Friday, Apr 1

Discrete Marginal Effects

Consider a regression model with (without loss of generality) two explanatory variables, X_1 and X_2. A *discrete marginal effect* is the change in the expected response when we change an explanatory variable.

For example, if we have a regression model where $E(Y)$ is a function of X_1 and X_2, the discrete marginal effect of changing X_1 from x_b to x_a is

$$E(Y|X_1 = x_a, X_2 = x_2) - E(Y|X_1 = x_b, X_2 = x_2).$$

That is, the change in the expected response when X_1 is changed from x_b to x_a. (Note: When we talk about a change in the expected response or the "effect" of a change in an explanatory variable, we do not necessarily mean that this is a *causal* relationship.)

In a linear model a discrete marginal effect is basically what is done by `contrast`.

Example: Recall our model for the `whiteside` data. The function `margeff` in the **trtools** package will estimate a discrete marginal effect.

```
m <- lm(Gas ~ Insul + Temp + Insul:Temp, data = MASS::whiteside)
summary(m)$coefficients
```

```
                Estimate Std. Error t value  Pr(>|t|)
(Intercept)       6.8538    0.13596  50.409 7.997e-46
InsulAfter       -2.1300    0.18009 -11.827 2.316e-16
Temp             -0.3932    0.02249 -17.487 1.976e-23
InsulAfter:Temp   0.1153    0.03211   3.591 7.307e-04
```

The model is

$$E(Y_i) = \beta_0 + \beta_1 a_i + \beta_2 t + \beta_3 a_i t_i,$$

where Y_i is gass consumption,

$$a_i = \begin{cases} 1, & \text{if the } i\text{-th observation is after insulation,} \\ 0, & \text{otherwise.} \end{cases}$$

So the marginal effect of increasing temperature from $t_b = 2$ to $t_a = 7$ after insulation is

$$E(Y|a = 1, t = 7) - E(Y|a = 1, t = 2) = 5(\beta_2 + \beta_3).$$

Before insulation it is

$$E(Y|a = 0, t = 7) - E(Y|a = 0, t = 2) = 5\beta_2.$$

We can estimate this using the `lincon` or `contrast` functions.

```
library(trtools)
lincon(m, a = c(0,0,5,5)) # marginal effect after insulation
```

```
            estimate     se lower upper tvalue df    pvalue
(0,0,5,5),0    -1.39 0.1146 -1.62 -1.16 -12.12 52 8.936e-17
```

```r
lincon(m, a = c(0,0,5,0)) # marginal effect after insulation
```

```
            estimate     se  lower  upper tvalue df    pvalue
(0,0,5,0),0   -1.966 0.1124 -2.192 -1.741 -17.49 52 1.976e-23
```

```r
contrast(m, cnames = c("Before","After"),
  a = list(Temp = 7, Insul = c("Before","After")),
  b = list(Temp = 2, Insul = c("Before","After")))
```

```
       estimate     se  lower  upper tvalue df    pvalue
Before   -1.966 0.1124 -2.192 -1.741 -17.49 52 1.976e-23
After    -1.390 0.1146 -1.620 -1.160 -12.12 52 8.936e-17
```

The function `margeff` (also from the **trtools** package) is specifically designed to estimate marginal effects (and other things) and works similarly to `contrast`.

```r
margeff(m, cnames = c("Before","After"),
  a = list(Temp = 7, Insul = c("Before","After")),
  b = list(Temp = 2, Insul = c("Before","After")))
```

```
       estimate     se  lower  upper tvalue df    pvalue
Before   -1.966 0.1124 -2.192 -1.741 -17.49 52 1.976e-23
After    -1.390 0.1146 -1.620 -1.160 -12.12 52 8.936e-17
```

We can also estimate the discrete marginal effect of adding insulation at different temperatures.

```r
contrast(m, cnames = c("0C","5C","10C"),
  a = list(Temp = c(0,5,10), Insul = "After"),
  b = list(Temp = c(0,5,10), Insul = "Before"))
```

```
    estimate      se  lower  upper  tvalue df    pvalue
0C   -2.1300 0.18009 -2.491 -1.769 -11.827 52 2.316e-16
5C   -1.5535 0.08777 -1.730 -1.377 -17.699 52 1.155e-23
10C  -0.9769 0.18583 -1.350 -0.604  -5.257 52 2.784e-06
```

```r
margeff(m, cnames = c("0C","5C","10C"),
  a = list(Temp = c(0,5,10), Insul = "After"),
  b = list(Temp = c(0,5,10), Insul = "Before"))
```

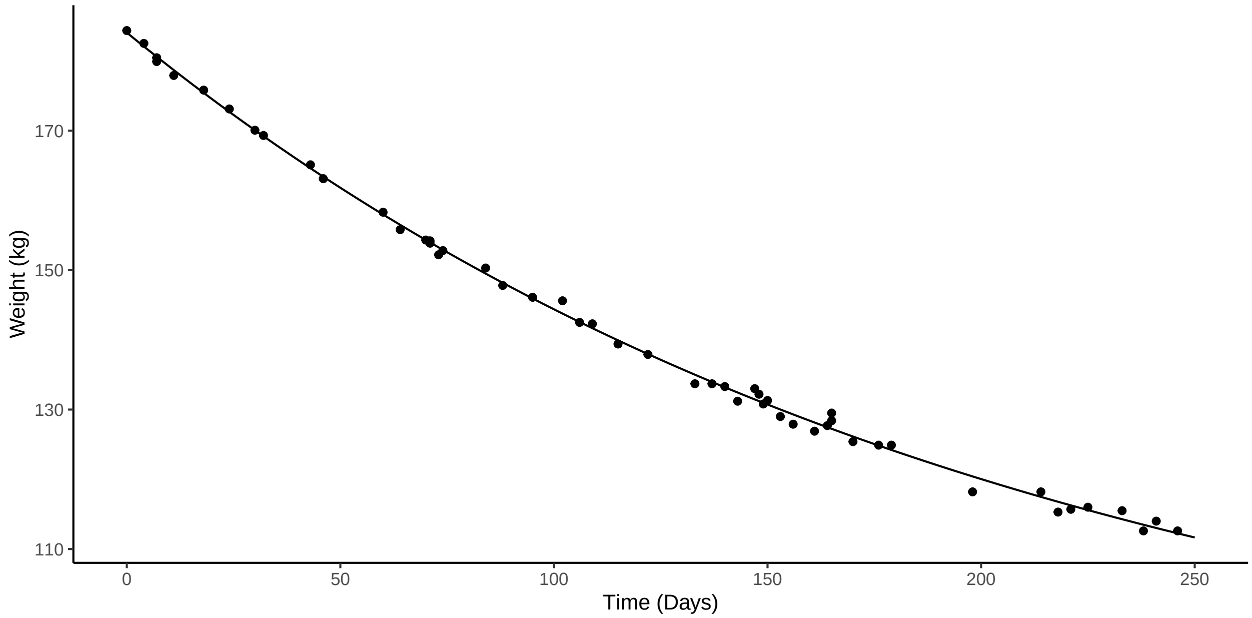
```
    estimate      se  lower  upper  tvalue df    pvalue
0C   -2.1300 0.18009 -2.491 -1.769 -11.827 52 2.316e-16
5C   -1.5535 0.08777 -1.730 -1.377 -17.699 52 1.155e-23
10C  -0.9769 0.18583 -1.350 -0.604  -5.257 52 2.784e-06
```

So what's the use of `margeff`? The `contrast` and `lincon` functions can only handle *linear* functions of the model parameters. But in some cases the marginal effect is not a linear function of the model parameters. This is where the `margeff` function is useful.

Example: Consider the following nonlinear model for the change in expected weight over time.

```r
m <- nls(Weight ~ t1 + t2*2^(-Days/t3), data = MASS::wtloss,
  start = list(t1 = 90, t2 = 95, t3 = 120))

d <- data.frame(Days = seq(0, 250, by = 1))
d$yhat <- predict(m, newdata = d)

p <- ggplot(MASS::wtloss, aes(x = Days, y = Weight)) +
  geom_point() + theme_classic() +
  labs(y = "Weight (kg)", x = "Time (Days)") +
```

```
  geom_line(aes(y = yhat), data = d)
plot(p)
```



The model is

$$E(Y) = \theta_1 + \theta_2 2^{-d/\theta_3},$$

where Y is weight and d is days. The discrete marginal effect of going from 50 to 100 days is

$$\underbrace{\theta_1 + \theta_2 2^{-100/\theta_3}}_{E(Y|d=100)} - (\underbrace{\theta_1 + \theta_2 2^{-50/\theta_3}}_{E(Y|d=50)}) = \theta_2(2^{-100/\theta_3} - 2^{-50/\theta_3}).$$

This is *not* a linear function of the model parameters, so we cannot use the usual methods like `contrast` or `lincon`. But we can make (approximate) inferences using the *delta method* (more on that later). The `margeff` function makes implementing this method relatively straight forward.

```
margeff(m, a = list(Days = 100), b = list(Days = 50))
```

```
 estimate     se  lower  upper tvalue df    pvalue
   -17.43 0.1292 -17.69 -17.17 -134.9 49 1.182e-64
```

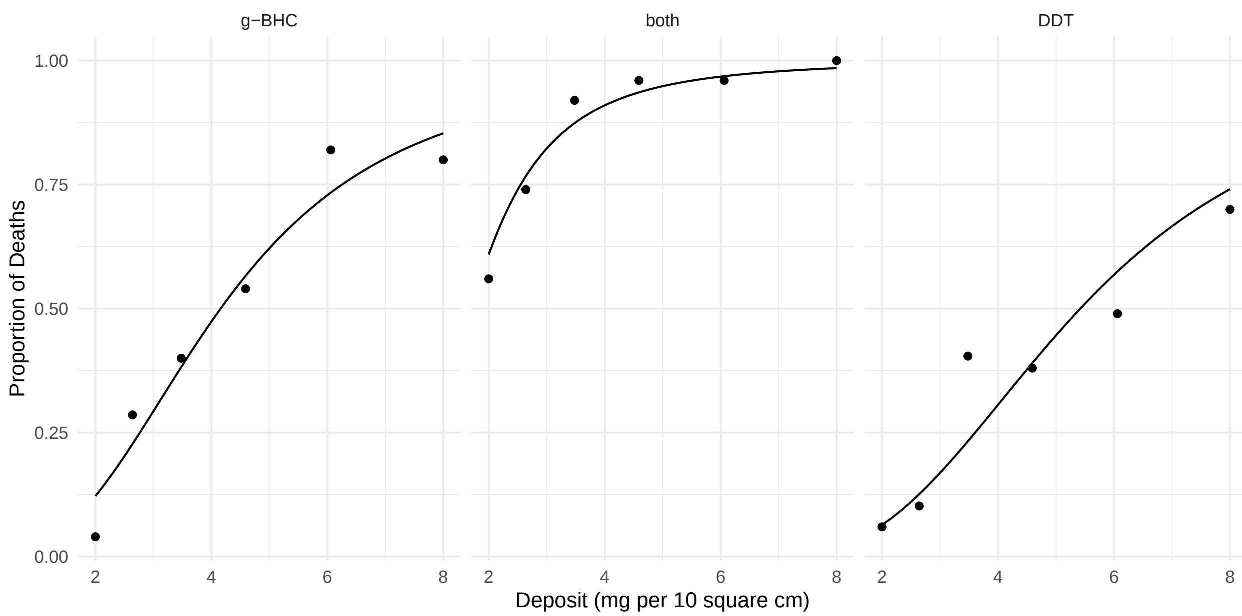
```
margeff(m,
  a = list(Days = c(50,100,150,200)),
  b = list(Days = c(0,50,100,150)),
  cnames = c("0->50", "50->100", "100->150", "150->200"))
```

```
         estimate     se  lower  upper tvalue df    pvalue
0->50      -22.25 0.3291 -22.91 -21.59  -67.6 49 4.838e-50
50->100    -17.43 0.1292 -17.69 -17.17 -134.9 49 1.182e-64
100->150   -13.65 0.1033 -13.86 -13.44 -132.2 49 3.109e-64
150->200   -10.69 0.1606 -11.02 -10.37  -66.6 49 1.001e-49
```

Example: Consider the following model for the `insecticide` data.

```
m <- glm(cbind(deaths, total-deaths) ~ log2(deposit) + insecticide, family = binomial, data = insecticid

d <- expand.grid(deposit = seq(2, 8, length = 100),
```

```
  insecticide = unique(insecticide$insecticide))
d$phat <- predict(m, newdata = d, type = "response")

p <- ggplot(insecticide, aes(x = deposit, y = deaths/total)) +
  geom_point() + facet_wrap(~ insecticide) +
  geom_line(aes(y = phat), data = d) + theme_minimal() +
  labs(x = "Deposit (mg per 10 square cm)",
    y = "Proportion of Deaths")
plot(p)
```



We know how to interpret the effects using *odds ratios*. Here are the odds ratios for the effect of doubling the deposit from 2 to 4 units.

```
contrast(m, tf = exp,
  a = list(deposit = 4, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 2, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate lower upper
g-BHC    6.479 4.833 8.685
both     6.479 4.833 8.685
DDT      6.479 4.833 8.685
```

And here are the odds ratios for the effect of insecticide (g-BHC versus DDT).

```
contrast(m, tf = exp,
  a = list(deposit = c(2,4,6,8), insecticide = "g-BHC"),
  b = list(deposit = c(2,4,6,8), insecticide = "DDT"),
  cnames = c("2","4","6","8"))
```

```
  estimate lower upper
2     2.04 1.383 3.007
4     2.04 1.383 3.007
6     2.04 1.383 3.007
8     2.04 1.383 3.007
```

But with odds ratios we have to interpret effects in terms of *odds*. What if we want to interpret the effect on the *probability*? The discrete marginal effect is in terms of the *expected response* (here the expected proportion or, equivalently, the probability of death).

```
margeff(m,
  a = list(deposit = 4, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 2, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate      se  lower  upper tvalue  df    pvalue
g-BHC   0.3517 0.02470 0.3033 0.4001 14.240 Inf 5.183e-46
both    0.3007 0.03650 0.2292 0.3723  8.239 Inf 1.737e-16
DDT     0.2424 0.02188 0.1995 0.2853 11.076 Inf 1.633e-28
```

Here are some discrete marginal effects of insecticide (g-BHC versus DDT).

```
margeff(m,
  a = list(deposit = c(2,4,6,8), insecticide = "g-BHC"),
  b = list(deposit = c(2,4,6,8), insecticide = "DDT"),
  cnames = c("2","4","6","8"))
```

```
  estimate      se   lower   upper tvalue  df    pvalue
2  0.05821 0.01773 0.02346 0.09295  3.283 Inf 0.0010257
4  0.16753 0.04565 0.07806 0.25700  3.670 Inf 0.0002425
6  0.16034 0.04395 0.07420 0.24647  3.648 Inf 0.0002638
8  0.11275 0.03225 0.04955 0.17596  3.496 Inf 0.0004717
```

The appeal of the marginal effect here is that for many people probabilities are more intuitive than odds.

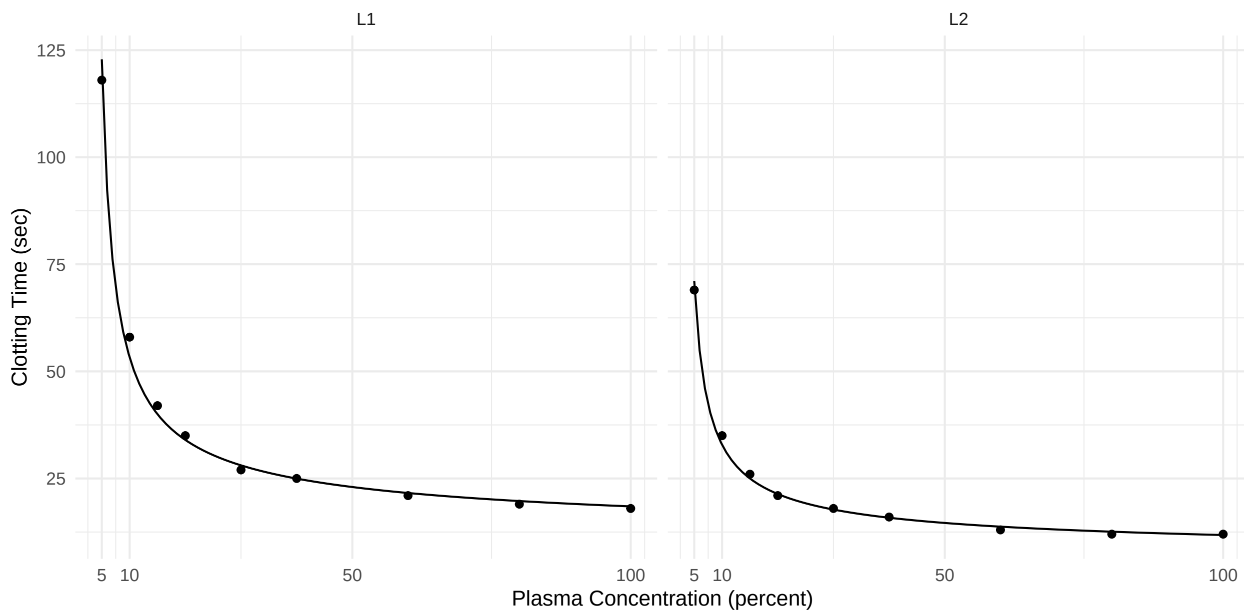
Example: Consider the following model for data from a study of the effect of blood plasma concentration/dilution on clotting time.

```
clotting <- data.frame(
  conc = rep(c(5,10,15,20,30,40,60,80,100), 2),
  time = c(118,58,42,35,27,25,21,19,18,69,35,26,21,18,16,13,12,12),
  lot = rep(c("L1","L2"), each = 9)
)
head(clotting)
```

```
  conc time lot
1    5  118  L1
2   10   58  L1
3   15   42  L1
4   20   35  L1
5   30   27  L1
6   40   25  L1
```

```
m <- glm(time ~ lot + log(conc) + lot:log(conc),
  family = Gamma(link = inverse), data = clotting)

d <- expand.grid(conc = seq(5, 100, length = 100), lot = c("L1","L2"))
d$yhat <- predict(m, newdata = d, type = "response")

p <- ggplot(clotting, aes(x = conc, y = time)) + theme_minimal() +
  geom_point() + facet_wrap(~ lot) + facet_wrap(~ lot) +
  labs(x = "Plasma Concentration (percent)", y = "Clotting Time (sec)") +
  scale_x_continuous(breaks = c(5,10,50,100)) +
  geom_line(aes(y = yhat), data = d)
```

```
plot(p)
```




```
summary(m)$coefficients
```

```
                    Estimate Std. Error t value  Pr(>|t|)
(Intercept)      -0.016554  0.0008655 -19.127 1.967e-11
lotL2            -0.007354  0.0016780  -4.383 6.252e-04
log(conc)         0.015343  0.0003872  39.626 8.851e-16
lotL2:log(conc)   0.008256  0.0007353  11.228 2.184e-08
```

This generalized linear model can be written as

$$\frac{1}{E(T_i)} = \beta_0 + \beta_1 l_i + \beta_2 \log_2 c_i + \beta_3 l_i \log_2 c_i,$$

or, equivalently,

$$E(T_i) = \frac{1}{\beta_0 + \beta_1 l_i + \beta_2 \log_2 c_i + \beta_3 l_i \log_2 c_i},$$

where T_i is clotting time, c_i is plasma concentration, and l_i is an indicator variable such that $l_i = 1$ if the i-th observation is from the second lot, and $l_i = 0$ otherwise.

Marginal effects of increasing the plasma concentration from 5 to 10 in each lot.

```
margeff(m,
  a = list(conc = 10, lot = c("L1","L2")),
  b = list(conc =  5, lot = c("L1","L2")),
  cnames = c("L1,5->10","L2,5->10"))
```

```
         estimate    se  lower  upper tvalue df    pvalue
L1,5->10    -69.6 4.808 -79.91 -59.28 -14.47 14 8.149e-10
L2,5->10    -38.2 2.711 -44.01 -32.38 -14.09 14 1.160e-09
```

Marginal effects of increasing from 5 to 10, 10 to 50, and 50 to 100 in the first lot.

```
margeff(m,
  a = list(conc = c(10,50,100), lot = "L1"),
```

```r
  b = list(conc = c(5,10,50), lot = "L1"),
  cnames = c("L1,5->10","L1,10->50","L1,50->100"))
```

```
           estimate      se   lower   upper tvalue df    pvalue
L1,5->10    -69.595 4.80805 -79.907 -59.283 -14.47 14 8.149e-10
L1,10->50   -30.259 0.71242 -31.787 -28.731 -42.47 14 3.376e-16
L1,50->100   -4.522 0.06961  -4.671  -4.373 -64.96 14 9.064e-19
```

Marginal effects for plasma concentration for the *second* lot.

```r
margeff(m,
  a = list(conc = c(10,50,100), lot = "L2"),
  b = list(conc = c(5,10,50), lot = "L2"),
  cnames = c("L2,5->10","L2,10->50","L2,50->100"))
```

```
           estimate     se   lower   upper tvalue df    pvalue
L2,5->10    -38.197 2.7107 -44.010 -32.383 -14.09 14 1.160e-09
L2,10->50   -18.244 0.4595 -19.230 -17.259 -39.71 14 8.606e-16
L2,50->100   -2.821 0.0436  -2.914  -2.727 -64.69 14 9.613e-19
```

Marginal effects for lot at three plasma concentrations.

```r
margeff(m,
  a = list(conc = c(25,50,75), lot = c("L1")),
  b = list(conc = c(25,50,75), lot = c("L2")),
  cnames = c("25","50","75"))
```

```
   estimate     se  lower  upper tvalue df    pvalue
25   11.246 0.5809 10.000 12.492  19.36 14 1.672e-11
50    8.388 0.4810  7.356  9.420  17.44 14 6.835e-11
75    7.301 0.4394  6.359  8.244  16.62 14 1.304e-10
```

"Instantaneous" Marginal Effects

Consider a regression model with (without loss of generality) two explanatory variables, X_1 and X_2. Assuming that X_1 is *continuous*, the "instantaneous" marginal effect of X_1 at x_1 when $X_2 = x_2$ is

$$\lim_{\delta \to 0} \frac{E(Y|X_1 = x_1 + \delta, X_2 = x_2) - E(Y|X_1 = x_1, X_2 = x_2)}{\delta}.$$

This can also be written as

$$\left.\frac{\partial E(Y|X_1 = z, X_2 = x_2)}{\partial z}\right|_{z=x_1}$$

i.e., the partial derivative of $E(Y|X_1 = x_1, X_2 = x_2)$ with respect to and evaluated at x_1.

Intuitively, this is the rate of change in the expected response at a specific value of the explanatory variable — i.e., the slope of the function at a specific point.

To compute this marginal effect we can either find the partial derivative analytically or approximate it numerically using

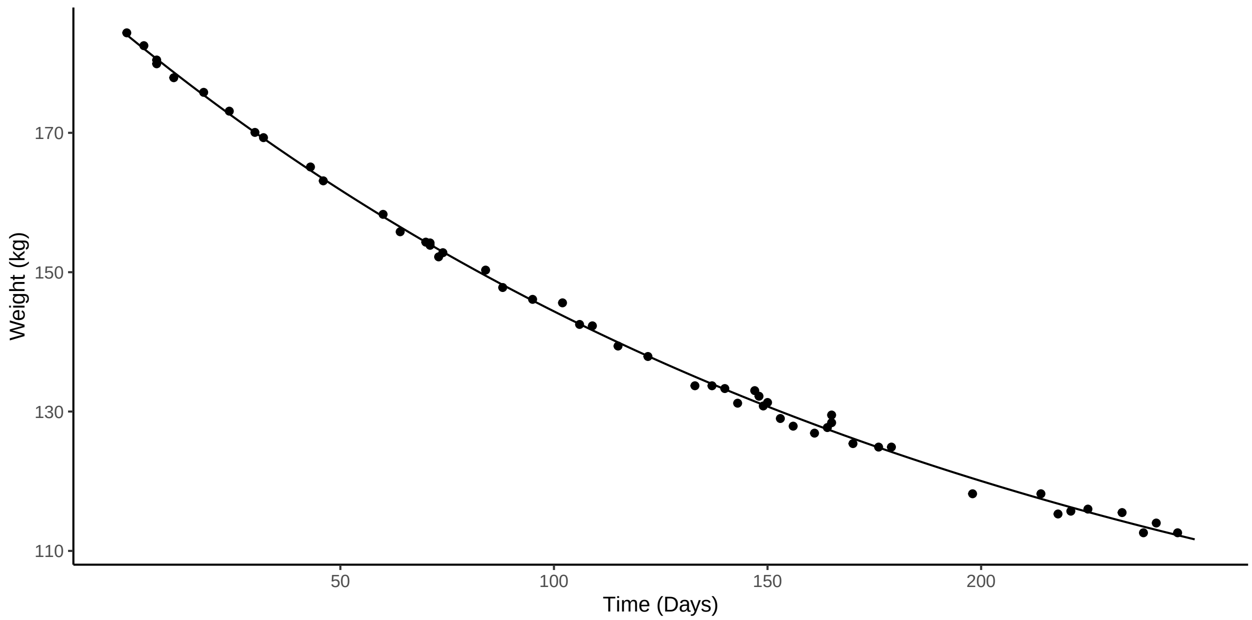
$$\frac{E(Y|X_1 = x_1 + \delta, X_2 = x_2) - E(Y|X_1 = x_1, X_2 = x_2)}{\delta}$$

where δ set to a small value relative to x_1 (this is called *numerical differentiation*).

Note that instantaneous marginal effects are only defined for *continuous quantitative variables*.

Example: Consider again the nonlinear regression model for expected weight as a function of days.

```
m <- nls(Weight ~ t1 + t2*2^(-Days/t3), data = MASS::wtloss,
  start = list(t1 = 90, t2 = 95, t3 = 120))

d <- data.frame(Days = seq(0, 250, by = 1))
d$yhat <- predict(m, newdata = d)

p <- ggplot(MASS::wtloss, aes(x = Days, y = Weight)) +
  geom_point() + theme_classic() +
  labs(y = "Weight (kg)", x = "Time (Days)") +
  geom_line(aes(y = yhat), data = d) +
  scale_x_continuous(breaks = c(50,100,150,200))
plot(p)
```



We can estimate the instantaneous marginal effects at 50, 100, 150, and 200 days.

```
margeff(m, delta = 0.001,
  a = list(Days = c(50,100,150,200) + 0.001),
  b = list(Days = c(50,100,150,200)),
  cnames = c("@50", "@100", "@150", "@200"))
```

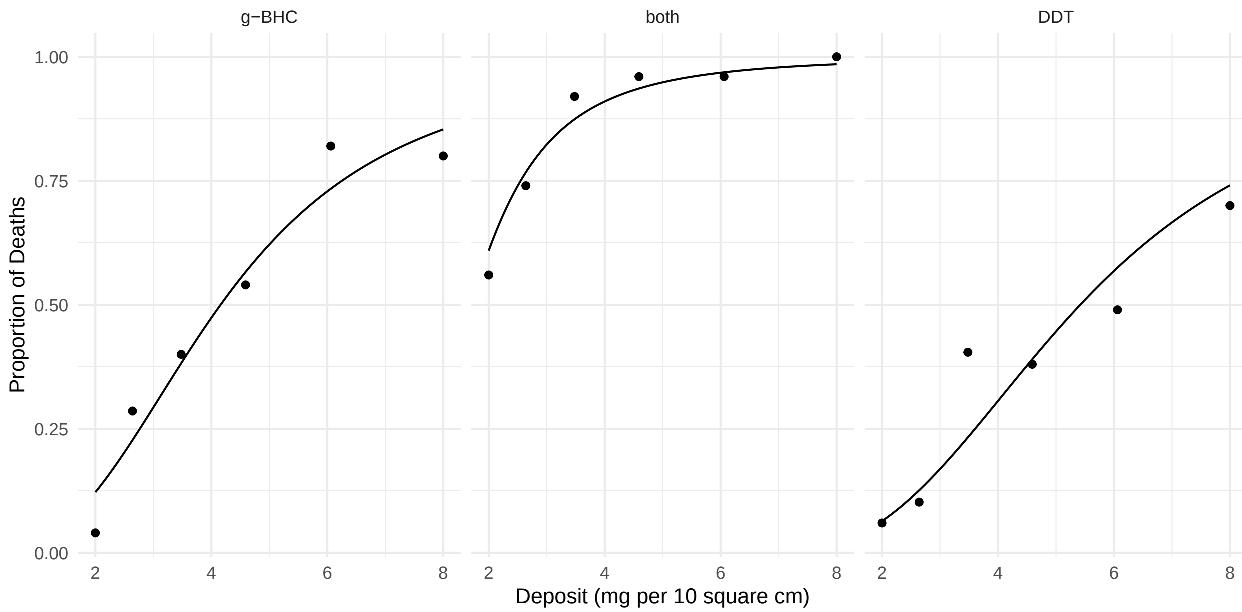
```
     estimate       se   lower   upper  tvalue df    pvalue
@50   -0.3929 0.004173 -0.4013 -0.3845  -94.14 49 4.929e-57
@100  -0.3077 0.001832 -0.3114 -0.3041 -168.03 49 2.529e-69
@150  -0.2411 0.002685 -0.2465 -0.2357  -89.79 49 4.944e-56
@200  -0.1888 0.003675 -0.1962 -0.1814  -51.39 49 2.760e-44
```

Note: To estimate an instantaneous marginal effect, add a relatively small value of δ to the `a` variable, and also specify this amount to the `delta` argument.

Example: Consider again the model for the `insecticide` data.

```
m <- glm(cbind(deaths, total-deaths) ~ log2(deposit)
 + insecticide, family = binomial, data = insecticide)

d <- expand.grid(deposit = seq(2, 8, length = 100),
```

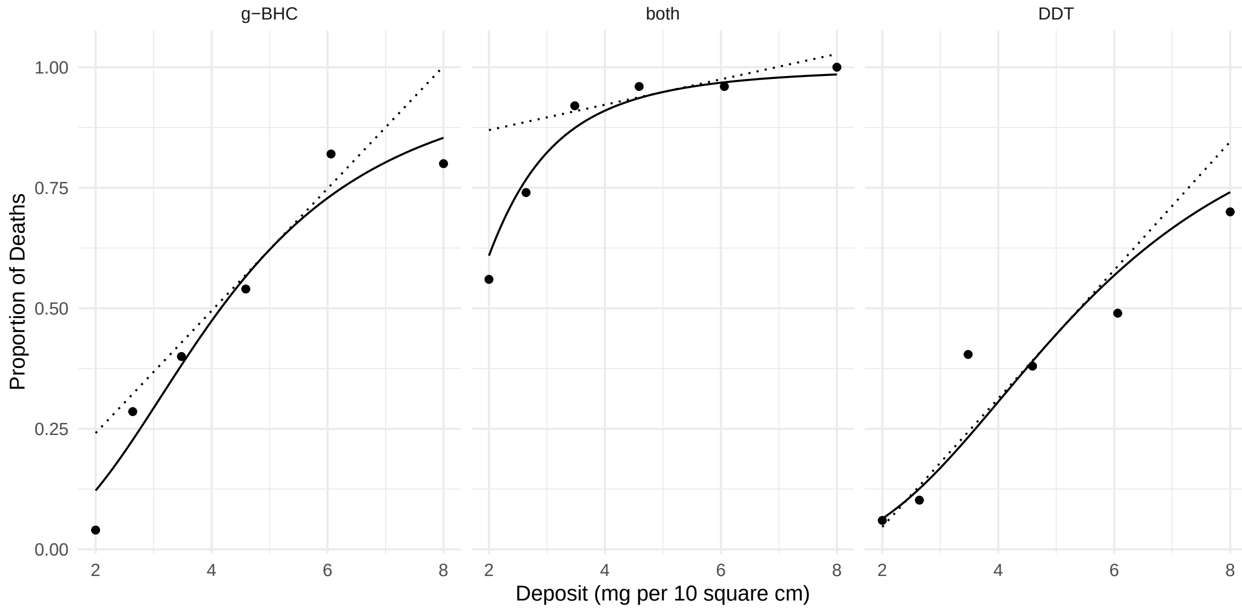
```r
  insecticide = unique(insecticide$insecticide))
d$phat <- predict(m, newdata = d, type = "response")

p <- ggplot(insecticide, aes(x = deposit, y = deaths/total)) +
  geom_point() + facet_wrap(~ insecticide) +
  geom_line(aes(y = phat), data = d) + theme_minimal() +
  labs(x = "Deposit (mg per 10 square cm)", y = "Proportion of Deaths")
plot(p)
```



We can estimate the instantaneous marginal effect of deposit at a given amount of deposit, say 5 mg per 10 square cm.

```r
margeff(m, delta = 0.001,
  a = list(deposit = 5 + 0.001, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 5, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

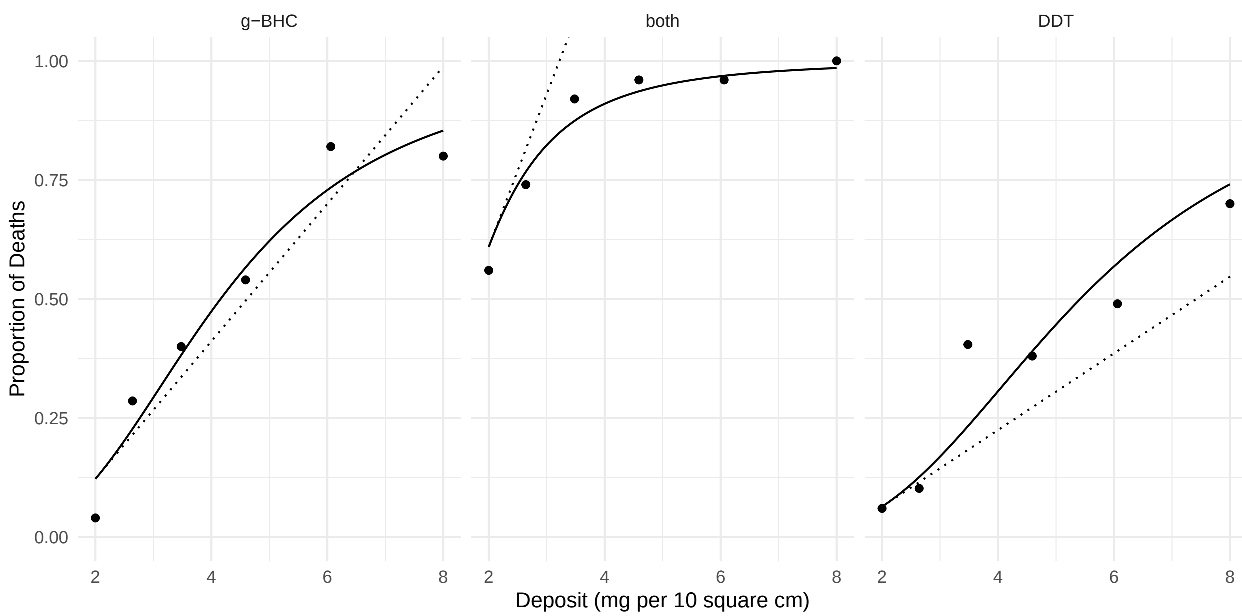
```
      estimate       se   lower  upper tvalue  df    pvalue
g-BHC  0.12680 0.009750 0.10769 0.1459 13.005 Inf 1.149e-38
both   0.02631 0.004281 0.01792 0.0347  6.146 Inf 7.941e-10
DDT    0.13321 0.011056 0.11154 0.1549 12.048 Inf 1.978e-33
```



Note that the instantaneous effect of deposit *depends on the deposit* because the probability is not a linear function of deposit.

```
margeff(m, delta = 0.001,
  a = list(deposit = 2 + 0.001, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 2, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate      se   lower  upper tvalue  df    pvalue
g-BHC  0.14439 0.01575 0.11352 0.1753  9.168 Inf 4.839e-20
both   0.32078 0.03323 0.25565 0.3859  9.654 Inf 4.750e-22
DDT    0.08049 0.01180 0.05737 0.1036  6.824 Inf 8.878e-12
```



Instantaneous Marginal Effects for Generalized Linear Models

Recall that in a GLM that $E(Y) = g^{-1}(\eta)$ where $\eta = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k$. Consider a GLM where $\eta_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}$. The instantaneous marginal effect of X_1 at x_1 is

$$\frac{\partial E(Y|X_1 = x_1, X_2 = x_2)}{\partial x_1} = \frac{\partial g^{-1}(\eta)}{\partial x_1} = \frac{\partial g^{-1}(\eta)}{\partial \eta}\frac{\partial \eta}{\partial x_1} = \frac{\partial g^{-1}(\eta)}{\partial \eta}\beta_1$$

by the "chain rule" for (partial) derivatives.

Suppose that $E(Y) = e^{\eta}$ (i.e., log link function) where $\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2$. Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\beta_1 = \frac{\partial e^{\eta}}{\partial \eta}\beta_1 = e^{\eta}\beta_1 = E(Y)\beta_1.$$

Suppose now that $E(Y) = e^{\eta}/(1 + e^{\eta})$ (i.e., logit link function). Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\beta_1 = \frac{\partial e^{\eta}/(1 + e^{\eta})}{\partial \eta}\beta_1 = \frac{e^{\eta}}{(1 + e^{\eta})^2}\beta_1 = E(Y)[1 - E(Y)]\beta_1.$$

Suppose now that $E(Y) = \eta$ (e.g., identity link function). Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\beta_1 = \frac{\partial \eta}{\partial \eta}\beta_1 = \beta_1.$$

Things get a little more complicated if X_1 is a *transformed* explanatory variable or represents an interaction.

Suppose $E(Y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2$. Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\frac{\partial \eta}{\partial x_1} = \frac{\partial \eta}{\partial x_1} = \beta_1/x_1.$$

Suppose $E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$. Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\frac{\partial \eta}{\partial x_1} = \frac{\partial \eta}{\partial x_1} = \beta_1 + 2\beta_2 x_1.$$

Suppose $E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$. Then

$$\frac{\partial g^{-1}(\eta)}{\partial \eta}\frac{\partial \eta}{\partial x_1} = \frac{\partial \eta}{\partial x_1} = \beta_1 + \beta_3 x_2.$$

Fortunately, `margeff` does the calculus!

Discrete Marginal Effects as Percent Change

Consider a regression model with (without loss of generality) two explanatory variables, X_1 and X_2. The *percent change* in the expected response when changing X_1 from x_b to x_a when $X_2 = x_2$ is

$$\frac{E(Y|X_1 = x_a, X_2 = x_2) - E(Y|X_1 = x_b, X_2 = x_2)}{E(Y|X_1 = x_b, X_2 = x_2)} \times 100\%.$$

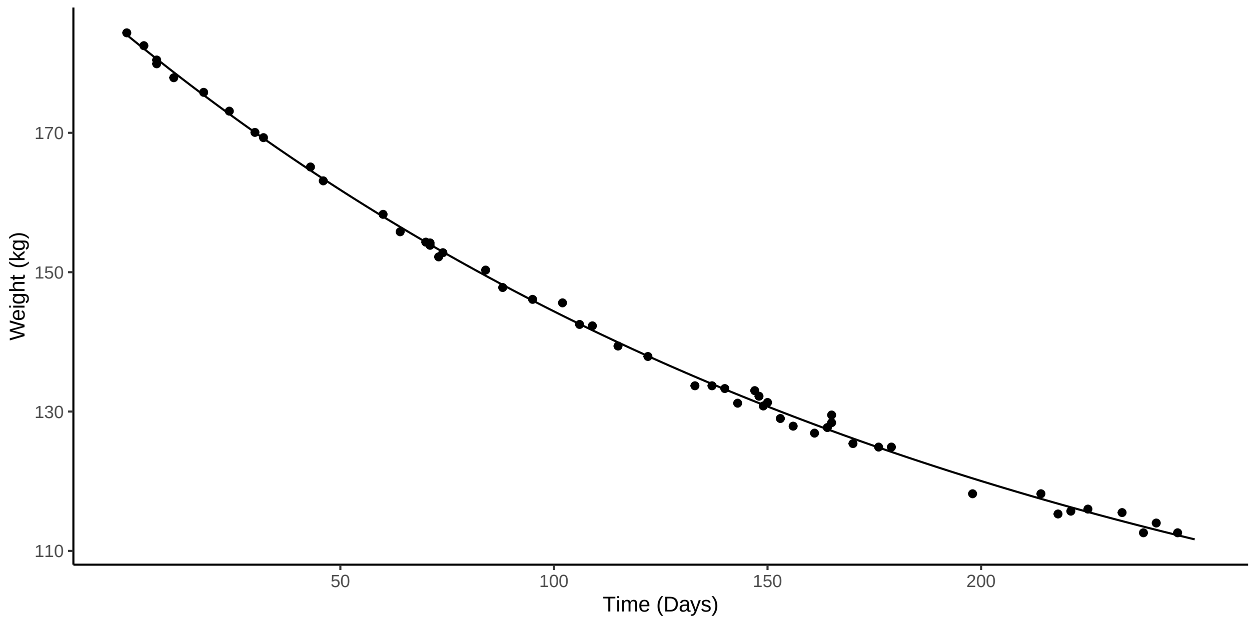
or, equivalently,

$$\left[\frac{E(Y|X_1 = x_a, X_2 = x_2)}{E(Y|X_1 = x_b, X_2 = x_2)} - 1\right] \times 100\%.$$

Note that the *sign* indicates if it is a percent increase or decrease.

Example: Consider again the weight loss model.

```r
m <- nls(Weight ~ t1 + t2*2^(-Days/t3), data = MASS::wtloss,
  start = list(t1 = 90, t2 = 95, t3 = 120))

d <- data.frame(Days = seq(0, 250, by = 1))
d$yhat <- predict(m, newdata = d)

p <- ggplot(MASS::wtloss, aes(x = Days, y = Weight)) +
  geom_point() + theme_classic() +
  labs(y = "Weight (kg)", x = "Time (Days)") +
  geom_line(aes(y = yhat), data = d) +
  scale_x_continuous(breaks = c(50,100,150,200))
plot(p)
```



Consider the percent change in expected weight from 50 to 100 days. This is

$$\frac{\theta_1 + \theta_2 2^{-100/\theta_3} - \theta_1 - \theta_2 2^{-50/\theta_3}}{\theta_1 + \theta_2 2^{-50/\theta_3}} = \frac{\theta_2 2^{-100/\theta_3} - \theta_2 2^{-50/\theta_3}}{\theta_1 + \theta_2 2^{-50/\theta_3}}.$$

We can estimate the percent change in expected weight from 50 to 100 days as follows.

```r
margeff(m, a = list(Days = 100), b = list(Days = 50), type = "percent")
```

```
 estimate      se  lower  upper tvalue df    pvalue
   -10.77 0.07673 -10.93 -10.62 -140.4 49 1.666e-65
```

We can do it for several 50 day increments as well.

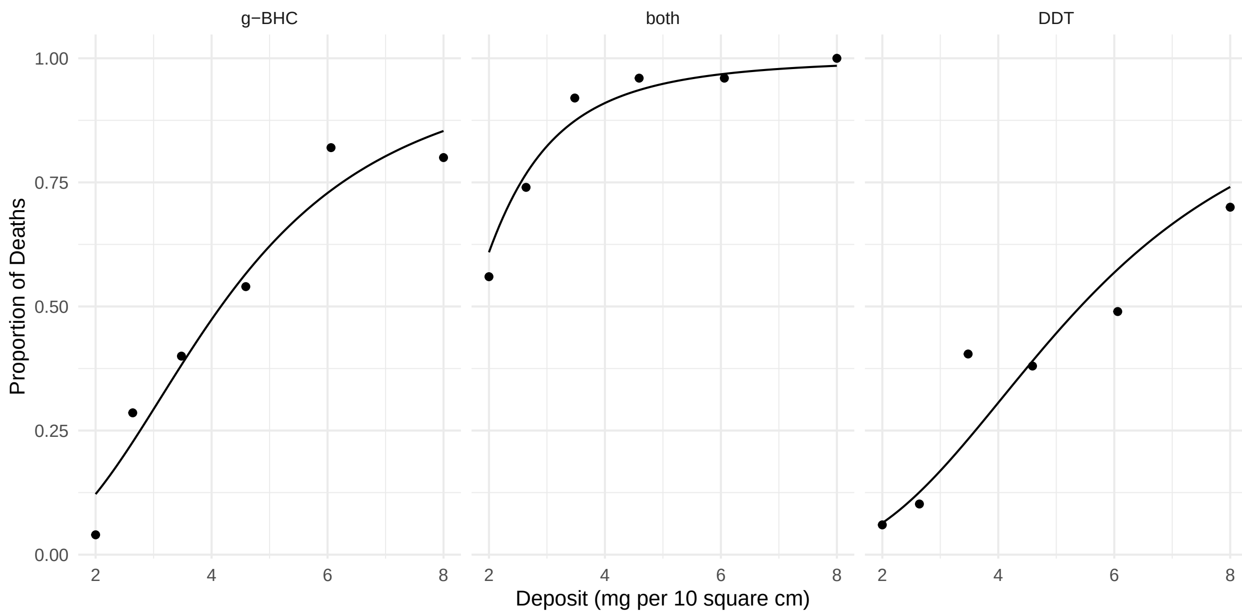
```r
margeff(m, type = "percent",
  a = list(Days = c(50,100,150,200)),
  b = list(Days = c(0,50,100,150)),
  cnames = c("0->50", "50->100", "100->150", "150->200"))
```

```
         estimate      se   lower   upper  tvalue df    pvalue
0->50     -12.089 0.15795 -12.406 -11.771  -76.53 49 1.171e-52
50->100   -10.771 0.07673 -10.926 -10.617 -140.37 49 1.666e-65
100->150   -9.456 0.06627  -9.589  -9.323 -142.69 49 7.488e-66
```

```
150->200   -8.180 0.12091  -8.423  -7.937  -67.66 49 4.660e-50
```

Example: Consider again the model for the `insecticide` data.

```
m <- glm(cbind(deaths, total-deaths) ~ log2(deposit) + insecticide,
  family = binomial, data = insecticide)

d <- expand.grid(deposit = seq(2, 8, length = 100),
  insecticide = levels(insecticide$insecticide))
d$phat <- predict(m, newdata = d, type = "response")

p <- ggplot(insecticide, aes(x = deposit, y = deaths/total)) +
  geom_point() + facet_wrap(~ insecticide) +
  geom_line(aes(y = phat), data = d) + theme_minimal() +
  labs(x = "Deposit (mg per 10 square cm)", y = "Proportion of Deaths")
plot(p)
```



We can estimate the percent change in the probability of death from 4 to 6 mg per 10 square cm.

```
margeff(m, type = "percent",
  a = list(deposit = 6, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 4, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate     se  lower   upper tvalue  df    pvalue
g-BHC   53.821  6.567 40.950  66.692  8.196 Inf 2.488e-16
both     6.372  1.111  4.195   8.548  5.738 Inf 9.604e-09
DDT     85.621 11.031 63.999 107.242  7.761 Inf 8.394e-15
```

Note that here the percent change depends on where we make the increment.

```
margeff(m, type = "percent",
  a = list(deposit = 8, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 6, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate     se  lower  upper tvalue  df    pvalue
```

```
g-BHC   17.153 1.9971 13.238 21.067  8.589 Inf 8.798e-18
both     1.764 0.3625  1.053  2.474  4.866 Inf 1.140e-06
DDT     30.364 3.2940 23.908 36.820  9.218 Inf 3.021e-20
```

We can also estimate the percent change in the probability of death between two insecticides.

```
margeff(m, type = "percent",
  a = list(deposit = c(2,4,6,8), insecticide = "g-BHC"),
  b = list(deposit = c(2,4,6,8), insecticide = "DDT"),
  cnames = c("2","4","6","8"))
```

```
  estimate     se  lower  upper tvalue  df   pvalue
2    91.29 34.752 23.173 159.40  2.627 Inf 0.008620
4    54.72 19.037 17.406  92.03  2.874 Inf 0.004049
6    28.21  9.132 10.315  46.11  3.090 Inf 0.002005
8    15.22  4.904  5.607  24.83  3.103 Inf 0.001914
```

Discrete Marginal Effects as Multiplicative Factors

Consider a regression model with (without loss of generality) two explanatory variables, X_1 and X_2. A multiplicative factor to describe the effect of changing X_1 from x_b to x_a when $X_2 = x_2$ is

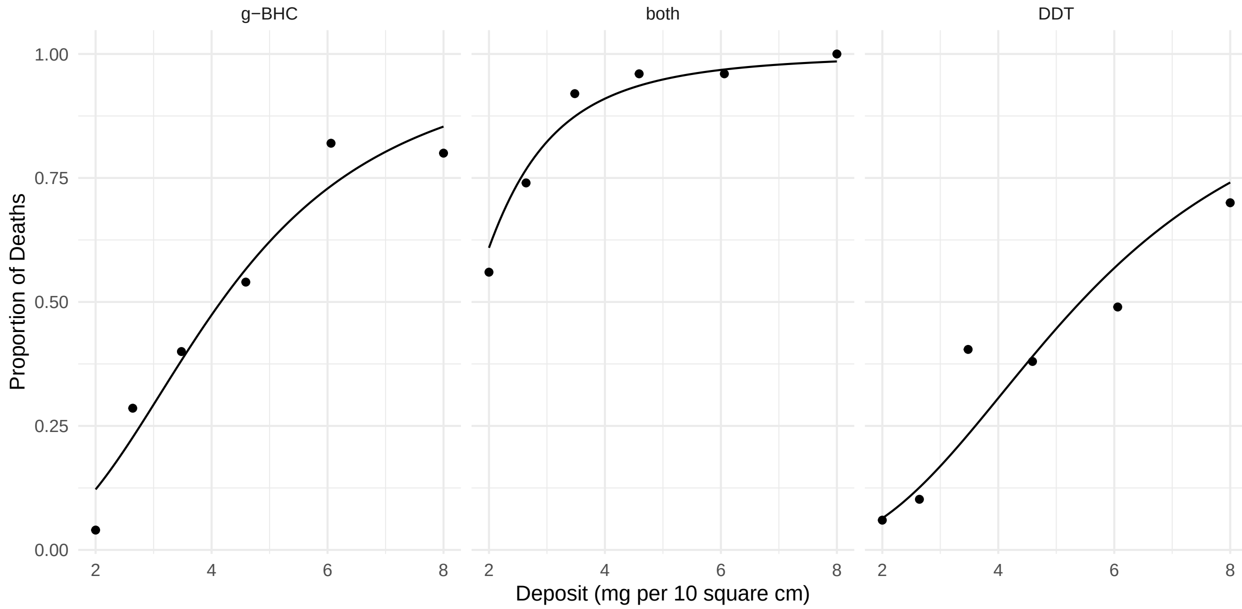
$$f = \frac{E(Y|X_1 = x_a, X_2 = x_2)}{E(Y|X_1 = x_b, X_2 = x_2)},$$

meaning that

$$E(Y|X_1 = x_a, X_2 = x_2) = f \times E(Y|X_1 = x_b, X_2 = x_2).$$

Example: Consider again the model for the `insecticide` data.

```
m <- glm(cbind(deaths, total-deaths) ~ log2(deposit) + insecticide,
  family = binomial, data = insecticide)

d <- expand.grid(deposit = seq(2, 8, length = 100),
  insecticide = levels(insecticide$insecticide))
d$phat <- predict(m, newdata = d, type = "response")

p <- ggplot(insecticide, aes(x = deposit, y = deaths/total)) +
  geom_point() + facet_wrap(~ insecticide) +
  geom_line(aes(y = phat), data = d) + theme_minimal() +
  labs(x = "Deposit (mg per 10 square cm)", y = "Proportion of Deaths")
plot(p)
```



We can estimate the factor by which we increase probability by increasing deposit from 4 to 6 mg per 10 square cm.

```
margeff(m, type = "factor",
  a = list(deposit = 6, insecticide = c("g-BHC","both","DDT")),
  b = list(deposit = 4, insecticide = c("g-BHC","both","DDT")),
  cnames = c("g-BHC","both","DDT"))
```

```
      estimate      se lower upper tvalue  df     pvalue
g-BHC    1.538 0.06567 1.410 1.667  23.42 Inf 2.436e-121
both     1.064 0.01111 1.042 1.085  95.78 Inf  0.000e+00
DDT      1.856 0.11031 1.640 2.072  16.83 Inf  1.562e-63
```

We can also estimate the factor for comparing both insecticides with g-BHC only.

```
margeff(m, type = "factor",
  a = list(deposit = c(2,4,6,8), insecticide = "both"),
  b = list(deposit = c(2,4,6,8), insecticide = "g-BHC"),
  cnames = c("2","4","6","8"))
```

```
  estimate      se lower upper tvalue  df     pvalue
2    4.994 0.87192 3.285 6.703  5.728 Inf  1.016e-08
4    1.921 0.14204 1.642 2.199 13.523 Inf  1.139e-41
6    1.328 0.05464 1.221 1.435 24.312 Inf 1.469e-130
8    1.154 0.03098 1.093 1.215 37.242 Inf 1.444e-303
```

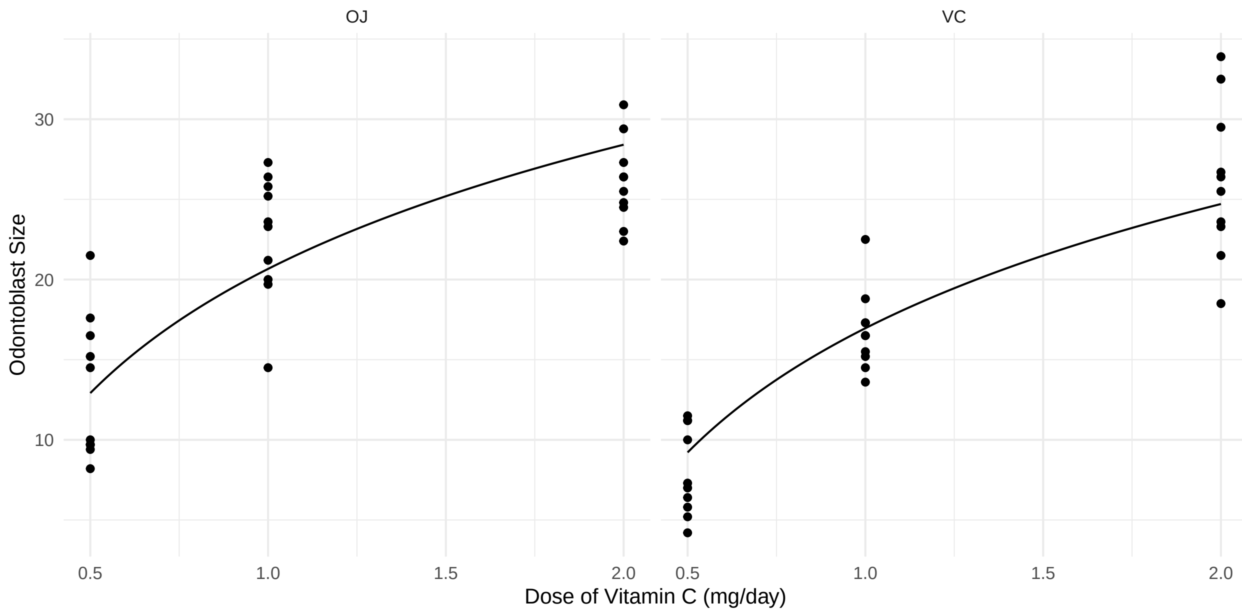
Using Different Kinds of Marginal Effects

Marginal effects give us a variety of ways to summarize the statistical relationship between a response variable and an explanatory variable.

Example: Consider the following model for the `ToothGrowth` data.

```
m <- lm(len ~ log2(dose) + supp, data = ToothGrowth)

d <- expand.grid(dose = seq(0.5, 2, length = 100), supp = c("OJ","VC"))
d$yhat <- predict(m, d)
```

```r
p <- ggplot(ToothGrowth, aes(x = dose, y = len)) +
  geom_point() + facet_wrap(~ supp) +
  geom_line(aes(y = yhat), data = d) +
  labs(x = "Dose of Vitamin C (mg/day)", y = "Odontoblast Size") +
  theme_minimal()
plot(p)
```



We can use discrete marginal effects, such as when increasing dose from 0.5 to 1 mg/day.

```r
margeff(m, cnames = c("OJ","VC"),
  a = list(dose = 1.0, supp = c("OJ","VC")),
  b = list(dose = 0.5, supp = c("OJ","VC")))
```

```
   estimate     se lower upper tvalue df    pvalue
OJ    7.748 0.6091 6.528 8.967  12.72 57 2.736e-18
VC    7.748 0.6091 6.528 8.967  12.72 57 2.736e-18
```

We can use instantaneous effects, such as the instantaneous effect at 1 mg/day.

```r
margeff(m, cnames = c("OJ","VC"), delta = 0.001,
  a = list(dose = 1 + 0.001, supp = c("OJ","VC")),
  b = list(dose = 1, supp = c("OJ","VC")))
```

```
   estimate     se lower upper tvalue df    pvalue
OJ    11.17 0.8783 9.413 12.93  12.72 57 2.736e-18
VC    11.17 0.8783 9.413 12.93  12.72 57 2.736e-18
```

We can use the percent change, such as when increasing dose from 0.5 to 1 mg/day.

```r
margeff(m, cnames = c("OJ","VC"), type = "percent",
  a = list(dose = 1.0, supp = c("OJ","VC")),
  b = list(dose = 0.5, supp = c("OJ","VC")))
```

```
   estimate     se lower  upper tvalue df    pvalue
OJ    59.98  8.222 43.52  76.45  7.296 57 1.022e-09
VC    84.07 13.754 56.53 111.61  6.112 57 9.411e-08
```

We can use a multiplicative factor, such as when increasing dose form 0.5 to 1 mg/day.

```
margeff(m, cnames = c("OJ","VC"), type = "factor",
  a = list(dose = 1.0, supp = c("OJ","VC")),
  b = list(dose = 0.5, supp = c("OJ","VC")))
```

```
   estimate      se lower upper tvalue df    pvalue
OJ    1.600 0.08222 1.435 1.764  19.46 57 7.556e-27
VC    1.841 0.13754 1.565 2.116  13.38 57 3.081e-19
```