Example #1

Verify the indicated function is a solution of the differential equation in the interval $(-\infty, \infty)$

$$\frac{dy}{dx} = xy^{\frac{1}{2}}$$
$$y = \frac{1}{16}x^4$$

Solution:

Given the first order differential equation,

$$\frac{dy}{dx} = xy^{\frac{1}{2}}$$

Deriving the solution will yield to,

$$y = \frac{1}{16}x^4$$
$$\frac{dy}{dx} = \frac{1}{4}x^3$$

Substituting and simplifying the derived equation to the differential equation results to,

$$\frac{1}{4}x^3 = x\left[\frac{1}{16}x^4\right]^{\frac{1}{2}}$$

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$$\frac{1}{4}x^3 = x\left[\frac{x^2}{4}\right]$$

$$\frac{1}{4}x^3 = \frac{1}{4}x^3$$

It is a solution

Example #2

Verify the indicated function is a solution of the differential equation in the interval $(-\infty, \infty)$

$$y'' - 2y' + y = 0$$
$$y = xe^x$$

Solution:

Given the second order differential equation,

$$y'' - 2y' + y = 0$$

Deriving the solution will yield to,

$$y = xe^x$$
$$y' = xe^x + e^x$$

$$y'' = xe^x + e^x + e^x$$

$$y'' = xe^x + 2e^x$$

Substituting and simplifying the derived equation to the differential equation results to,

$$xe^x + 2e^x - 2(xe^x + e^x) + xe^x = 0$$

$$0 = 0$$

$$\boxed{\text{It is a solution}}$$

Example #3

Find the value of m so that the function $y = e^{mx}$ is a solution of the given differential equation.

$$y' + 2y = 0$$

Solution:

Given the first order differential equation,

$$y' + 2y = 0$$

Deriving the solution will yield to,

$$y = e^{mx}$$
$$y' = me^{mx}$$

Substituting and factoring e^{mx} the derived equation to the differential equation results to the value of m,

$$me^{mx} + 2e^{mx} = 0$$
$$e^{mx}(m+2) = 0$$
$$m = -2$$

and thus the solution of the differential equation is

$$\boxed{y = e^{-2x}}$$

Example #4

Find the value of m so that the function $y = x^m$ is a solution of the given differential equation.

$$xy'' + 2y' = 0$$

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Solution:

Given the second order differential equation,

$$xy'' + 2y' = 0$$

Deriving the solution will yield to,

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)x^{m-2}$$

Substituting the derivatives and simplifying the equation will yield to

$$x[m(m-1)x^{m-2}] + 2mx^{m-1} = 0$$
$$(m^2 - m)x^{m-1} + 2mx^{m-1} = 0$$

factoring out x^{m-1} and m will yield to the values of m as,

$$x^{m-1}(m^2 + m) = 0$$
$$x^{m-1}m(m+1) = 0$$
$$m = 0 \;\&\; m = -1$$

and thus the solution of the differential equation are

$$\boxed{y = x^0 = 1 \quad \& \quad y = x^{-1} = \frac{1}{x}}$$

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