() + geom_line(aes(group = transition)) + ylim(0, 0.5) +
   geom_label_repel(aes(label = round(yhat,2)),
      box.padding = 0.75, show.legend = FALSE) +
   labs(x = "Grade", y = "Hazard Rate", color = "Parenting\nTransition") +
   theme(legend.position = c(0.2,0.8))
plot(p)
```




```r
# odds ratio
contrast(m, tf = exp,
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate lower upper
Grade 7     2.396 1.564 3.668
Grade 8     2.396 1.564 3.668
Grade 9     2.396 1.564 3.668
Grade 10    2.396 1.564 3.668
Grade 11    2.396 1.564 3.668
Grade 12    2.396 1.564 3.668
```

```r
# discrete marginal effect
margeff(m,
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate      se    lower   upper tvalue  df    pvalue
Grade 7   0.05942 0.01848 0.023189 0.09565  3.214 Inf 1.307e-03
Grade 8   0.03178 0.01315 0.006002 0.05757  2.416 Inf 1.568e-02
Grade 9   0.10393 0.02824 0.048575 0.15928  3.680 Inf 2.334e-04
Grade 10  0.13997 0.03622 0.068983 0.21095  3.865 Inf 1.112e-04
Grade 11  0.15365 0.04059 0.074099 0.23320  3.786 Inf 1.533e-04
Grade 12  0.18901 0.04802 0.094884 0.28313  3.936 Inf 8.293e-05
```

```r
# discrete marginal effect (factor)
margeff(m, type = "factor",
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate     se lower upper tvalue  df    pvalue
```

```
Grade 7     2.246 0.4595 1.346 3.147  4.888 Inf 1.017e-06
Grade 8     2.318 0.4886 1.360 3.275  4.744 Inf 2.099e-06
Grade 9     2.121 0.4077 1.322 2.920  5.203 Inf 1.957e-07
Grade 10    2.006 0.3606 1.299 2.713  5.564 Inf 2.642e-08
Grade 11    1.957 0.3396 1.291 2.623  5.763 Inf 8.279e-09
Grade 12    1.804 0.2793 1.256 2.351  6.458 Inf 1.058e-10
```

```
# marginal effect (percent)
margeff(m, type = "percent",
    a = list(transition = "yes", t = c("7","8","9","10","11","12")),
    b = list(transition = "no", t = c("7","8","9","10","11","12")),
    cnames = paste("Grade", 7:12))
```

```
         estimate    se lower upper tvalue  df   pvalue
Grade 7    124.61 45.95 34.55 214.7  2.712 Inf 0.006688
Grade 8    131.75 48.86 36.00 227.5  2.697 Inf 0.007001
Grade 9    112.12 40.77 32.22 192.0  2.750 Inf 0.005954
Grade 10   100.61 36.06 29.94 171.3  2.790 Inf 0.005267
Grade 11    95.71 33.96 29.15 162.3  2.818 Inf 0.004830
Grade 12    80.35 27.93 25.62 135.1  2.877 Inf 0.004009
```