IT-University of Copenhagen

Automated software analysis

Analysis-2

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Relations and Partial-Orders:

Signature:

$$S \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{R}$$

Relation:

$$P(x+1, 2*y, z/3) = \{\emptyset, \{x+1\}, \{2*y\}, \{z/3\}, \{x+1, 2*y\}, \{x+1, z/3\}, \{2*y, z/3\}, \{x+1, 2*y, z/3\}\}$$

An example of a member of the relation:

$$(2, 2, 6) \in S$$

$$(2, 2, 6)$$

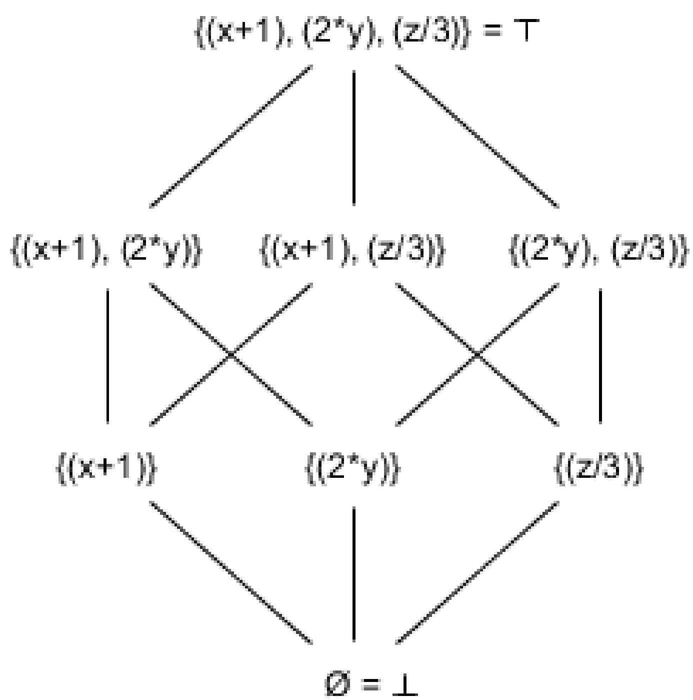
An example of a non-member of the relation:

$$(2, 3, 6) \notin S$$

$$(2, 3, 6)$$

Does the set S and relation form a partial-order? Yes. The relation is:

- Reflexive, every element is related to itself. Example $\{x+1\} \sqsubseteq \{x+1\}$
- Antisymmetric, two elements must not be related in both directions. Example $\{x+1\} \sqsubseteq \{x+1, 2*y\}$ but $\{x+1\} \not\sqsubseteq \{x+1, 2*y\}$
- Transitive, if the first element is related to the second element, and the second is related to the third, then the first is related to the third. Example $\{x+1\} \sqsubseteq \{x+1, 2*y\}$ and $\{x+1, 2*y\} \sqsubseteq \{x+1, 2*y, z/3\}\}$ then $\{x+1\} \sqsubseteq \{x+1, 2*y, z/3\}\}$

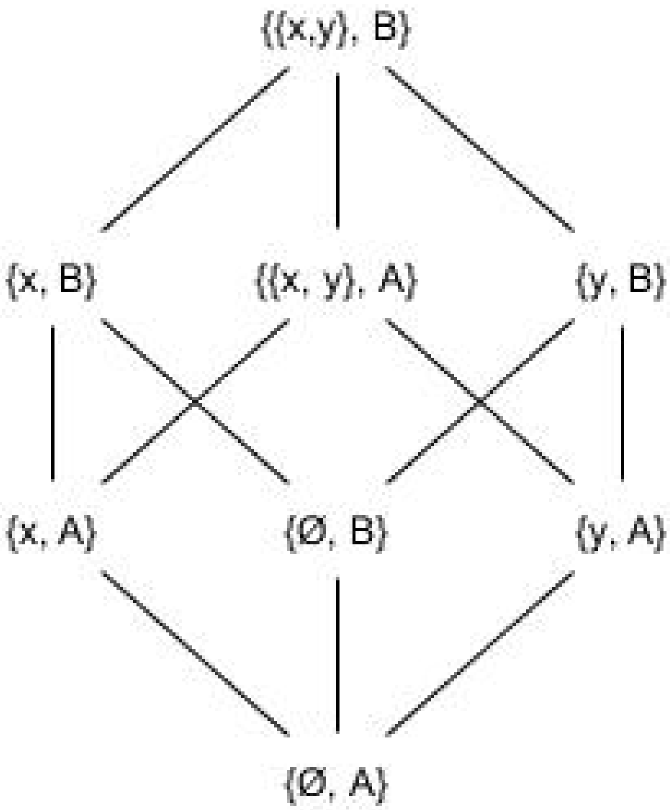


Draw a Hasse diagram:

Greatest-Lower-Bound:

Given a lattice $L = (S, \sqsubseteq)$; what do the elements $\sqcup S$ and $\sqcap S$ correspond to? Assuming S from above, $\sqcup S$ is $\{x+1, 2*y, z/3\}\}$, and $\sqcap S$ is $\emptyset$

Lattices:



A $L_1 \times L_2$ lattice will have $|L_1| * |L_2|$ points. So the resulting lattice will in this case have $2 * 4 = 8$ points.