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Area Vector $= \hat{n} \cdot S = \vec{S}$ where $\hat{n}$ is vector normal to plane with area S.
ONLY BOUNDED SURFACE

λ linear charge density C/M

σ surface charge density C/m²

ρ Volumetric charge density C/m³

Bounded surface ≠ Closed surface

Formula Sheet

Physics 260/Fall 2021

Electricity

Coulomb's Law

$$N = F_e = k\frac{|q_1||q_2|}{r^2} \quad (1)$$

Electric Field

$\vec{E} = \frac{N}{C}$

$$\vec{E} = \frac{\vec{F_e}}{q_0} \; \frac{N}{C} \quad (2)$$

Electric Field due to a point charge

$\int \frac{KQ}{r^2} dr = +KQ_0 \frac{1}{r} = V$

$$\vec{E} = \frac{kq}{r^2}\hat{r} \quad (3)$$

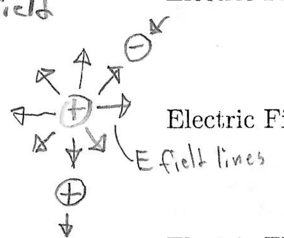
Electric Field due to a continuous charge distribution

⊕ charge moves in the dirrection of electric field

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} \quad (4)$$

Electric Field due to an infinite plane of charge

$$E = \frac{\sigma}{2\varepsilon_0} \quad (5)$$



Electric Field just outside of a conductor

$$E = \frac{\sigma}{\varepsilon_0} \quad (6)$$

Electric Flux

ϕ V·M $= \frac{N \cdot m^2}{C}$ SCALER

$$\phi_E = \oint \vec{E}.d\vec{A} \quad (7)$$

Gauss's Law

Gauss Law applied to find ELECRIC Field ONLY in Sphere INFINITE PLANE and INFINITE LINE

$$\phi_E = \frac{q_{enc}}{\varepsilon_0} \quad (8)$$

Electric Potential Energy

$$\Delta U = -q_0 \int \vec{E}.\vec{ds} \quad (9)$$

Electric Potential

$V = 1\frac{J}{C} = \frac{N \cdot M}{C}$ SCALER

$$\Delta V = \frac{\Delta U}{q_0} = -\int_{\infty}^{r} \vec{E}.\vec{ds} \quad (10)$$

Electric Potential due to a Point Charge

$$V = k\frac{q}{r} \quad (11)$$

Electric Potential due to a continuous charge distribution

$$V = k\int \frac{dq}{r} \quad (12)$$

$$U = k\frac{q_1 q_2}{r_{12}} \quad (13)$$

$$\vec{E} = -\vec{\nabla}V \quad (14)$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} \quad (15)$$

Capacitance

$$C = \frac{q}{V} \quad \frac{\text{Coulombs}}{\text{Volt}} \quad (16)$$

Capacitance for parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d} \quad (17)$$

Capacitors in parallel

$$C_{eq} = C_1 + C_2 + ... \quad (18)$$

Capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... \quad (19)$$

Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV \quad (20)$$

Electrical energy density in an electric field

$$u_E = \frac{1}{2}\epsilon_0 E^2 \quad (21)$$

Capacitor with dielectric

$$C = \kappa C_0 \quad (22)$$

Dielectric constant of a dielectric material with permittivity ϵ

$$\kappa = \frac{\epsilon}{\epsilon_0} \quad (23)$$

Electric Dipole Moment for a symmetric dipole

$$\vec{p} = |q|\vec{d} \quad (24)$$

$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{Farad}}{m}, \frac{C}{Nm^2}$

$K = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} = 8.99 \times 10^9 \, Nm^2/C^2$

divergence theorem

$\phi = \oint_A \vec{F} \cdot \vec{da} = \int_V \nabla \cdot \vec{F} \, dv$

$E = N/C$

$V = \frac{N \cdot M}{C}$

$B = \frac{Kg}{A \cdot s^2} = \frac{N \cdot s}{C \cdot m} = T$

$\phi_E = \frac{N}{C} \cdot m^2$

$\phi_B = \frac{Kg \cdot M^2}{A \cdot s^2}$

1

Α α	alpha	
Β β	beta	
Γ γ	gamma	
Δ δ	delta	
Ε ε	epsilon	
Ζ ζ	zeta	
Η η	eta	
Θ θ	theta	
Ι ι	iota	
Κ κ	kappa	
Λ λ	lambda	
Μ μ	mu	
Ν ν	nu	
Ξ ξ	xi	
Ο ο	omicron	
Π π	pi	
Ρ ρ	rho	
Σ σ	sigma	
Τ τ	tau	
Υ υ	upsilon	
Φ φ	phi	
Χ χ	chi	
Ψ ψ	psi	
Ω ω	omega	

Prefix		Base 10
Name	Symbol	
quetta	Q	10^{30}
ronna	R	10^{27}
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
—	—	10^{0}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}
ronto	r	10^{-27}
quecto	q	10^{-30}

$V = IR$

$I \circ I \circ R$

Electrostatic Conductor 1. $E_{inside} = 0$

2. All excess charges are located on the surface

3. E Just outside is normal to surface see (6)

Positive charge direction of field

$I(t) = C \cdot \frac{dV}{dt}$ $\quad V(t) = V_\infty + (V_0 - V_\infty) \cdot e^{-t/\tau}$

Center of charge

$$\vec{x}_{CC} = \frac{\sum_{i=1}^{N} |q_i| \vec{x}_i}{\sum_{i=1}^{N} |q_i|} \tag{25}$$

Torque on an electric Dipole

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{26}$$

Potential Energy of Electric Dipole

$$U = -\vec{p}.\vec{E} \tag{27}$$

Electric Current

$$I = \frac{dq}{dt} \tag{28}$$

Current Density
vector

$$J = \frac{I}{A} = nqv_d \tag{29}$$

$$\vec{J} = \sigma \vec{E} \tag{30}$$

Resistance

$$R = \frac{V}{I} \tag{31}$$

$$R = \frac{\rho L}{A} \tag{32}$$

Temperature dependence

$$\rho = \rho_0(1 + \alpha(T - T_0)) \tag{33}$$

Resistors in series

$$R_{eq} = R_1 + R_2 + ... \tag{34}$$

Resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \tag{35}$$

Electric Power

$$P = IV = I^2 R = \frac{V^2}{R} \tag{36}$$

Kirchhoff's Junction and Loop Rules:

$$\Sigma I_{in} = \Sigma I_{out} \tag{37}$$

$$\Sigma_{closed} \Delta V = 0 \tag{38}$$

RC Circuit - Charging Capacitor

$\frac{dq}{dt} = C \cdot \frac{dV}{dt}$

$$q(t) = Q(1 - e^{-\frac{t}{RC}}) \tag{39}$$

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} \tag{40}$$

RC Circuit - Discharging Capacitor

$$q(t) = Qe^{-\frac{t}{RC}} \tag{41}$$

$$I(t) = -\frac{Q}{RC} e^{-\frac{t}{RC}} \tag{42}$$

Time Constant

$$\tau = RC \tag{43}$$

Magnetism

Magnetic Force on a moving charge

$$\vec{F_B} = q\vec{v} \times \vec{B} \tag{44}$$

$$F_B = |q| vB \sin\theta \tag{45}$$

Magnetic Force on a current carrying wire

$$I \cdot L \cdot B \cdot \sin\theta = \vec{F_B} = I\vec{L} \times \vec{B} \tag{46}$$

Magnetic Dipole Moment

$$\vec{\mu} = I\vec{A} \tag{47}$$

Torque on Current Loop

$$\underset{\text{Turns}}{N} \cdot \overset{\text{cr}}{I} \cdot \underset{\text{AREA}}{A} \cdot \overset{\text{Field}}{B} = \vec{\tau} = \vec{\mu} \times \vec{B} \tag{48}$$

Magnetic Dipole Potential Energy

$$U = -\vec{\mu}.\vec{B} \tag{49}$$

Biot-Savart Law

$\frac{\mu_0 \cdot I \cdot \theta}{4\pi \cdot R} = B$ in center arc

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \tag{50}$$

Small L approx $= \frac{\mu_0 \cdot I}{4\pi} \cdot \frac{\Delta L \cdot \sin\theta}{r^2}$

infinite thin $B = \frac{\mu_0 I}{2\pi R}$

Ampere's Law

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enc} \tag{51}$$

Magnetic Flux

$$\phi_B = N \int \vec{B}.d\vec{A} \tag{52}$$

CAPACITOR

$V = \frac{1}{C} \int I \, dt$

$\frac{dV}{dt} = \frac{1}{C} I$

$\frac{d^2V}{dt^2} = \frac{1}{C} \frac{dI}{dt}$

INDUCTOR

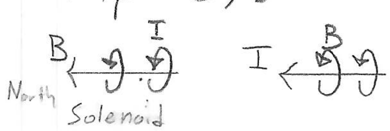
$I = \frac{1}{L} \int V \, dt$

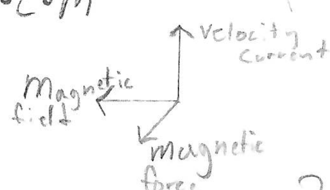
$\frac{dI}{dt} = \frac{1}{L} V$

$\frac{d^2I}{dt^2} = \frac{1}{L} \frac{dV}{dt}$

Tesla $= Kg \cdot s^{-2} \cdot A^{-1} = N \cdot s \cdot C^{-1} M^{-1}$

Amp $= C/s$





Thevin Notes

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

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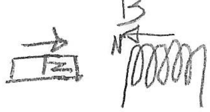

When Φ_B is decreasing ⊙ ⊙
When Φ_B is increasing ⊙ ⊗

Review **Lenz's Law**
The magnetic field induced will "resist" the magnetic field ← IMPORTANT FOR CONSERVATION ENERGY

Back Emf: When power is cut to a solenoid, it will can be induced by it's own magnetic field (polarity reversal)

Gauss's Law of Magnetism

$$\oint \vec{B}.d\vec{A} = 0 \tag{53}$$

Faraday's Law

$$-N\frac{d\Phi_B}{dt} = \varepsilon = -\frac{d\phi_B}{dt} = \oint \vec{E}.d\vec{s} \tag{54}$$

Magnetic Field due to a long, straight wire

$$B = \frac{\mu_0 I}{2\pi r} \tag{55}$$

Magnetic Field of solenoid

$$B = \mu_0 I n \tag{56}$$

$$n = \frac{N}{L} \tag{57}$$

Magnetic field in a material with magnetic susceptibility χ

$$B = (1+\chi)B_0 \tag{58}$$

Magnetic permeability of a material with magnetic susceptibility χ

$$\mu = (1+\chi)\mu_0 \tag{59}$$

Inductance and Inductors
Self Inductance

$$L = \frac{N\phi_B}{I} \tag{60}$$

Inductance of a solenoid

solenoid self inductance $L = \frac{\mu_0 \cdot n^2 \cdot \text{Area}}{\text{length}}$

$$\frac{L}{l} = \mu_0 n^2 A \tag{61}$$

Self-induced emf inductance in henries

ε_L opposes change in current

$$\varepsilon_L = -L\frac{dI}{dt} \tag{62}$$

RL Circuit

$$L\frac{dI}{dt} + RI = \varepsilon \tag{63}$$

Current in RL Circuit

$I(t) = I_\infty + (I_0 - I_\infty)e^{-t/\tau}$ EVERY

$$I = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}}) \tag{64}$$

Current Decay in RL Circuit

$$I = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}} = I_0 \cdot e^{-\frac{t}{\tau}} \tag{65}$$

RL time constant

$$\tau = \frac{L}{R} \tag{66}$$

Energy stored in an inductor

$$U = \frac{1}{2}LI^2 \tag{67}$$

Magnetic energy density in a magnetic field

$$u_m = \frac{B^2}{2\mu_0} \tag{68}$$

Mutual inductance

$$\varepsilon_2 = -M\frac{dI_1}{dt} \tag{69}$$

LC Circuit

$$Q(t) = Q_{max}\cos(\omega t + \phi) \tag{70}$$

$$I(t) = \frac{dQ}{dt} = -\omega Q_{max}\sin(\omega t + \phi) \tag{71}$$

LC angular frequency
rad/s

$$\omega = \frac{1}{\sqrt{LC}} \tag{72}$$

LC energy

$$U = U_C + U_L = constant \tag{73}$$

AC Circuits

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}; V_{rms} = \frac{V_{max}}{\sqrt{2}} \tag{74}$$

Inductive Reactance

$$X_L = \omega L j \tag{75}$$

Capacitive Reactance

$$X_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} \tag{76}$$

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rectangular toroid self inductance

$L = \frac{\mu_0 \cdot n^2 \cdot h}{2\pi} \cdot \ln\frac{R_2}{R_1}$

$V(t) = L \cdot \frac{dI}{dt}$

$\frac{I_1}{I_2} = \frac{L_2}{L_1}$

Energy stored in a capacitor

$U_c = \frac{1}{2}\cdot C \cdot V^2 = \frac{Q^2}{2 \cdot C} = \frac{1}{2}Q \cdot V$

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{77}$$

$$\phi = \tan^{-1}(\frac{X_L - X_C}{R}) \tag{78}$$

Average Power

$$P_{ave} = I_{rms} V_{rms} \cos\phi \tag{79}$$

$$P_{ave} = I_{rms}^2 R \tag{80}$$

RLC Circuit

$$I_{rms} = \frac{V_{rms}}{Z} \tag{81}$$

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{82}$$

Transformers

$$V_2 = \frac{N_2}{N_1} V_1 \tag{83}$$

$$I_1 V_1 = I_2 V_2 \tag{84}$$

EM Waves
Displacement current

$$I_{\mathrm{d}} = \epsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} \tag{85}$$

Ampere-Maxwell law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_{\mathrm{E}}}{dt} \tag{86}$$

1D wave equation for plane EM waves

$$\frac{\partial^2 E_y}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2} \tag{87}$$

Speed of EM waves

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{88}$$

E/B field ratio in an EM wave

$$c = \frac{E}{B} \tag{89}$$

Poynting (energy flux) vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times B \tag{90}$$

Average intensity of an EM wave

$$I = S_{\mathrm{avg}} = \frac{c\epsilon_0 E_0^2}{2} = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0} \tag{91}$$

Radiation pressure felt by a perfect absorber

$$p = \frac{I}{c} \tag{92}$$

Radiation pressure felt by a perfect reflector

$$p = \frac{2I}{c} \tag{93}$$

Charge Densities
Linear charge density λ

$$Q = \int \lambda ds \tag{94}$$

Surface charge density σ

$$Q = \int \sigma dA \tag{95}$$

Volume charge density ρ

$$Q = \int \rho dV \tag{96}$$

Constants

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 Nm^2/C^2 \tag{97}$$

more constants on back

Permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2 \tag{98}$$

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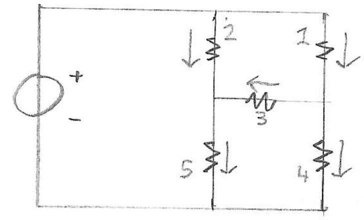
whet bridge



$I_4 = I_1 - I_3$

$I_5 = I_2 + I_3$

$0 = R_1 \cdot I_1 + R_3 \cdot I_3 - R_2 \cdot I_2$

$0 = R_4 \cdot I_4 - R_3 \cdot I_3 - R_5 \cdot I_5$

$V_T = R_1 \cdot I_1 + R_3 \cdot I_3 + R_5 \cdot I_5$

Solve I_1, I_2, I_3

Permeability of free space 1.257E-6

$$\mu_0 = 4\pi \times 10^{-7} Tm/A \tag{99}$$

Elementary Charge

$$e = 1.602 \times 10^{-19} C \tag{100}$$

Mass of an Electron

$$m_e = 9.11 \times 10^{-31} kg \tag{101}$$

Mass of a Proton

$$m_p = 1.67 \times 10^{-27} kg \tag{102}$$

Speed of Light

$$c = 3.00 \times 10^{8} m/s \tag{103}$$

Maxwell equations

$\text{div } E = \frac{\rho}{\varepsilon_0}, \oint_A \vec{E} \cdot da = Q/\varepsilon_0 = \Phi_E$ gauss's Law

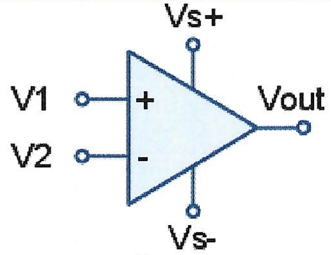
$\text{div } B = 0, \oint_A B \cdot da = 0, \int_V \nabla \cdot B \, dv = 0$ magnetic monopole DNE

$\text{curl } E = -\frac{\partial B}{\partial t}, \oint_L \vec{E} \cdot dl = -\frac{\partial}{\partial t} \oint_A B \cdot da = \text{Voltage}$ Faradays law of induction Lenz's Law

$\text{curl } B = \mu_0 (J + \varepsilon_0 \cdot \frac{\partial E}{\partial t}), \oint_A \nabla \times B \cdot da = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int_A E \cdot da$ Magnetic field around a current carrying wire

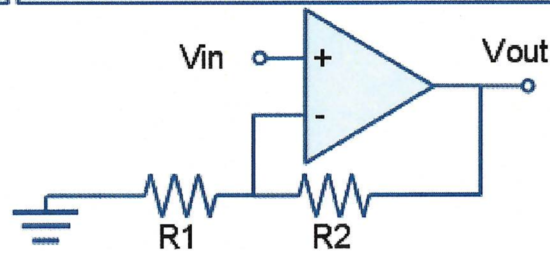
Basic Operational Amplifier Configurations

Voltage Comparator



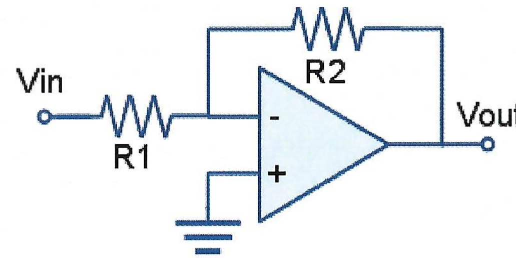
$$Vout = \begin{cases} Vs+ & V1 > V2 \\ Vs- & V1 < V2 \end{cases}$$

Non-Inverting Amplifier



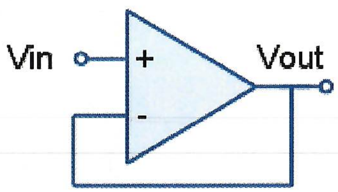
$$Vout = Vin * \left(1 + \frac{R2}{R1}\right)$$

Inverting Amplifier



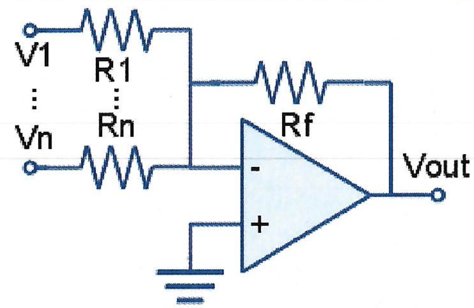
$$Vout = -Vin * \left(\frac{R2}{R1}\right)$$

Voltage Follower



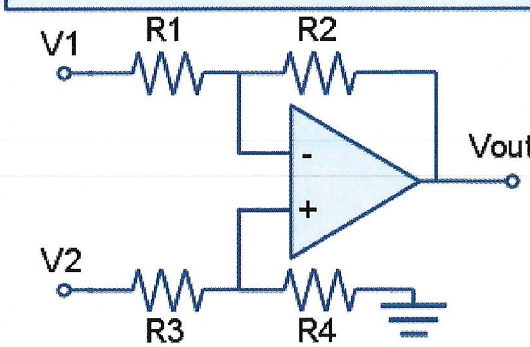
$$Vout = Vin$$

Inverting Summing Amplifier



$$Vout = -Rf * \left(\frac{V1}{R1} + \dots + \frac{Vn}{Rn}\right)$$

Differential Amplifier

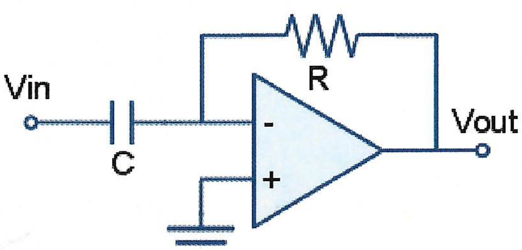


$$Vout = \left(1 + \frac{R2}{R1}\right)\left(\frac{R4}{R3 + R4}\right) * V2 - \left(\frac{R2}{R1}\right) * V1$$

If R1 = R3 and R2 = R4 Then

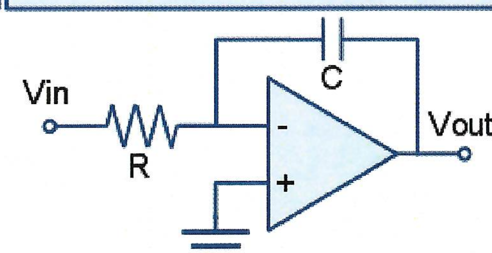
$$Vout = \left(\frac{R2}{R1}\right)(V2 - V1)$$

Differentiator Amplifier



$$Vout = -R * C * \left(\frac{dVin}{dt}\right)$$

Integrator Amplifier



$$Vout = -\left(\frac{1}{R * C}\right) Vin\ dt$$

RLC

OVERDAMPED

ALL REAL ROOTS

$$x(t) = K_1 e^{s_1 \cdot t} + K_2 e^{s_2 \cdot t} + x_\infty$$

~~$\omega_0 = \frac{1}{\sqrt{LC}}$~~

ω_0 = undamped frequency

α = damping coeff

$\zeta = \frac{\alpha}{\omega_0}$

$I = C \cdot \frac{dV}{dt}$, $V = L \cdot \frac{dI}{dt}$

SOLVE INITIAL COND

UNDER DAMPED

$-\alpha \pm i \cdot \omega_d = r \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$$x(t) = e^{-\alpha t}\left(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d \cdot t)\right) \; x_\infty$$

CRITICALLY DAMPED REAL IDENTICAL ROOTS

IDENTICAL REAL ROOTS

$$x(t) = B_1 e^{-\alpha t} + B_2 \cdot t e^{-\alpha t} + x_\infty$$