IT University
of Copenhagen

IT-UNIVERSITY OF COPENHAGEN

AUTOMATED SOFTWARE ANALYSIS

Analysis-2

Tróndur Høgnason (thgn@itu.dk)

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Monotone Functions and Fixed-Points:

i)

$$X = \{a,\ b\}$$
$$Y = X \cup Y$$

In equation form:

$$X = f(X,Y) = \{a,\ b\}$$
$$Y = g(X,Y) = X \cup Y$$

Can be written as:

$$X = f() = \{a,\ b\}$$
$$Y = g(X,Y) = X \cup Y$$

The function $f()$ is monotone, as it is constant. Therefore $f() \sqsubseteq f()$ will always hold. The function $g(X,\ Y)$ is also monotone, as no matter which sets use give the function it will combine them and the resulting set will always be greater or equal to the input sets.

ii)

$$X = \{a,\ b\} \cup Y$$
$$Y = X \setminus \{b\}$$

In equation form:

$$X = f(X,Y) = \{a,\ b\} \cup Y$$
$$Y = g(X,Y) = X \setminus \{b\}$$

Can be written as:

$$X = f(Y) = \{a,\ b\} \cup Y$$
$$Y = g(X) = X \setminus \{b\}$$

The function $f(Y)$ is monotone, as it return $\{a,\ b\} \cup Y$ which is always a superset of Y. The function $g(X)$ is also monotone. It is hard to give an example of why, however intuitively removing a constant element preserves monotonicity, as the same element is removed on both sides of $\sqsubseteq$.

iii)

$$X = \{a,\ b\} \cup Z$$
$$Y = \{a,\ c\} \setminus X$$
$$Z = X^C$$

In equation form:

$$X = f(X,\ Y,\ Z) = \{a,\ b\} \cup Z$$
$$Y = g(X,\ Y,\ Z) = \{a,\ c\} \setminus X$$
$$Z = h(X,\ Y,\ Z) = X^C$$

Can be written as:

$$X = f(Z) = \{a,\ b\} \cup Z$$
$$Y = g(X) = \{a,\ c\} \setminus X$$
$$Z = h(X) = X^C$$

The function $f(Z)$ is monotone, as it return $\{a,\ b\} \cup Z$ which is always a superset of Z. If $g(X)$ is monotone then $\forall\, Y,\ Y'\ P(\{a,\ b,\ c\}) : Y\ \sqsubseteq\ Y'$. However this is not the case:

$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ c\} \not\sqsubseteq \emptyset$$

$g(X)$ is therefore not monotone. $h(X)$ is not monotone either, as:

$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ b\ c\} \not\sqsubseteq \emptyset$$