# Week 02 For 316

## DD1 Individual Quiz

1. Match the terms with the definitions. (This is a matching question. The definitions are next to the term that it goes with. In the quiz, don't let them match up. Thank you!)

   (a) Carrying Capacity $\Longrightarrow$ the number of people, other living organisms, or crops that a region can support without environmental degradation.

   (b) Joint Probability $\Longrightarrow$ The statistical measure where the likelihood of two events occurring together at the same point in time are calculated.

   (c) Separable Differential Equation $\Longrightarrow$ A differential equations where the types of variables can be separated with an equal sign.

2. Write a differential equation that fits the following description: The rate of change of the mass $A$ of salt at time $t$, is proportional to the square of the mass of salt present at time $t$.

   (a) $\frac{dA}{dt} = kA^2 \Longrightarrow$ **Correct**

   (b) $\frac{dA}{dt} = A^2 \Longrightarrow$ **Feedback: What does proportional mean?**

   (c) $\frac{dt}{dA} = kA^2 \Longrightarrow$ **Feedback: What are you taking the derivative of?**

   (d) $\frac{dA}{dt} = k\sqrt{A} \Longrightarrow$ **Feedback: What does "the square" mean?**

3. Consider the differential equation: $\frac{dy}{dx} = \frac{x}{y}$. Sketch the directional field and select the graph that appropriately represents your results. (NOTE: keep the order and the answers the same. Just add the feedback.)

   (a) **Feedback: Check your equation in DField**

   (b) **Feedback:Correct**

   (c) **Feedback: Check your equation in DField**

   (d) **Feedback: Check your equation in DField**

4. For your previous differential equation, if your initial value is $y(0) = 3$ draw the solution curve with this initial value and select the curve that best fits with your result. (NOTE: Keep the answers in that order, just add feedback.)

   (a) **Feedback: Correct**

   (b) **Feedback: Check your original equation and your initial condition.**

   (c) **Feedback: Check your original equation.**

   (d) **Feedback: Should it be that steep? Look at your bounds**

5. Solve the following differential equation: $t\frac{dy}{dx} = \frac{1}{y^3}$.

   (a) $y = \pm\sqrt[4]{4t^2 + C} \Longrightarrow$ **Feedback: Check your integration.**

   (b) $y = \pm\sqrt{4t + C} \Longrightarrow$ **Feedback: Do you want a square root or some other type of root? Check your derivative.**

   (c) $y = \pm\sqrt[4]{4\ln(t) + C} \Longrightarrow$ **Correct**

   (d) $y = \pm\sqrt[4]{4\ln(t)} \Longrightarrow$ **Feedback: You are missing an important piece.**

6. Solve the following differential equation: $\frac{dy}{dt} = \frac{5t^3}{t(t-2)(t+4)}$

   (a) $y = \frac{1}{8}t^2 + \frac{19}{8}(t-4)^2 + C \Longrightarrow$ **Check your integral.**

   (b) $y = \frac{1}{4}\ln(t) + \frac{19}{4}\ln(t-4) + C \Longrightarrow$ **Correct**

   (c) $y = -\frac{4}{5}\ln(t) - \frac{1}{5}\ln(t-4) + C \Longrightarrow$ **Feedback: Check your partial fraction decomposition.**

   (d) $y = 5\ln(t) + C \Longrightarrow$ **Feedback: Check your math on the partial fraction decomposition.**

## DD2 Group Quiz

1. Match the terms with the definitions. (This is a matching question. The definitions are next to the term that it goes with. In the quiz, don't let them match up. Thank you!)

   (a) Equilibrium Solution $\Longrightarrow$ Constant solutions to a differential equation.

   (b) Unstable Solution $\Longrightarrow$ Solutions that move away from the equilibrium solution.

   (c) Stable Solution $\Longrightarrow$ Solutions that move toward the equilibrium solution.

   (d) First Order Linear Differential Equation $\Longrightarrow$ A differential equation in the form $a_1(t)y' + a_0(t)y = b(t)$.

2. Solve: $\frac{dy}{dt} = \frac{5t^3}{t(t-2)(t+4)}$

   (a) $y = 5t - \ln(t-2) - \ln(t+4) + C$

   (b) $y = 5t + \frac{10}{3}\ln(t-2) - \frac{40}{3}\ln(t+4) + C$ $\Longrightarrow$ **Correct**

   (c) $y = \ln(t-2) + \ln(t+4) + C$

   (d) $y = 5t + \frac{10}{3}\ln(t-2) + \frac{40}{3}\ln(t+4)$

3. Where do the equilibrium points exist for the differential equation: $\frac{dy}{dt} = 1 - y^2$.

   (a) $y = \pm 1$ $\Longrightarrow$ **Correct**

   (b) $y = 1$

   (c) $y = -1$

   (d) $y = \pm 2$

4. Use dfield to analyze the equilibrium for the previous question.

   (a) At $y = 0$ there is a stable solution while at $y = 1$ is an unstable solution.

   (b) At $y = 1$ there is an unstable solution.

   (c) At $y = 1$ there is a stable solution, while at $y = -1$ is an unstable solution. $\Longrightarrow$ **Correct**

   (d) At $y = 1$ there is an unstable solution, while at $y = -1$ is a stable solution.

5. Where do the equilibrium points exist for the differential equation: $\frac{dy}{dt} = -y$?

   (a) $y = 3$

   (b) $y = 1$

   (c) $y = -1$

   (d) $y = 0$ $\Longrightarrow$ **Correct**

6. Use dfield to analyze the equilibrium for the previous question.

   (a) At $y = 0$ there is an unstable solution.

   (b) At $y = 0$ there is a stable solution. $\Longrightarrow$ **Correct**

   (c) At $y = 1$ there is an unstable solution.

   (d) At $y = 0$ there is no solution.

7. Solve the following differential equation: $ty' + 2y = \frac{1}{t^3}$.

   (a) $y = -\frac{1}{t} - \frac{C}{t}$

   (b) $y = \frac{1}{t^2} + C$

   (c) $y = -\frac{1}{t^3} + C$

   (d) $y = -\frac{1}{t^3} + \frac{C}{t^2}$ $\Longrightarrow$ **Correct**

## Weekly Quiz

1. Choose the most accurate model for the following scenario: The rate of change in the temperature $T$ of a cup of hot chocolate at time $t$ is proportional to the difference between the temperature $M$ of air at time $t$ and the temperature of the hot Chocolate at time $t$.

   (a) $\frac{dT}{dt} = k(M - T) \implies$ **Correct.**

   (b) $\frac{dT}{dt} = -k(M - T)$

   (c) $\frac{dT}{dt} = k^2(M - T)$

   (d) $\frac{dT}{dt} = k(M + T)$

2. Choose the most accurate model for the following scenario: The change in population P of the U.S. over the change in time t is proportional to the time t divided by the 1 less than the population P.

   (a) $\frac{dP}{dt} = P(1 - \frac{1}{P})$

   (b) $\frac{dP}{dt} = kt(1 - P)$

   (c) $\frac{dP}{dt} = k\frac{t}{P-1} \implies$ **Correct.**

   (d) $\frac{dP}{dt} = k\frac{t}{P} - 1$

3. Find and classify the equilibria for the following differential equation: $\frac{dy}{dt} = (y^2 - 1)(y - 3)^2$.

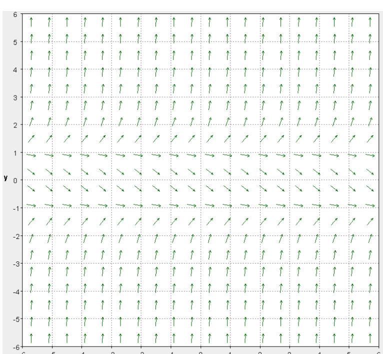
   (a) $y = \pm 1, 3$ where all are unstable.

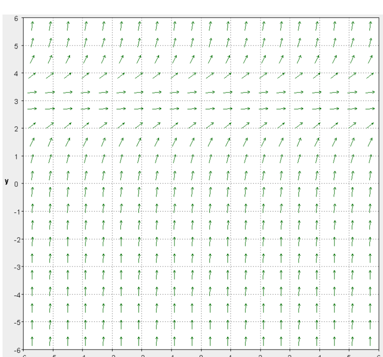
   (b) $y = \pm 1, 3$ where -1 is stable, while 1 and 3 are unstable. $\implies$ **Correct**

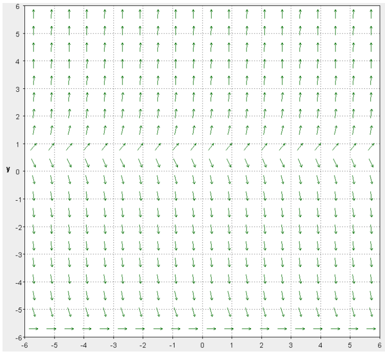
   (c) $y = 1, 3$ where 1 and 3 are unstable.

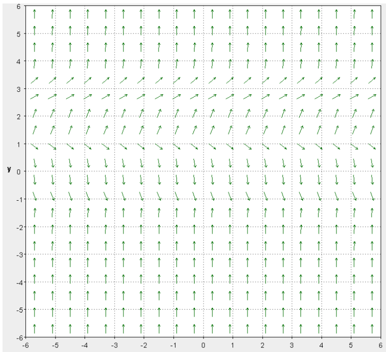
   (d) $y = 1, 3$ where 1 is stable and 3 is unstable.

4. Choose the correct Direction field for the differential equation above: $\frac{dy}{dt} = (y^2 - 1)(y - 3)^2$.

   (a) 

   (b) 

(c) 

(d)  $\Longrightarrow$ **Correct**

5. Solve the following differential equation: $t\frac{dy}{dt} = \frac{2}{3y^4}$.

    (a) $y = \sqrt[5]{10\ln(t) + C}$

    (b) $y = \sqrt{\frac{10}{3}\ln(t)}$

    (c) $y = \sqrt[5]{\frac{10}{3}\ln(t) + C}$ $\Longrightarrow$ **Correct**

    (d) $y = \sqrt{\frac{2}{3}\ln(t) + C}$

6. Solve the following differential equation: $\frac{dy}{dt} = \frac{2t-1}{t(t-2)}$

    (a) $y = \frac{1}{2}\ln(t) + \frac{3}{2}\ln(t-2) + C$ $\Longrightarrow$ **Correct**

    (b) $y = \frac{3}{2}\ln(t-2) + C$

    (c) $y = \ln(t) + \ln(t-2) + C$

    (d) $y = \frac{1}{2}\ln(t) + \frac{3}{2} + C$

7. Solve the following differential equation: $y' + \cos(t)y = 0$.

    (a) $y = 0$

    (b) $y = e^{-\cos(t)}$

    (c) $y = Ce^{-\sin(t)}$ $\Longrightarrow$ **Correct**

    (d) $y = Ce^{\cos(t)}$

8. Solve the following differential equation: $\frac{dy}{dt} = \frac{2yt}{1+t^2}$.

    (a) $y = C(t^2+1)$ $\Longrightarrow$ **Correct**

    (b) $y = \ln(C(t^2+1))$

    (c) $y = (t^2+1) + C$

    (d) $y = C\frac{(t^2+1)}{2}$

9. (Essay Question)Discuss how to recognize the types of equilibrium solutions, stable or unstable, when you visually see the Direction Field.

(a) Answer: The answer will vary. Students need to describe that a stable solution will be drawn toward the equilibrium solution, while the unstable solution will be pushed away or one side will pull while the other will push.

10. Solve the following differential equation on paper and upload or type your steps: $y' - y - e^{3t} = 0$.

   (a) The answer is $y = e^t\left(\frac{e^{2t}}{2} + C\right)$. The work is as follows: let $p(t) = -1$. Then $P(t) = -t$. Using the integration factor, we see that

   $$y = e^{-(-t)} \int e^{-t}(-e^{3t})dt$$

   Solving, we get the above answer. Make sure their steps are logical and can easily be followed.