2.3:

$$
\dfrac{\dfrac{}{\langle \mathtt{x} := 3,\ [] \rangle \to [x \mapsto 3]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{\dfrac{}{\langle \mathtt{y} := 0,\ [x \mapsto 3] \rangle \to [x \mapsto 3,\ y \mapsto 0]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{\dfrac{\dfrac{}{\langle \mathtt{y} := \mathtt{y} + \mathtt{x},\ [x \mapsto 3,\ y \mapsto 0] \rangle \to [x \mapsto 3,\ y \mapsto 3]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ [x \mapsto 3,\ y \mapsto 3] \rangle \to [x \mapsto 2,\ y \mapsto 3]}\,[\mathsf{ass}_{\mathsf{u}}]}{\langle D,\ [x \mapsto 3,\ y \mapsto 0] \rangle \to [x \mapsto 2,\ y \mapsto 3]}\,[\mathsf{comp}_{\mathsf{u}}] \quad \dfrac{\dfrac{\dfrac{}{\langle \mathtt{y} := \mathtt{y} + \mathtt{x},\ [x \mapsto 2,\ y \mapsto 3] \rangle \to [x \mapsto 2,\ y \mapsto 5]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ [x \mapsto 2,\ y \mapsto 5] \rangle \to [x \mapsto 1,\ y \mapsto 5]}\,[\mathsf{ass}_{\mathsf{u}}]}{\langle D,\ [x \mapsto 2,\ y \mapsto 3] \rangle \to [x \mapsto 1,\ y \mapsto 5]}\,[\mathsf{comp}_{\mathsf{u}}] \quad \dfrac{\dfrac{\dfrac{}{\langle \mathtt{y} := \mathtt{y} + \mathtt{x},\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 1,\ y \mapsto 6]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ [x \mapsto 1,\ y \mapsto 6] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{ass}_{\mathsf{u}}]}{\langle D,\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{comp}_{\mathsf{u}}] \quad \dfrac{}{\langle C,\ [x \mapsto 0,\ y \mapsto 6] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{ff}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 0,\ y \mapsto 6]} = \mathtt{ff}}{\langle C,\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 1,\ y \mapsto 5]} = \mathtt{tt}}{\langle C,\ [x \mapsto 2,\ y \mapsto 3] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 2,\ y \mapsto 3]} = \mathtt{tt}}{\langle C,\ [x \mapsto 3,\ y \mapsto 0] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 3,\ y \mapsto 0]} = \mathtt{tt}}{\langle B,\ [x \mapsto 3] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{comp}_{\mathsf{u}}]}{\langle A,\ [] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{comp}_{\mathsf{u}}]
\qquad T_3 \quad = \quad [\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \dfrac{\dfrac{\dfrac{\dfrac{}{\langle \mathtt{y} := \mathtt{y} + \mathtt{x},\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 1,\ y \mapsto 6]}\,[\mathsf{ass}_{\mathsf{u}}] \quad \dfrac{}{\langle \mathtt{x} := \mathtt{x} - 1,\ [x \mapsto 1,\ y \mapsto 6] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{ass}_{\mathsf{u}}]}{\langle D,\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{comp}_{\mathsf{u}}] \quad \dfrac{}{\langle C,\ [x \mapsto 0,\ y \mapsto 6] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{ff}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 0,\ y \mapsto 6]} = \mathtt{ff}}{\langle C,\ [x \mapsto 1,\ y \mapsto 5] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 1,\ y \mapsto 5]} = \mathtt{tt}}{\langle C,\ [x \mapsto 2,\ y \mapsto 3] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 2,\ y \mapsto 3]} = \mathtt{tt}}{\langle C,\ [x \mapsto 3,\ y \mapsto 0] \rangle \to [x \mapsto 0,\ y \mapsto 6]}\,[\mathsf{while}^{\mathtt{tt}}_{\mathsf{u}}]\ \mathcal{B}[\![\neg(\mathtt{x} \leq 0)]\!]_{[x \mapsto 3,\ y \mapsto 0]} = \mathtt{tt}
$$

2.4:

Do the following statements terminate:
while $\neg(\mathtt{x} = 1)$ **do** $(\mathtt{y} := y * x;\ \mathtt{x} := x - 1)$

while $1 \leq \mathtt{x}$ **do** $(\mathtt{y} := y * x;\ \mathtt{x} := x - 1)$

while $true$ **do skip**

2.6:

Show that $S1; (S2; S3)$ and $(S1; S2); S3$ are semantically equivalent:

Show that $S1; S2$ and $S2; S1$ are not always semantically equivalent:

2.7:

While can be extended with the: **repeat** S **until** b statement:

2.8: