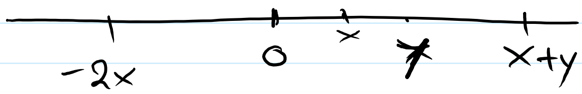
Lab 3 (new)

Πέμπτη, 1 Μαρτίου 2018 8:41 πμ

Νόρμες διανυσμάτων

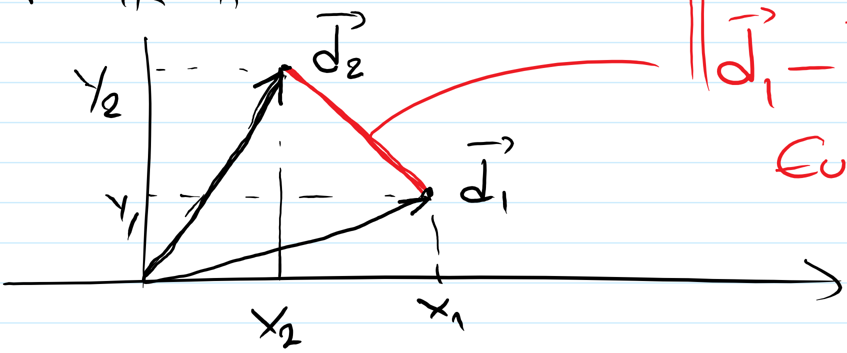
"Απόσταση" στο $\mathbb{R}^n$

$n=1$, $\mathbb{R}^n$:



$a, b \in \mathbb{R} \longrightarrow |a-b|$ = απόσταση των a, b.

$n=2$, $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R}$



$\|\vec{d_1} - \vec{d_2}\|_2$

Ευκλείδια νόρμα

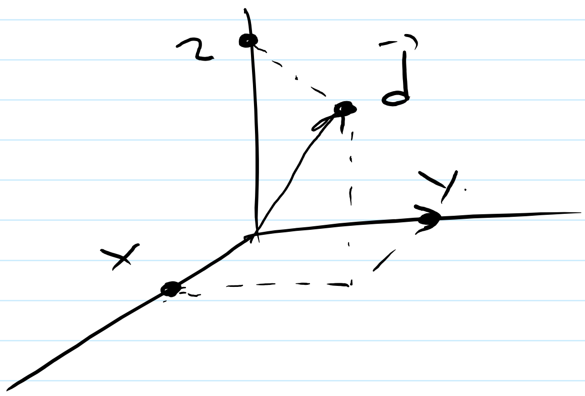
απόσταση

μέτρο

$$\|\vec{v}\|_2 = \sqrt{v_1^2 + v_2^2}$$

$$\vec{v} = (v_1, v_2)$$

$n=3$ $\mathbb{R}^3 = \mathbb{R}\times\mathbb{R}\times\mathbb{R}$



$$\|\vec{d}\|_3 = \sqrt{x^2+y^2+z^2}$$

$n=100$, $\mathbb{R}^{100}$, $\vec{v} = (v_1, v_2, \ldots, v_{100})$

$n = 100$, $\mathbb{R}^{100}$, $v = (v_1, v_2, \dots, v_{100})$

$$\|\vec{v}\|_{100} = \sqrt{\sum_{i=1}^{100} v_i^2}$$

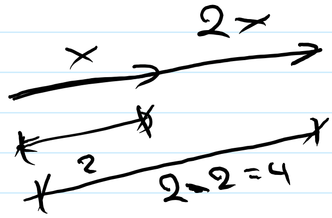
$$\|\cdot\| : X \longrightarrow \mathbb{R}_+ \qquad (\|x\| , x \in X)$$

$X = \mathbb{R}^2, \mathbb{R}^3$

$\mathbb{R}^n$

Ιδιότητες της νόρμας

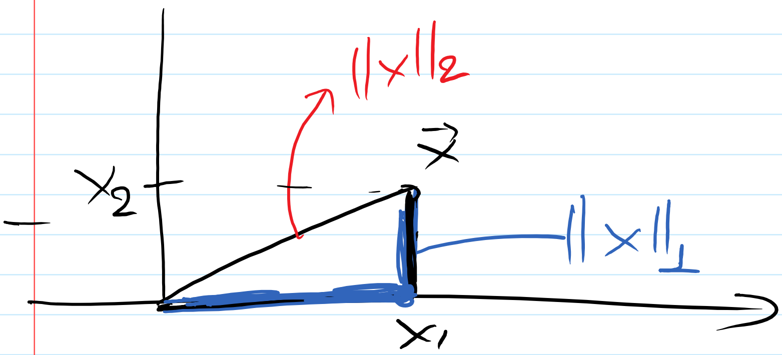
$$\begin{cases} \|x\| \geq 0 \quad \forall x \in X \\ \|x\| = 0 \iff x = 0 \\ \|\lambda x\| = |\lambda| \cdot \|x\| \\ \|x+y\| \leq \|x\| + \|y\| \end{cases}$$

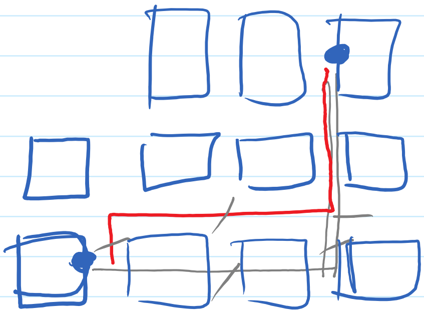


$$\|\cdot\|_1 : \mathbb{R}^n \longrightarrow \mathbb{R}_+$$

$n = 2$, $x = (x_1, x_2) \Rightarrow \|x\|_1 = |x_1| + |x_2|$

- $\|x\|_1 \geq 0 \quad \forall x \in \mathbb{R}^2$
- $\|x\|_1 = 0 \iff |x_1| + |x_2| = 0 \iff x_1 = x_2 = 0 \iff x = 0$
- $\|\lambda x\|_1 = |\lambda x_1| + |\lambda x_2| = |\lambda| \cdot |x_1| + |\lambda| |x_2| = |\lambda| \left(|x_1| + |x_2| \right) = |\lambda| \cdot \|x\|_1$
- $\|x+y\|_1 = |x_1 + y_1| + |x_2 + y_2| \leq |x_1| + |y_1| + |x_2| + |y_2| = \left(|x_1| + |x_2|\right) + \left(|y_1| + |y_2|\right) = \|x\|_1 + \|y\|_1$





$$\|\cdot\|_\infty : \mathbb{R}^n \longrightarrow \mathbb{R}_+$$

$$x\in\mathbb{R}^n \quad \|x\|_\infty = \max_{1\le i\le n} |x_i|$$

$$n=2, \quad x=(-3,2) \longrightarrow \|x\|_\infty = 3$$

$$\|x\|_1 = 5$$

$$\|x\|_2 = \sqrt{13}.$$

Άσκηση.

Αποδείξτε ότι η $\|\cdot\|_\infty$ είναι νόρμα.