MATH 316D W11

DD1 Individual Quiz

1. Match the terms with the definitions. (This is a matching question. The definitions are next to the term that it goes with. In the quiz, don't let them match up. Thank you!)

   (a) Equilibrium Solutions $\Longrightarrow$ $x(t)$ is constant for all values of t.

   (b) Stable equilibrium $\Longrightarrow$ every non-constant solution approaches the equilibrium.

   (c) Unstable equilibrium $\Longrightarrow$ every non-constant solution flows away from the equilibrium.

   (d) Repelling Node $\Longrightarrow$ the eigenvalues are positive and all solutions flow away from the equilibrium.

   (e) Attracting Node $\Longrightarrow$ the eigenvalues are negative and all solutions approach the equilibrium.

   (f) Saddle Point $\Longrightarrow$ the eigenvalues have opposite signs and some solutions are moving away while others are moving toward the equilibrium.

2. Let $A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$.

   Then $\vec{x}' = A\vec{x}$ is best represented by

   (a) $x_1' = -2x_1 + x_2$ $\Longrightarrow$ **This is the correct answer.**
   $x_2' = 0x_1 - 3x_2$

   (b) $x_1' = 0x_1 - 3x_2$ $\Longrightarrow$ **Feedback: Make sure you have the correct rows.**
   $x_2' = -2x_1 + x_2$

   (c) $x_1' = x_1 - 2x_2$ $\Longrightarrow$ **Feedback: Check the columns of your matrix.**
   $x_2' = -3x_1 + 0x_2$

   (d) $x_1' = 2x_1 - x_2$ $\Longrightarrow$ **Feedback: Check the signs of your matrix.**
   $x_2' = 0x_1 + 3x_2$

3. The eigenvalues for A in problem 2 are:

   (a) both positive and real. $\Longrightarrow$ **Feedback: Don't drop a negative.**

   (b) both negative and real. $\Longrightarrow$ **This is the correct answer.**

   (c) opposite signs. $\Longrightarrow$ **Feedback: Watch your signs.**

   (d) one equals zero. $\Longrightarrow$ **Feedback: Check your work once more.**

4. Mark all that apply if the eigenvalues for matrix A in $\vec{x}' = A\vec{x}$ are both positive real numbers.

   (a) $\vec{0}$ is a stable equilibrium. $\Longrightarrow$ **Feedback: Check your definition.**

   (b) $\vec{0}$ is an unstable equilibrium. $\Longrightarrow$ **Feedback: Correct**

   (c) $\vec{0}$ is a repelling node. $\Longrightarrow$ **Feedback: Correct.**

   (d) $\vec{0}$ is an attracting node. $\Longrightarrow$ **Feedback: Check your definition.**

   (e) $\vec{0}$ is a saddle point. $\Longrightarrow$ **Feedback: Check your definition.**

5. If a 2×2 matrix A has one eigenvalue equal to zero while the other is real, then $\vec{x}' = A\vec{x}$ has: (Mark all that apply)

   (a) more than one equilibrium solution. $\Longrightarrow$ **Feedback: Correct.**

   (b) exactly one equilibrium solution. $\Longrightarrow$ **Feedback: Check your definition.**

   (c) exactly one straight line solution. $\Longrightarrow$ **Feedback: Check your definition.**

   (d) more than one straight line solution. $\Longrightarrow$ **Feedback: Correct.**

6. Use the Wronksian Theorem to determine the linearity of $\vec{v_1} = [e^{-t} \quad -e^{-t} \quad e^{-t}]^T$, $\vec{v_2} = [3e^{2t} \quad e^{2t} \quad -2e^{2t}]^T$, and $\vec{v_3} = [e^{5t} \quad e^{5t} \quad e^{5t}]^T$. Mark the true statement.

   (a) The Wronksian is $10e^{6t}$ and thus v_1, v_2, and v_3 are linearly independent. $\Longrightarrow$ **Feedback: Correct.**

   (b) The Wronksian is 0 and thus v_1, v_2, and v_3 are linearly independent. $\Longrightarrow$ **Feedback: Check your determinate.**

   (c) The Wronksian is $10e^{6t}$ and thus v_1, v_2, and v_3 are linearly dependent. $\Longrightarrow$ **Feedback: Check the Wronksian.**

   (d) The Wronksian is 0 and thus v_1, v_2, and v_3 are linearly dependent. $\Longrightarrow$ **Feedback: Check your determinate.**

DD2 Group Quiz

1. Classify the stability of the orgin as an equilibrium for $\vec{x}' = A\vec{x}$ if $p(\lambda)$ is the characteristic equation determined by $\det(A - \lambda I)$.
   $p(\lambda) = \lambda^2 + \lambda + 1$.

   (a) Unstable repelling node

   (b) Unstable saddle

   (c) Stable center

   (d) Stable spiral sink $\Longrightarrow$ **Correct**

   (e) Unstable spiral source

2. Classify the stability of the orgin as an equilibrium for $\vec{x}' = A\vec{x}$ if $p(\lambda)$ is the characteristic equation determined by $\det(A - \lambda I)$.
   $p(\lambda) = \lambda^2 - 4$.

   (a) Unstable repelling node

   (b) Unstable saddle $\Longrightarrow$ **Correct**

   (c) Stable center

   (d) Stable spiral sink

   (e) Unstable spiral source

Instructions: For questions 3 - 6, let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, with initial values of $x_1(0) = 1$ and $x_2(0) = -1$.

3. Determine the solution to $\vec{x}' = A\vec{x}$.

   (a) $\vec{x}(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (b) $\vec{x}(t) = c_1 e^{3it} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-3it} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (c) $\vec{x}(t) = c_1 \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} -\cos(2t) \\ -\sin(2t) \end{bmatrix}$

   (d) $\vec{x}(t) = c_1 \begin{bmatrix} -\sin(2t) \\ \cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}$ $\Longrightarrow$ **Correct**

   (e) None of the above.

4. Now, consider the IVP $\vec{x}' = A\vec{x}$ where $x_1(0) = 1$ and $x_2(0) = -1$. Solve.

   (a) $\vec{x}(t) = e^{3t}\begin{bmatrix}1\\0\end{bmatrix} - e^{-3t}\begin{bmatrix}0\\1\end{bmatrix}$

   (b) $\vec{x}(t) = -1\begin{bmatrix}-\sin(2t)\\ \cos(2t)\end{bmatrix} + \begin{bmatrix}\cos(2t)\\ \sin(2t)\end{bmatrix} \implies$ **Correct**

   (c) $\vec{x}(t) = -1\begin{bmatrix}\cos(2t)\\ \sin(2t)\end{bmatrix} + e^{-3t}\begin{bmatrix}-\cos(2t)\\ -\sin(2t)\end{bmatrix}$

   (d) $\vec{x}(t) = -2\begin{bmatrix}-\sin(2t)\\ \cos(2t)\end{bmatrix} + 2\begin{bmatrix}\cos(2t)\\ \sin(2t)\end{bmatrix}$

   (e) None of the above.

5. Classify the stability of the equilibrium solution $\begin{bmatrix}0\\0\end{bmatrix}$.

   (a) Unstable repelling node

   (b) Stable attracting node

   (c) Unstable spiral source

   (d) Stable center $\implies$ **Correct**

   (e) Stable spiral sink

6. How many straight line solutions?

   (a) None $\implies$ **Correct**

   (b) One

   (c) Two

   (d) Infinite

   (e) Cannot be determined

DD3 Weekly Quiz

This quiz needs to be completely redone so please delete all the current questions in i-Learn and replace them with the following, thank you.

1. "In a system of three tanks of saltwater interconnected with pipes of inflow and outflow to and from each, the following information is given.

	Tank A	Tank B	Tank C
Tank volume	400 liters	800 liters	500 liters
Rate of inflow to tank	5 L/min.	10 L/min.	5 L/min.
Concentration of salt in inflow	25 g/L	15 g/L	40 g/L
Rate of drain outflow	4 L/min.	7 L/min.	9 L/min.
Rates of outflows to other tanks	to B: 6 L/min.	to C: 5 L/min.	to A: 4 L/min.
Rates of outflows to other tanks	to C: 4 L/min.	to A: 5 L/min.	to B: 1 L/min.

   Assume that, initially, there is a concentration of 10 g/L of salt in each of the three tanks. Set up an IVP of the system of tanks using the information given, then choose the answer that best represents your work."

   (a) $\begin{bmatrix}A'\\B'\\C'\end{bmatrix} = \begin{bmatrix}-7/200 & 1/160 & 1/125\\ 3/200 & -17/800 & 1/500\\ 1/100 & 1/160 & -7/250\end{bmatrix}\begin{bmatrix}A\\B\\C\end{bmatrix}, \begin{bmatrix}A(0)\\B(0)\\C(0)\end{bmatrix} = \begin{bmatrix}4000\\8000\\5000\end{bmatrix}$

(b) $\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} -7/200 & 1/160 & 1/125 \\ 3/200 & -17/800 & 1/500 \\ 1/100 & 1/160 & -7/250 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} + \begin{bmatrix} 125 \\ 150 \\ 200 \end{bmatrix}, \begin{bmatrix} A(0) \\ B(0) \\ C(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$

(c) $\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} -7/200 & 1/160 & 1/125 \\ 3/200 & -17/800 & 1/500 \\ 1/100 & 1/160 & -7/250 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} + \begin{bmatrix} 125 \\ 150 \\ 200 \end{bmatrix}, \begin{bmatrix} A(0) \\ B(0) \\ C(0) \end{bmatrix} = \begin{bmatrix} 4000 \\ 8000 \\ 5000 \end{bmatrix}$ $\Longrightarrow$ **Correct**

(d) $\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} -7/200 & 1/160 & 1/125 & 125 \\ 3/200 & -17/800 & 1/500 & 150 \\ 1/100 & 1/160 & -7/250 & 200 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}, \begin{bmatrix} A(0) \\ B(0) \\ C(0) \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$

(e) None of the above.

2. "Let us consider a system of two masses attached to two springs in parallel, where the mass m_1 is attached to the spring with spring constant k_1, and m_2 to the spring with constant k_2. Assuming that the surface they are on is frictionless the equations that govern the systems motion is:

$$x_1'' = -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}(x_2 - x_1)$$
$$x_2'' = -\frac{k_2}{m_2}(x_2 - x_1).$$

Suppose that k_1=2, m_1=1, k_2=4 and m_2=.5 . Using these constants and the following substitutions: $y_1 = x_1$, $y_2 = y_1' = x_1'$, $y_3 = x_2$, $y_4 = y_3 = x_2'$, convert the system of two second-order equations to a system of the form $\vec{y}' = \mathbb{A}\vec{y}$."

(a) $\begin{bmatrix} y_2' \\ y_4' \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_3 \end{bmatrix}$

(b) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ 8 & -8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

(c) $\begin{bmatrix} y_2' \\ y_4' \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ 8 & -8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_3 \end{bmatrix}$ $\Longrightarrow$ **Correct**

(d) None of the above.

3. "Suppose we have an RLC circuit that has an inductor of L=1 H, resistor of R=16 Ω, and capacitor of C=.01 F. Assume that $I(0)$=100 A and $I'(0)$=0, and that the system is provided a voltage source of $E(t) = 100\sin 10\,t$."

"State a second-order IVP whose solution is $I(t)$, the current in the circuit at time t."

(a) $I''(t) + 16\,I'(t) + 100\,I(t) = 1000\cos 10\,t;\ I'(0) = 0,\ I(0) = 100$ $\Longrightarrow$ **Correct**

(b) $I''(t) + 16\,I'(t) + .01\,I(t) = 1000\cos 10\,t;\ I'(0) = 100,\ I(0) = 0$

(c) $I'(t) + 16\,I(t) + 100\,Q(t) = 100\sin 10\,t;\ I'(0) = 0,\ I(0) = 100$

(d) None of the above.

4. "Using your answer from the previous problem, create a system of first-order IVPs using a standard substitution and in the standard form $\vec{x}' = \mathbb{A}\vec{x}$."

(a) $\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -16 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1000\cos 10\,t \end{bmatrix};\ \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$ $\Longrightarrow$ **Correct**

(b) $\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -.01 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1000\cos 10\,t \end{bmatrix};\ \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \end{bmatrix}$

(c) $\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -16 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 100\sin 10\,t \end{bmatrix};\ \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$

(d) None of the above.

5. "From your model, as defined in question four, state what the eigenpairs are."

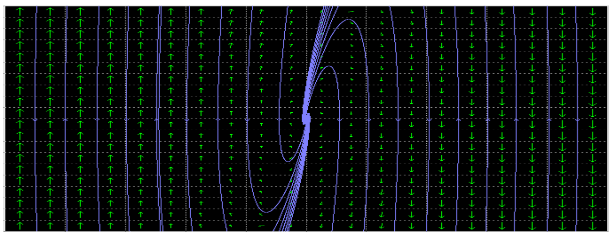
   (a) $\left\{\lambda_1 = 0, \vec{v}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$ and $\left\{\lambda_2 = -16, \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$

   (b) $\left\{\lambda_1 = -8 + 6i, \vec{v}_1 = \begin{bmatrix} -(4 - 3i) \\ 50 \end{bmatrix}\right\}$ and $\left\{\lambda_2 = -8 - 6i, \vec{v}_2 = \begin{bmatrix} -(4 + 3i) \\ 50 \end{bmatrix}\right\}$

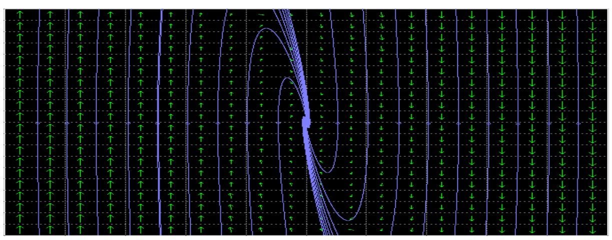
   (c) $\left\{\lambda_1 = -.005 + 10i, \vec{v}_1 = \begin{bmatrix} -(5 \cdot 10^{-5} + .1i) \\ 1 \end{bmatrix}\right\}$ and $\left\{\lambda_2 = -.005 - 10i, \vec{v}_2 = \begin{bmatrix} -(5 \cdot 10^{-5} - .1i) \\ 1 \end{bmatrix}\right\}$

   (d) $\left\{\lambda_1 = -8 + 6i, \vec{v}_1 = \begin{bmatrix} -(4 + 3i) \\ 50 \end{bmatrix}\right\}$ and $\left\{\lambda_2 = -8 - 6i, \vec{v}_2 = \begin{bmatrix} -(4 - 3i) \\ 50 \end{bmatrix}\right\}$ $\Longrightarrow$ **Correct**

6. "Which of the following graphs represents the direction field of the eigenpair from the system you defined in problem four?"

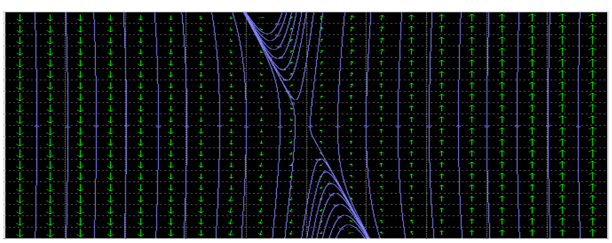


(a)

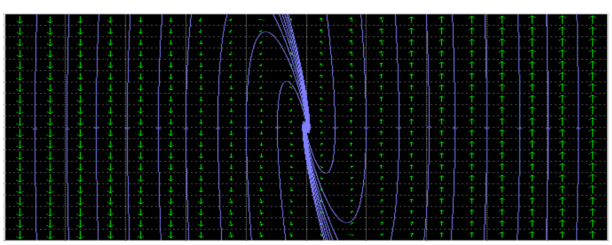


$\Longrightarrow$ **Correct**

(b)



(c)



(d)

7. "Solve the system that you defined in problem four to determine the current $I(t)$ at time t. You may use Mathematica to help find the solution but you may not use the commands `DSolve` or `NDSolve`."

(a) $\vec{x}_g(t) = -\left(\frac{5}{8} - \frac{95i}{6}\right) e^{(-8+6i)t} \begin{bmatrix} -(4+3i) \\ 50 \end{bmatrix} - \left(\frac{5}{8} + \frac{95i}{6}\right) e^{-(8+6i)t} \begin{bmatrix} -(4-3i) \\ 50 \end{bmatrix} + \begin{bmatrix} -\frac{5}{8}\cos 10\,t \\ \frac{25}{4}\sin 10\,t \end{bmatrix}$

(b) $\vec{x}_g(t) = -\left(\frac{5}{8} - \frac{95i}{6}\right) e^{(-8+6i)t} \begin{bmatrix} -(4+3i) \\ 50 \end{bmatrix} - \left(\frac{5}{8} + \frac{95i}{6}\right) e^{-(8+6i)t} \begin{bmatrix} -(4-3i) \\ 50 \end{bmatrix} + \begin{bmatrix} \frac{25}{4}\sin 10\,t \\ \frac{125}{2}\cos 10\,t \end{bmatrix} \Longrightarrow$

Correct

(c) $\vec{x}_g(t) = -\left(\frac{5}{8} - \frac{95i}{6}\right) e^{(-8+6i)t} \begin{bmatrix} -(4-3i) \\ 50 \end{bmatrix} - \left(\frac{5}{8} + \frac{95i}{6}\right) e^{-(8+6i)t} \begin{bmatrix} -(4+3i) \\ 50 \end{bmatrix} + \begin{bmatrix} \frac{25}{4}\sin 10\,t \\ \frac{125}{2}\cos 10\,t \end{bmatrix}$

(d) None of the above.

8. **(Essay Question)** "Consider a system of two tanks that are connected in such a way that each of the tanks has an independent inflow that delivers salt solution to it, each has an independent outflow (i.e. drain), and each tank is connected to the other with an outflow and an inflow. The relevant information is as follows:

	Tank A	Tank B
Tank volume	100 liters	200 liters
Rate of inflow to tank	5 L/min.	9 L/min.
Concentration of salt in inflow	7 g/L	3 g/L
Rate of drain outflow	4 L/min.	10 L/min.
Rates of outflows to other tanks	to B: 3 L/min.	to A: 2 L/min.

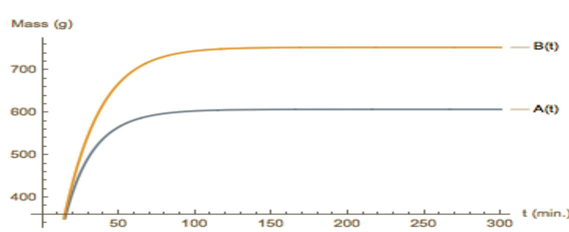
With Tank A initially having 20 g of salt present in solution, and Tank B having 75 g of salt present in solution."

"Set up and solve an IVP whose solution will determine the amount of salt in each tank at time t. You may use Mathematica to help you solve the IVP, but you may not use `DSolve` or `NDSolve` to help you find the solution in Mathematica. Once you have found the answer please upload all your work here including your Mathematica notebook or screen shot." The answer should be along the lines of: 1) the IVP set up as $\begin{bmatrix} A'(t) \\ B'(t) \end{bmatrix} = \begin{bmatrix} -\frac{7}{100} & \frac{1}{100} \\ \frac{3}{100} & -\frac{3}{50} \end{bmatrix} \begin{bmatrix} A(t) \\ B(t) \end{bmatrix} + \begin{bmatrix} 35 \\ 27 \end{bmatrix}$ with $\begin{bmatrix} A(0) \\ B(0) \end{bmatrix} = \begin{bmatrix} 20 \\ 75 \end{bmatrix}$ and 2) the answer

$$\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = -\frac{5}{26}\left(\left(1765 + 841\sqrt{13}\right) e^{\frac{(-13+\sqrt{13})t}{200}} \begin{bmatrix} -\frac{1}{6}(1-\sqrt{13}) \\ 1 \end{bmatrix} + \left(1765 - 841\sqrt{13}\right) e^{-\frac{(13+\sqrt{13})t}{200}} \begin{bmatrix} -\frac{1}{6}(1+\sqrt{13}) \\ 1 \end{bmatrix} \right) + \begin{bmatrix} \frac{7900}{13} \\ \frac{9800}{13} \end{bmatrix}$$

9. **(Essay Question)**

(a) "Using the solution that you calculated in problem nine create a graph of the behavior of the solution over time, and upload the graph or a screen shot of the graph here." See the solution on the following page.



(b) "Is there an equilibrium solution to the system?" Correct: $\Longrightarrow$ **Yes**

(c) "If so, what is the equilibrium solution?" Correct: $\Longrightarrow$ $\left(\frac{7900}{13}, \frac{9800}{13}\right)$