

IT-UNIVERSITY OF COPENHAGEN

AUTOMATED SOFTWARE ANALYSIS

Analysis-2

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Relations and Partial-Orders:

Signature:

$$:) \subseteq S \times S$$

Members of the relation:

$$\{(\emptyset, \emptyset), (\emptyset, \{x+1\}), (\emptyset, \{2*y\}), (\emptyset, \{z/3\}), (\emptyset, \{x+1, 2*y\}), (\emptyset, \{x+1, z/3\}), (\emptyset, \{2*y, z/3\}), (\emptyset, \{x+1, 2*y, z/3\}), (\{x+1\}, \{x+1\}), (\{x+1\}, \{x+1, 2*y\}), (\{x+1\}, \{x+1, z/3\}), (\{x+1\}, \{x+1, 2*y, z/3\}), (\{2*y\}, \{2*y\}), (\{2*y\}, \{x+1, 2*y\}), (\{2*y\}, \{2*y, z/3\}), (\{2*y\}, \{x+1, 2*y, z/3\}), (\{z/3\}, \{z/3\}), (\{z/3\}, \{x+1, z/3\}), (\{z/3\}, \{2*y, z/3\}), (\{z/3\}, \{x+1, 2*y, z/3\}), (\{x+1, 2*y\}, \{x+1, 2*y\}), (\{x+1, 2*y\}, \{x+1, 2*y, z/3\}), (\{2*y, z/3\}, \{2*y, z/3\}), (\{2*y, z/3\}, \{x+1, 2*y, z/3\}), (\{x+1, z/3\}, \{x+1, z/3\}), (\{x+1, z/3\}, \{x+1, 2*y, z/3\}), (\{x+1, 2*y, z/3\}, \{x+1, 2*y, z/3\})\}$$

An example of a member of the relation:

$$\begin{aligned} (\{x+1, z/3\}, \{x+1, z/3\}) &\in :) \\ \{x+1, z/3\} &:) \{x+1, z/3\} \end{aligned}$$

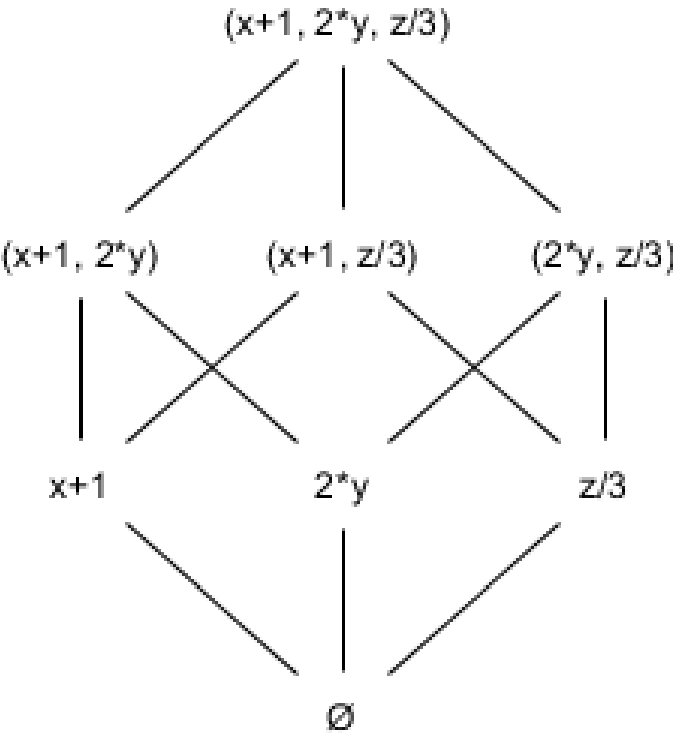
An example of a non-member of the relation:

$$\begin{aligned} (\{x+1, z/3\}, \{x+1, z/3\}) &\notin :) \\ \{x+1, z/3\} &:) \not\{x+1, z/3\} \end{aligned}$$

Do the set S and relation :) form a partial-order? Yes, because The relation is:

- Reflexive, every element is related to itself.
- Antisymmetric, two elements must not be related in both directions (unless they are the same element).
- Transitive, if the first element is related to the second element, and the second is related to the third, then the first is related to the third.

Draw a Hasse diagram:



Greatest-Lower-Bound:

A lower bound x for a set S is defined as:

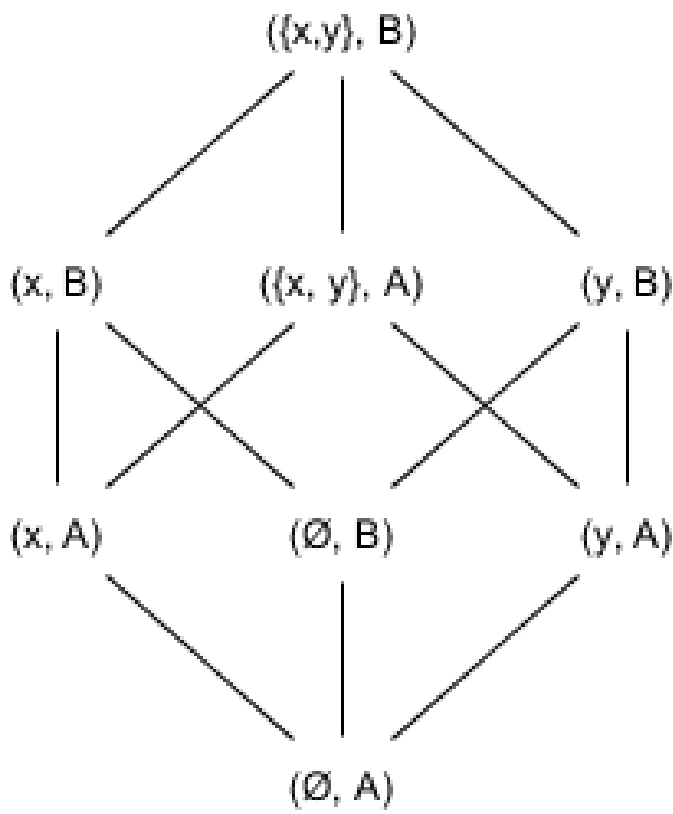
$$\forall s \in S : s \supseteq x$$

If we want to define the greatest lower bound the following must hold:

$$S \supseteq x \wedge \forall y : S \supseteq y \implies x \supseteq y$$

Given a lattice $L = (S, \sqsubseteq)$; what do the elements $\sqcup S$ and $\sqcap S$ correspond to? Assuming S from above, $\sqcap S$ is $\{x + 1, 2 * y, z/3\}$, and $\sqcup S$ is \emptyset

Lattices:



A $L_1 \times L_2$ lattice will have $|L_1| * |L_2|$ points. So the resulting lattice will in this case have $2 * 4 = 8$ points.