Example #1

Classify the following differential equations as ordinary differential equations or partial differential equations.

A. $(1-y)y' + 2y = e^x$

B. $\frac{d^2y}{dr^2} + \frac{dy}{du} + y = \cos(r+u)$

C. $\frac{d^2R}{dt^2} = \frac{-k}{R^2}$

Answers:

A. $(1-y)y' + 2y = e^x$

Ordinary Differential Equation

Since it involves only one independent variable x

B. $\frac{d^2y}{dr^2} + \frac{dy}{du} + y = \cos(r+u)$

Partial Differential Equation

Because it involves two independent variables r and u.

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C. $\frac{d^2R}{dt^2} = \frac{-k}{R^2}$

Ordinary Differential Equation

Since it involves only one independent variable x

Example #2

Classify the following differential equations according to order and degree.

A. $\frac{d^2y}{dx^2} + siny = 0$

B. $\frac{d^2y}{dr^2} + \left(\frac{dy}{dr}\right)^3 + y = \cos(r+u)$

C. $\left(\frac{d^2y}{dx^2}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Answers:

A. $\frac{d^2y}{dx^2} + siny = 0$

2^{nd} order and 1^{st} degree

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The equation only involves a second derivative whose power is one, thus, the order is second and degree is 2

B. $\frac{d^2y}{dr^2} + \left(\frac{dy}{dr}\right)^3 + y = \cos(r + u)$

$\boxed{2^{nd}\text{ order and }1^{st}\text{ degree}}$

The highest derivative in the equation is second derivative, thus its order is second order. Although the highest power is 3 however that is not the power of the highest derivative. In this case the power of second derivative is 1, therefore its degree is one.

C. $\left(\frac{d^2y}{dx^2}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$

$\boxed{2^{nd}\text{ order and }2^{nd}\text{ degree}}$

The equation must be free from rational exponents. Squaring both sides will transform the equation into,

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$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

This makes the equation order 2 and degree 2.

Example #3

Classify the following differential equations as either linear or non linear.

A. $y' + 2y = e^y + x$

B. $\frac{d^4y}{dx^4} + y^2 = 0$

C. $x^5y''' - x^3y'' + 6y' = 0$

D. $\frac{d^2y}{dr^2} + \frac{dy}{dr} + y = \cos(r + u)$

Answers:

A. $y' + 2y = e^y + x$

Non linear differential equation

The equation involves a transcendental function of the dependent variable y which is the e^y, thus, it is non linear.

B. $\frac{d^4y}{dx^4} + y^2 = 0$

Non linear differential equation

The equation contains a dependent variable with a power more than one which is 2 for y^2, therefore it is non linear.

C. $x^5y''' - x^3y'' + 6y' = 0$

Linear differential equation

All derivatives have a power of one, no transcendental functions involved, and the existence of product is between derivatives and independent variable only. Thus, the equation is linear.

D. $\frac{d^2y}{dr^2} + \frac{dy}{dr} + y = \cos\,(r + u)$

Linear differential equation

All derivatives have a power of one, there is transcendental function involved which is $\cos(r+u)$, however it is a transcendental function involving independent variable r and constant u and lastly the product between derivatives and independent variable does not exist. Thus, the equation is linear.