A glance into Arithmetic Dynamics

Sebastian Troncoso

January 30, 2018.

Overview:

1. PART I:
   - Basic definitions and Notations
   - Examples
   - Conjectures
2. PART II:
   - Extra hypothesis: Good reduction
   - Results
3. PART III:
   - Current research
   - Projects

Definitions:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

Definitions:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

Definition

A dynamical system is a pair (S, ϕ) consisting of a set S and a self-map $\phi : S \to S$.

Definitions:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

Definition

A dynamical system is a pair (S, ϕ) consisting of a set S and a self-map $\phi : S \to S$.

For example consider

- $(\mathbb{C}, x^6 - 1)$
- $(\mathbb{R}, \dfrac{x^3 + 2}{x^4 + 7})$
- $(\mathbb{Q}, x^3 + 2)$
- $(\mathbb{Z}, x^2 - 1)$.

Definitions:

The goal of dynamics is to study the behavior of points in S as ϕ is applied repeatedly.

Definitions:

The goal of dynamics is to study the behavior of points in S as ϕ is applied repeatedly.

The nth iterate of ϕ is denoted by

$$\phi^n(x) = \phi \circ \phi \circ \ldots \circ \phi(x)$$

Definitions:

The goal of dynamics is to study the behavior of points in S as ϕ is applied repeatedly.

The nth iterate of ϕ is denoted by

$$\phi^n(x) = \phi \circ \phi \circ \ldots \circ \phi(x)$$

The orbit of $x \in S$ is the set of points obtained by applying the iterates of ϕ to x and it is denoted by

$$\mathcal{O}_\phi(x) = \mathcal{O}(x) = \{x, \phi(x), \phi^2(x), \phi^3(x), \ldots\}$$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0$

Examples

Consider the dynamical system $(\mathbb{Q}, x^2 - 2)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

$$0 \longrightarrow -2 \longrightarrow 2 \circlearrowleft$$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

$$0 \longrightarrow -2 \longrightarrow 2 \circlearrowleft$$

$\mathcal{O}(3) = \{3$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

$$0 \longrightarrow -2 \longrightarrow 2 \circlearrowleft$$

$\mathcal{O}(3) = \{3, 7, 47$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

$$0 \longrightarrow -2 \longrightarrow 2 \circlearrowleft$$

$\mathcal{O}(3) = \{3, 7, 47, 2207, 4870847, 23725150497407, \ldots\}$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 2\right)$. Then

$\mathcal{O}(1) = \{1, -1\}$

$$1 \longrightarrow -1 \circlearrowleft$$

$\mathcal{O}(0) = \{0, -2, 2\}$

$$0 \longrightarrow -2 \longrightarrow 2 \circlearrowleft$$

$\mathcal{O}(3) = \{3, 7, 47, 2207, 4870847, 23725150497407, \ldots\}$

$$3 \rightarrow 7 \longrightarrow 47 \longrightarrow 2207 \longrightarrow 4870847 \longrightarrow 23725150497407 \longrightarrow \ldots$$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 3\right)$. Then

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 3\right)$. Then

$\mathcal{O}(1) = \{1, -2\}$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 3\right)$. Then

$\mathcal{O}(1) = \{1, -2\}$

$$1 \longleftrightarrow -2$$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 3\right)$. Then

$\mathcal{O}(1) = \{1, -2\}$

$$1 \longleftrightarrow -2$$

$\mathcal{O}(0) = \{0, -3, 6, 33, 1086, \ldots\}$

Examples

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 3\right)$. Then

$\mathcal{O}(1) = \{1, -2\}$

$$1 \longleftrightarrow -2$$

$\mathcal{O}(0) = \{0, -3, 6, 33, 1086, \ldots\}$

$$0 \longrightarrow -3 \longrightarrow 6 \longrightarrow 33 \longrightarrow 1086 \longrightarrow \ldots$$

Definition

Given a dynamical system (S, ϕ). There are two possibilities for the orbits of a point $P \in S$

- If the orbit $\mathcal{O}_\phi(P)$ is infinite, we say that P is a **wandering point**.

Definition

Given a dynamical system (S, ϕ). There are two possibilities for the orbits of a point $P \in S$

- If the orbit $\mathcal{O}_\phi(P)$ is infinite, we say that P is a **wandering point**.
- If the orbit $\mathcal{O}_\phi(P)$ is finite, we say that P is a **preperiodic point**.

Definition

Given a dynamical system (S, ϕ). There are two possibilities for the orbits of a point $P \in S$

- If the orbit $\mathcal{O}_\phi(P)$ is infinite, we say that P is a **wandering point**.

- If the orbit $\mathcal{O}_\phi(P)$ is finite, we say that P is a **preperiodic point**.

A important subset of the preperiodic points consists of those points whose orbit eventually return to its starting point.
These points are called **periodic points**.

Example

Consider the dynamical system $\left(\mathbb{Q}, x^2 - 1\right)$. Then

$$1 \longrightarrow 0 \longleftrightarrow -1$$

- 1 is preperiodic.
- 0 and -1 are periodic.
- Every other element of $\mathbb{Q}$ is a wandering point.

Example

Consider the dynamical system $(\mathbb{Q}, x^2 - 1)$. Then

$$1 \longrightarrow 0 \longleftrightarrow -1$$

- 1 is preperiodic.
- 0 and -1 are periodic.
- Every other element of $\mathbb{Q}$ is a wandering point.

SEE SAGE

Definition:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

Definition:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

From now on we will always consider the dynamical system $(\mathbb{Q}, f)$ where f is a polynomial of degree d with coefficients in $\mathbb{Q}$.

Definition:

Arithmetic Dynamics is the study of arithmetic properties of dynamical systems.

From now on we will always consider the dynamical system $(\mathbb{Q}, f)$ where f is a polynomial of degree d with coefficients in $\mathbb{Q}$.

Notation

The set of preperiodic and periodic points of a map $f : \mathbb{Q} \to \mathbb{Q}$ are denoted respectively by

$$PrePer(f, \mathbb{Q}) \quad \text{and} \quad Per(f, \mathbb{Q}.)$$

Question:

- Are the sets $\mathrm{Per}(f, \mathbb{Q})$ and $\mathrm{PrePer}(f, \mathbb{Q})$ finite?

Question:

- Are the sets $\mathrm{Per}(f, \mathbb{Q})$ and $\mathrm{PrePer}(f, \mathbb{Q})$ finite?

Yes.

Question:

- Are the sets $\mathrm{Per}(f, \mathbb{Q})$ and $\mathrm{PrePer}(f, \mathbb{Q})$ finite?

Yes.

Theorem (Northcott 1950)

Let $f : \mathbb{Q} \to \mathbb{Q}$ be an polynomial of degree ≥ 2 with coefficients on $\mathbb{Q}$. Then f has only finitely many preperiodic points in $\mathbb{Q}$.

Question:

- Are the sets $\mathrm{Per}(f, \mathbb{Q})$ and $\mathrm{PrePer}(f, \mathbb{Q})$ finite?

Yes.

Theorem (Northcott 1950)

Let $f : \mathbb{Q} \to \mathbb{Q}$ be an polynomial of degree ≥ 2 with coefficients on $\mathbb{Q}$. Then f has only finitely many preperiodic points in $\mathbb{Q}$.

- How large is the set $\mathrm{PrePer}(f, \mathbb{Q})$?
- Can you give an explicit bound?
- How is the bound depending on f?

Uniform Boundedness Conjecture:

Consider $f(x) = (x-1)(x-2)(x-3)\dots(x-d)+x$.
Can you see that f has at least d periodic points?

Uniform Boundedness Conjecture:

Consider $f(x) = (x-1)(x-2)(x-3)\dots(x-d) + x$.
Can you see that f has at least d periodic points?

$$1 \circlearrowleft$$

$$2 \circlearrowleft$$

$$\vdots$$

$$d \circlearrowleft$$

Uniform Boundedness Conjecture:

Consider $f(x) = (x-1)(x-2)(x-3)\dots(x-d) + x$.
Can you see that f has at least d periodic points?

$$1 \circlearrowleft$$

$$2 \circlearrowleft$$

$$\vdots$$

$$d \circlearrowleft$$

Conjecture (Uniform Boundedness Conjecture - Morton–Silverman 1994)

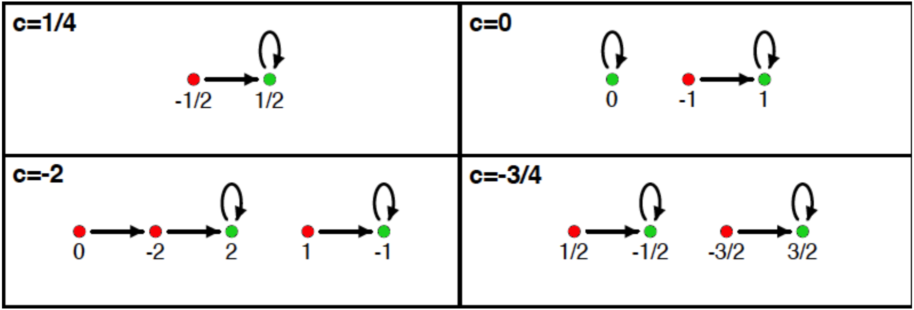
Let $f : \mathbb{Q} \to \mathbb{Q}$ be a polynomial of degree $d \geq 2$ with coefficients in $\mathbb{Q}$. Then there exists a bound $B = B(d)$ such that

$$|\mathrm{PrePer}(\phi, \mathbb{Q})| \leq B.$$

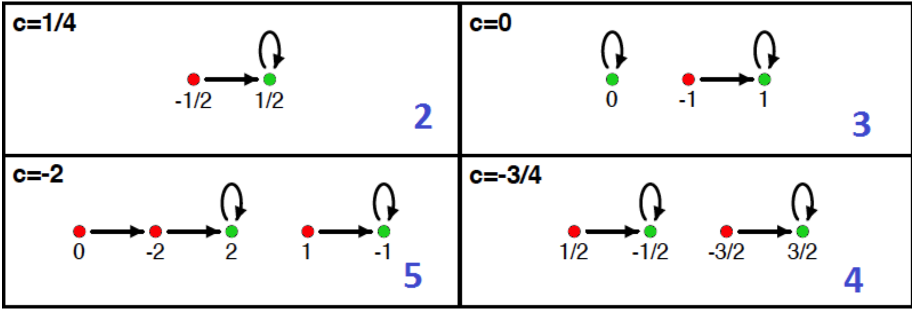
Quadratic functions:

Lets study quadratic Polynomials.

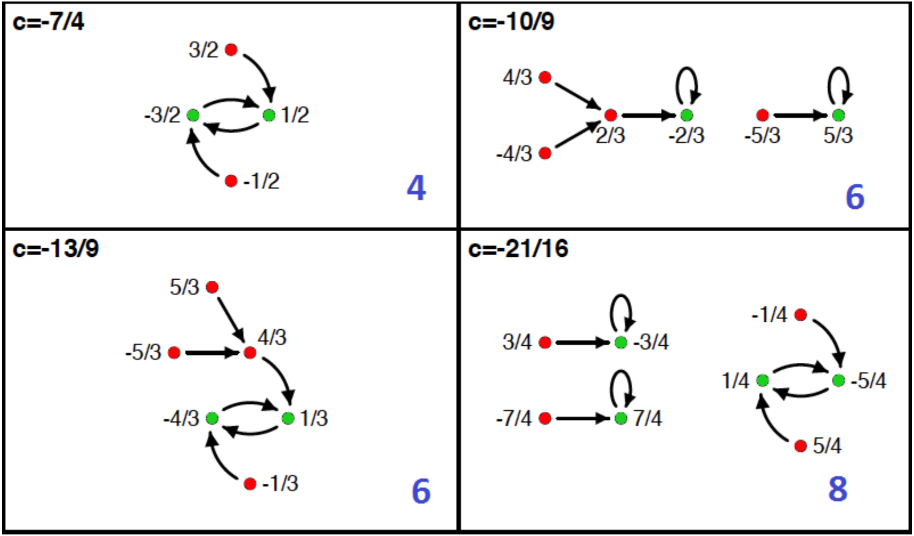
Even more, assume $f_c(x) = x^2 + c$ with $c \in \mathbb{Q}$.



Diagrams of purely preperiodic points (red) and periodic points (green) of $f_c(z) = z^2 + c$ in $\mathbb{Q}$.



Diagrams of purely preperiodic points (red) and periodic points (green) of $f_c(z) = z^2 + c$ in $\mathbb{Q}$.



Diagrams of purely preperiodic points (red) and periodic points (green) of $f_c(z) = z^2 + c$ in $\mathbb{Q}$.

Conjecture:
Let f be a quadratic polynomial of the form $f_c(z) = z^2 + c$ with $c \in \mathbb{Q}$. Then

$$|PrePer(f_c, \mathbb{Q})| \leq 8$$

~~Conjecture:~~ Question:

Let f be a quadratic polynomial of the form $f_c(z) = z^2 + c$ with $c \in \mathbb{Q}$. Then

$$|PrePer(f_c, \mathbb{Q})| \leq 8$$

~~**Conjecture:**~~ **Question:**
Let f be a quadratic polynomial of the form $f_c(z) = z^2 + c$ with $c \in \mathbb{Q}$. Then

$$|PrePer(f_c, \mathbb{Q})| \leq 8$$

USE SAGE

Poonen's Conjecture

Conjecture (Poonen's Conjecture 1998)

Let $f : \mathbb{Q} \to \mathbb{Q}$ be a quadratic polynomial with coefficients in $\mathbb{Q}$. Then

$$|\mathrm{PrePer}(\phi, \mathbb{Q})| \leq 8.$$

If $f = x^2 + d$ then B. Hutz and P. Ingram have shown that Poonen's conjecture holds when the numerator and denominator of d don't exceed 10^8.

PART II:

- Extra hypothesis: Good/Bad Reduction.
- Results

Good/Bad Reduction

Let $\mathfrak{p}$ be a prime number in $\mathbb{Z}$ and $f(x) = \frac{a_d}{b_d}x^d + \ldots + \frac{a_1}{b_1}x + \frac{a_0}{b_0}$ be a polynomial with coefficients in $\mathbb{Q}$.

Good/Bad Reduction

Let $\mathfrak{p}$ be a prime number in $\mathbb{Z}$ and $f(x) = \frac{a_d}{b_d}x^d + \ldots + \frac{a_1}{b_1}x + \frac{a_0}{b_0}$ be a polynomial with coefficients in $\mathbb{Q}$.

- We say that f has **good reduction** at $\mathfrak{p}$ if $\mathfrak{p} \nmid a_d$ and $\mathfrak{p} \nmid b_d$.

Good/Bad Reduction

Let $\mathfrak{p}$ be a prime number in $\mathbb{Z}$ and $f(x) = \frac{a_d}{b_d}x^d + \ldots + \frac{a_1}{b_1}x + \frac{a_0}{b_0}$ be a polynomial with coefficients in $\mathbb{Q}$.

- We say that f has **good reduction** at $\mathfrak{p}$ if $\mathfrak{p} \nmid a_d$ and $\mathfrak{p} \nmid b_d$.
- Otherwise, we say that f has **bad reduction** at $\mathfrak{p}$.

Good/Bad Reduction

Let $\mathfrak{p}$ be a prime number in $\mathbb{Z}$ and $f(x) = \frac{a_d}{b_d}x^d + \ldots + \frac{a_1}{b_1}x + \frac{a_0}{b_0}$ be a polynomial with coefficients in $\mathbb{Q}$.

- We say that f has **good reduction** at $\mathfrak{p}$ if $\mathfrak{p} \nmid a_d$ and $\mathfrak{p} \nmid b_d$.
- Otherwise, we say that f has **bad reduction** at $\mathfrak{p}$.

Definition

Let S be a finite set of primes of $\mathbb{Z}$ and f a polynomial of degree d with coefficients in $\mathbb{Q}$. We say that f has **good reduction outside** S if f has good reduction at $\mathfrak{p}$ for every $\mathfrak{p} \notin S$.

Example

1. Consider $f(x) = 15x^3 + 5x - 4$.
   Can you find the primes of bad reduction?

Example

1. Consider $f(x) = 15x^3 + 5x - 4$.
   Can you find the primes of bad reduction?
   Yes, 3 and 5 are primes of bad reduction for f.

   If we set $S = \{3, 5\}$ then f has good reduction outside S.

Example

1. Consider $f(x) = 15x^3 + 5x - 4$.
   Can you find the primes of bad reduction?
   Yes, 3 and 5 are primes of bad reduction for f.

   If we set $S = \{3, 5\}$ then f has good reduction outside S.

2. Consider $f(x) = x^7 + 2x - 2$.
   Can you find the primes of bad reduction?

Example

1. Consider $f(x) = 15x^3 + 5x - 4$.
   Can you find the primes of bad reduction?
   Yes, 3 and 5 are primes of bad reduction for f.

   If we set $S = \{3, 5\}$ then f has good reduction outside S.

2. Consider $f(x) = x^7 + 2x - 2$.
   Can you find the primes of bad reduction?
   No.
   If we set $S = \emptyset$ then f has good reduction outside S.

Example

1. Consider $f(x) = 15x^3 + 5x - 4$.
   Can you find the primes of bad reduction?
   Yes, 3 and 5 are primes of bad reduction for f.

   If we set $S = \{3, 5\}$ then f has good reduction outside S.

2. Consider $f(x) = x^7 + 2x - 2$.
   Can you find the primes of bad reduction?
   No.
   If we set $S = \emptyset$ then f has good reduction outside S.

If we allow the number of primes of bad reduction as a parameter, much more is known for the cardinality of the set of preperiodic points.

Bound on the set of preperiodic point

Theorem

Let $f : \mathbb{Q} \to \mathbb{Q}$ be a polynomial of degree $d \geq 2$ with coefficients in $\mathbb{Q}$. Suppose f has good reduction outside a finite set of primes S and denote $s = |S|$. Then

Bound on the set of preperiodic point

Theorem

Let $f : \mathbb{Q} \to \mathbb{Q}$ be a polynomial of degree $d \geq 2$ with coefficients in $\mathbb{Q}$. Suppose f has good reduction outside a finite set of primes S and denote $s = |S|$. Then

- $|PrePer(f, \mathbb{Q})| \leq d^{2^{16s}(s \log(s))}$
  J.K. Canci and L. Paladino (2015).

Bound on the set of preperiodic point

Theorem

Let $f : \mathbb{Q} \to \mathbb{Q}$ be a polynomial of degree $d \geq 2$ with coefficients in $\mathbb{Q}$. Suppose f has good reduction outside a finite set of primes S and denote $s = |S|$. Then

- $|PrePer(f, \mathbb{Q})| \leq d^{2^{16s}(s\log(s))}$
  J.K. Canci and L. Paladino (2015).
- $|PrePer(f, \mathbb{Q})| \leq 5\left(2^{16sd^3}\right) + 3$
  S. Troncoso (2017).

Bound on the set of preperiodic point

Theorem

Let $f : \mathbb{Q} \to \mathbb{Q}$ be a polynomial of degree $d \geq 2$ with coefficients in $\mathbb{Q}$. Suppose f has good reduction outside a finite set of primes S and denote $s = |S|$. Then

- $|PrePer(f, \mathbb{Q})| \leq d^{2^{16s}(s\log(s))}$
  J.K. Canci and L. Paladino (2015).
- $|PrePer(f, \mathbb{Q})| \leq 5\left(2^{16sd^3}\right) + 3$
  S. Troncoso (2017).
- $|PrePer(f, \mathbb{Q})| \leq \alpha d^2 + \beta d + \gamma$
  where α, β and γ are roughly 2^{78s}.
  J.K. Canci, S. Troncoso and S. Vishkautsan (submitted).

Bounds for Preperiodic points

The previous theorem use some important results like:

Bounds for Preperiodic points

The previous theorem use some important results like:

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula
- Baker's Theorem on existence of periodic points

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula
- Baker's Theorem on existence of periodic points
- Kisaka's analysis on Baker's Theorem

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula
- Baker's Theorem on existence of periodic points
- Kisaka's analysis on Baker's Theorem
- Logarithmic p-adic chordal distance.

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula
- Baker's Theorem on existence of periodic points
- Kisaka's analysis on Baker's Theorem
- Logarithmic p-adic chordal distance.
- Study the distance between periodic and purely preperiodic points.

Bounds for Preperiodic points

The previous theorem use some important results like:

- Riemann-Hurwitz formula
- Baker's Theorem on existence of periodic points
- Kisaka's analysis on Baker's Theorem
- Logarithmic p-adic chordal distance.
- Study the distance between periodic and purely preperiodic points.
- Number of solution of the S-unit equation.

More general setting

$\mathbb{Q}$

More general setting

$$\mathbb{Q} \Longrightarrow \mathbb{P}^1(\mathbb{Q}) = \mathbb{Q} \cup \{\infty\}$$

More general setting

$$\mathbb{Q} \Longrightarrow \mathbb{P}^1(\mathbb{Q}) = \mathbb{Q} \cup \{\infty\} \Longrightarrow \mathbb{P}^1(K)$$

where K is a number field *i.e* a finite field extension of $\mathbb{Q}$.

More general setting

$$\mathbb{Q} \Longrightarrow \mathbb{P}^1(\mathbb{Q}) = \mathbb{Q} \cup \{\infty\} \Longrightarrow \mathbb{P}^1(K)$$

where K is a number field *i.e* a finite field extension of $\mathbb{Q}$.

$f(x) = a_d x^d \ldots + a_1 x + a_0$ a polynomial with coefficients in $\mathbb{Q}$.

More general setting

$$\mathbb{Q} \Longrightarrow \mathbb{P}^1(\mathbb{Q}) = \mathbb{Q} \cup \{\infty\} \Longrightarrow \mathbb{P}^1(K)$$

where K is a number field *i.e* a finite field extension of $\mathbb{Q}$.

$f(x) = a_d x^d \ldots + a_1 x + a_0$ a polynomial with coefficients in $\mathbb{Q}$.

$$\Downarrow$$

$\phi(x) = \frac{p(x)}{q(x)}$ where f and g are polynomials with coefficients in $\mathbb{Q}$.

More general setting

$$\mathbb{Q} \Longrightarrow \mathbb{P}^1(\mathbb{Q}) = \mathbb{Q} \cup \{\infty\} \Longrightarrow \mathbb{P}^1(K)$$

where K is a number field *i.e* a finite field extension of $\mathbb{Q}$.

$f(x) = a_d x^d \ldots + a_1 x + a_0$ a polynomial with coefficients in $\mathbb{Q}$.

$$\Downarrow$$

$\phi(x) = \dfrac{p(x)}{q(x)}$ where f and g are polynomials with coefficients in $\mathbb{Q}$.

$$\Downarrow$$

$\phi(x) = \dfrac{p(x)}{q(x)}$ where f and g are polynomials with coefficients in K.

PART III:

- Current Research
- Projects

Current project

Arithmetic dynamics in higher dimension

Notation of preperiodic hypersurfaces

Let ϕ be a function (polynomial in several variables) and H an irreducible hypersurface.

Notation of preperiodic hypersurfaces

Let ϕ be a function (polynomial in several variables) and H an irreducible hypersurface.

Periodic hypersurface: $\phi^n(H) = H$ for some $n \geq 1$.

Preperiodic hypersurface: $\exists m \geq 0$ such that $\phi^m(H)$
The set of preperiodic hypersurface is denoted by $HPrePer(\phi, \mathbb{Q})$.
The set of preperiodic hypersurface of degree e is denoted by $HPrePer(\phi, \mathbb{Q}, e)$.

Question:

- Is the set $HPrePer(\phi, \mathbb{Q})$ finite?

Question:

- Is the set $HPrePer(\phi, \mathbb{Q})$ finite?

No.

Question:

- Is the set $HPrePer(\phi, \mathbb{Q})$ finite?

No.

- Is the set $HPrePer(\phi, \mathbb{Q}, e)$ finite?

Question:

- Is the set $HPrePer(\phi, \mathbb{Q})$ finite?

No.

- Is the set $HPrePer(\phi, \mathbb{Q}, e)$ finite?

Yes.

Question:

- Is the set $HPrePer(\phi, \mathbb{Q})$ finite?

No.

- Is the set $HPrePer(\phi, \mathbb{Q}, e)$ finite?

Yes.

Theorem (B. Hutz 2016)

Let ϕ be a function. Then there are only finitely many preperiodic hypersurfaces of degree e.

Project 1: Data base of preperiodic hypersurfaces

- Use Sage to provide examples of preperiodic hypersurfaces.

Project 1: Data base of preperiodic hypersurfaces

- Use Sage to provide examples of preperiodic hypersurfaces.

- The goal is to state "Uniform Boundedness Conjecture" and provide evidence for such a conjecture.

Project 2: S-unit equations and applications

Let S be a finite set of primes of $\mathbb{Z}$ and $\mathbb{Z}_S^*$ be the group of S-units *i.e.* $\mathbb{Z}_S^*$ is the set of fraction of the form $\dfrac{a}{b}$ where a and b are not divisible by primes not in S.

Project 2: S-unit equations and applications

Let S be a finite set of primes of $\mathbb{Z}$ and $\mathbb{Z}_S^*$ be the group of S-units *i.e.* $\mathbb{Z}_S^*$ is the set of fraction of the form $\frac{a}{b}$ where a and b are not divisible by primes not in S.

- A linear relation of the form

$$u + v = 1$$

  where $(u, v) \in (\mathbb{Z}_S^*)^2$ is called a S-unit equation.

Project 2: S-unit equations and applications

Let S be a finite set of primes of $\mathbb{Z}$ and $\mathbb{Z}_S^*$ be the group of S-units *i.e.* $\mathbb{Z}_S^*$ is the set of fraction of the form $\frac{a}{b}$ where a and b are not divisible by primes not in S.

- A linear relation of the form

$$u + v = 1$$

  where $(u, v) \in (\mathbb{Z}_S^*)^2$ is called a S-unit equation.
- Beukers and Schlickewei give an explicit bound for the number of solution of a S-unit equation.

Project 2: S-unit equations and applications

Let S be a finite set of primes of $\mathbb{Z}$ and $\mathbb{Z}_S^*$ be the group of S-units *i.e.* $\mathbb{Z}_S^*$ is the set of fraction of the form $\frac{a}{b}$ where a and b are not divisible by primes not in S.

- A linear relation of the form

$$u + v = 1$$

  where $(u, v) \in (\mathbb{Z}_S^*)^2$ is called a S-unit equation.
- Beukers and Schlickewei give an explicit bound for the number of solution of a S-unit equation.
- An interesting project will be to understand application in mathematics of Beukers and Schlickewei's result.

THANK YOU