IT-UNIVERSITY OF COPENHAGEN

INTELLIGENT SYSTEMS PROGRAMMING

Mandatory exercise 9

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1)

A point in n-dimensions has n numbers that indicate where it is located. In one dimension a point has the form (x), in two dimension (x, y), in three (x, y, z) and so on. A unit hypercube in n dimension has corner points in all permutations of the coordinates $(x, y, z...n)$ where the valid domain is $\{0, 1\}$.

The corner points for a 2-hypercube are therefore:

$$(0, 0), (0, 1), (1, 0), (1, 1)$$

and the corner points for a 3-hypercube are:

$$(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)$$

The number of possible permutations for a point in n-dimensions where the valid domain is $\{0, 1\}$ is 2^n. A hypercube in n dimensions therefore has 2^n corner points as other hypercubes are just a scaled unit hypercube.

2)

Wikipedia defines a hypercube as: "a closed, compact, convex figure whose 1-skeleton consists of groups of opposite parallel line segments aligned in each of the space's dimensions, perpendicular to each other and of the same length[1]"

Therefore a 2-hypercube, a hypercube in two dimensions, has two pairs of parallel lines.

Example of line coordinates in a 2-hypercube where C_x is a constant and x and y are variables:
(C_1, y)
$(C_1 + 1, y)$
(x, C_2)
$(x + 1, C_2)$

Example of line coordinates in a 3-hypercube where C_x is a constant and x, y and z are variables:

1. (C_1, C_2, z)
2. $(C_1 + 1, C_2, z)$
3. $(C_1, C_2 + 1, z)$
4. $(C_1 + 1, C_2 + 1, z)$
5. (C_1, y, C_3)
6. $(C_1 + 1, y, C_3)$
7. $(C_1, y, C_3 + 1)$
8. $(C_1 + 1, y, C_3 + 1)$

[1] https://en.wikipedia.org/wiki/Hypercube

9. (x, C_2, C_3)
10. $(x, C_2 + 1, C_3)$
11. $(x, C_2, C_3 + 1)$
12. $(x, C_2 + 1, C_3 + 1)$

3)

I don't really understand the question, so i will try to answer both my interpretations of them.

Let us consider two corner points in a n-hypercube $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$

If the two corners are adjacent, only one pivot is needed to reach to other corner, as increasing a slack variable to its maximum is equivalent to traveling along one of the inequalities from one corner point to another.

If on the other hand the two corner points are as far as possible from each other a minimum of n pivots are needed to travel from A to B. This can be observed when we consider a unit hypercube, where we need to travel at least once along every dimension to reach the corner point furthest away.

4)

It *is* possible to define an objective, maximize $\sum_{j=1}^{\infty} c_j x_j$. An example of this is when a degenerate problem creates a cycle, and the simplex algorithm does not terminate. It is however possible to implement cycle breaking in simplex, but it is often not done, as cyclic degenerate problems are rare. A degenerate example:[2]

$x_5 = -0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4$
$x_6 = -0.5x_1 + 1.5x_2 + 0.5x_3 - x_4$
$x_7 = 1 - x_1$
$z = 10x_1 - 57x_2 - 9x_3 - 24x_4$

[2]The example is taken from the slides from lecture 12