Week 04 For 316

DD1 Individual Quiz

1. Keep

2. keep

3. keep

4. keep

5. Use Mathematica to find a general solution to the following second-order differential equation: $y''+8y'+16y = 0$.

   (a) $y[x] = -2 + e^(- 8x)C[1]$
   (b) $y[x] = e^{-4x}C[1] + e^{-4x}xC[2] \implies$ **Correct**
   (c) $y[x] = C[1]\cos(2\sqrt{6}x) + C[2]\sin(2\sqrt{6}x)$
   (d) $y[x] = 5/16 + e^{-4x}C[1] + e^{-4x}xC[2]$

DD2 Group Quiz

1. Match the following terms with the definitions.

   (a) Kirchhoff's Current Law $\implies$ At each point of a circuit, the sum of the currents flowing into a point equals the sum of the currents flowing out.
   (b) Kirchhoff's law $\implies$ A conservation law that tells us what we can expect for the voltage drops across various parts of a circuit.
   (c) Voltage $\implies$ Energy force associated with the RLC circuit.
   (d) Resistance $\implies$ Proportional to the current flowing through the resistor.
   (e) Inductance $\implies$ The voltage drop across an inductor.
   (f) Capacitance $\implies$ The voltage drop across a capacitor.
   (g) Charge $\implies$ The flow of electrons measured with units of coulombs.
   (h) Current $\implies$ In a circuit at time t, this is proportional to the number of positive charge carriers that move past any given point per second in the conductor.
   (i) Laplace Transform $\implies$ On a function $f(t)$ that is defined on $[0, \infty)$ this is equal to $\int_0^\infty e^{-st}f(t)dt$.

2. W04 Group Quiz Question 1

3. W04 Group Quiz Question 2

4. W04 Group Quiz Question 3

5. Us the Laplace Transform definition to transform the following function: $f(t) = t$.

   (a) $F(s) = 1/s$
   (b) $F(s) = 1/s^2 \implies$ **Correct**
   (c) $F(s) = 2/s^2$
   (d) $F(s) = sin(t)$

6. W04 Group Quiz Question 5

Weekly Quiz

1. An RLC series circuit has a voltage source given by $E(t) = 30\cos(2t)V$, a resistor of 4Ω, and inductor of $1/2H$, and a capacitor of $1/30F$. If both the initial current and the initial charge are zero, which of the following matches this IVP?

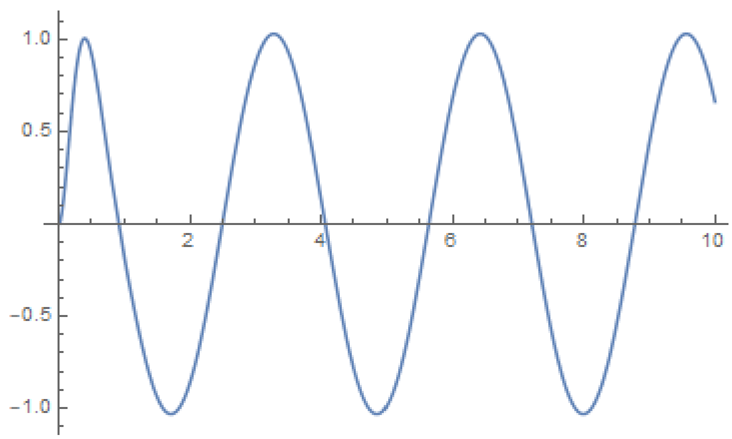
   (a) $\frac{1}{2}y'' + 30y' + 4y = \cos(2t); y(1) = 0, y'(1) = 0$

   (b) $4y'' + \frac{1}{2}y' + 30y = 30\cos(2t); y(0) = 0, y'(0) = 0$

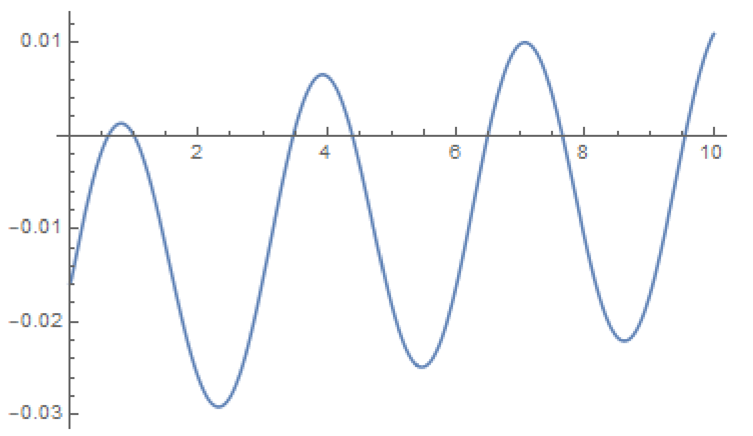
   (c) $\frac{1}{2}y'' + 4y' + 30y = 30\cos(2t); y(0) = 0, y'(0) = 0 \implies$ **CORRECT**

   (d) $\frac{1}{30}y'' + 4y' + \frac{1}{2}y = \cos(2t); y(0) = 0, y'(0) = 0$

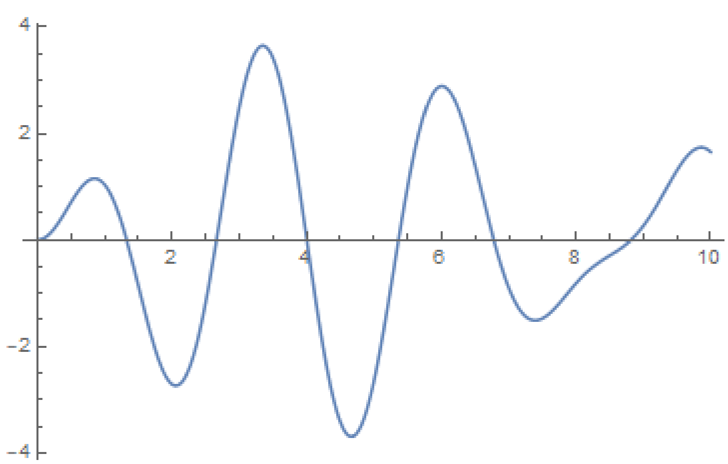
2. Use Mathematica to solve and plot your IVP from question 1. Which of the following best represents your answer?



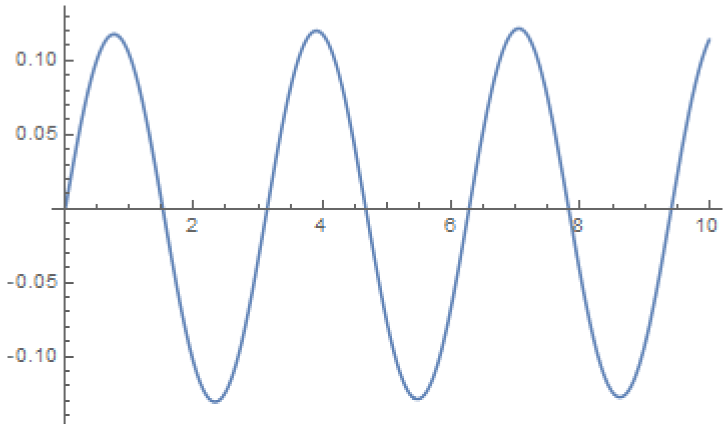
(a) $\implies$ **Correct is a**



(b)



(c)

(d) 

3. A spring with a mass of 2 kg has a spring constant of 6 N/m with a damping constant of 14 N-sec/m and an external force of $f(t) = 20\cos(t)N$. Which of the following represents this scenario when the spring is displaced by 0.25 m and has no initial velocity?

   (a) $2y'' + 14y' + 6y = \cos(t); y(0) = 0, y'(0) = 0.25$

   (b) $2y'' + 14y' + 6y = 20\cos(t); y(0) = 0.25, y'(0) = 0 \implies$ **Correct**

   (c) $14y'' + 6y' + 2y = 20\cos(t); y(0) = 0.25, y'(0) = 0$

   (d) $y'' + y' + y = 20\cos(t); y(0) = 0, y'(0) = 0.25$

4. Rewrite the following homogeneous function $y'' - 4y' + 5y = 0$ as a system of linear, first order differential equations. Notice that this function has no forcing function.

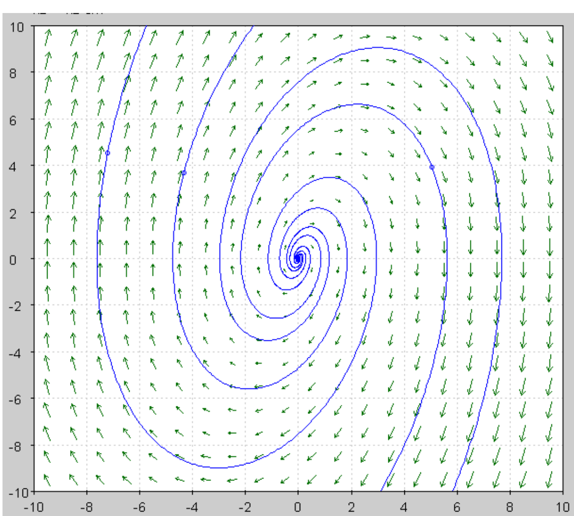
   (a) $x_1' = x_2$
   $x_2' = -2x_2 + 5x_1$

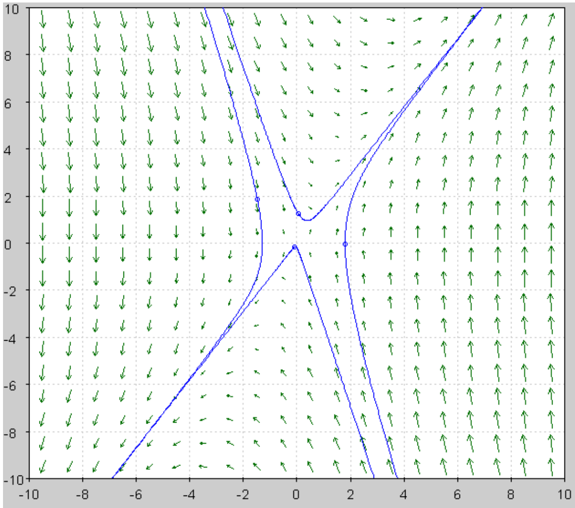
   (b) $x_1' = x_2$
   $x_2' = 1x_2 - 3x_1$

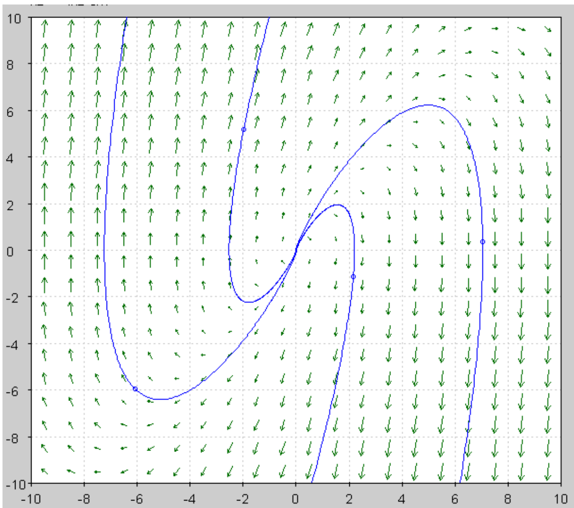
   (c) $x_1' = x_1$
   $x_2' = -4x_1 - 5x_2$

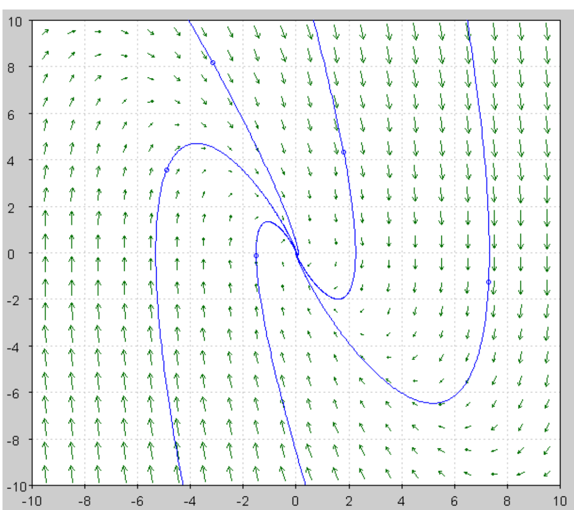
   (d) $x_1' = x_2$
   $x_2' = 4x_2 - 5x_1 \implies$ **Correct**

5. Which of the following best represents your system of equations that you found above? (Use pplane). The graphs have some trajectories. This is not the solution, it will just help you see what is happening with the pplane.

   (a) 

(b) 

(c)  $\Longrightarrow$ **Correct is c**

(d) 

6. What is the Laplace Transform of $f(t) = e^{3t}\sin(2t)$?

   (a) $L(s) = \frac{1}{(s-3)^2+1}$

   (b) $L(s) = \frac{2}{(s-3)^2+4}$ $\Longrightarrow$ **Correct.**

   (c) $L(s) = \frac{2}{s^2+4}$

   (d) $L(s) = \frac{1}{(s-3)}$

7. Use question 2 from the current W04 Weekly Quiz.

8. Solve the following differential equation: $y' + 2y = 10t$.

   (a) $y = e^{t}(5te^{t} - \frac{5}{2}e^{t} + C)$
   (b) $y = e^{-2t}(te^{2t} - e^{2t} + C)$
   (c) $y = e^{-2t}(5te^{2t} - \frac{5}{2}e^{2t} + C)$ $\Longrightarrow$ **Correct**
   (d) $y = e^{-2t}(5te^{2t} + C)$

9. (Essay Question) What information do we get from a phase plane (pplane)? What information does the orbit (or trajectory) in the pplane give us about the differential equation? Explain as if you are discussing with someone who has never seen a pplane.

   - This answer should be along the lines: the pplane describes the movement of the solution to the differential equation with its derivative. The orbit tells us how the particular solution is changing with it's derivative according to time t.

10. Find the Laplace transform using the definition for the following function: $f(t) = 1$.

    - They should include most of the following steps:

$$\begin{aligned} L[1] &= \int_0^\infty e^{-st}dt \\ &= \lim_{r\to\infty}\int_0^r e^{-st}dt \\ &= \lim_{r\to\infty} -\frac{1}{s}e^{-st}\Big|_0^r \\ &= \lim_{r\to\infty}\Big[-\frac{1}{s}e^{-sr} + \frac{1}{s}\Big] \\ &= \frac{1}{s} \end{aligned}$$

    because $e^{-sr} \to 0$ as $r \to \infty$.