Monday, Jan 30

The purpose of this exercise is to familiarize you with the specification of linear models and contrasts within R. You will need to have the following packages installed: **abd**, **cowplot**, **ggplot2**, and **trtools**. You should already have **ggplot2** and **trtools** installed. The other packages can be installed using `install.packages("abd")` and `install.packages("cowplot")`, or `install.packages(c("abd","cowplot"))`.

This exercise features data from a study published in *Nature* in 2006.[1] The data can be found in the data frame `MoleRats`, included in the package **abd**. You can inspect the data frame by typing `MoleRats` at the prompt after you load the package with `library(abd)`, and you can look at a summary of the data with `summary(MoleRats)` or the structure of the variables with `str(MoleRats)`. The study concerned two "castes" of Damaraland mole-rats (*Cryptomys damarensis*): frequent workers (denoted here as "worker") and infrequent workers (denoted here as "lazy"). The researchers observed the body mass (g) and daily energy expenditure (kJ/day) of samples mole-rats from both castes. Our objective is to model the difference between the two castes in terms of daily energy expenditure. However, mole-rats in the two castes differ by body mass, which may also be related to energy expenditure.

Here are plots of the raw data, showing the distribution of log energy expenditure for lazy and worker caste rats, and then again but also taking into account the (log) mass of the rats.

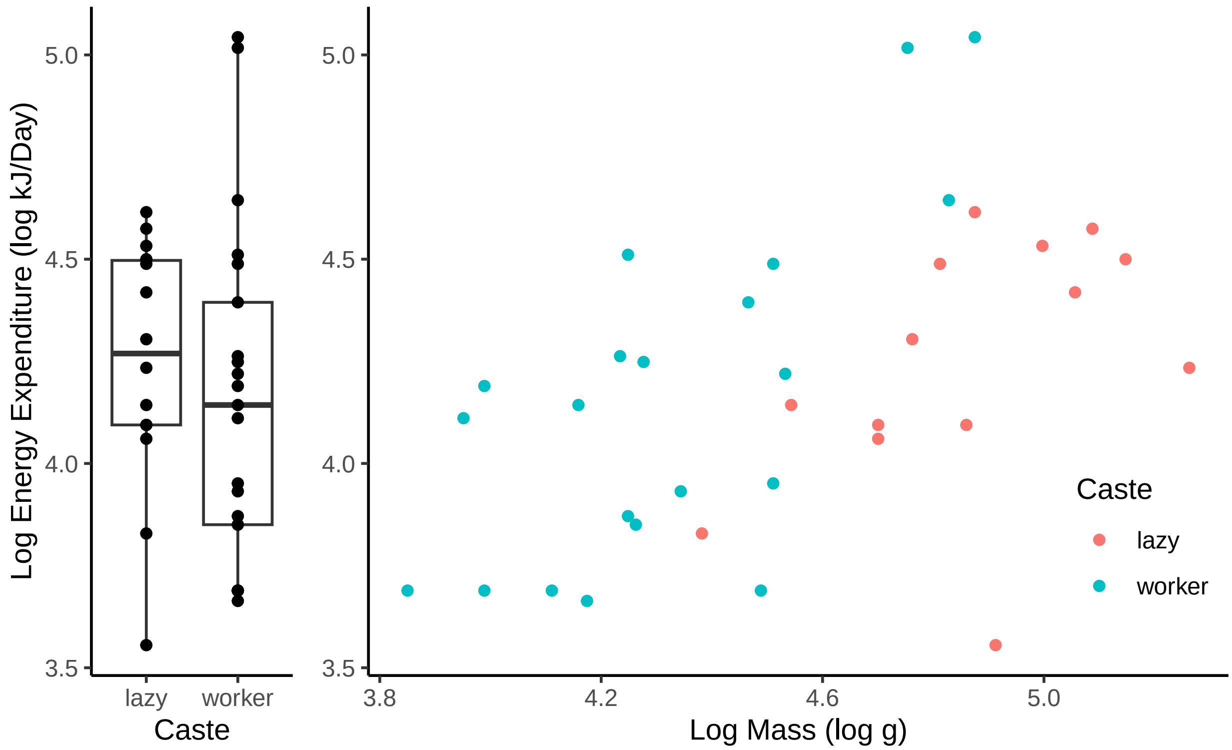
```
library(abd)     # for the data
library(ggplot2) # for plotting
library(cowplot) # for use of the plot_grid function

# boxplot
p1 <- ggplot(MoleRats, aes(x = caste, y = ln.energy)) +
   geom_boxplot() + geom_point() + theme_classic() +
   labs(x = "Caste", y = "Log Energy Expenditure (log kJ/Day)")

# scatterplot
p2 <- ggplot(MoleRats, aes(x = ln.mass, y = ln.energy, color = caste)) +
   geom_point() + theme_classic() + theme(legend.position = c(0.9, 0.2)) +
   labs(x = "Log Mass (log g)", y = NULL, color = "Caste")

# plot both plots side-by-side
plot_grid(p1, p2, align = "h", rel_widths = c(1,3))
```

[1] Scantlebury, M., Speakman, J. R., Oosthuizen, M. K., Roper, T. J., & Bennett, N. C. (2006). Energetics reveals physiological distinct castes in a eurosical mammal. *Nature*, *440*, 795-797.



Note that both energy expenditure and mass are recorded on the (natural) log scale. We will be modeling both of these variables on the log scale.

1. First consider a model where expected log energy expenditure is modeled as a function of only caste. Use the `lm` function to estimate this model. We should see in the output of the `summary` function applied to the model object that R will, by default, parameterize this model as

$$E(Y_i) = \beta_0 + \beta_1 d_i,$$

where Y_i is the log of energy expenditure of the i-th observation and

$$d_i = \begin{cases} 1, & \text{if the } i\text{-th observation is of a worker,} \\ 0, & \text{otherwise,} \end{cases}$$

so that the model can also be written as

$$E(Y_i) = \begin{cases} \beta_0, & \text{if the } i\text{-th observation is from the lazy caste,} \\ \beta_0 + \beta_1, & \text{if the } i\text{-th observation is from the worker caste.} \end{cases}$$

2. Consider the following three quantities and how they can be expressed as functions of the model parameters: the expected log energy expenditure of mole-rats from the *lazy* caste (β_0), the expected log energy expenditure of mole-rats from the *worker* caste ($\beta_0 + \beta_1$), and the difference in expected log energy expenditure between the two castes (β_1 if we subtract lazy from worker). Note that using `summary` provides inferences for β_0 and β_1, but not $\beta_0 + \beta_1$. All three quantities can be written as a linear combination of the form

$$\ell = a_0\beta_0 + a_1\beta_1 + b.$$

How can we use the `lincon` and `contrast` functions to estimate these three quantities? Note that inferences for two of these quantities — i.e., the expected log expenditure of mole-rats of the *lazy* caste and the *difference* in the expected log expenditures between the two castes — should match those given by using `summary` and `confint` applied to the model object.

3. The model we considered above compares the two castes of mole-rats, but it does not account for differences in their sizes. The lazy mole-rats tend to be larger than the worker mole-rats. It may be useful to compare rats in the two castes while "controlling for size" meaning that comparisons can be made between mole-rats in different castes but of the same size. We can use `lm` to estimate a linear model that includes both caste and the log of mass as explanatory variables. If we list `caste` first in the right-hand side of the model formula then the model will be parameterized as

$$E(Y_i) = \beta_0 + \beta_1 d_i + \beta_2 \ln(m_i),$$

   where d_i is defined as before and $\log(m_i)$ is the log of mass of the i-th observation (i.e., the `ln.mass` variable). Note that this model can also be written as

$$E(Y_i) = \begin{cases} \beta_0 + \beta_2 \ln(m_i), & \text{if the } i\text{-th observation is from the lazy caste,} \\ \beta_0 + \beta_1 + \beta_2 \ln(m_i), & \text{if the } i\text{-th observation is from the worker caste.} \end{cases}$$

4. Using the code given above, add lines to the plot to show the estimated expected log energy expenditure as a function of log mass and caste.

5. With the model now including the log of mass as an explanatory variable, comparisons between the castes must also consider mass. How can we use `contrast` to estimate the expected log energy consumption for a mole-rat from each of the two castes with a log mass of 4.5? Also how can we estimate the *difference* in expected log energy expenditure between the two castes for a mole-rat with log mass 4.5? How can we do the same for log masses of 4.0 and 5.0? Our estimates should be consistent with the figure (i.e., they should look reasonable based on "eyeball" estimates from the figure). Also because the model specifies that the two lines are parallel, the estimated difference in expected log energy consumption between the castes will not depend on mass.

6. The parameter β_2 is the rate of change of the expected log energy consumption per unit increase in the log of mass. Inferences concerning this quantity can be obtained simply using `summary` and `confint` applied to the model object, but as an exercise how can we estimate this rate of change for each caste using the `contrast` function? Naturally we should find that the estimated rate is the same for each caste.

7. A test to determine if there is a statistically significant difference in expected log energy expenditure between the two castes uses the null hypothesis $H_0\colon \beta_1 = 0$ based on how the model was parameterized earlier. How can we test this null hypothesis using the full-null model approach (i.e., by specifying a null model based on the full model but with $\beta_1 = 0$) and using the `anova` function? We should find that the resulting F test statistic is the square of the t test statistic from `summary`, and that the p-values from `summary` and `anova` are the same.

8. Repeat 3-6 with both `caste` and `ln.mass` as explanatory variables, but now with an "interaction" between the them by including `caste:ln.energy` in your model formula. Note that this model can be written case-wise as

$$E(Y_i) = \begin{cases} \beta_0 + \beta_2 \ln(m_i), & \text{if the } i\text{-th observation is from the lazy caste,} \\ \beta_0 + \beta_1 + (\beta_2 + \beta_3) \ln(m_i), & \text{if the } i\text{-th observation is from the worker caste.} \end{cases}$$