AERO483/ENGR6471 Integration of Avionics Systems

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Homework 1

Problem 1 (Mortgage Model):
Assume that you want to buy a house and need a mortgage. The price of the house is $375,000 and you will ask the bank for a mortgage of 80% of the cost of the house. The interest rate of the mortgage is 2% per year. Answer the following questions:

1. Write the difference equations for this system based on monthly payments.
2. Compute how much the monthly payment should be assuming that it is constant and that you would like to pay out the mortgage in 10 years.
3. Repeat item 2. for: 20 years, 25 years, and 30 years mortgage.
4. Plot the interest paid to the bank as a function of the number of years (1-30 years).
5. Plot the constant monthly payment value as a function of the number of years it takes for you to pay out the mortgage.
6. What can you conclude ?

Problem 2 (Covid-19 Epidemic Model):
Pick your favorite country/state/province to study the covid-19 epidemic (you cannot pick the province of Quebec because it was already done in class). You must tell me your choice by email so that I make sure that each student has a different set of data to study. For the country/state/province you picked collect the available data on covid-19 for the last two weeks of December 2022 and the first three weeks of January 2023. You will also need either the number of tested people (better option) or the total population of the country. As we did in class use a model with three classes: susceptible, infected, and recovered. Assume that once a person is infected it stays infected for two weeks. After these two weeks the person is recovered and will not be susceptible again for an entire month. The number of recovered people for a given day will thus be the total number of cases two weeks prior to that day. The susceptible people will be the total number of people tested (or the total population) minus the number of infected people minus the number of recovered. You can find the data you need either on the website of the Health Institute of the Country/State/Province or o
n the World Health Organization (WHO) website. Answer the following questions:

1. Using least squares fit a line to the log of the total number of cases.
2. Find the *A* matrix for the epidemic S-I compartmental model that was discussed in class. What are the values of the parameters beta and gamma discussed in class ?
3. Compare the predictions for the total number of cases in the last two weeks of January from both models that you found in 1. and 2. What can you conclude ?

Problem 3 (Kinematics of Rotation at Constant Angular Velocity as Linear Model)
The kinematics of rotational motion of a point in 3-space with respect to the origin at a constant angular velocity ω is described by

$$\dot{x} = \omega \times x$$

Answer the following questions:

1. Determine the matrix A so that you can express the kinematics in the form $\dot{x} = Ax$
2. Compute e^{At}

Problem 4 (Zero-order-hold Equivalent Model)
Find A, B for the zero-order hold equivalent of the linear continuous-time system

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$

for the following cases:

1. $A_c = \begin{bmatrix} 0 & 1 \\ -g & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$
2. $A_c = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}, \quad B_c = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$
3. $A_c = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}, \quad B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$
4. $A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -b \\ 0 & c & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$