Classification of Differential Equations

Objectives:

1. To classify the type, order, degree, and linearity of a given differential equation.

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Differential Equations

Definition:

Equations containing the derivatives of one or more unknown functions (or dependent variables) with respect to one or more independent variables are called differential equations.

Classification of Differential Equations

Classification:

1. Type
2. Order
3. Degree
4. Linearity

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Type:

- Ordinary Differential Equations (ODE)

  DE's containing only ordinary derivatives of one or more unknown functions with respect to a single independent variable.

- Partial Differential Equations (PDE)

  DE's involving partial derivative of one or more unknown functions of two or more independent variables.

Examples:

ODE:

$$\frac{dy}{dx} + 5y = e^x$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0$$

PDE:

$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Order:

- The order of differential equation is the order of the highest derivative in the equation.

Examples:

First Order: $\frac{dy}{dx} + 5y = e^x$

Second Order: $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0$

Second Order: $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$

Degree:

- The degree of the differential equation is the same as the exponent of the highest ordered derivative in the given equation after the equation has be rationalized or cleared of fractions with respect to the derivatives.

Examples:

First Degree: $\frac{dy}{dx} + 5y = e^x$

First Degree: $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$

Third Degree: $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{\frac{1}{3}}$

Linearity:

- An ODE of order n is called linear if it may be written in the form.

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Properties of Linear ODEs:

1. Dependent variable (y) and all its derivatives $(y', y'' \ldots, y^n)$ are of the first degree. The power of each term involving y is 1.
2. No transcendental function of the dependent variable (y) is present in any term of the differential equation.
3. No product of the dependent variable (y) and/or any of its derivatives exist.