IT-University of Copenhagen

Automated software analysis

Analysis-3

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Putting it all Together:

Recursive equations:
$a = A()$
$b = B(a)$
$c = C(b)$
$d = D(c,\ e)$
$e = E(d)$
$f = F(e)$

Transfer functions:
$A = [\bot,\ \bot]$
$B = \lambda E.\ E[x \mapsto +]$
$C = \lambda E.\ E[y \mapsto 0]$
$D = \lambda E.\ \lambda E'.\ E \sqcup E'$
$E = \lambda E.\ E[x \mapsto E[y],\ y \mapsto E[x]]$
$F = \lambda E.\ E[y \mapsto E[y] \oplus\ +]$

Big transfer function:

$$T(a, b, c, d, e, f) \ = \ ([\bot,\ \bot],\ a[x \mapsto +],\ b[y \mapsto 0],\ c \sqcup e,\ (d[x \mapsto d[y],\ y \mapsto d[x]]),\ e[y \mapsto e[y] \oplus\ +])$$

Finding least fixed:

1. $([\bot,\ \bot], [\bot,\ \bot], [\bot,\ \bot], [\bot,\ \bot], [\bot,\ \bot], [\bot,\ \bot])$
2. $([\bot,\ \bot], [+,\ \bot], [\bot,\ 0], [\bot,\ \bot], [\bot,\ \bot], [\bot,\ \bot])$
3. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [\bot,\ 0], [\bot,\ \bot], [\bot,\ \bot])$
4. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [+,\ 0], [0,\ \bot], [0,\ \bot])$
5. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [0+,\ 0], [0,\ +], [0,\ \bot])$
6. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [0+,\ 0+], [0,\ 0+], [0,\ +])$
7. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [0+,\ 0+], [0+,\ 0+], [0,\ 0+])$
8. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [0+,\ 0+], [\bot,\ \bot], [\bot,\ \bot])$
9. $([\bot,\ \bot], [+,\ \bot], [+,\ 0], [0+,\ 0+], [\bot,\ \bot], [\bot,\ \bot])$

Monotone Functions and Fixed-Points:

i)

$$\begin{aligned} X &= \{a,\ b\} \\ Y &= X \cup Y \end{aligned}$$

In equation form:

$$\begin{aligned} X &= f(X, Y) = \{a,\ b\} \\ Y &= g(X, Y) = X \cup Y \end{aligned}$$

Can be written as:

$$\begin{aligned} X &= f() = \{a,\ b\} \\ Y &= g(X, Y) = X \cup Y \end{aligned}$$

The function $f()$ is monotone, as it is constant. Therefore $f() \sqsubseteq f()$ will always hold. The function $g(X,\ Y)$ is also monotone, as no matter which sets use give the function it will combine them and the resulting set will always be greater or equal to the input sets.

ii)

$$X = \{a,\ b\} \cup Y$$
$$Y = X \setminus \{b\}$$

In equation form:

$$X = f(X, Y) = \{a,\ b\} \cup Y$$
$$Y = g(X, Y) = X \setminus \{b\}$$

Can be written as:

$$X = f(Y) = \{a,\ b\} \cup Y$$
$$Y = g(X) = X \setminus \{b\}$$

The function $f(Y)$ is monotone, as it return $\{a,\ b\} \cup Y$ which is always a superset of Y. The function $g(X)$ is also monotone. It is hard to give an example of why, however intuitively removing a constant element preserves monotonicity, as the same element is removed on both sides of $\sqsubseteq$.

iii)

$$X = \{a,\ b\} \cup Z$$
$$Y = \{a,\ c\} \setminus X$$
$$Z = X^C$$

In equation form:

$$X = f(X,\ Y,\ Z) = \{a,\ b\} \cup Z$$
$$Y = g(X,\ Y,\ Z) = \{a,\ c\} \setminus X$$
$$Z = h(X,\ Y,\ Z) = X^C$$

Can be written as:

$$X = f(Z) = \{a,\ b\} \cup Z$$
$$Y = g(X) = \{a,\ c\} \setminus X$$
$$Z = h(X) = X^C$$

The function $f(Z)$ is monotone, as it return $\{a,\ b\} \cup Z$ which is always a superset of Z. If $g(X)$ is monotone then $\forall\, Y,\ Y'\ P(\{a,\ b,\ c\}) : Y\ \sqsubseteq\ Y'$. However this is not the case:

$$\emptyset \sqsubseteq \{a,\ b,\ c\}$$
$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ c\} \not\sqsubseteq \emptyset$$

$g(X)$ is therefore not monotone. $h(X)$ is not monotone either, as:

$$\emptyset \sqsubseteq \{a,\ b,\ c\}$$
$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ b\ c\} \not\sqsubseteq \emptyset$$