2.3:

$\mathtt{z} := 0;\ \textbf{while}\ \mathtt{y} \leq \mathtt{x}\ \textbf{do}\ (\mathtt{z} := \mathtt{z}+1;\ \mathtt{x} := \mathtt{x}-\mathtt{y})$

$$\begin{array}{l} s_0 = [x \mapsto 17 y \mapsto 5] \\ s_1 = [x \mapsto 17,\ y \mapsto 5; y \mapsto 0] \\ s_2 = [x \mapsto 17,\ y \mapsto 5; y \mapsto 1] \\ s_3 = [x \mapsto 12,\ y \mapsto 5; y \mapsto 1] \\ s_4 = [x \mapsto 12,\ y \mapsto 5; y \mapsto 2] \\ s_5 = [x \mapsto 7,\ y \mapsto 5; y \mapsto 2] \\ s_6 = [x \mapsto 7,\ y \mapsto 5; y \mapsto 3] \\ s_7 = [x \mapsto 2,\ y \mapsto 5; y \mapsto 3] \end{array} \qquad \begin{array}{rcl} b &=& \mathtt{y} \leq \mathtt{x} \\ A &=& \mathtt{z} := 0;\ B \\ B &=& \textbf{while}\ b\ \textbf{do}\ C \\ C &=& \mathtt{z} := \mathtt{z}+1;\ D \\ D &=& \mathtt{x} := \mathtt{x}-\mathtt{y} \end{array}$$

$$[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{z} := 0,\ s\rangle \to s_1} \quad [\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{z} := \mathtt{z}+1,\ s_1\rangle \to s_2} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x}-y,\ s_2\rangle \to s_3}}{\langle C,\ s_1\rangle \to s_3} \quad [\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{z} := \mathtt{z}+1,\ s_3\rangle \to s_4} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x}-y,\ s_4\rangle \to s_5}}{\langle C,\ s_3\rangle \to s_5} \quad [\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{z} := \mathtt{z}+1,\ s_5\rangle \to s_6} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x}-y,\ s_6\rangle \to s_7}}{\langle C,\ s_5\rangle \to s_7} \quad [\mathsf{while}^{\mathsf{ff}}_{\mathsf{ns}}]\ \dfrac{}{\langle B,\ s_7\rangle \to s_7}\ \mathcal{B}[b]_{s_7} = \mathbf{ff}}{\langle B,\ s_5\rangle \to s_7}\ \mathcal{B}[b]_{s_5} = \mathbf{tt}}{\langle B,\ s_3\rangle \to s_7}\ \mathcal{B}[b]_{s_3} = \mathbf{tt}}{\langle B,\ s_1\rangle \to s_7}\ \mathcal{B}[b]_{s_1} = \mathbf{tt}}{\langle A,\ s_0\rangle \to s_7}$$

2.4:

Program one will not terminate, because $\neg(\mathtt{x} = 1)$ will never be false when the initial state of x is 0. So the inference tree will expand forever.
$\textbf{while}\ \neg(\mathtt{x} = 1)\ \textbf{do}\ (\mathtt{y} := y * x;\ \mathtt{x} := x - 1)$

$$\begin{array}{l} s_0 = [x \mapsto 0, y \mapsto 0] \\ s_1 = [x \mapsto 0, y \mapsto -1] \\ s_2 = [x \mapsto 0, y \mapsto -1] \\ s_3 = [x \mapsto 0, y \mapsto -2] \\ s_4 = [x \mapsto 0, y \mapsto -2] \end{array} \qquad \begin{array}{rcl} b &=& \neg(\mathtt{x} = 1) \\ A &=& \textbf{while}\ b\ \textbf{do}\ B \\ B &=& \mathtt{y} := y * x;\ C \\ C &=& \mathtt{x} := x - 1 \end{array}$$

$$[\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{y} := \mathtt{y} * \mathtt{x},\ s_0\rangle \to s_1} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ s_1\rangle \to s_2}}{\langle B,\ s_0\rangle \to s_2} \quad [\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{y} := \mathtt{y} * \mathtt{x},\ s_2\rangle \to s_3} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ s_3\rangle \to s_4}}{\langle B,\ s_2\rangle \to s_4} \quad \langle \textbf{while}\ b\ \textbf{do}\ B,\ s_4\rangle \to s_x}{\langle A,\ s_2\rangle \to s_x}\ \mathcal{B}[b]_{s_2} = \mathbf{tt}}{\langle A,\ s_0\rangle \to s_x}\ \mathcal{B}[b]_{s_0} = \mathbf{tt}$$

Program three will never terminate because **true** will never be false, so the inference tree will expand forever.
$\textbf{while}\ 1 \leq \mathtt{x}\ \textbf{do}\ (\mathtt{y} := y * x;\ \mathtt{x} := x - 1)$

$$\begin{array}{l} s_0 = [x \mapsto 0, y \mapsto 0] \\ s_1 = [x \mapsto 0, y \mapsto 0] \\ s_2 = [x \mapsto -1, y \mapsto 0] \end{array} \qquad \begin{array}{rcl} b &=& 1 \leq x \\ A &=& \textbf{while}\ b\ \textbf{do}\ B \\ B &=& \mathtt{y} := y * x;\ C \\ C &=& \mathtt{x} := x - 1 \end{array}$$

$$[\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{y} := \mathtt{y} * \mathtt{x},\ s_0\rangle \to s_1} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ s_1\rangle \to s_2}}{\langle B,\ s_0\rangle \to s_2} \quad \langle \textbf{while}\ b\ \textbf{do}\ B,\ s_2\rangle \to s_x}{\langle A,\ s_0\rangle \to s_x}\ \mathcal{B}[b]_{s_0} = \mathbf{tt}$$

$\textbf{while}\ \mathit{true}\ \textbf{do}\ \textbf{skip}$

$$s_0 = [] \qquad \begin{array}{rcl} b &=& \mathbf{tt} \\ A &=& \textbf{while}\ b\ \textbf{do}\ \textbf{skip} \end{array}$$

$$[\mathsf{while}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{\langle \textbf{skip},\ s_0\rangle \to s_0, \qquad \langle \textbf{while}\ b\ \textbf{do}\ \textbf{skip},\ s_0\rangle \to s_x}{\langle A,\ s_0\rangle \to s_x}\ \mathcal{B}[b]_{s_0} = \mathbf{tt}$$

2.6:

Show that $S1; (S2; S3)$ and $(S1; S2); S3$ are semantically equivalent:

$$\begin{array}{rcl} C_1 &=& S_1;\ C_2 \\ C_2 &=& S_2;\ S_3 \\ C_3 &=& C_4;\ S_3 \\ C_4 &=& S_1;\ S_2 \end{array}$$

$$[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{\langle S_1,\ s_0\rangle \to s_1 \quad [\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{\langle S_2,\ s_1\rangle \to s_2 \qquad \langle S_3,\ s_2\rangle \to s_3}{\langle C_2,\ s_1\rangle \to s_3}}{\langle S_1;\ C_2,\ s_0\rangle \to s_3}$$

$$[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{\langle S_1,\ s_0\rangle \to s_1 \qquad \langle S_2,\ s_1\rangle \to s_2}{\langle C_4,\ s_0\rangle \to s_2} \quad \langle S_3,\ s_2\rangle \to s_3}{\langle C_3;\ S_1,\ s_0\rangle \to s_3}$$

Show that $S1; S2$ and $S2; S1$ are not always semantically equivalent:

$$\begin{array}{l} s_0 = [] \\ s_1 = [x \mapsto 3] \\ s_2 = [x \mapsto 5] \\ s_3 = [x \mapsto 5] \\ s_4 = [x \mapsto 3] \end{array} \qquad \begin{array}{rcl} C_1 &=& S_1;\ S_2 \\ C_2 &=& S_2;\ S_1 \\ S_1 &=& \mathtt{x} := 3 \\ S_2 &=& \mathtt{x} := 5 \end{array}$$

$$[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := 3,\ s_0\rangle \to s_1} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := 5,\ s_1\rangle \to s_2}}{\langle C_1,\ s_0\rangle \to s_2}$$

$$[\mathsf{comp}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := 5,\ s_0\rangle \to s_3} \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{x} := 3,\ s_3\rangle \to s_4}}{\langle C_2,\ s_0\rangle \to s_4}$$

2.7:

The While language can be extended with the $\textbf{repeat}\ S\ \textbf{until}\ b$ statement:

$$[\mathsf{repeat}^{\mathsf{ff}}_{\mathsf{ns}}]\ \dfrac{\textbf{skip}}{\langle \textbf{repeat}\ S\ \textbf{until}\ b,\ s_0\rangle \to s_0}\ \mathcal{B}[b]_{s_0} = \mathbf{ff}$$

$$[\mathsf{repeat}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{\langle S,\ s_0\rangle \to s_1, \qquad \langle \textbf{repeat}\ S\ \textbf{until}\ b,\ s_1\rangle \to s_x}{\langle \textbf{repeat}\ S\ \textbf{until}\ b,\ s_0\rangle \to s_x}\ \mathcal{B}[b]_{s_1} = \mathbf{tt}$$

2.8:

The While language can be extended with the: $\textbf{for}\ \mathtt{A} := a_1\ \textbf{to}\ a_2\ \textbf{do}\ S$, statement:

$$[\mathsf{for}^{\mathsf{ff}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{A} := a_0,\ s_0\rangle \to s_1} \quad \langle S,\ s_1\rangle \to s_2 \quad [\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{A} := \mathsf{a}_0 + 1,\ s_2\rangle \to s_3} \quad \langle \textbf{for}\ \mathtt{A} := a_1\ \textbf{to}\ a_2\ \textbf{do}\ S,\ s_3\rangle \to s_4}{\langle \textbf{for}\ \mathtt{A} := a_1\ \textbf{to}\ a_2\ \textbf{do}\ S,\ s_0\rangle \to s_1}\ \mathcal{B}[a_1 = a_2]_{s_1} = \mathbf{ff}$$

$$[\mathsf{for}^{\mathsf{tt}}_{\mathsf{ns}}]\ \dfrac{[\mathsf{ass}_{\mathsf{ns}}]\ \dfrac{}{\langle \mathtt{A} := a_0,\ s_0\rangle \to s_1} \quad \langle \textbf{skip},\ s_1\rangle \to s_1}{\langle \textbf{for}\ \mathtt{A} := a_1\ \textbf{to}\ a_2\ \textbf{do}\ S,\ s_0\rangle \to s_4}\ \mathcal{B}[a_1 = a_2]_{s_1} = \mathbf{tt}$$