

## IT-UNIVERSITY OF COPENHAGEN

INTELLIGENT SYSTEMS PROGRAMMING

## Mandatory exercise 9

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## 1)

A point in n-dimensions has n numbers that indicate where it is located. In one dimension a point has the form (x), in two dimension (x, y), in three (x, y, z) and so on. A unit hypercube in n dimension has corner points in all permutations of the coordinates (x, y, z...n) where the valid domain is  $\{0, 1\}$ .

The corner points for a 2-hypercube are therefore:

and the corner points for a 3-hypercube are:

$$(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)$$

The number of possible permutations for a point in n-dimensions where the valid domain is  $\{0,1\}$  is  $2^n$ . A hypercube in n dimensions therefore has  $2^n$  corner points as other hypercubes are just a scaled unit hypercube.

## 2)

Wikipedia defines a hypercube as: "a closed, compact, convex figure whose 1-skeleton consists of groups of opposite parallel line segments aligned in each of the space's dimensions, perpendicular to each other and of the same length<sup>1</sup>"

Therefore a 2-hypercube, a hypercube in two dimensions, has two pairs of parallel lines.

Example of line coordinates in a 2-hypercube where  $C_x$  is a constant and x and y are variables:

```
(C_1, y)

(C_1 + 1, y)

(x, C_2)

(x + 1, C_2)
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Example of line coordinates in a 3-hypercube where  $C_x$  is a constant and x, y and z are variables:

- 1.  $(C_1, C_2, z)$
- 2.  $(C_1+1,C_2,z)$
- 3.  $(C_1, C_2 + 1, z)$
- 4.  $(C_1+1,C_2+1,z)$
- 5.  $(C_1, y, C_3)$
- 6.  $(C_1+1,y,C_3)$
- 7.  $(C_1, y, C_3 + 1)$
- 8.  $(C_1+1,y,C_3+1)$

<sup>&</sup>lt;sup>1</sup>https://en.wikipedia.org/wiki/Hypercube

- 9.  $(x, C_2, C_3)$
- 10.  $(x, C_2 + 1, C_3)$
- 11.  $(x, C_2, C_3 + 1)$
- 12.  $(x, C_2 + 1, C_3 + 1)$
- 3)
- 4)