IT-University of Copenhagen

Automated software analysis

Analysis-2

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October 26, 2017

Relations and Partial-Orders:

Signature:

$$:) \subseteq S \times S \times \mathbb{R}$$

Relation: $\{\emptyset, \emptyset\}, \{\emptyset, \{x+1\}\}, \{\emptyset, \{2*y\}\}, \{\emptyset, \{z/3\}\}, \{\emptyset, \{x+1, 2*y\}\}, \{\emptyset, \{x+1, z/3\}\}, \{\emptyset, \{2*y, z/3\}\}, \{\emptyset, \{x+1, 2*y, z/3\}\}, \{\{x+1\}, \{x+1\}\}, \{\{x+1\}, \{x+1, 2*y\}\}, \{\{x+1\}, \{x+1, z/3\}\}, \{\{x+1\}, \{x+1, 2*y, z/3\}\}, \{\{2*y\}, \{2*y\}\}, \{\{2*y\}, \{x+1, 2*y\}\}, \{\{2*y\}, \{2*y, z/3\}\}, \{\{2*y\}, \{x+1, 2*y, z/3\}\}, \{\{z/3\}, \{z/3\}\}, \{\{z/3\}, \{x+1, z/3\}\}, \{\{z/3\}, \{2*y, z/3\}\}, \{\{z/3\}, \{x+1, 2*y, z/3\}\}, \{\{x+1, 2*y\}, \{x+1, 2*y\}\}, \{\{x+1, 2*y\}, \{x+1, 2*y, z/3\}\}, \{\{2*y, z/3\}, \{2*y, z/3\}\}, \{\{2*y, z/3\}, \{x+1, 2*y, z/3\}\}, \{\{x+1, z/3\}, \{x+1, z/3\}\}, \{\{x+1, z/3\}, \{x+1, 2*y, z/3\}\}, \{\{x+1, 2*y, z/3\}, \{x+1, 2*y, z/3\}\},$

An example of a member of the relation:

$$(\{x+1, z/3\}, \{x+1, z/3\}) \in :)$$

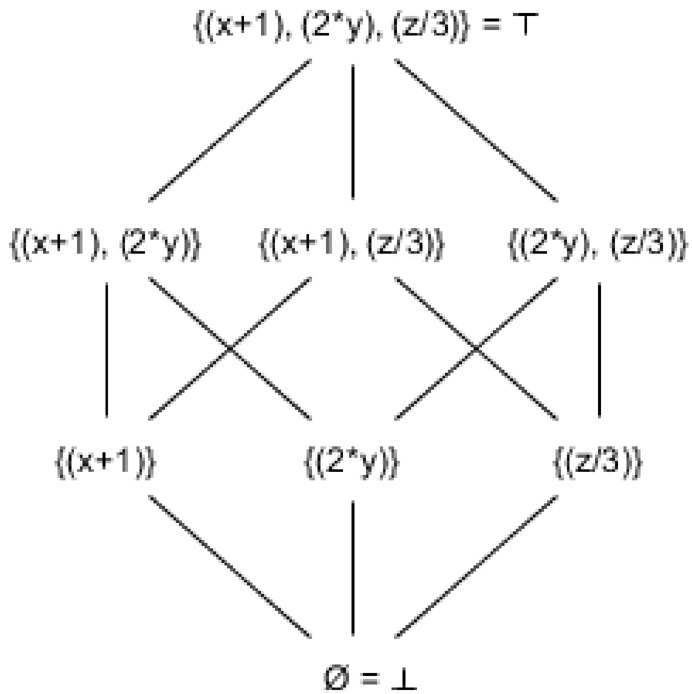
$$\{x+1, z/3\} :) \{x+1, z/3\}$$

An example of a non-member of the relation:

$$(\{x+1, z/3\}, \{x+1, z/3\}) \notin :)$$

$$\{x+1, z/3\} \not{:)} \{x+1, z/3\}$$

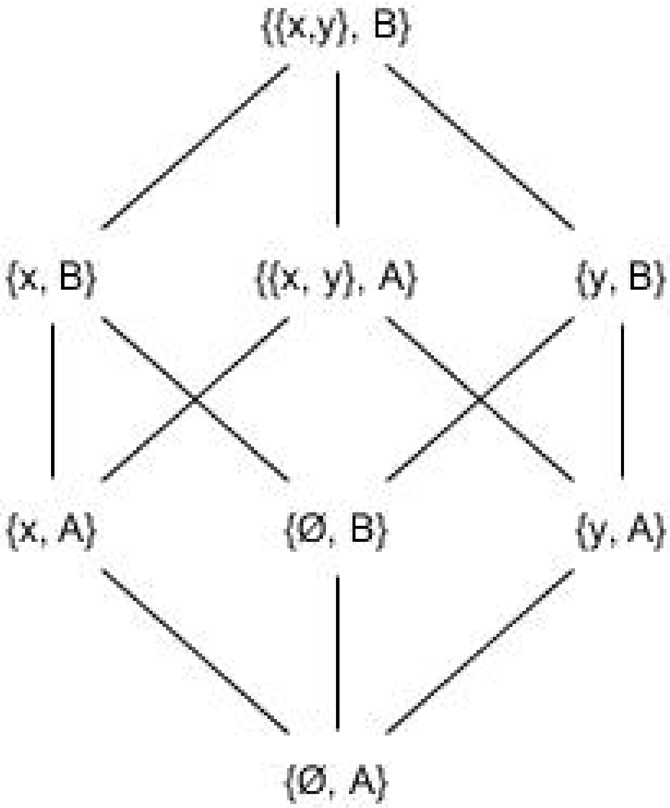
Does the set S and relation form a partial-order?



Yes Draw a Hasse diagram:

Greatest-Lower-Bound:

Lattices:



A $L_1 \times L_2$ lattice will have $|L_1| * |L_2|$ points. So the resulting lattice will in this case have $2 * 4 = 8$ points.

Monotone Functions and Fixed-Points:

i)

$$
\begin{aligned}
X &= \{a,\ b\} \\
Y &= X \cup Y
\end{aligned}
$$

In equation form:

$$
\begin{aligned}
X &= f(X, Y) = \{a,\ b\} \\
Y &= g(X, Y) = X \cup Y
\end{aligned}
$$

Can be written as:

$$
\begin{aligned}
X &= f() = \{a,\ b\} \\
Y &= g(X, Y) = X \cup Y
\end{aligned}
$$

The function $f()$ is monotone, as it is constant. Therefore $f() \sqsubseteq f()$ will always hold. The function $g(X,\ Y)$ is also monotone, as no matter which sets use give the function it will combine them and the resulting set will always be greater or equal to the input sets.

ii)

$$
\begin{aligned}
X &= \{a,\ b\} \cup Y \\
Y &= X \setminus \{b\}
\end{aligned}
$$

In equation form:

$$
\begin{aligned}
X &= f(X, Y) = \{a,\ b\} \cup Y \\
Y &= g(X, Y) = X \setminus \{b\}
\end{aligned}
$$

Can be written as:

$$X = f(Y) = \{a,\ b\} \cup Y$$
$$Y = g(X) = X \setminus \{b\}$$

The function $f(Y)$ is monotone, as it return $\{a,\ b\} \cup Y$ which is always a superset of Y. The function $g(X)$ is also monotone. It is hard to give an example of why, however intuitively removing a constant element preserves monotonicity, as the same element is removed on both sides of $\sqsubseteq$.

iii)

$$X = \{a,\ b\} \cup Z$$
$$Y = \{a,\ c\} \setminus X$$
$$Z = X^C$$

In equation form:

$$X = f(X,\ Y,\ Z) = \{a,\ b\} \cup Z$$
$$Y = g(X,\ Y,\ Z) = \{a,\ c\} \setminus X$$
$$Z = h(X,\ Y,\ Z) = X^C$$

Can be written as:

$$X = f(Z) = \{a,\ b\} \cup Z$$
$$Y = g(X) = \{a,\ c\} \setminus X$$
$$Z = h(X) = X^C$$

The function $f(Z)$ is monotone, as it return $\{a,\ b\} \cup Z$ which is always a superset of Z. If $g(X)$ is monotone then $\forall\, Y,\ Y'\ P(\{a,\ b,\ c\}) : Y\ \sqsubseteq\ Y'$. However this is not the case:

$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ c\} \not\sqsubseteq \emptyset$$

$g(X)$ is therefore not monotone. $h(X)$ is not monotone either, as:

$$g(\emptyset) \not\sqsubseteq g(\{a,\ b,\ c\})$$
$$\{a,\ b\ c\} \not\sqsubseteq \emptyset$$