Effect of Street Geometry on the Vehicular Traffic Throughput and its Impact on Smart Cities Mapping Design

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Abstract—Traffic congestion, the main constituent of vehicular extrinsic flow resistance, has been an important factor in the design of routing algorithms. There are important intrinsic factors that can also aid these algorithms. This study proposes a new vehicular intrinsic flow resistance metric referred as sinuousness. To obtain this metric several scenarios are designed. Each scenario has a unique topology with varying amount of curves, curve-radii lengths, total path lengths, and other features. Simulated vehicles flow through these scenarios and velocity data is gathered. Simulations show that the velocity, and hence the sinuousness, depends on several street aspects, such as distance between curves, curve radii, and aggregated arc circumference. A sinuousness expression is derived. Results show that the proposed metric of sinuousness is proportional and in agreement with the inverse of the average velocity of the vehicles.

Index Terms—Congestion; Routing; Smart City; Sinuousness; Traffic; Urban Traffic Model; Vehicular Safety

I. Introduction

The geometric design of streets is an important aspect in the deploy of Smart Cities. It is concerned with various aspects, such as: positioning of physical elements (traffic lights, signs, speed bump,etc.), reducing costs, reducing environmental damage, livability, safety, accessibility, and flow. The aspect that this work focuses on is flow. Apart from traffic congestion, which is an extrinsic quality of street design, the most important intrinsic factor in vehicular flow is the street alignment. Streets are a series of horizontal tangents connected to arc-like curves. A highly flexuous street can significantly hinder the traffic flow.

Vehicular routing is generally computed based on an ideal mapping, where intersections are seeing as nodes and streets are seen as node connections. There is little or no physical information about the streets on the models. The flow or street throughput is learned from observation (or rather a resistance metric such as congestion). This creates a design conundrum, if anything changes (e.g. a street direction, a road block, etc.) this learned model will likely fail to predict the outcome of the altered scenario. To get more accurate estimations of the flow effect under changing scenarios, it is necessary to study all the intrinsic factors that contribute to the flow impairment or improvement. One important metric that affects the traffic flow is what will be referred to as sinuousness. Sinuousness is an indicator of the street curvilinearity. Though, there are more intrinsic factors, such as roughness, crosswalks, etc., in this study the sinuousness effect is single out.

To put this in perspective, the mathematical contribution is described in more detail. To compute the route between two points many mapping tools use Dijkstra's shortest path algorithm. As just mentioned, the nodes are equivalent to the intersections and the streets are the connections or paths between nodes. Just as in data routing, the connections (i.e. streets) can be unidirectional or bidirectional. Each street is assigned a weight, for example a congestion-based time metric, and the best path is the one that minimizes the time to reach the destination. Some algorithms attempt to minimize distance, nevertheless, the same method can be used using a distance metric. If time is used, the metric can have two main components: an intrinsic and extrinsic parameter. Each parameter could also have a confidence ratio, which is initially biased toward the intrinsic parameter and as the amount of observation increases it weighs more the extrinsic factors. If the topology is modified, the confidence value resets and again weighs more the intrinsic factors. The flow resistance can be expressed as: $R_F = R_e\, C_e + R_i\,(1 - C_e)$, where R_e is the extrinsic flow resistance (i.e. congestion), R_i is the intrinsic flow resistance (i.e. sinuousness), and C_e is the extrinsic confidence weight. This study focuses on determining the sinuousness metric and how this can benefit the street mapping design of Smart Cities.

To efficiently present the information in an organized manner, the paper outline is as follows. Section II discusses studies related to city traffic modelling, vehicular congestion avoidance, and other similar topics beneficial to the context of this work. Section III includes mathematical expressions useful for the understanding of the core work, needed for self-containment of the document. Section IV presents the techniques used to gather the data from the simulated scenarios. Section V explains the intrinsic resistance flow metric, which is the core work of this study. Section VI describes the initial conditions and other simulation parameters. Results are found in Section VII. Finally, Section VIII summarizes the conclusions of this work.

II. Related Work

The performance of a vehicle driving in flexuous conditions depends on a variety of factors like: the radius, vehicle velocity, acceleration, friction characteristics, angle of curvature, neighboring vehicles, etc. That is the reason the research of traffic management and road safety have many objectives, some propose to study the road geometry effect on the velocity and, consequently, reduction/increment of the driving velocity [1], [2], others propose to analyze the drive behavior to develop operating velocity prediction models [2], [3] or guidelines for the coordination of drivers [4]–[6]. In others cases, which share a common trait with the proposed work, these are based on detailed dynamical model alongside the implementation of a cautiousness feature in the car-following behavior [7].

III. Background

A. Maximum Curve Velocity

The curves of the path have an inherent maximum velocity that is determined by the coefficient of friction μ, the radius curve R, and the gravitational acceleration g (9.81 m/s^2).

$$V_{max} = \sqrt{\mu\, g\, R} \tag{1}$$

This is the main factor of the intrinsic flow resistance. The static coefficient of friction for rubber on asphalt is 0.9. The coefficient of friction is unitless.

B. Vehicle Behavior Algorithm

To avoid vehicle collisions and to do so gradually (cautiously), a Car-following Mechanism is implemented considering a safe distance. Let safe distance of vehicle j at time n ($Ds_j[n]$) be the length covered by the vehicle j in t_{safe} time. This safe time (t_{safe}) is a lag time generated to improve safety. Safe distance varies according to the vehicle's velocity and length and is calculated at every n as:

$$Ds_j[n] = t_{safe} V_j[n-1] + L_v j \tag{2}$$

where $L_v j$ is the length of the vehicle j in meters. The length of the vehicle is required since the reference point to compute the distance between vehicles is their center (i.e. front bumper to rear bumper plus car length) . Then, the safe distance threshold position of vehicle j is given by:

$$Xs_j[n] = X_{j+1}[n] - Ds_j[n] \tag{3}$$

A collision avoidance mechanism is initialized only if vehicle j crosses the safe distance threshold position j <u>and</u> the velocity of the vehicle j is larger than the velocity of vehicle $j+1$.This mechanism execute as follows: first, a estimated coalition point Xc_j is calculated based on the position and velocity of both vehicles as:

$$Xc_j[n] = \frac{X_{j+1}[n-1]V_j[n-1] - X_j[n-1]V_{j+1}[n-1]}{V_j[n-1] - V_{j+1}[n-1]} \tag{4}$$

Then, to avoid the collision, it is necessary to decelerate the vehicle j. The amount of deceleration is proportional to a deceleration factor fd, which is computed as follows:

$$fd_j[n] = \frac{Xc_j[n] - X_j[n]}{Xc_j[n] - Xs_j[n]} \tag{5}$$

To apply the deceleration factor, the position of the vehicle j at time n is recomputed:

$$X_j[n] = X_j[n-1] + fd_j[n](X_j[n] - X_j[n-1]) \tag{6}$$

Finally, being consistent with the kinematic equations, setting this new position requires to recompute acceleration of j at time $n-1$ as:

$$a_j[n-1] = 2(X_j[n] - X_j[n-1] - V_j[n-1]) \tag{7}$$

IV. Path Parametrization

The kinetic behavior of the vehicles is based on the model described in [8]. This model is one-dimentional, hence, some modifications are needed to represent the 1D data in a 2D map. For this path parametrization is used. Every path that has curves can be represented by interleaved lines and circular arcs. [Note: It was noticed, during this study, that curved streets always follow a circular pattern, not just elliptical. It is believed this is because it is easier to assess the necessary velocity needed to take the turn (the safe velocity is constant throughout the turn). If the street is elliptical it is harder to assess the velocity needed to safely drive through it as the safe velocity varies throughout the curve.] This implies that the path can be simplified using the circle and straight line parametric equations. The computations are straight forward and basic, for the reproducibility of the results we include a brief description of the steps used to compute the street path parametric equations. For clarity and redundancy we mention that there is one equation per dimension when parameterizing. The position parameters x and y will depend on the distance travelled. Each paths begins and ends with a straight line and interleaved portions of circle sections (arcs) and straight lines. All the straight lines are tangential to the circle sections that connect them. See Figure 1 for an example of two straight lines connected by an arc. Our path-building code first computes the latitudinal slope for the straight line that crosses points 1 and 2, which is called line 2. Similarly, for points 3 and 4, the line is called line 3. Points 2 and 3 are strategic points for the curve computation and naming the lines based on these values simplifies notation in the formulas, as seen further ahead in this section. The latitudinal slopes are then $m_2 = \frac{y_2-y_1}{x_2-x_1}$ and $m_3 = \frac{y_4-y_3}{x_4-x_3}$. It follows to compute the y-intercepts using the expressions $b_2 = y_2 - m_2 x_2$ and $b_3 = y_3 - m_3 x_3$. The intersection between the two lines is computed, this is needed because point 2 or 3 will be rewritten such that both are equally distant from the intersecting point (but still are contained in their respective lines). These points will be the intersections of the circle tangents and lines 2 and 3. If points 2 and 3 where not equally distant from the intersecting point, the curve would result in an elliptical shape. To compute the intersecting points of the line we use the expressions:

$$I_x = \frac{b_3 - b_2}{m_2 - m_3} \quad \text{and} \quad I_y = m_2 I_x + b_2 \tag{8}$$

The points should be placed the closest possible to the circular arc, without placing it inside. The code will then choose the closest of the two points, using:

$$\check{p} = \underset{i \in [2,3]}{\arg\min} \left((I_x - x_i)^2 + (I_y - y_i)^2 \right) \tag{9}$$

Once the minimum distance is established, it is necessary to compute the new position for point 2 or 3, whichever is farthest from the intersection point. The farthest point can be expressed, similarly to (9), as: $\hat{p} = \arg\max_{i \in [2,3]} \left((I_x - x_i)^2 + (I_y - y_i)^2 \right)$. Then the minimum distance is computed as:

$$D = (I_x - x_{\check{p}})^2 + (I_y - y_{\check{p}})^2 \tag{10}$$

This yields the new position of point p_{max}, which has coordinates:

$$x_{\hat{p}} = sign(x_{\hat{p}} - I_x) \sqrt{\frac{D^2}{m_{\hat{p}}^2 + 1}} + I_x \tag{11}$$

$$y_{\hat{p}} = sign(m_{\hat{p}}) sign(y_{\hat{p}} - I_y) m_{\hat{p}} \sqrt{\frac{D^2}{m_{\hat{p}}^2 + 1}} + I_y \tag{12}$$

At this stage we have set up the points to obtain the circle equation. To find the center of the circle it is necessary to first compute the equation of two lines: a. the lines that is orthogonal to line 2 and intersects point 2, which is referred as line $\bar{2}$; and, b. the line that is orthogonal to line 3 and intersects point 3, referred as line $\bar{3}$. The latitudinal slopes are simply $\bar{m_2} = -1/m_2$ and $\bar{m_3} = -1/m_3$. Then the y-intercepts are $\bar{b_2} = y_2 - \bar{m_2}\, x_2$ and $\bar{b_3} = y_2 - \bar{m_3}\, x_3$. The parametric equations are in terms of the travelled distance d. The curve's arc center is then computed as follows:

$$C_x = \frac{b_3 - b_2}{m_2 - m_3} \quad \text{and} \quad C_y = m_2\, I_x + b_2 \tag{13}$$

The curve radius is then:

$$R = \sqrt{(C_x - x_{\check{p}})^2 + (I_y - y_{\check{p}})^2} \tag{14}$$

The resulting parametric equations for the straight line are:

$$d_{line} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{15}$$

$$x_{path}(d) = \frac{x_2 - x_1}{d_{line}} d + x_1 \quad \text{for} \quad d < d_{line} \tag{16}$$

$$y_{path}(d) = \frac{y_2 - y_1}{d_{line}} d + y_1 \quad \text{for} \quad d < d_{line} \tag{17}$$

To compute the parametric equations for the arc, it is necessary to compute first the direction, clockwise (CW) or counter-clockwise (CCW), and the angle ranges. To determine if the vehicle direction is turning CW the following condition must be true $sign(x_1 - x_2) = sign(C_y - y_2)$ and $sign(y_1 - y_2) = sign(C_x - x_2)$. If the condition is not true it is not necessarily CCW, but the cases that are not CCW are impossible to occur, so for simplicity the else condition then corresponds to the CCW case. The angle ranges are computed in the following way:

For the CW case: If the $\phi_{end} > \phi_{start}$, then $\phi_{start} = \phi_{start} + 2\pi$; and, for the CCW case: If the $\phi_{start} > \phi_{end}$, then $\phi_{end} = \phi_{end} + 2\pi$. For simplification, the variable cw takes the value of 1, when the turn is CW and -1 when it is CCW. Then the distance can be expressed as:

$$d_{arc} = cw(\phi_{start} - \phi_{end})R \tag{18}$$

And the parametric equations are then:

$$x_{path} = R\,cos(\phi_{start} + \frac{cw}{R} d) + C_x \quad \text{for} \quad d < d_{line} + d_{arc} \tag{19}$$

$$y_{path} = R\,sin(\phi_{start} + \frac{cw}{R} d) + C_y \quad \text{for} \quad d < d_{line} + d_{arc} \tag{20}$$

Fig. 1: Map used as an example to show path parametrization

V. Curve Deceleration Mechanism

The curve deceleration is the main factor for the inherent flow resistance. To determine the sinuousness metric it is essential that the kinetic model of the vehicle takes in account the maximum curve velocities. If the vehicle travels at a velocity greater than the maximum velocity of the curve, it will skid. To avoid having abrupt velocities changes when the vehicle enters the curve, a safety distance is established. Once the vehicle enters the safety distance it will slow down gradually until the velocity of the vehicle matches the maximum velocity of the curve times a safety margin factor, which avoids having the model compute velocities near the maximum, which is dangerous in practice.

The safety time is computed based on time, which implies that if the vehicle is travelling fast it should look "father ahead" than a vehicle that is travelling slower. The curve safety margin time is empirically determined to be 10 s. If the curve is at a distance of $v_{j,n}\, t_C$, where $v_{j,n}$ is the velocity of vehicle j at time n and t_C is the curve safety margin time (i.e., curve awareness time frame, vehicle does not reach to curves father than t_C away considering the current velocity). Defining $D_S(c)$ and $D_E(c)$, as the start and end distance of the curve c (resp.), $V_{max}(c)$ as the physical maximum velocity (if surpassed, the vehicle skids), and f_{Vmax} as the safe maximum velocity factor (fraction of V_max defined for this study); then, the kinematics of the vehicle are described by the following expressions: [Note: The time resolution is $dt = 1$ for the expressions below]

$$\Delta t = 2\frac{D_S(c) - x_{j,n}}{v_{j,n} + V_{max}(c) f_{Vmax}} \tag{21}$$

$$a_{j,n} = \frac{V_{max}(c) f_{Vmax} - v_{j,n}}{\Delta t} \tag{22}$$

If the acceleration $a_{j,n}$ is lower than zero, the vehicle is required to slow down, then:

$$x_{j,n+1} = x_{j,n} + v_{j,n} + \frac{1}{2} a_{j,n} \tag{23}$$

If the kinematics involved to slow down, due to the approach of a leading vehicle (travelling slower than the reference vehicle), are more restrictive then this case takes precedence.

VI. Simulation Setup

To study the effect of sinuousness it is necessary to simulate various street topologies. Each topology has a unique set of curves, street lengths, and structure in general. A parametric equation is obtained to trace the path, as described in Section IV. A set of N vehicles travel through the path exposed to the kinematics, vehicle behavior, and curve deceleration explained in previous sections (III-B, and V). The initial conditions and other simulation parameters are found in Table I.

TABLE I: Summary of the simulation variables

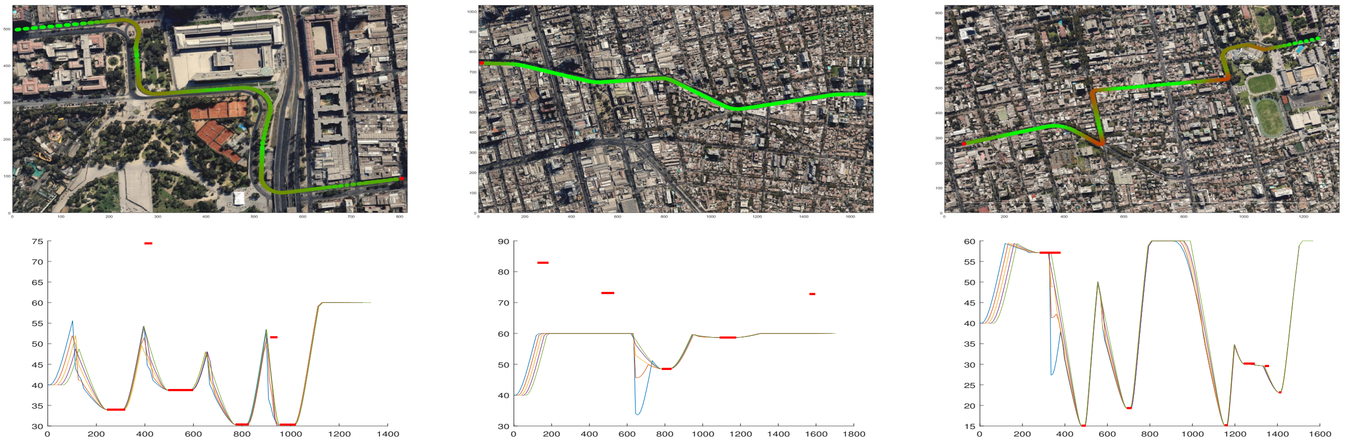
Number of Vehicles N	5
Vehicle length $L_v[m]$	4
Initial distance between vehicles $[m]$	10
vehicle safety margin time $t_S[s]$	4
curve safety margin time $t_C[s]$	10
Initial velocity of vehicles $[km/h], [m/s]$	40, 11.11
Maximum velocity $[km/h], [m/s]$	60, 16.67
Minimum velocity $[km/h], [m/s]$	10, 2.78
Initial acceleration $[m/s^2]$	0
Safe maximum velocity factor f_{Vmax}	0.5

VII. Results

To determine the sinuousness metric, it is necessary to observe the intrinsic qualities that affect in a greater degree the flow of vehicles. Figure 2 shows the different scenarios studied in this work. For each case there is a two-dimentional visual aid of a street map with a heatmap of the velocities of the simulated vehicles flowing through the selected path. Under the map there is a distance versus velocity graph, which has red lines that represent the physical maximum velocity (times a safe maximum velocity factor as described in Section V) as given by the physics of curves, discussed in Section III-A. The width of the lines represents the distance of the curves. $E_n\{v_{j,n}\}$ is the mean velocity of vehicle j for all n. $E_j\{\cdot\}$ is the mean considering all vehicles. Therefore, $E_j\{E_n\{v_{j,n}\}\}$ is the mean velocity for all time of all the vehicles, i.e., the overall mean velocity. ς is the sinuousness metric.

For the first scenario it can be observed that the maximum allowable velocity (imposed by city regulations) was not reached by any car except after the last curve. Based on this it is clear that the standard deviation of the distances between the curves should have an impact on the sinuousness metric we wish to design. Observing scenario 4, which has a similar amount of curves, if we remove the curves that do not have a significant impact (i.e. the curves with radius $R > V_M^2/\mu g$, where V_M is the maximum allowed velocity, not V_{max}) there is a large gap between two curves. This will yield a greater standard deviation of the distance between two curves, hence agreeing with the previous assumption. The process of removing the radius with negligible impact is key. If scenario 2 is observed it can be noticed that the curves are relatively evenly spread out, but only one curve meets the $R < V_M^2/\mu g$ criteria. If the others are not discarded the standard deviation of the distance between two curves will not yield the desired result. It should be clarified that the maximum regulated velocity V_M is considered an intrinsic quality of the street.

Another factor that has an impact on the flow of vehicles is the radii of the curves. Observing scenario 6 we notice that all curve radii are small, yielding small V_{max} curve velocities. In contrast, scenario 9 has various curves but all with large radii. None of the curves had an impact on



Scenario 1
$E_j\{E_n\{v_{j,n}\}\} = 10.14\,m/s$
$\varsigma = 2.743\,1/km^2$

Scenario 2
$E_j\{E_n\{v_{j,n}\}\} = 15.56\,m/s$
$\varsigma = 1.038\,1/km^2$

Scenario 3
$E_j\{E_n\{v_{j,n}\}\} = 8.65\,m/s$
$\varsigma = 4.541\,1/km^2$

Scenario 4
$E_j\{E_n\{v_{j,n}\}\} = 12.32\,m/s$
$\varsigma = 1.035\,1/km^2$

Scenario 5
$E_j\{E_n\{v_{j,n}\}\} = 9.53\,m/s$
$\varsigma = 1.311\,1/km^2$

Scenario 6
$E_j\{E_n\{v_{j,n}\}\} = 8.02\,m/s$
$\varsigma = 17.550\,1/km^2$

Scenario 7
$E_j\{E_n\{v_{j,n}\}\} = 9.47\,m/s$
$\varsigma = 3.996\,1/km^2$

Scenario 8
$E_j\{E_n\{v_{j,n}\}\} = 13.98\,m/s$
$\varsigma = 0.397\,1/km^2$

Scenario 9
$E_j\{E_n\{v_{j,n}\}\} = 16.67\,m/s$
$\varsigma = 0.918\,1/km^2$

Fig. 2: Street scenarios used to demonstrate the inherent flow resistance

the velocity of the simulated vehicles. The only factor that had an impact in this scenario was the imposed maximum velocity. There is one last element that has an effect on the intrinsic vehicular traffic flow and this is the aggregated curve distance (or aggregated arc circumsference) relative to the straight distances. For example, scenarios 3, 6, and 7 have a large amount of curves in relation to the straight portions. While scenarios 8 and 9, for example, has many straight portions. This will yield a small curve-distance to straight-distance ratio. This factor can be combined with the previous factor, in the context of creating a metric, as we can have a weighted average of the radii with respect to the total length of the path where the weights are the distances of the radii. This takes in account for the radii size and the curve distance.

The sinuousness metric ς is then considered to be:

$$\varsigma = \frac{1}{\sigma_{\Delta X_c}} \frac{D_T}{\sum_{c=1}^{C} R_c \, D_c} \tag{24}$$

where $\sigma_{\Delta X_c}$ is the standard deviation of the distance between two curves, R_c is the radius of curve c, D_c is the circumference of the arc that forms curve c, and D_T is the total path length. Observing Figure 3, it is seen that the proposed sinuousness metric is inversely proportional to average velocity of the vehicles. Since $\varsigma \propto \frac{1}{E\{v\}}$, the data points show agreement with this relation. Because the metric is the inverse of the standard deviation of the distances between curves [m] times the inverse of the weighted radii [m], the final metric is in $1/m^2$ (or similarly $1/km^2$).

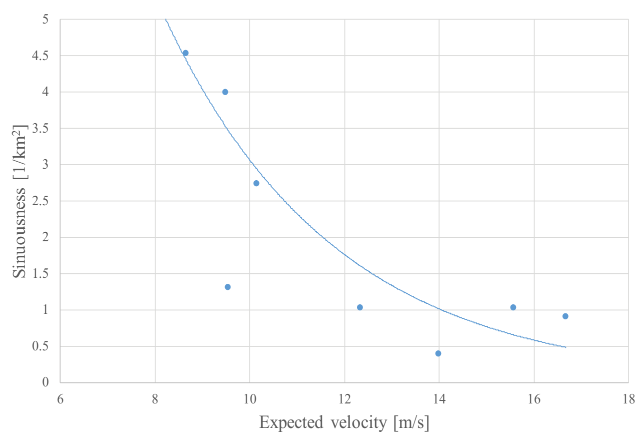


Fig. 3: Sinuousness exponential regression versus expected velocity

VIII. Conclusions

Routing protocols can benefit from having intrinsic flow information as well as extrinsic. The steady-state (long period) extrinsic flow resistance information (i.e. congestion information) is accurate in static conditions. If there is a change in street topology or directionality, the effects cannot be accurately estimated/predicted using only extrinsic information. This is because the main constituent of extrinsic vehicular flow resistance is traffic congestion, which changes with changing conditions. In this case, the intrinsic flow resistance model can provide more insight. The natural vehicular traffic flow is affected by two main reasons: the distance between curves, the radii of the curves, and the aggregated distance of the curves. Having curves evenly distributed along a path does not allow the driver to reach maximum velocities, if the curves are stacked together connected by long straight paths the flow will increase. This last statement assumes the curves are absolutely necessary, as the ideal case is to eliminate the curves, which is not always possible. Another factor is the radii of the curves. If the curves have large radii, the restraint will be the regulatory maximum velocity, not the street geometry. In contrast, if the radii are short, more precisely $R < \frac{V_M^2}{\mu g}$, then it will restrict the traffic flow to certain degree. Finally, the remaining factor is the aggregated curve distance. The longer vehicles remain inside the curves the innate velocity of the vehicular flow will decrease. In summary, sinuousness — or inherent flow resistance metric — is proportional and in agreement with the inverse of the velocity data.

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