AERO483/ENGR6461 – Homework 2

Problem 1

For the system in figure 1, where all coordinates are in kilometers, the satellites A, B, and C have pseudorange measurements of:

$$\rho_A = 5Km$$
$$\rho_B = 7Km$$
$$\rho_C = 6Km$$

1. Using the linearized GPS equations, compute the position of the user assuming that initially the user is 4Km away from the origin on the positive y-axis.
2. For the same conditions as in item 1. compute the regularized least squares solution with $L = 2I, S = 10I$. Compare the solution to the one obtained in 1. How different would be the affine minimum variance estimator solution?
3. Assuming a GPS measurement covariance matrix of $R = 100I$ compute the affine minimum variance estimate of the user's position if the position of the user is a random vector with a-priori mean and covariance matrix given by $\delta\bar{r} = \begin{bmatrix} 0 & 4 & 0 \end{bmatrix}^T, E\left[\left(\delta r - \delta\bar{r}\right)\left(\delta r - \delta\bar{r}\right)^T\right] = 100I$

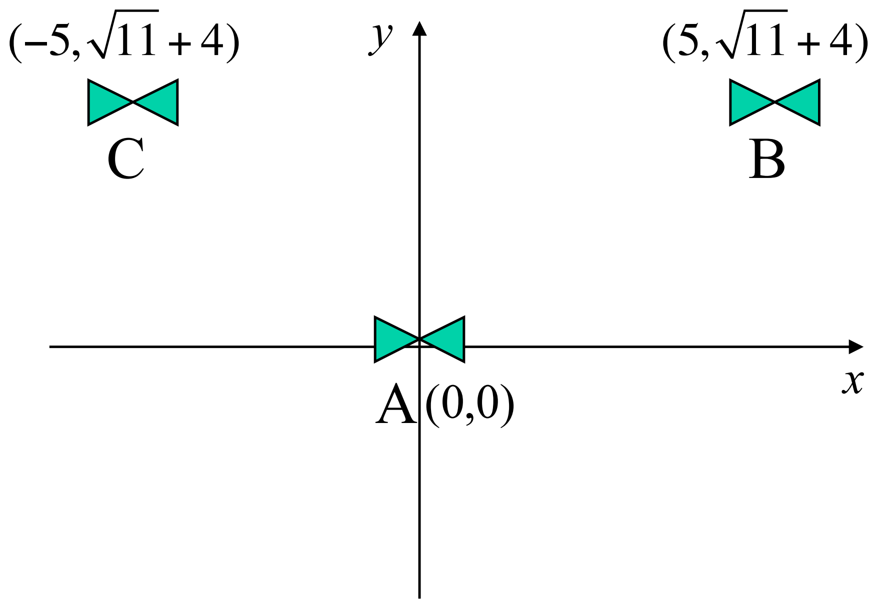

Figure 1 - Problem 1

Problem 2

Consider an INS for a navigation platform on top of a vehicle flying above a curved Earth as depicted in figure 2. Neglect the rotational motion of the Earth. Attach an inertial reference frame to Earth and a body reference frame to the vehicle as shown in the figure. Assume that x represents the distance travelled by the vehicle, v represents the speed and a represents the tangential acceleration. Define $\psi = \phi - \theta$. Let r be the radius of the Earth and let h be the constant altitude above the Earth. Answer the following questions:

1. Write an equation for $\dot{x}$, an equation for $\dot{v}$, and an equation for $\dot{\psi} = \dot{\varphi} - \dot{\theta}$
2. Write the mechanization equations for $\dot{\hat{x}}$, $\dot{\hat{v}}$, $\dot{\hat{\varphi}}$ as a function of the measurements f_{meas} and $\dot{\psi}_{meas}$
3. Write the equation for the acceleration a_{xb} along the longitudinal x-body axis
4. Assume that the measurements of the acceleration and gyro have a bias. Assume you have an estimate of both biases. Under these assumptions, write down the equations for the measured quantities f_{meas} and $\dot{\psi}_{meas}$.
5. Define the errors as $\delta x = \hat{x} - x, \delta v = \hat{v} - v, \delta\varphi = \hat{\varphi} - \varphi$ and write down the linearized error equations for each one of these errors.

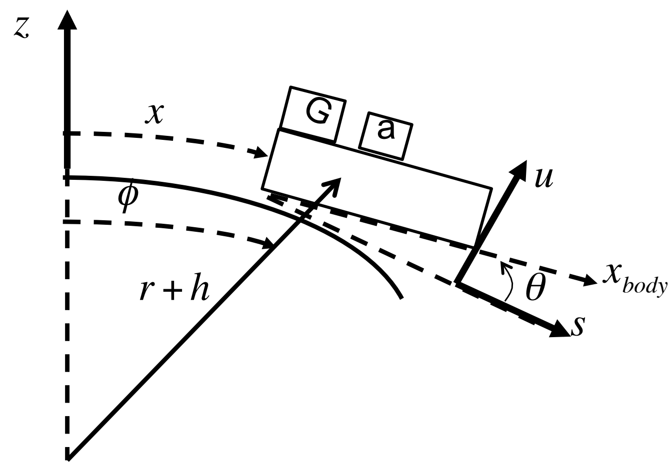

Figure 2 - Problem 2

Problem 3
Let a measurement be described by the equation

$$y = x + b + n = f(a) + b + n$$

where a is a constant vector, b is a zero mean Gaussian random vector with covariance matrix P_b, n is a zero mean Gaussian random vector with covariance matrix P_n, and the two random vectors are uncorrelated with each other. The function f is an invertible and continuously differentiable function. Let $g(y) = f^{-1}(y)$.

1. Compute the mean and the covariance matrix of y.
2. Linearize $g(y)$ around the point $f(a)$ and use this linearization to obtain an expression for a as a function of $g(y)$.
3. Let $\delta a = a - \hat{a} = a - g(y)$. Compute the expected value and the covariance matrix of δa using the expression from item 1.
4. Compute the expected value and covariance matrix of the vector $\delta c = \begin{bmatrix} \delta a^T & \delta b^T \end{bmatrix}^T$.
5. Let $\rho = Aa$. Compute the expected value and the covariance matrix of ρ.