IT-UNIVERSITY OF COPENHAGEN

AUTOMATED SOFTWARE ANALYSIS

Analysis-2

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Relations and Partial-Orders:

Signature:

$$:) \subseteq S \times S$$

Members of the relation:
$\{(\emptyset,\emptyset),(\emptyset,\{x+1\}),(\emptyset,\{2*y\}),(\emptyset,\{z/3\}),(\emptyset,\{x+1,2*y\}),(\emptyset,\{x+1,z/3\}),(\emptyset,\{2*y,z/3\}),(\emptyset,\{x+1,2*y,z/3\}),(\{x+1\},\{x+1\}),(\{x+1\},\{x+1,2*y\}),(\{x+1\},\{x+1,z/3\}),(\{x+1\},\{x+1,2*y,z/3\}),(\{2*y\},\{2*y\}),(\{2*y\},\{x+1,2*y\}),(\{2*y\},\{2*y,z/3\}),(\{2*y\},\{x+1,2*y,z/3\}),(\{z/3\},\{z/3\}),(\{z/3\},\{x+1,z/3\}),(\{z/3\},\{2*y,z/3\}),(\{z/3\},\{x+1,2*y,z/3\}),(\{x+1,2*y\},\{x+1,2*y\}),(\{x+1,2*y\},\{x+1,2*y,z/3\}),(\{2*y,z/3\},\{2*y,z/3\}),(\{2*y,z/3\},\{x+1,2*y,z/3\}),(\{x+1,z/3\},\{x+1,z/3\}),(\{x+1,z/3\},\{x+1,2*y,z/3\}),(\{x+1,2*y,z/3\},\{x+1,2*y,z/3\})\}$

An example of a member of the relation:

$$(\{x+1,z/3\},\{x+1,z/3\}) \in :)$$

$$\{x+1,z/3\} :) \{x+1,z/3\}$$

An example of a non-member of the relation:

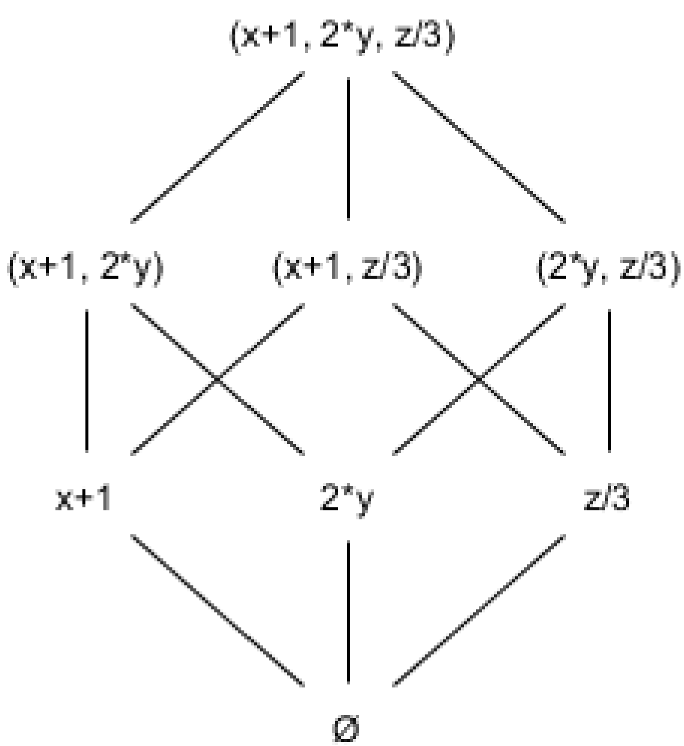
$$(\{x+1,z/3\},\{x+1,z/3\}) \notin :)$$

$$\{x+1,z/3\} \not{:)} \{x+1,z/3\}$$

Do the set S and relation :) form a partial-order? Yes, because The relation is:

- Reflexive, every element is related to itself.
- Antisymmetric, two elements must not be related in both directions (unless they are the same element).
- Transitive, if the first element is related to the second element, and the second is related to the third, then the first is related to the third.

Draw a Hasse diagram:



Greatest-Lower-Bound:

A lower bound x for a set S is defined as:

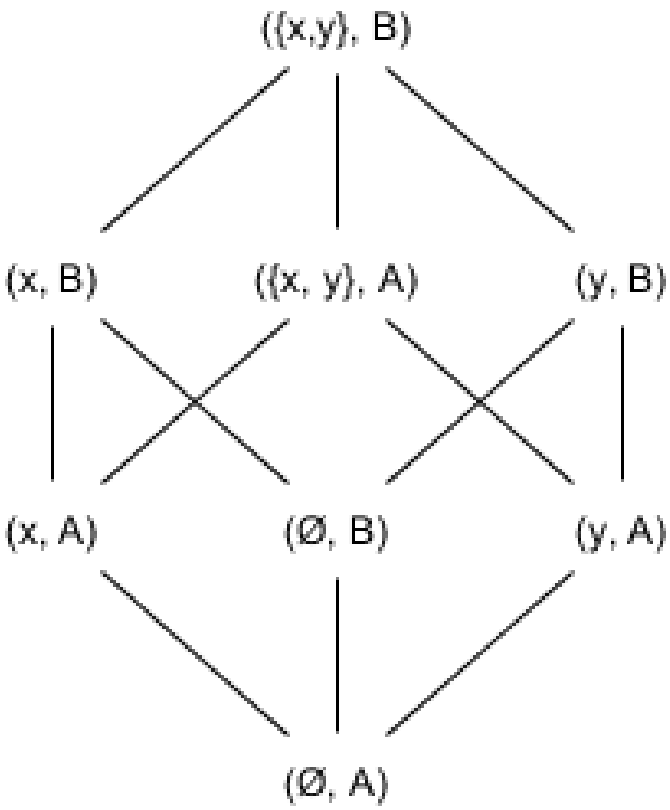
$$\forall s \in S : \; s \sqsupseteq x$$

If we want to define the greatest lower bound the following must hold:

$$S \sqsupseteq x \wedge \forall y : \; S \sqsupseteq y \implies x \sqsupseteq y$$

Given a lattice $L = (S, \sqsubseteq)$; what do the elements $\sqcup S$ and $\sqcap S$ correspond to? Assuming S from above, $\sqcap S$ is $\{x + 1, 2 * y, z/3\}\}$, and $\sqcup S$ is $\emptyset$

Lattices:



A $L_1 \times L_2$ lattice will have $|L_1| * |L_2|$ points. So the resulting lattice will in this case have $2 * 4 = 8$ points.