MATH 316D W09

DD1 Individual Quiz

1. **Keep**.
2. **Keep**.
3. **Change** to, "Let $\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$; one of the eigenvalues of $\mathbf{A}$ is:

   (a) 6
   (b) 0 $\Longrightarrow$ **Correct**
   (c) -2
   (d) None of the above.

4. **Keep**.
5. **Change** to, "Given the definition of an eigenpair as, $\mathbf{A}\vec{v} = \lambda\vec{v}$, where $\mathbf{A}$ is an $n \times n$ matrix and $\vec{v}$ is the non-zero eigenvector. How is the action of $\mathbf{A}$ equivalent to the scalar multiplication of λ, the eigenvalue?"

   (a) It is the same geometrically in that it will flip, stretch, or shrink the vector equivalently. $\Longrightarrow$ **Correct**
   (b) λ is equivalent to the determinant of $\mathbf{A}$.
   (c) All of the above.
   (d) The scalar multiplication of λ is not equivalent to the action of $\mathbf{A}$ but depends on the column vector $\vec{v}$.

DD2 Group Quiz (*Exam III Review*)

1. **Keep**.
2. **Change** to, "Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

   a. What is the span of the columns of $\mathbf{A}$?
   b. Does $\mathbf{A}^{-1}$ exist? If it does, find it. If it does not, explain why it does not exist.
   c. What is the volume of the parallelepiped spanned by the columns of $\mathbf{A}$?
   d. Write out the linear system represented by $\mathbf{A}\vec{x} = \vec{b}$, if $\vec{b} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$.
   e. Is $\vec{b}$ in the span of the columns of $\mathbf{A}$? If it is, write $\vec{b}$ as a linear combination of the columns of $\mathbf{A}$.

3. **Change** to, "Mark each statement True or False. Justify your answer. Let S be a set of n vectors in $\mathbb{R}^m$."

   (a) If $n > m$ the elements of S are linearly independent.

4. **Keep**.
5. **Keep** but please make sure that the questions reads as follows: "Compute the eigenvalues and eigenvectors for $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$. Describe the action of $\mathbf{A}$ on a vector $\vec{x}$ in $\mathbb{R}^2$."
6. **Keep**.
7. **Keep** but please make sure that the question reads as follows: "Determine all values of h such that the augmented system is consistent, $\begin{bmatrix} 1 & h & 3 \\ 2 & h & 6 \end{bmatrix}$."
8. **Keep**.
9. **Keep**.
10. **Keep**.

KEY - Exam III Review

1. $\lambda = 1$
2. a. $Span(\mathbf{A}) = c_1 \begin{bmatrix}1\\0\\0\end{bmatrix} + c_2 \begin{bmatrix}1\\1\\0\end{bmatrix} + c_3 \begin{bmatrix}1\\1\\1\end{bmatrix}$

   b. Yes. $\mathbf{A}^{-1} = \begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}$

   c. $V = 1$

   d.
   $$\begin{aligned} x_1 + x_2 + x_3 &= -1 \\ x_2 + x_3 &= 2 \\ x_3 &= 3 \end{aligned}$$

   e. Yes.
3. (a) $\Longrightarrow$ **False**
   (b) $\Longrightarrow$ **True**
   (c) $\Longrightarrow$ **True**
   (d) $\Longrightarrow$ **True**
4. a. -2
   b. -8
   c. -2
   d. $-\frac{1}{2}$
   e. This operation is not possible because...
5. The eigenvalues of $\mathbf{A}$ are $\lambda_1 = \sqrt{2}$ and $\lambda_2 = -\sqrt{2}$, with corresponding eigenvectors of $\vec{x}_1 = \begin{bmatrix}-1-\sqrt{2}\\1\end{bmatrix}$, and $\vec{x}_2 = \begin{bmatrix}-1+\sqrt{2}\\1\end{bmatrix}$. The action of $\mathbf{A}$ on $\vec{x}$ is to stretch the vector $\vec{x}$ by a factor of $\sqrt{2}$.
6. $\vec{r}(t) = \begin{bmatrix}-\frac{5}{2}\\2\end{bmatrix} t + \begin{bmatrix}\frac{1}{6}\\\frac{1}{3}\end{bmatrix}$ or $\vec{r}(t) = \begin{bmatrix}-5\\4\end{bmatrix} t + \begin{bmatrix}\frac{1}{6}\\\frac{1}{3}\end{bmatrix}$
7. $\Longrightarrow$ **All reals**.
8. a. An appropriate answer will be along the lines of: "A pivot point is a point in a given matrix, $\mathbf{A}$, that corresponds to a leading 1 when that matrix is in RREF." An appropriate equation will be of the form: $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$.

   b. An appropriate answer is along the lines of: "Two matrices are row equivalent when any number of elementary steps can be taken to transform one matrix into the other." Some elementary steps that may be mentioned are: 1) *replacement*, where a row is put back after being added to or subtracted by another row times a constant, 2) *interchangeability*, rows can be swapped as long as their columns do not shift position, and 3) *scaling*, every entry in each row can be multiplied by a non-zero constant.
9. An appropriate answer to this question may be, "Yes. A 2×3 linear system, when augmented, can be inconsistent because..."
10. An example of a matrix with complex eigenvalues is, $\mathbf{T} = \begin{bmatrix}4 & -4\\5 & -4\end{bmatrix}$. The eigenvalues and eigenvectors of this matrix are as follows: $\lambda_1 = 2\mathrm{i}$ and $\lambda_2 = -2\mathrm{i}$, with the corresponding eigenvectors being $\vec{v}_1 = \begin{bmatrix}4+2\mathrm{i}\\5\end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix}4-2\mathrm{i}\\5\end{bmatrix}$.