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Introduction

The Official Guide for GMAT Review has been designed and written by the staff of Educational Testing Service (ETS), which prepares the Graduate Management Admission Test used by many graduate schools of business and management as one criterion in considering applications for admission to the schools' graduate programs. This book is intended to be a general guide to the kinds of questions likely to appear in the GMAT. All questions used to illustrate the various types of verbal and mathematical multiple-choice questions are taken from actual administrations of the test.*

The questions that appear in this *Guide* are presented in the format used in the paper-based version of the GMAT. The GMAT is now administered as a computer-adaptive test (CAT) in most countries. The GMAT includes the question types found in this *Guide*, but the format and presentation of the questions is different on the computer.

- Only one question at a time is presented on the computer screen.
- The answer choices for the multiple-choice questions are preceded by ovals rather than by letters.
- You select your answer using the computer mouse.
- You must choose an answer and confirm your choice before moving on to the next question; you cannot skip any question, and you cannot return to a question or change your answer once you have confirmed your answer and moved on to the next question.
- The questions in this *Guide* are arranged by question type, whereas in the GMAT the multiple-choice quantitative questions appear in one timed section and the multiple-choice verbal questions appear in another separately timed section; within these sections questions of different types may appear in any order.
- The two analytical writing sections are separately timed, and you write your responses for these sections using a word processor that is part of the computer-based test.

For more information about how you will respond to questions in the test, see chapter 2, Answering GMAT Questions.

The GMAT is not a test of knowledge in specific subjects — for example, it does not test knowledge specifically or uniquely acquired in accounting or economics courses. Rather, it is a test of certain skills and abilities that have been found to contribute to success in graduate programs in business and management. For this reason, it is useful to become familiar with the general types of questions likely to be found in GMAT and the reasoning skills, analytical writing skills, and problem-solving strategies that these types of questions demand. This book illustrates various types of questions that appear in the GMAT and explains in detail some of the most effective strategies for mastering these questions.

The most efficient and productive way to use this book is to read first through chapters 1 and 2. Each type of question is briefly described, the directions are given, and the skills each question type measures are outlined. You should pay particular attention to the directions for each question type. This is especially important for the data sufficiency questions, which have lengthy and complex directions, and for the Analytical Writing Assessment, which requires you to discuss the complexities of a given issue and to critique a given argument. In chapter 2 you will learn how to respond to questions in the GMAT.

You may find it useful to read through all of chapter 3, Math Review, before working through chapters 4, Problem Solving, and 5, Data Sufficiency, or you may wish to use chapter 3 as a reference as you work on chapters 4 and 5. However, because chapter 3 is intended to provide you with a comprehensive review of the basic mathematical concepts used in the quantitative section of the GMAT, you may find it valuable to read through the chapter as a whole.

Chapters 4-9 provide detailed illustrations and explanations of individual question types. You will find the most advantageous way to use the book to study for the multiple-choice sections of the test is to choose a chapter on a particular multiple-choice question type; read the introductory material carefully, and then do the sample questions in that chapter. As you do the sample questions, follow the directions and try to work as quickly and efficiently as possible. Then review the questions and explanations, spending as much time as is necessary to familiarize yourself with the range of questions or problems presented.

The chapter on the Analytical Writing Assessment (chapter 9) is somewhat different from those on the multiple-choice questions. It presents writing tasks that may appear on the test as well as a selection of actual examinee responses to two questions. Each response is followed by an explanation of why it was awarded a particular score. You will also see the general scoring guides that readers use to score the responses. Chapter 9, Analytical Writing Assessment, provides all the information you need to familiarize yourself thoroughly with the kinds of writing tasks that you will see in the GMAT as well as with the standards that will be used in judging your responses.

*The material in *The Official Guide for GMAT Review* is intended to familiarize you with the types of questions found in the GMAT. Although the sample questions in chapters 4-9 represent the general nature of the questions in the test, it is possible that a type of question not illustrated by and explained in the *Guide* may appear on the GMAT. It is also possible that material illustrated by and explained in the *Guide* may not appear on the test.

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Because a computer-adaptive test cannot be presented in paper form, practice versions of the GMAT have been developed to allow you to experience the computer-adaptive test and gauge your preparedness for the GMAT before you actually take it. *Test Preparation for the GMAT POWERPREP®* software is available for the personal computer and includes two GMAT tests as well as practice questions and information about the test. POWERPREP is now provided to all registrants for the GMAT, and is also available for download at www.gmat.org

NOTE: The tests in POWERPREP are made up of questions that appear in this *Guide*. If you are using the *Guide* and POWERPREP together, you may want to take the POWERPREP tests before reviewing the questions in this book. Prior familiarity with the questions you receive could make the POWERPREP tests easier for you than they would otherwise be and could artificially inflate your scores.

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1 Description of the Graduate Management Admission Test

The Graduate Management Admission Test is designed to help graduate schools assess the qualifications of applicants for advanced study in business and management The test can be used by both schools and students in evaluating verbal and mathematical skills as well as general knowledge and preparation for graduate study Note, however, that GMAT scores should be considered as only one of several indicators of ability.

Format

The GMAT consists of four separately timed sections (see table below) Each of the first two sections contains a 30-minute writing task; the other two sections are 75 minutes each and contain multiple-choice questions The first of these sections contains quantitative questions, and the second contains verbal questions

Every test contains trial multiple-choice questions needed for pretesting for future use These questions, however, are not identified and appear in varying locations within the test You should therefore do your best on all questions Answers to trial questions are not counted in the scoring of your test

In a computer-adaptive test, questions are chosen from a very large pool of test questions categorized by content and difficulty Only one question at a time is presented The test is constantly trying to target your individual ability level, this means that the questions you are presented with depend on your answers to all previous questions Consequently, you must enter an answer for each question and may not return to or change your answer to any previous question If you answer a question incorrectly by mistake — or correctly by lucky guess — your answer to subsequent questions will lead you back to questions that are at the appropriate level of difficulty for you

Your scores will depend on the statistical characteristics of the questions presented to you, including difficulty level; your answers to those questions, and the number of questions you answer Adaptive test score calculations do not assign any differential credit to questions depending on where they appear in the test The questions in an adaptive test are weighted according to their difficulty and other statistical properties, not according to their position in the test However, because the test is adaptive, the responses provided to early questions do influence the selection of later questions

Format of the GMAT

	Questions	Timing
Analytical Writing		
Analysis of an Issue	1	30 min
Analysis of an Argument	1	30 min
Optional Break		5 min
Quantitative	37	75 min
Problem Solving		
Data Sufficiency		
Optional Break		5 min
Verbal	41	75 min
Reading Comprehension		
Critical Reasoning		
Sentence Correction		
	Total Time	**4 hours (approx.)**

Content

It is important to recognize that the GMAT evaluates skills and abilities that develop over relatively long periods of time Although the sections are basically verbal or mathematical, the complete test provides one method of measuring overall ability The GMAT does not test specific knowledge obtained in college course work, and it does not seek to measure achievements in any specific areas of study

The Graduate Management Admission Council recognizes that questions arise concerning techniques for taking standardized examinations such as the GMAT, and it is hoped that the descriptions, sample questions, and explanations given here will give you a practical familiarity with the concepts and techniques required by GMAT questions

All of the multiple-choice questions in this book have appeared in the actual GMAT

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Quantitative Section

The quantitative section of the GMAT measures basic mathematical skills and understanding of elementary concepts, and the ability to reason quantitatively, solve quantitative problems, and interpret graphic data

Two types of multiple-choice questions are used in the quantitative section.

- problem solving
- data sufficiency

Problem solving and data sufficiency questions are intermingled throughout the section Both types of questions require knowledge of

- arithmetic
- elementary algebra
- commonly known concepts of geometry

Problem Solving Questions

Problem solving questions are designed to test basic mathematical skills, understanding of elementary mathematical concepts, and the ability to reason quantitatively and to solve quantitative problems

The directions for problem solving questions read as follows

Solve the problem and indicate the best of the answer choices given

Numbers: All numbers used are real numbers.

Figures. A figure accompanying a problem solving question is intended to provide information useful in solving the problem Figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. Straight lines may sometimes appear jagged All figures lie in a plane unless otherwise indicated

Data Sufficiency Questions

Each data sufficiency question consists of a question, often accompanied by some initial information, and two statements, labeled (1) and (2), containing additional information You must decide whether sufficient information to answer the question is given by either (1) or (2) individually or, if not, by both combined

Data sufficiency questions are designed to measure your ability to analyze a quantitative problem, to recognize which information is relevant, and to determine at what point there is sufficient information to solve the problem

These are the directions for data sufficiency questions Read them carefully.

This data sufficiency problem consists of a question and two statements, labeled (1) and (2), in which certain data are given You have to decide whether the data given in the statements are sufficient for answering the question Using the data given in the statements plus your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of *counterclockwise*), you must indicate whether

- statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked,
- BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient,
- EACH statement ALONE is sufficient to answer the question asked,
- statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed

Numbers All numbers used are real numbers

Figures. A figure accompanying a data sufficiency problem will conform to the information given in the question, but will not necessarily conform to the additional information given in statements (1) and (2)

Lines shown as straight can be assumed to be straight and lines that appear jagged can also be assumed to be straight

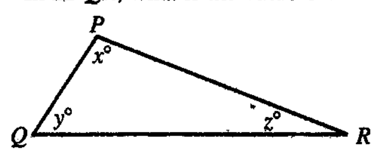
You may assume that the positions of points, angles, regions, etc., exist in the order shown and that angle measures are greater than zero.

All figures lie in a plane unless otherwise indicated

Note In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for the quantity

Example:

In ΔPQR, what is the value of x?



(1) $PQ = PR$
(2) $y = 40$

Explanation: According to statement (1), $PQ = PR$, therefore, ΔPQR is isosceles and $y = z$. Since $x + y + z = 180$, it follows that $x + 2y = 180$. Since statement (1) does not give a value for y, you cannot answer the question using statement (1) alone According to statement (2), $y = 40$, therefore, $x + z = 140$ Since statement (2) does not give a value for z, you cannot answer the question using statement (2) alone Using both statements together, since $x + 2y = 180$ and the value of y is given, you can find the value of x Therefore, BOTH statements (1) and (2) TOGETHER are sufficient to answer the question, but NEITHER statement ALONE is sufficient

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Verbal Section

The verbal section of the GMAT measures your ability to read and comprehend written material, to reason and evaluate arguments, and to correct written material to conform to standard written English

Three types of multiple-choice questions are used in the verbal section of the GMAT

- reading comprehension
- critical reasoning
- sentence correction

These question types are intermingled throughout the verbal section

Reading Comprehension Questions

Reading comprehension passages are accompanied by interpretive, applicative, and inferential questions The passages are up to 350 words long, and they discuss topics from the social sciences, the physical or biological sciences, and such business-related fields as marketing, economics, and human resource management Because the verbal section of the GMAT includes passages from several different content areas, you may be generally familiar with some of the material, however, neither the passages nor the questions assume detailed knowledge of the topics discussed

WHAT IS MEASURED

Reading comprehension questions measure your ability to understand, analyze, and apply information and concepts presented in written form All questions are to be answered on the basis of what is stated or implied in the reading material, and no specific knowledge of the material is required

Reading comprehension therefore evaluates your ability to

- understand words and statements in the reading passages (Questions of this type are not vocabulary questions These questions test your understanding of and ability to comprehend terms used in the passage as well as your understanding of the English language You may also find that questions of this type ask about the overall meaning of a passage)
- understand the logical relationships between significant points and concepts in the reading passages (For example, such questions may ask you to determine the strong and weak points of an argument or to evaluate the importance of arguments and ideas in a passage)
- draw inferences from facts and statements in the reading passages (The inference questions will ask you to consider factual statements or information and, on the basis of that information, reach a general conclusion)
- understand and follow the development of quantitative concepts as they are presented in verbal material (This may involve the interpretation of numerical data or the use of simple arithmetic to reach conclusions about material in a passage)

The directions for reading comprehension questions read as follows

The questions in this group are based on the content of a passage After reading the passage, choose the best answer to each question Answer all questions following the passage on the basis of what is <u>stated</u> or <u>implied</u> in the passage

Critical Reasoning Questions

Critical reasoning questions are designed to test the reasoning skills involved (1) in making arguments, (2) in evaluating arguments, and (3) in formulating or evaluating a plan of action The materials on which questions are based are drawn from a variety of sources. No familiarity with the subject matter of those materials is presupposed

WHAT IS MEASURED

Critical reasoning questions are designed to provide one measure of your ability to reason effectively in the areas of

- argument construction (Questions in this category may ask you to recognize such things as the basic structure of an argument, properly drawn conclusions, underlying assumptions, well-supported explanatory hypotheses, parallels between structurally similar arguments)
- argument evaluation (Questions in this category may ask you to analyze a given argument and to recognize such things as factors that would strengthen, or weaken, the given argument, reasoning errors committed in making that argument, aspects of the method by which the argument proceeds)
- formulating and evaluating a plan of action (Questions in this category may ask you to recognize such things as the relative appropriateness, effectiveness, or efficiency of different plans of action, factors that would strengthen, or weaken, the prospects of success for a proposed plan of action, assumptions underlying a proposed plan of action)

The directions for critical reasoning questions read as follows

For this question, select the best of the answer choices given

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Sentence Correction Questions

Sentence correction questions ask you which of the five choices best expresses an idea or relationship The questions will require you to be familiar with the stylistic conventions and grammatical rules of standard written English and to demonstrate your ability to improve incorrect or ineffective expressions

WHAT IS MEASURED

Sentence correction questions test two broad aspects of language proficiency

1 *Correct expression* A correct sentence is grammatically and structurally sound It conforms to all the rules of standard written English (for example noun-verb agreement, noun-pronoun agreement, pronoun consistency, pronoun case, and verb tense sequence) Further, a correct sentence will not have dangling, misplaced, or improperly formed modifiers, unidiomatic or inconsistent expressions, or faults in parallel construction

2 *Effective expression* An effective sentence expresses an idea or relationship clearly and concisely as well as grammatically This does not mean that the choice with the fewest and simplest words is necessarily the best answer It means that there are no superfluous words or needlessly complicated expressions in the best choice

In addition, an effective sentence uses proper diction (Diction refers to the standard dictionary meanings of words and the appropriateness of words in context) In evaluating the diction of a sentence, you must be able to recognize whether the words are well chosen, accurate, and suitable for the context

The directions for sentence correction questions read as follows

This question presents a sentence, part of which or all of which is underlined Beneath the sentence you will find five ways of phrasing the underlined part The first of these repeats the original, the other four are different If you think the original is best, choose the first answer, otherwise choose one of the others

This question tests correctness and effectiveness of expression In choosing your answer, follow the requirements of standard written English, that is, pay attention to grammar, choice of words, and sentence construction Choose the answer that produces the most effective sentence, this answer should be clear and exact, without awkwardness, ambiguity, redundancy, or grammatical error

Analytical Writing Assessment

The Analytical Writing Assessment consists of two 30-minute writing tasks, "Analysis of an Issue" and "Analysis of an Argument" For the Analysis of an Issue task, you will need to analyze a given issue or opinion and then explain your point of view on the subject by citing relevant reasons and/or examples drawn from your experience, observations, or reading For the Analysis of an Argument task, you will need to analyze the reasoning behind a given argument and then write a critique of that argument You may, for example, consider what questionable assumptions underlie the thinking, what alternative explanations or counterexamples might weaken the conclusion, or what sort of evidence could help strengthen or refute the argument

WHAT IS MEASURED

The Analytical Writing Assessment is designed as a direct measure of your ability to think critically and to communicate your ideas. More specifically, the Analysis of an Issue task tests your ability to explore the complexities of an issue or opinion and, if appropriate, to take a position informed by your understanding of those complexities The Analysis of an Argument task tests your ability to formulate an appropriate and constructive critique of a specific conclusion based upon a specific line of thinking

The issue and argument that you will find on the test concern topics of general interest, some related to business and some pertaining to a variety of other subjects It is important to note, however, that none presupposes any specific knowledge of business or of other specific content areas only your capacity to write analytically is being assessed.

College and university faculty members from various subject matter areas, including but not confined to management education, will evaluate how well you write To qualify as GMAT readers, they must first demonstrate their ability to evaluate a large number of sample responses accurately and reliably, according to GMAT standards and scoring criteria Once qualified, readers will consider both the overall quality of your ideas about the issue and argument presented and your overall ability to organize, develop, and express those ideas, to provide relevant supporting reasons and examples, and to control the elements of standard written English In addition, responses may be scored by e-rater™, an automated scoring program designed to reflect the judgment of expert readers *

* In considering the elements of standard written English, readers are trained to be sensitive and fair in evaluating the responses of English as a Second Language [ESL] examinees

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The directions for the two writing tasks in the Analytical Writing Assessment read as follows

ANALYSIS OF AN ISSUE

In this section, you will need to analyze the issue presented and explain your views on it There is no "correct" answer Instead, you should consider various perspectives as you develop your own position on the issue.

WRITING YOUR RESPONSE: Take a few minutes to think about the issue and plan a response before you begin writing Be sure to organize your ideas and develop them fully, but leave time to reread your response and make any revisions that you think are necessary

EVALUATION OF YOUR RESPONSE: College and university faculty members from various subject-matter areas, including management education, will evaluate the overall quality of your thinking and writing They will consider how well you

- organize, develop, and express your ideas about the issue presented
- provide relevant supporting reasons and examples
- control the elements of standard written English

ANALYSIS OF AN ARGUMENT

In this section you will be asked to write a critique of the argument presented *You are NOT being asked to present your own views on the subject*

WRITING YOUR RESPONSE. Take a few minutes to evaluate the argument and plan a response before you begin writing Be sure to organize your ideas and develop them fully, but leave time to reread your response and make any revisions that you think are necessary

EVALUATION OF YOUR RESPONSE. College and university faculty members from various subject-matter areas, including management education, will evaluate the overall quality of your thinking and writing They will consider how well you

- organize, develop, and express your ideas about the argument presented
- provide relevant supporting reasons and examples
- control the elements of standard written English

Examples of both types of writing tasks in the Analytical Writing Assessment can be found in chapter 9.

General Test-taking Suggestions

Specific test-taking strategies for individual question types are presented in chapters 4-9 The following are general suggestions to help you perform your best on the GMAT

1 Although the GMAT stresses accuracy more than speed, it is important to use the allotted time wisely You will be able to do so if you are familiar with the mechanics of the test and the kinds of materials, questions, and directions in the test Therefore, become familiar with the formats and requirements of each section of the test

2 After you become generally familiar with all question types, use the individual chapters on each question type in this book (chapters 4-9), which include sample questions and detailed explanations, to prepare yourself for the actual GMAT

3 Read all test directions carefully The directions explain exactly what is required in order to answer each question type If you read hastily, you may miss important instructions and seriously jeopardize your scores To review directions during the test, click on the Help icon

4 In the multiple-choice sections, it is important to try to answer all of the questions in the section If a question is too difficult for you, do not waste time on it, eliminate as many answer choices as possible, select the best answer from among the remaining choices, and move on to the next question **Keep moving through the test and try to finish each section. There is a chance that guessing at the end of the test can seriously lower your score. The best strategy is to pace yourself so that you have time to consider each test question, so you don't have to guess.**

5 The best way to approach the two writing tasks comprising the Analytical Writing Assessment is to take a few minutes to think about each question and plan a response before you begin writing Take care to organize your ideas and develop them fully, but leave time to reread your response and make any revisions that you think would improve it

6 On all sections of the test, make every effort to pace yourself Consult the on-screen timer periodically and note the time remaining during your testing session Work steadily and as rapidly as possible without being careless It is not wise to spend too much time on one question if that causes you to neglect other questions On the average, a verbal question takes about 1 3/4 minutes and a quantitative question takes about 2 minutes to answer Give yourself enough time to answer every question **If you do not finish in the allotted time, you will still get a score as long as you've worked on every section. However, your score will reflect the number of questions answered, and most test takers get higher scores when they finish each section.**

7 On all sections of the test, multiple-choice and writing, read each question carefully and thoroughly Before answering a question,

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determine exactly what is being asked Never skim a question or, in the case of a multiple-choice question, the possible answers Skimming may cause you to miss important information or nuances in the question

8 Do not become upset if you have to guess at a question in a multiple-choice section A person can do very well without answering every question correctly No one is expected to get a perfect score.

Test Development Process

Educational Testing Service professional staff responsible for developing the verbal and writing measures of the GMAT have backgrounds and advanced degrees in the humanities, in measurement, or in writing assessment Those responsible for the quantitative portion have advanced degrees in mathematics or related fields

Standardized procedures have been developed to guide the test-generation process, to assure high-quality test material, to avoid idiosyncratic questions, and to encourage development of test material that is widely appropriate

An important part of the development of test material is the review process Each question, whether writing task or multiple-choice question, as well as any stimulus material on which questions are based, must be reviewed by several independent critics Questions are also reviewed by experts outside ETS who can bring fresh perspectives to bear on the questions in terms of actual content or in terms of sensitivity to minority and women's concerns

After all the questions have been reviewed and revised as appropriate, the multiple-choice questions are assembled into clusters suitable for trial during actual administrations of the GMAT In this manner, new questions are tried out, under standard testing conditions, by representative samples of GMAT examinees. Questions being tried out do not affect examinees' scores but are themselves evaluated they are analyzed statistically for usefulness and weaknesses The questions that perform satisfactorily are added to the pool of questions from which each computer-adaptive test is constructed; those that do not are rewritten to correct the flaws and tried out again — or discarded

In contrast to the multiple-choice questions, the writing tasks are not tried out during actual administrations of the GMAT this would be impractical Instead, the writing tasks are pretested on first-year business school students — students who not so long ago were GMAT examinees themselves and who are therefore representative of the GMAT test-taking population. The responses are read at a pretest scoring session to determine which writing tasks are clear and accessible to examinees, which can be successfully completed within the allotted half-hour, and which discriminate fairly and reliably (i e , they are not skewed in some way so as to disadvantage certain examinees, and they produce scores all along the scoring scale) Only those tasks that perform well in the pretest scoring sessions become part of the pool used in the GMAT.

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2 Answering GMAT Questions

Before you arrive at the test center, it is very important that you familiarize yourself with the mechanics of taking a computer-adaptive test. Test tutorials have been developed for this purpose, they allow you to review the testing tools you will have for responding to the questions (both multiple-choice and writing) as well as gain experience using a mouse and scrolling These tutorials are part of the GMAT POWERPREP software, which is provided free to all test registrants, the tutorials can also be accessed by following the links provided at *www ets org/powerprep*

The tutorials are divided into specific areas

- How to Use a Mouse
- How to Answer
- How to Use the Testing Tools
- How to Scroll

This chapter highlights the "How to Use the Testing Tools" and "How to Answer" tutorial screens

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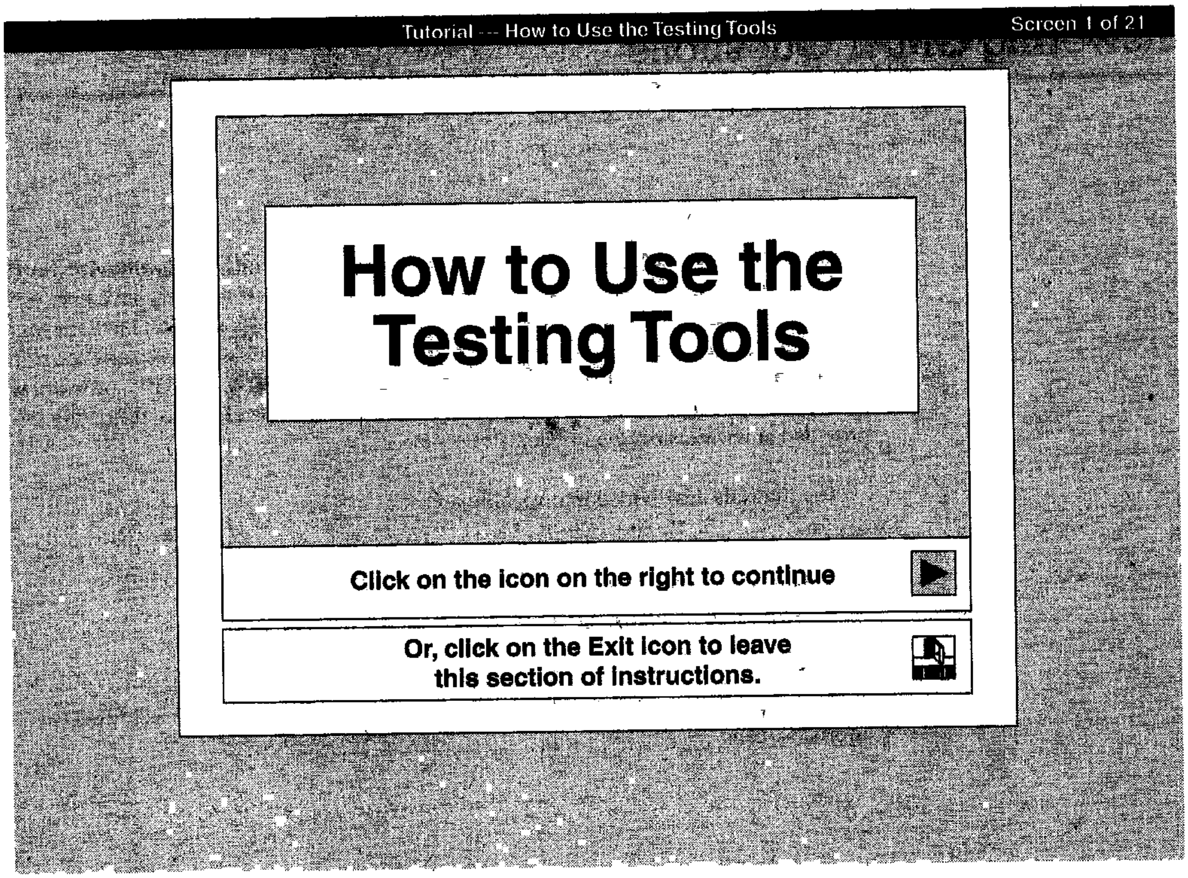

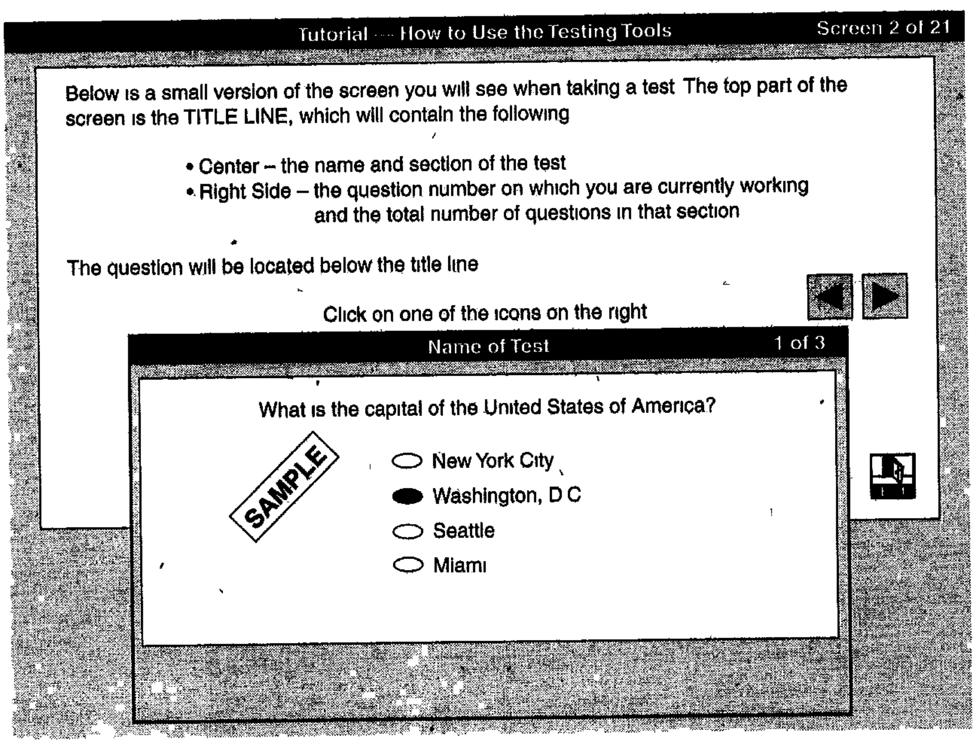

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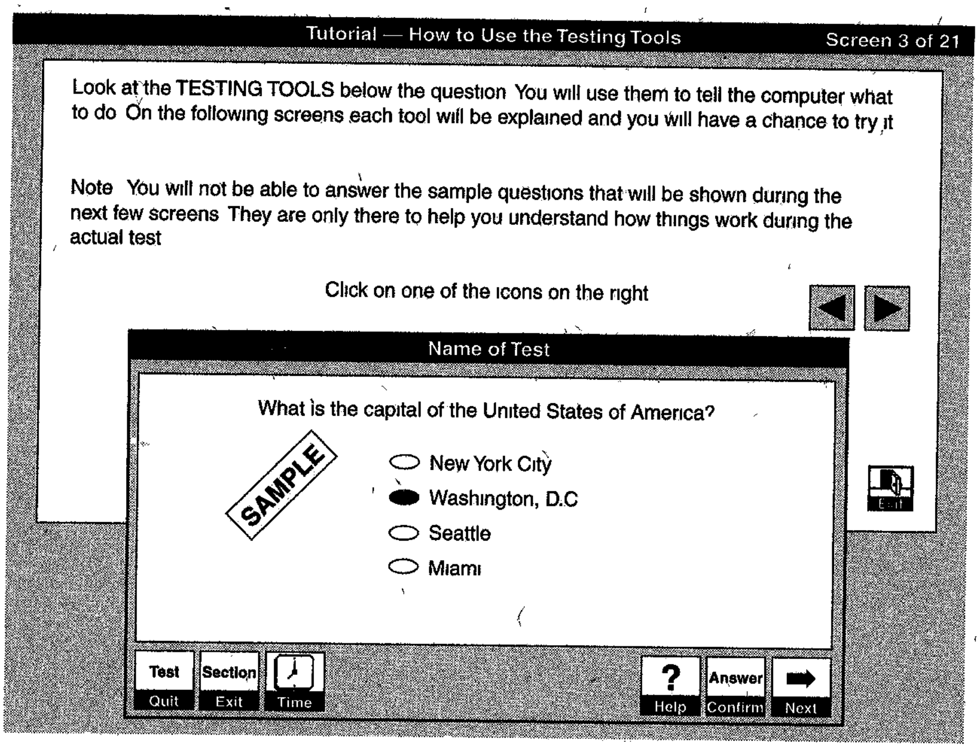

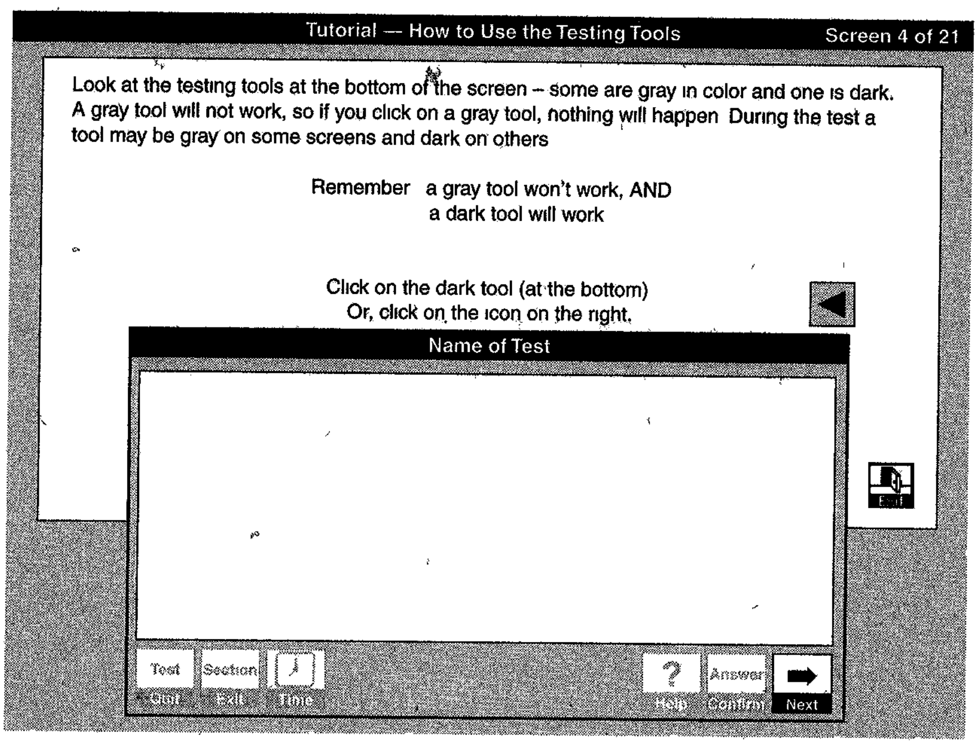

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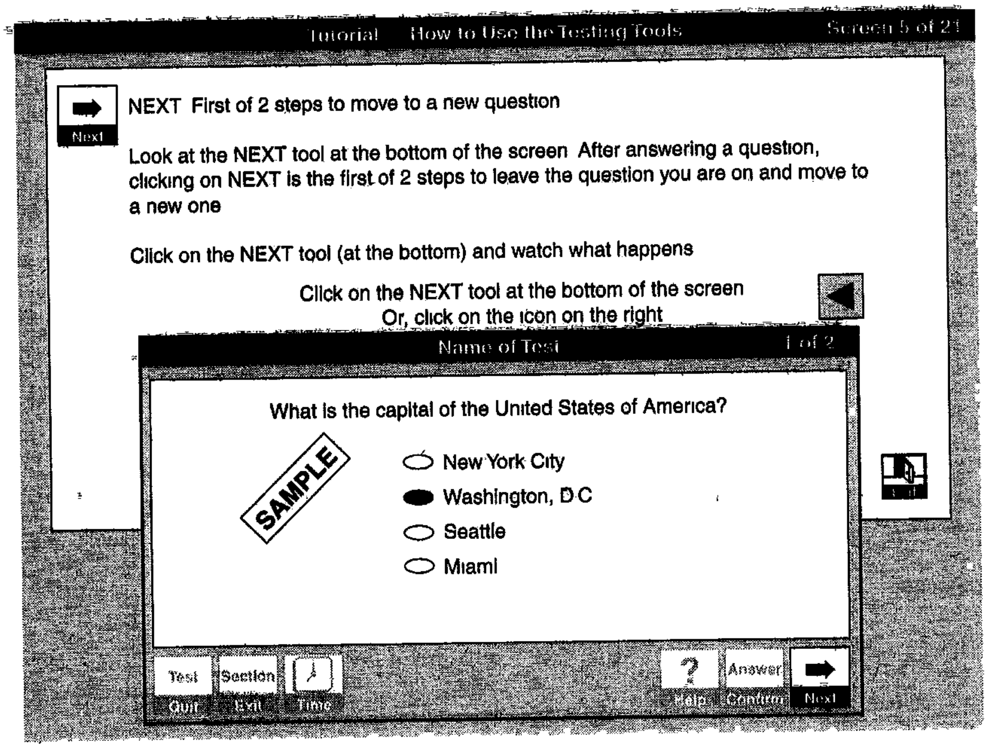

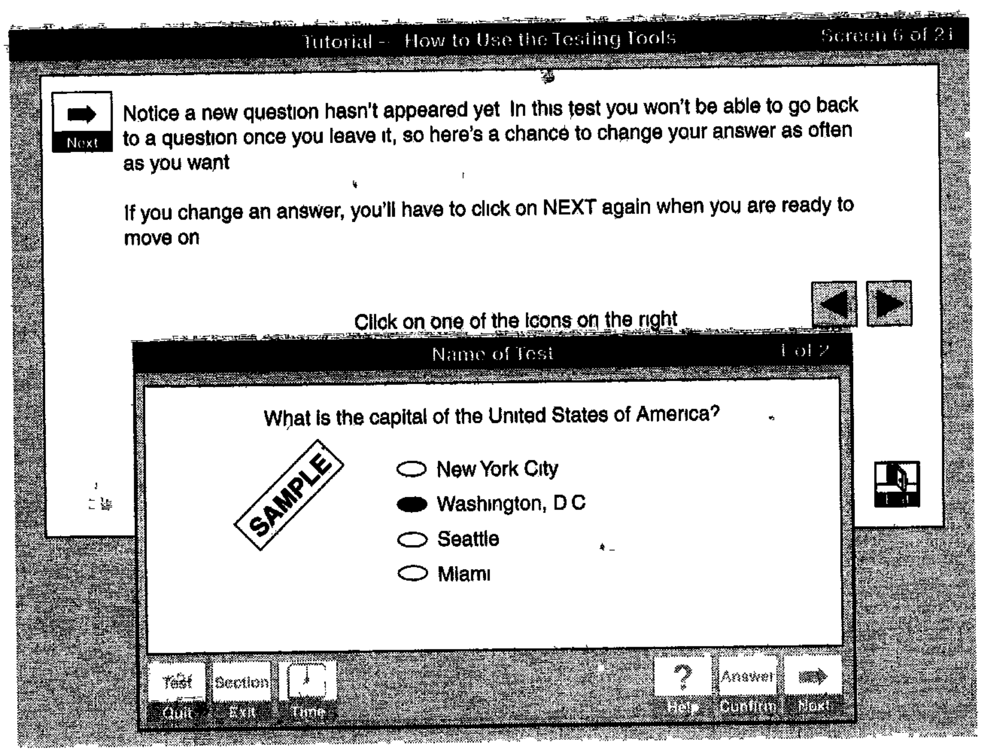

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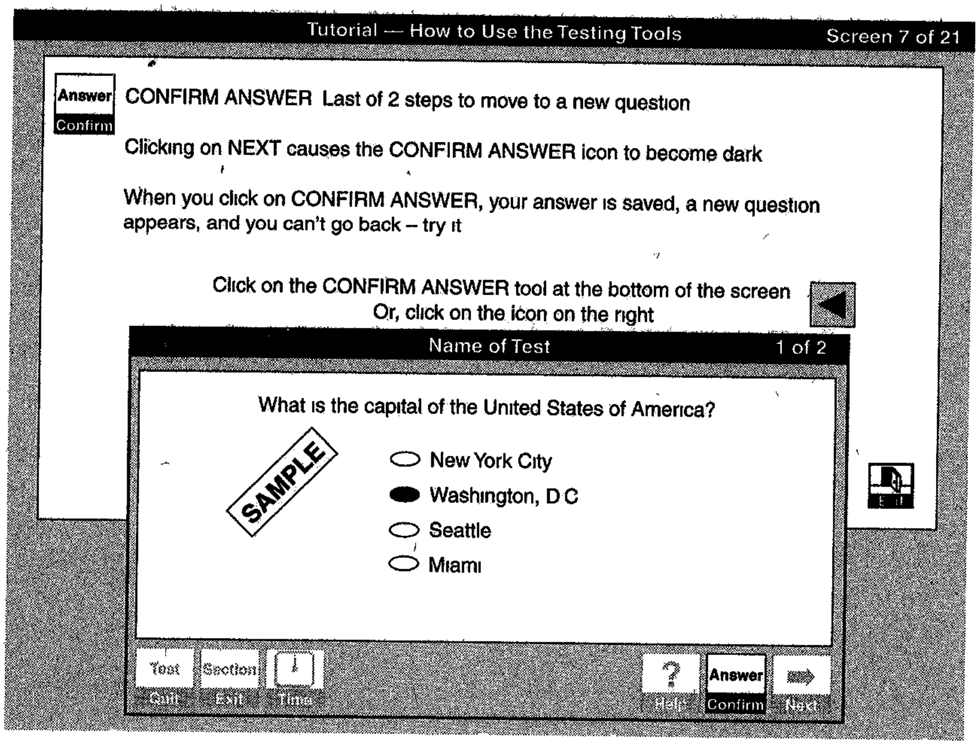

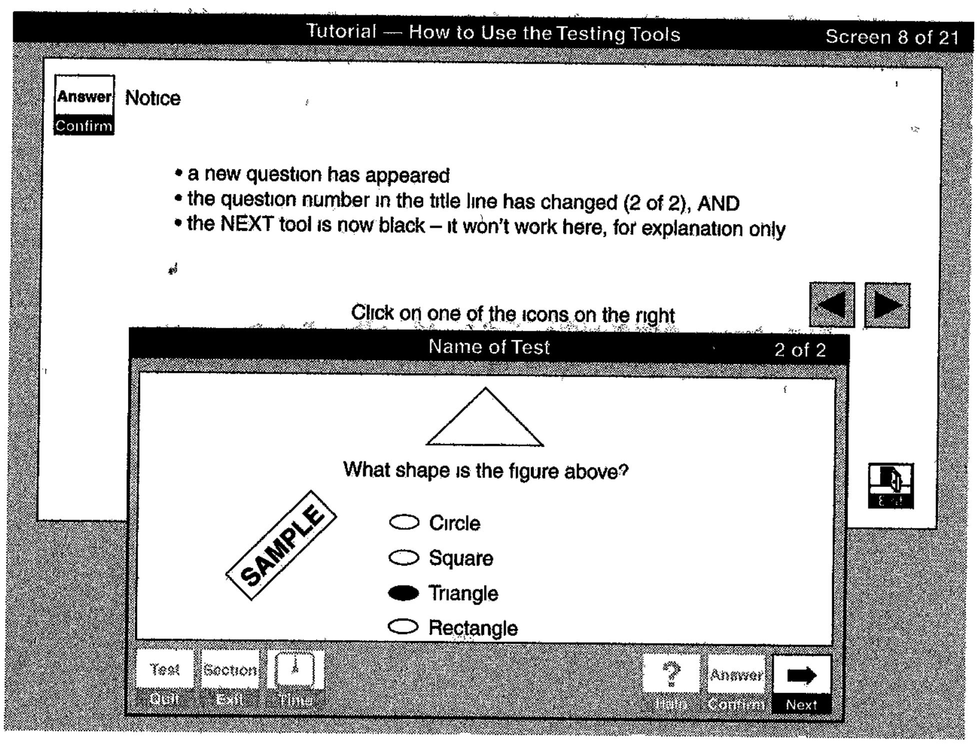

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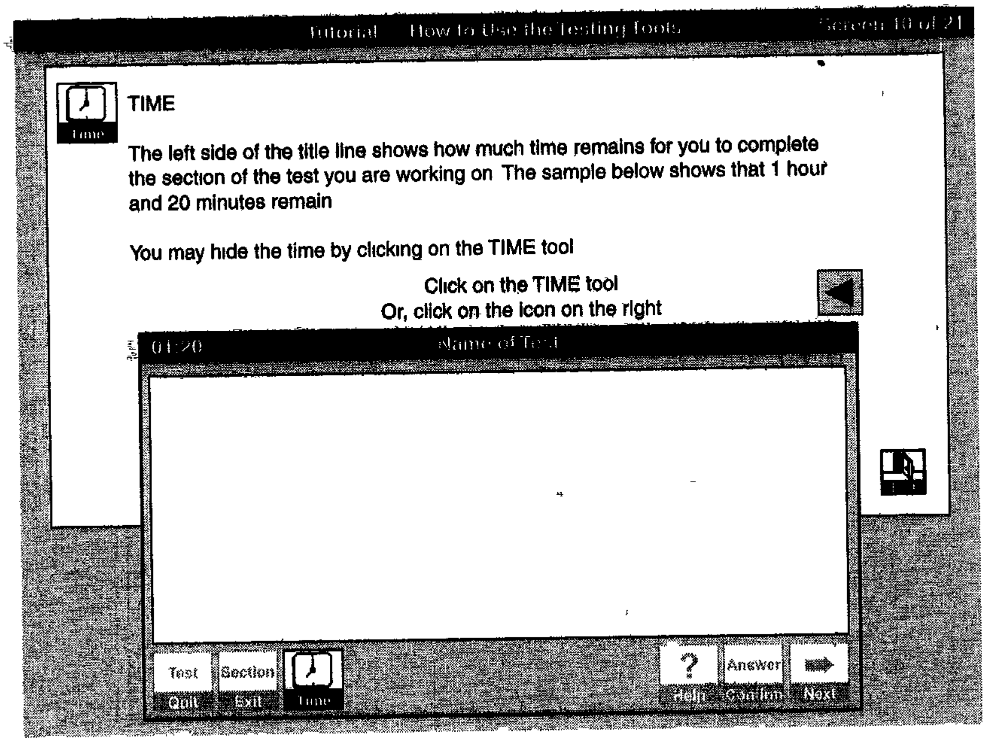

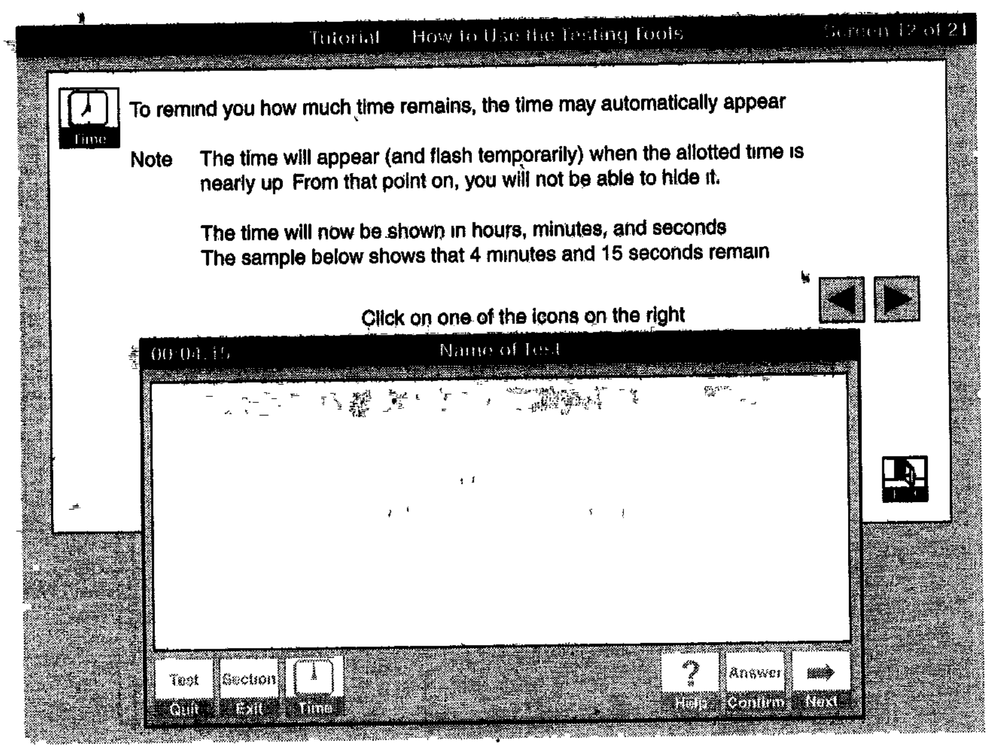

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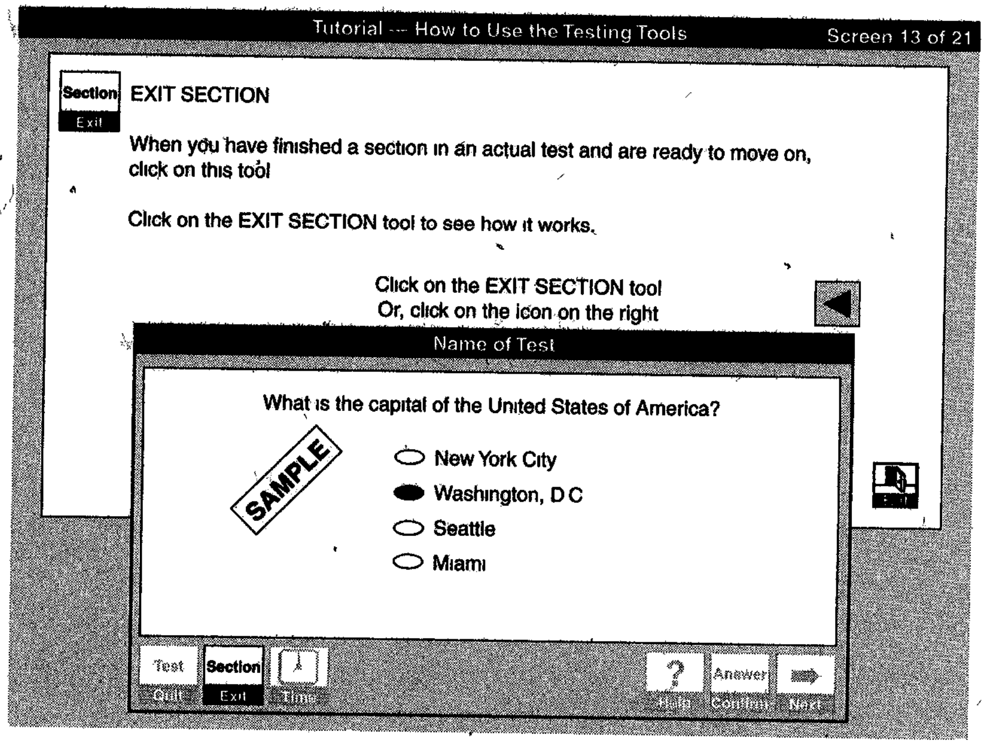

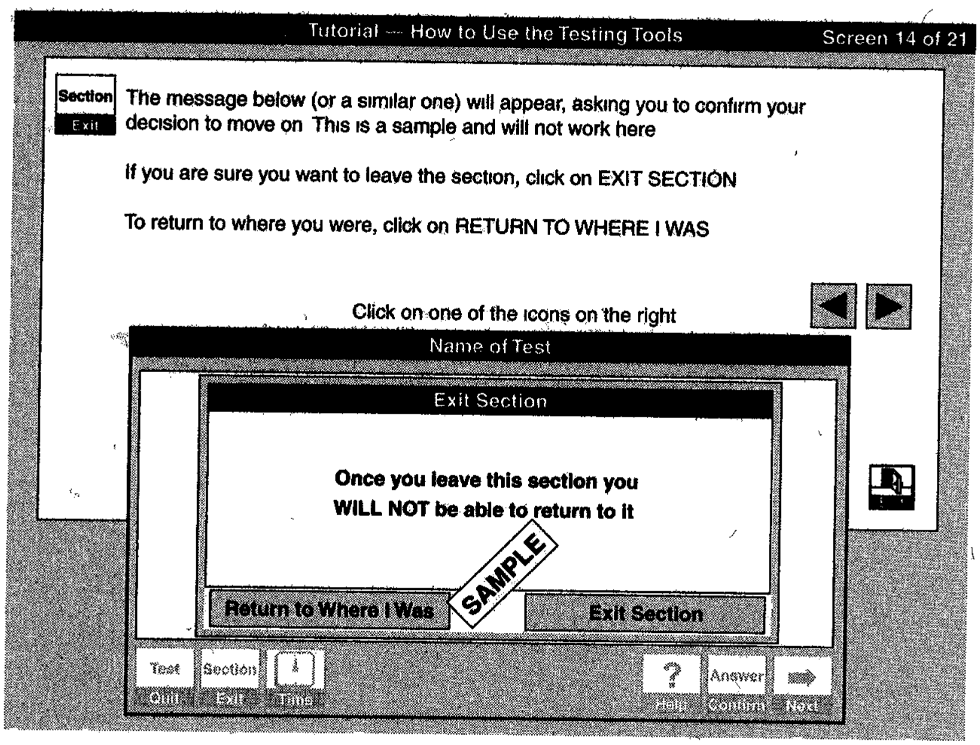

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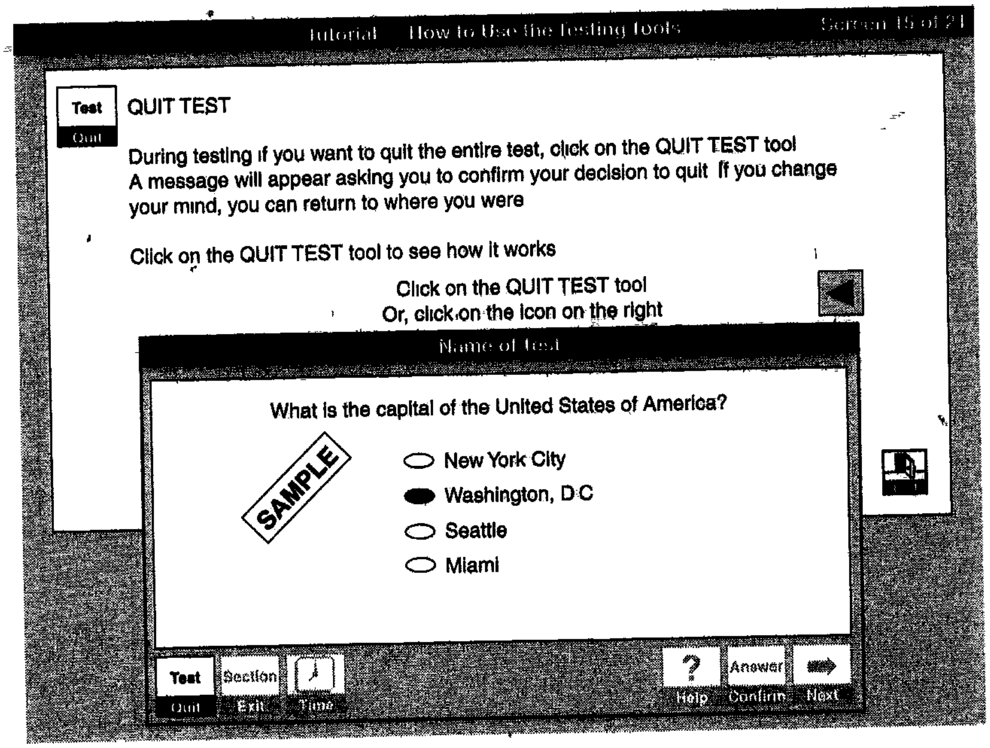

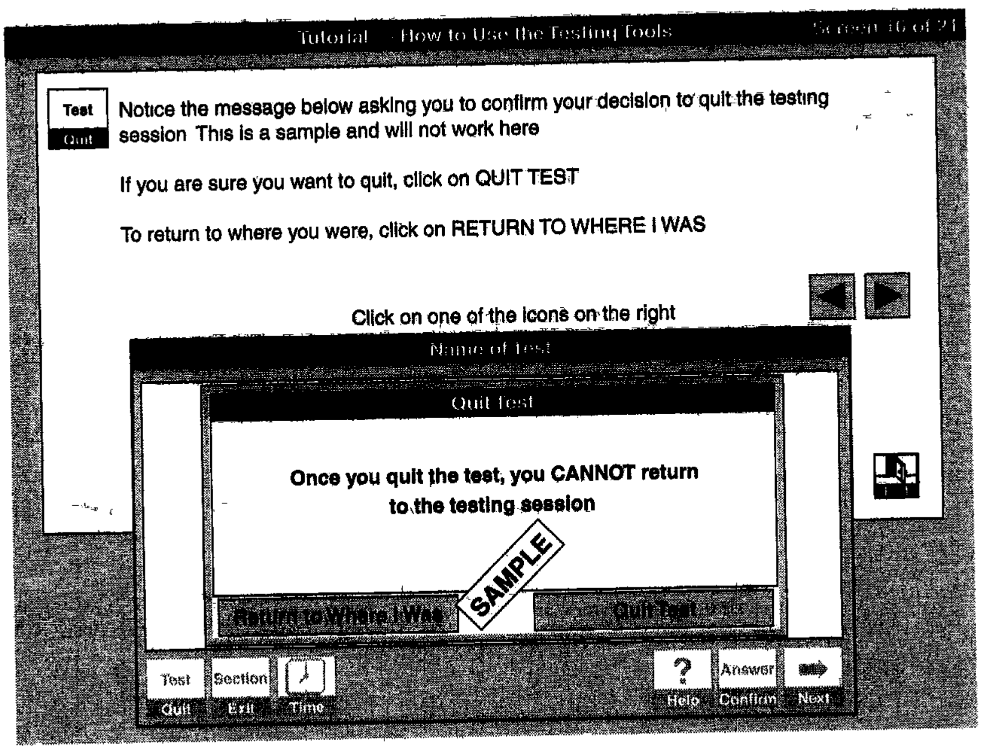

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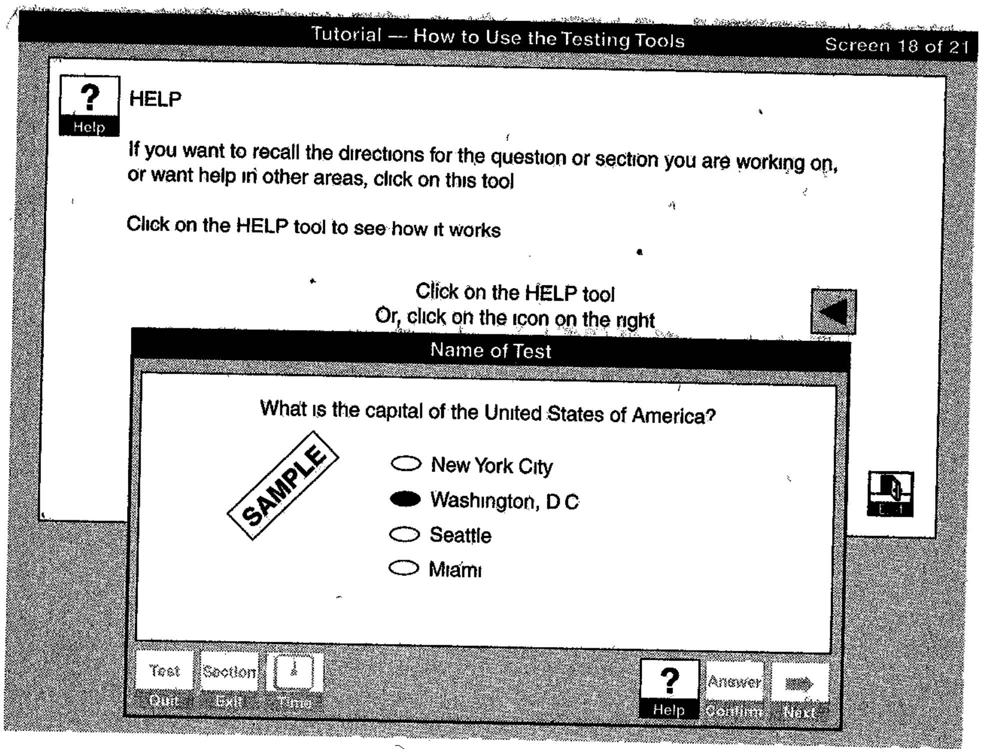

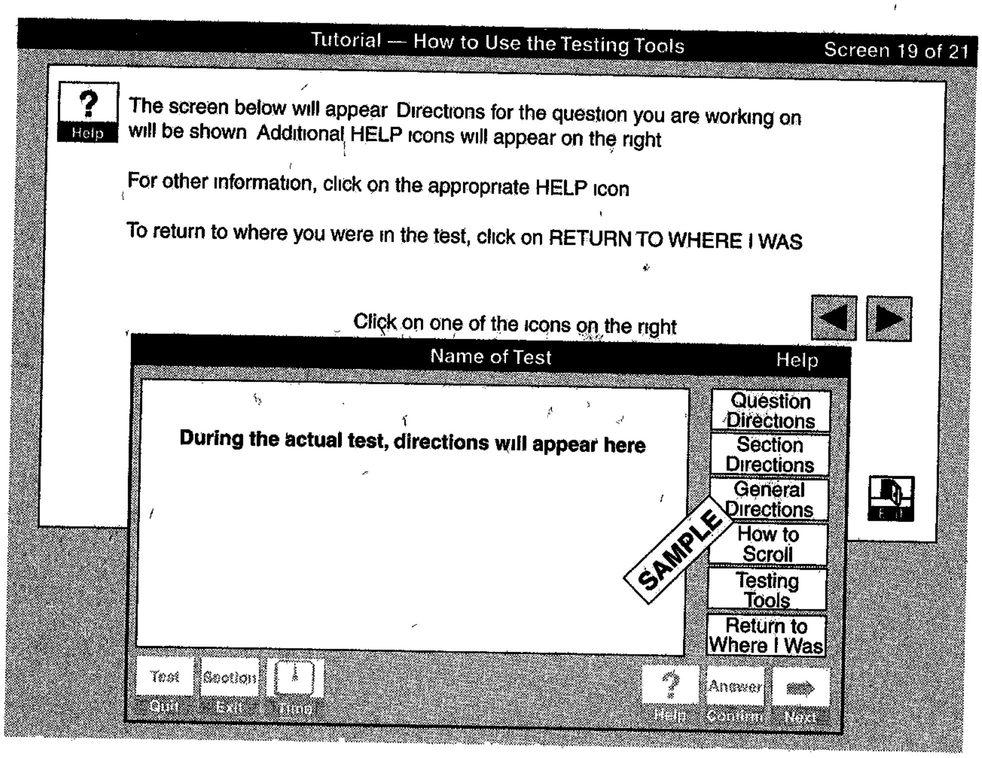

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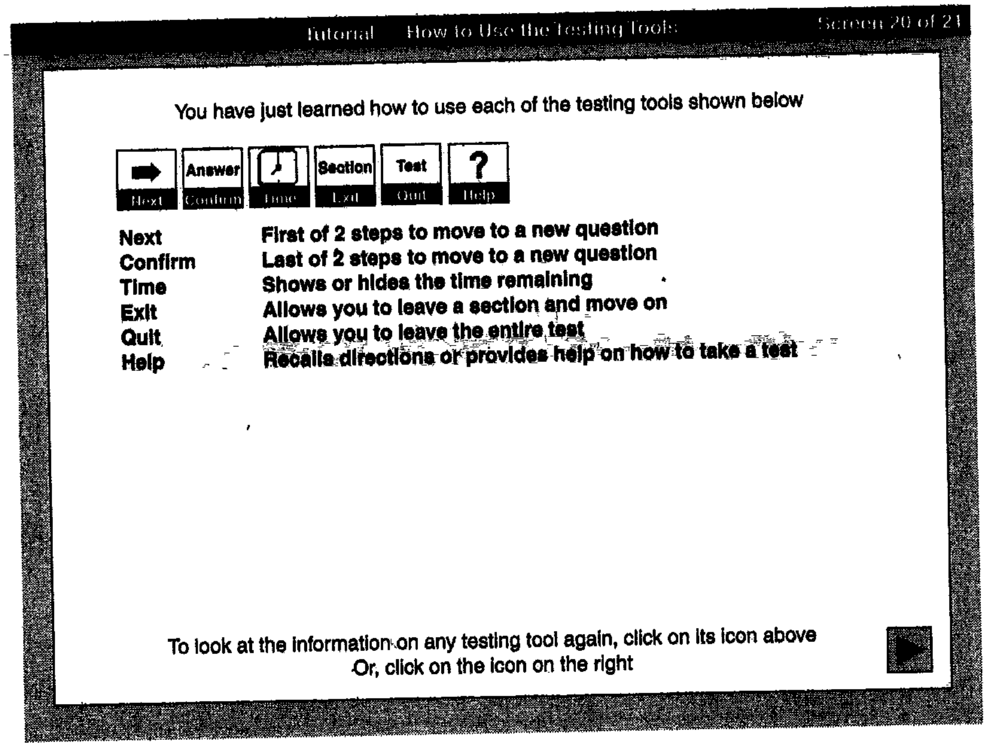

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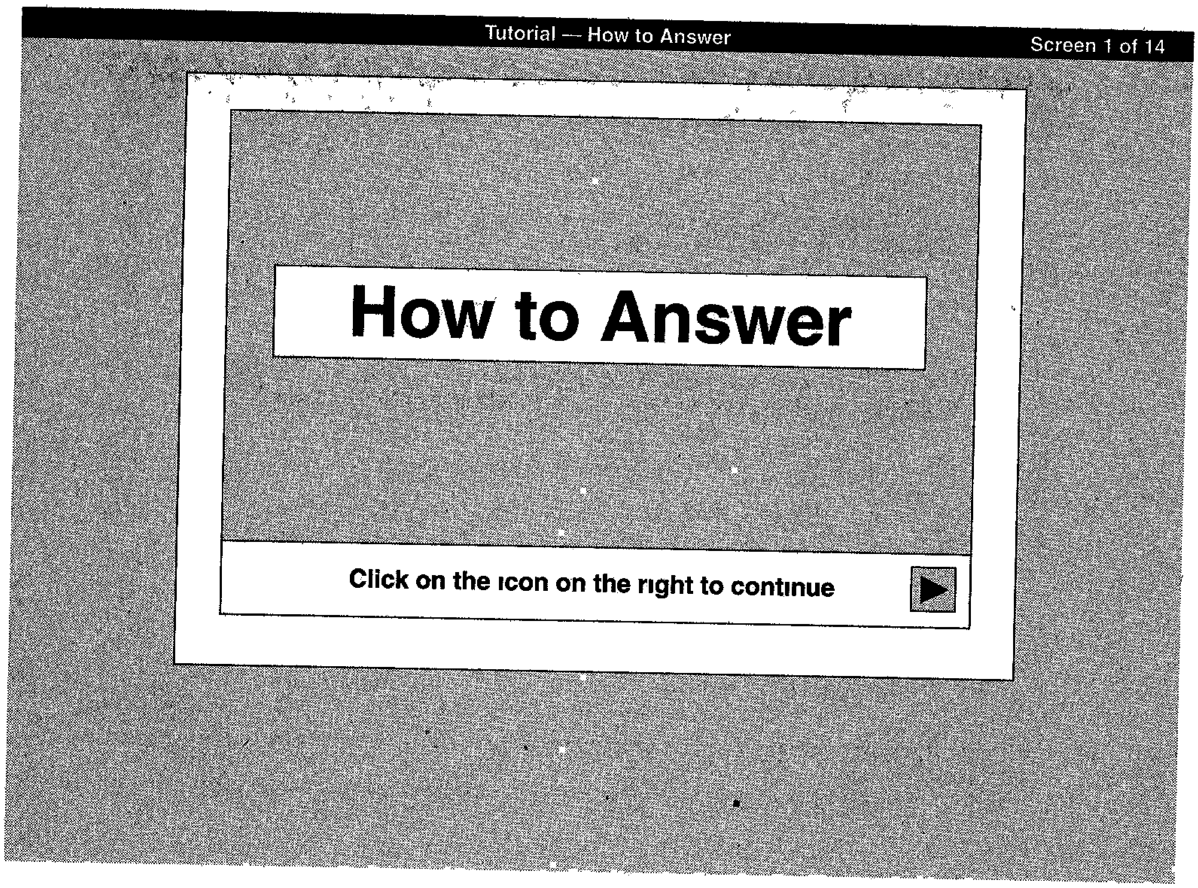

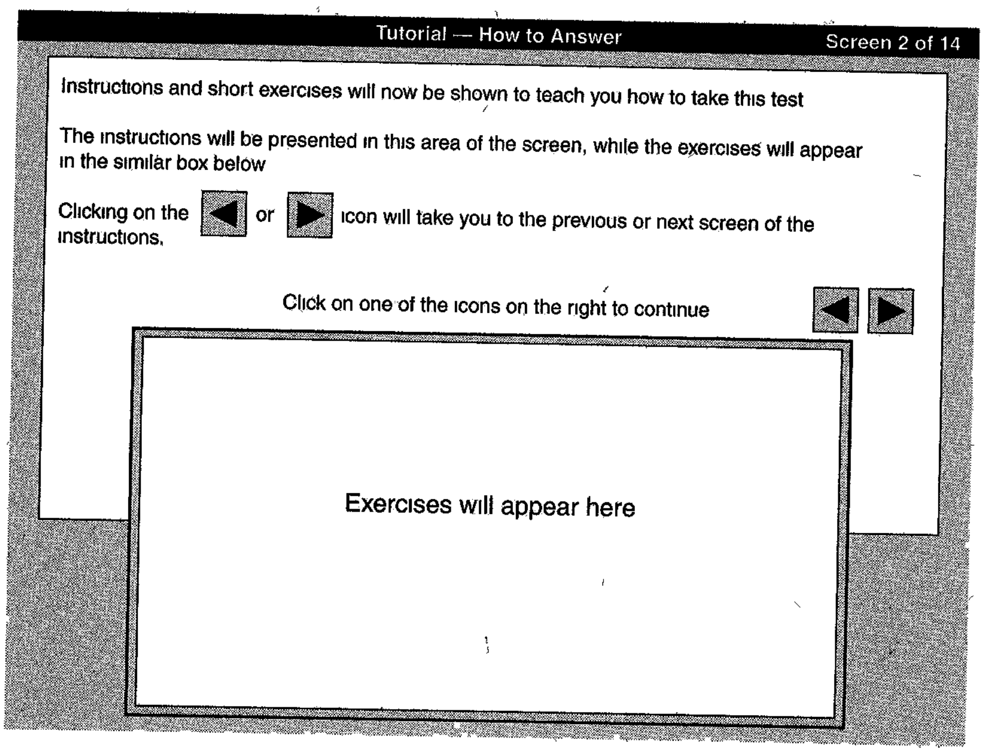

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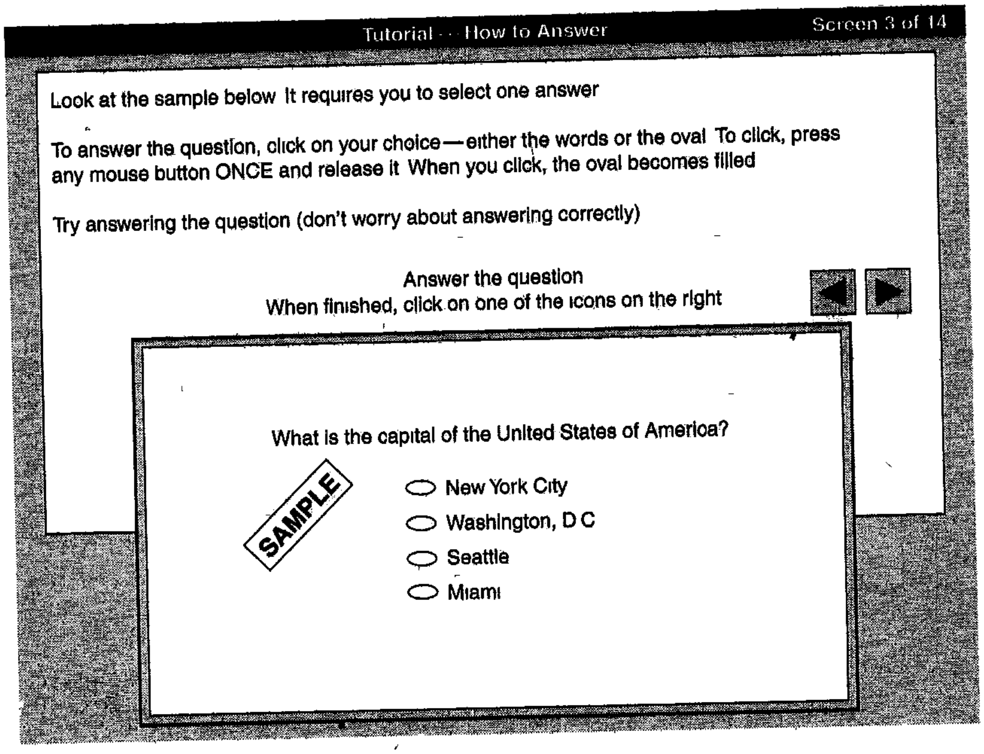

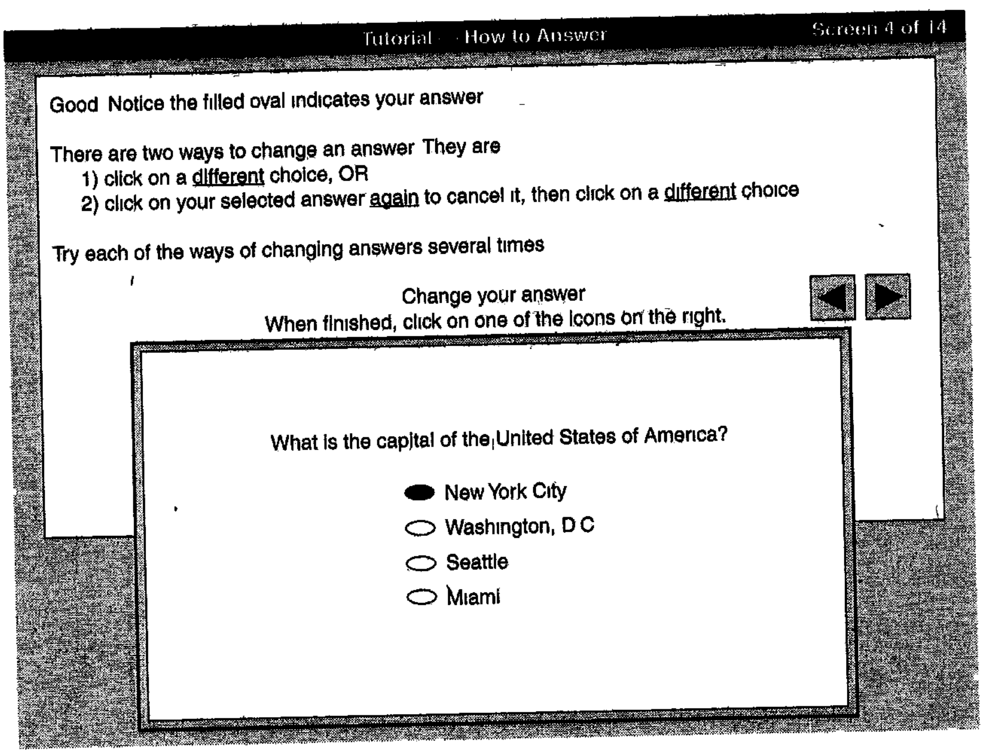

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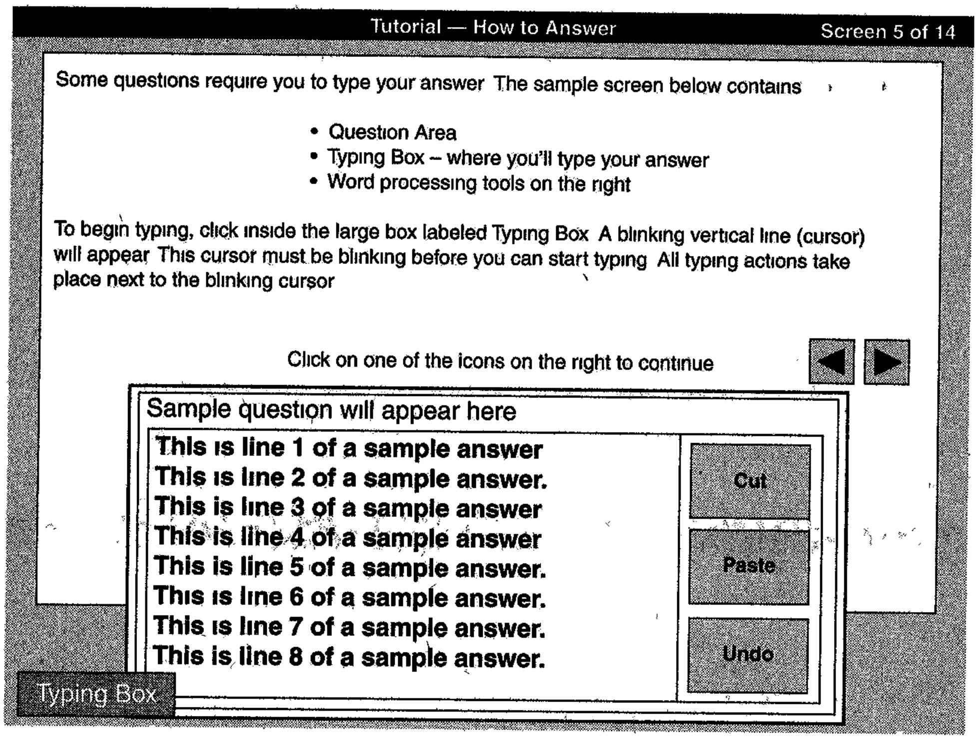
How to Answer the Analytical Writing Assessment (AWA)

The following screens of information from the "How to Answer" tutorial will help you to become familiar with the word processing tools available for responding to the AWA writing tasks



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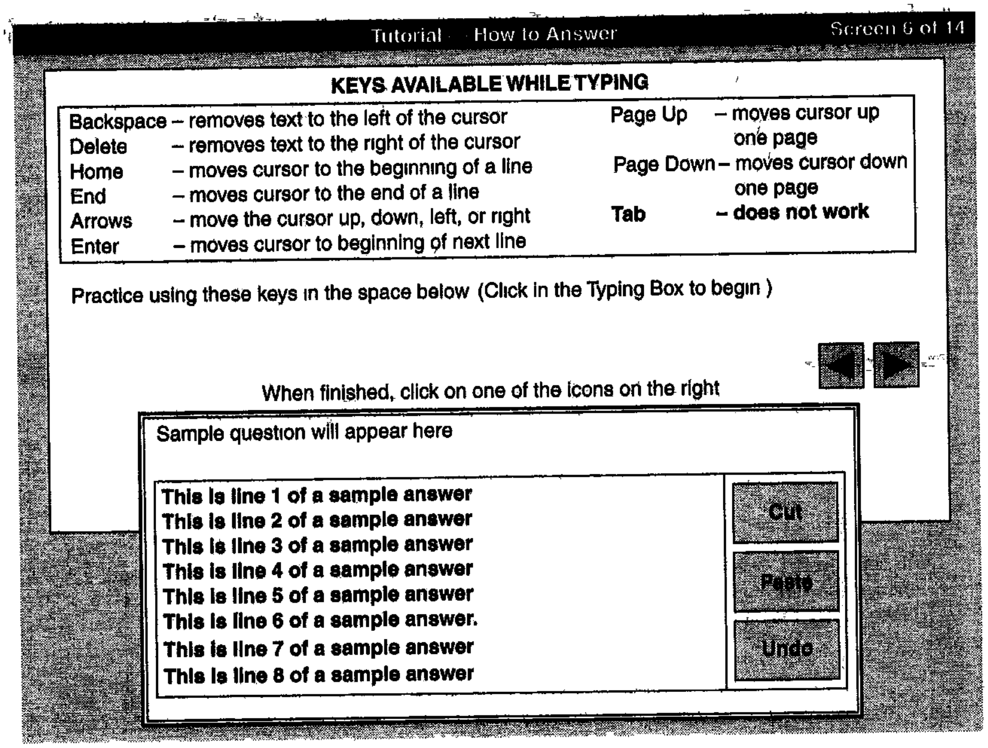

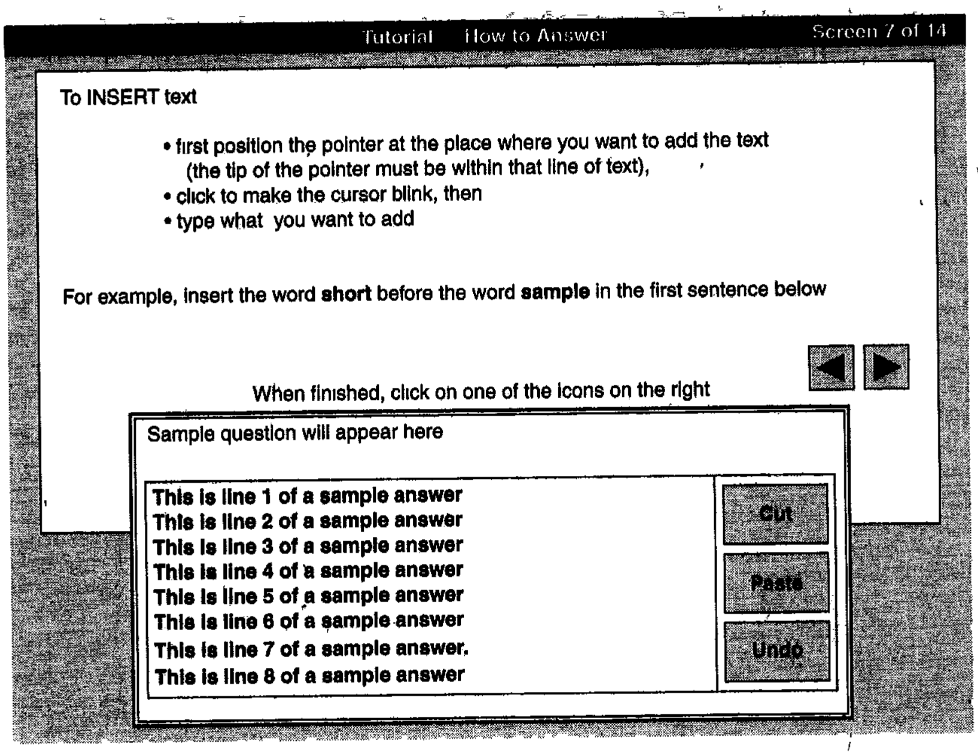

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You will need to HIGHLIGHT text to cut or paste it To highlight.

- position the pointer directly before the first letter you want to highlight,
- press the mouse button down and while HOLDING it down, drag to the place you want to stop, then
- release the mouse button — the text will be highlighted

To unhighlight, click again anywhere within the Typing Box

The tip of the pointer must stay in the MIDDLE of a line of text If the tip moves ABOVE or BELOW a line, text from another line will also be highlighted Moving the pointer to the left (or the right) highlights text to the left (or the right)

Try highlighting the words **sample answer** in a sentence below

When finished, click on one of the icons on the right

Sample question will appear here

This is line 1 of a sample answer
This is line 2 of a sample answer
This is line 3 of a sample answer
This is line 4 of a sample answer
This is line 5 of a sample answer
This is line 6 of a sample answer
This is line 7 of a sample answer
This is line 8 of a sample answer

Cut
Paste
Undo

The CUT icon cuts (removes) a block of text and stores it in the computer's memory The text remains in memory until you replace it with text from another cut

To CUT
- highlight the text, then
- click on the CUT icon

The PASTE icon inserts the block of text that you previously cut

To PASTE
- first CUT the text you want to paste,
- click where you want the text to appear (to make the cursor blink), then
- click on the PASTE icon

Try cutting the words **sample answer** in a sentence below Then paste them at the beginning of a sentence

When finished, click on one of the icons on the right

Sample question will appear here

This is line 1 of a sample answer
This is line 2 of a sample answer
This is line 3 of a sample answer
This is line 4 of a sample answer
This is line 5 of a sample answer
This is line 6 of a sample answer
This is line 7 of a sample answer
This is line 8 of a sample answer

Cut
Paste
Undo

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Tutorial How to Answer Screen 10 of 14

Clicking on UNDO reverses the previous action For instance, if you typed some text and clicked on UNDO, the text would disappear You can also UNDO a CUT or a PASTE For example, if you cut some text but decide you want it back, click on the UNDO icon to undo your cut (or bring it back)

Try typing the words **The grass is green,** then click on UNDO

Try cutting some text, then click on UNDO

Try pasting the text you just cut, then click on UNDO

Remember, UNDO will only work on your last action

When finished, click on one of the icons on the right

Sample question will appear here

This is line 1 of a sample answer
This is line 2 of a sample answer
This is line 3 of a sample answer
This is line 4 of a sample answer
This is line 5 of a sample answer
This is line 6 of a sample answer
This is line 7 of a sample answer.
This is line 8 of a sample answer

Cut
Paste
Undo

Tutorial How to Answer Screen 11 of 14

In the actual test if you need help while typing your answer, click on the HELP icon

? Help

When the Help screen appears, click on the HOW TO ANSWER icon

How to Answer

Here's your chance to practice all you've been taught on the sample below

Try using the keys — Delete, Backspace, Home, End, Arrows, etc
inserting new text
highlighting
cutting
pasting
undoing

When finished, click on one of the icons on the right

Sample question will appear here

This is line 1 of a sample answer
This is line 2 of a sample answer
This is line 3 of a sample answer
This is line 4 of a sample answer
This is line 5 of a sample answer
This is line 6 of a sample answer
This is line 7 of a sample answer
This is line 8 of a sample answer

Cut
Paste
Undo

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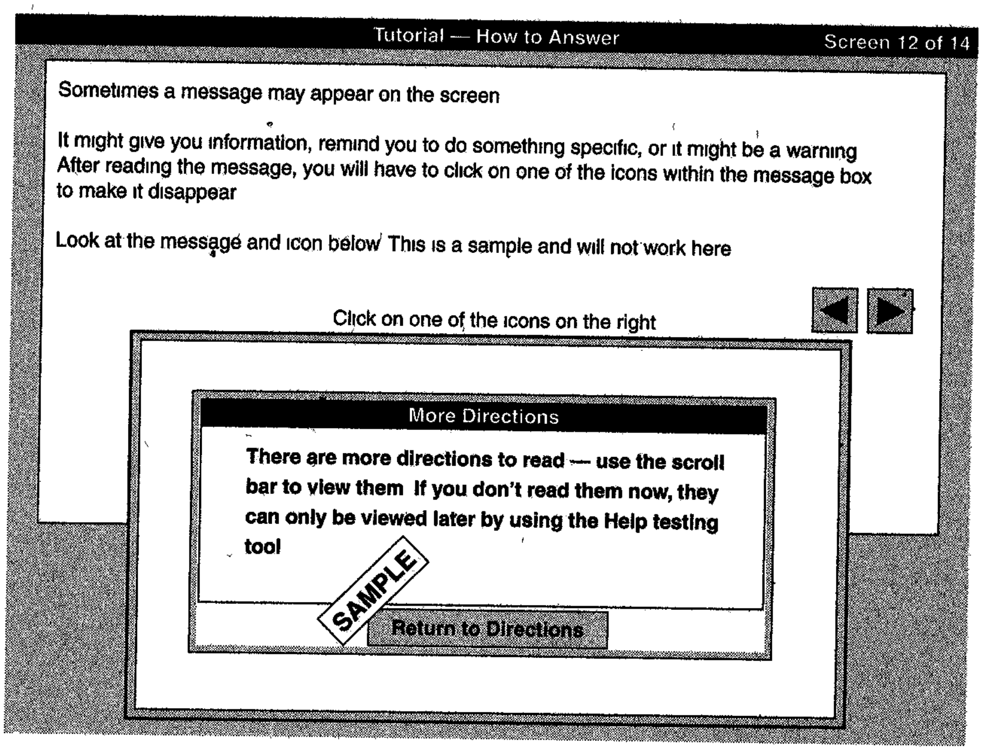

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3 Math Review

Although this chapter provides a review of some of the mathematical concepts of arithmetic, algebra, and geometry, it is not intended to be a textbook You should use this chapter to familiarize yourself with the kinds of topics that are tested in the GMAT You may wish to consult an arithmetic, algebra, or geometry book for a more detailed discussion of some of the topics

The topics that are covered in Section A, arithmetic, include

1 Properties of integers
2 Fractions
3 Decimals
4 Real numbers
5 Ratio and proportion
6 Percents
7 Powers and roots of numbers
8 Descriptive statistics
9 Sets
10 Counting methods
11 Discrete probability

The content of Section B, algebra, does not extend beyond what is usually covered in a first-year high school algebra course The topics included are·

1 Simplifying algebraic expressions
2 Equations
3 Solving linear equations with one unknown
4 Solving two linear equations with two unknowns
5. Solving equations by factoring
6 Solving quadratic equations
7 Exponents
8 Inequalities
9 Absolute value
10 Functions

Section C, geometry, is limited primarily to measurement and intuitive geometry or spatial visualization Extensive knowledge of theorems and the ability to construct proofs, skills that are usually developed in a formal geometry course, are not tested The topics included in this section are

1 Lines
2 Intersecting lines and angles
3 Perpendicular lines
4 Parallel lines
5 Polygons (convex)
6 Triangles
7 Quadrilaterals
8 Circles
9 Rectangular solids and cylinders
10 Coordinate geometry

Section D, word problems, presents examples of and solutions to the following types of word problems

1 Rate
2 Work
3 Mixture
4 Interest
5 Discount
6 Profit
7 Sets
8 Geometry
9 Measurement
10 Data interpretation

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A. Arithmetic

1. PROPERTIES OF INTEGERS

An *integer* is any number in the set {. . . −3, −2, −1, 0, 1, 2, 3, . . .} If x and y are integers and $x \neq 0$, then x is a *divisor* (*factor*) of y provided that $y = xn$ for some integer n In this case, y is also said to be *divisible* by x or to be a *multiple* of x For example, 7 is a divisor or factor of 28 since 28 = (7)(4), but 8 is not a divisor of 28 since there is no integer n such that 28 = 8n.

If x and y are positive integers, there exist unique integers q and r, called the *quotient* and *remainder*, respectively, such that $y = xq + r$ and $0 \leq r < x$ For example, when 28 is divided by 8, the quotient is 3 and the remainder is 4 since 28 = (8)(3) + 4 Note that y is divisible by x if and only if the remainder r is 0, for example, 32 has a remainder of 0 when divided by 8 because 32 is divisible by 8 Also, note that when a smaller integer is divided by a larger integer, the quotient is 0 and the remainder is the smaller integer. For example, 5 divided by 7 has the quotient 0 and the remainder 5 since 5 = (7)(0) + 5

Any integer that is divisible by 2 is an *even integer*; the set of even integers is {. . . −4, −2, 0, 2, 4, 6, 8, . . .} Integers that are not divisible by 2 are *odd integers*, {. . . −3, −1, 1, 3, 5, . . .} is the set of odd integers.

If at least one factor of a product of integers is even, then the product is even, otherwise the product is odd If two integers are both even or both odd, then their sum and their difference are even. Otherwise, their sum and their difference are odd

A *prime number* is a positive integer that has exactly two different positive divisors, 1 and itself For example, 2, 3, 5, 7, 11, and 13 are prime numbers, but 15 is not, since 15 has four different positive divisors, 1, 3, 5, and 15 The number 1 is not a prime number, since it has only one positive divisor Every integer greater than 1 is either prime or can be uniquely expressed as a product of prime factors For example, 14 = (2)(7), 81 = (3)(3)(3)(3), and 484 = (2)(2)(11)(11)

The numbers −2, −1, 0, 1, 2, 3, 4, 5 are *consecutive integers* Consecutive integers can be represented by n, $n + 1$, $n + 2$, $n + 3$, . . . , where n is an integer The numbers 0, 2, 4, 6, 8 are *consecutive even integers*, and 1, 3, 5, 7, 9 are *consecutive odd integers* Consecutive even integers can be represented by $2n$, $2n + 2$, $2n + 4$, . . . , and consecutive odd integers can be represented by $2n + 1$, $2n + 3$, $2n + 5$, . . . , where n is an integer.

Properties of the integer 1 If n is any number, then $1 \cdot n = n$, and for any number $n \neq 0$, $n \cdot \frac{1}{n} = 1$ The number 1 can be expressed in many ways; for example, $\frac{n}{n} = 1$ for any number $n \neq 0$ Multiplying or dividing an expression by 1, in any form, does not change the value of that expression

Properties of the integer 0 The integer 0 is neither positive nor negative If n is any number, then $n + 0 = n$ and $n \cdot 0 = 0$ Division by 0 is not defined

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2. FRACTIONS

In a fraction $\frac{n}{d}$, n is the *numerator* and d is the *denominator* The denominator of a fraction can never be 0, because division by 0 is not defined

Two fractions are said to be *equivalent* if they represent the same number For example, $\frac{8}{36}$ and $\frac{14}{63}$ are equivalent since they both represent the number $\frac{2}{9}$ In each case, the fraction is reduced to lowest terms by dividing both numerator and denominator by their *greatest common divisor* (gcd) The gcd of 8 and 36 is 4 and the gcd of 14 and 63 is 7

Addition and subtraction of fractions Two fractions with the same denominator can be added or subtracted by performing the required operation with the numerators, leaving the denominators the same For example, $\frac{3}{5}+\frac{4}{5}=\frac{3+4}{5}=\frac{7}{5}$, and $\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}=\frac{3}{7}$ If two fractions do not have the same denominator, express them as equivalent fractions with the same denominator For example, to add $\frac{3}{5}$ and $\frac{4}{7}$, multiply the numerator and denominator of the first fraction by 7 and the numerator and denominator of the second fraction by 5, obtaining $\frac{21}{35}$ and $\frac{20}{35}$, respectively,

$$\frac{21}{35}+\frac{20}{35}=\frac{41}{35}$$

For the new denominator, choosing the *least common multiple* (lcm) of the denominators usually lessens the work. For $\frac{2}{3}+\frac{1}{6}$, the lcm of 3 and 6 is 6 (not $3 \times 6 = 18$), so

$$\frac{2}{3}+\frac{1}{6}=\frac{2}{3}\times\frac{2}{2}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$$

Multiplication and division of fractions To multiply two fractions, simply multiply the two numerators and multiply the two denominators For example, $\frac{2}{3}\times\frac{4}{7}=\frac{2\times4}{3\times7}=\frac{8}{21}$

To divide by a fraction, invert the divisor (that is, find its *reciprocal*) and multiply For example $\frac{2}{3}\div\frac{4}{7}=\frac{2}{3}\times\frac{7}{4}=\frac{14}{12}=\frac{7}{6}$

In the problem above, the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$ In general, the reciprocal of a fraction $\frac{n}{d}$ is $\frac{d}{n}$ where n and d are not zero

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Mixed numbers A number that consists of a whole number and a fraction, for example, $7\frac{2}{3}$, is a mixed number. $7\frac{2}{3}$ means $7+\frac{2}{3}$

To change a mixed number into a fraction, multiply the whole number by the denominator of the fraction and add this number to the numerator of the fraction, then put the result over the denominator of the fraction For example,

$$7\frac{2}{3} = \frac{(3\times 7)+2}{3} = \frac{23}{3}$$

3. DECIMALS

In the decimal system, the position of the period or *decimal point* determines the place value of the digits For example, the digits in the number 7,654 321 have the following place values

Thousands		Hundreds	Tens	Ones or units		Tenths	Hundredths	Thousandths
7	,	6	5	4	.	3	2	1

Some examples of decimals follow

$$0\ 321 = \frac{3}{10}+\frac{2}{100}+\frac{1}{1{,}000} = \frac{321}{1{,}000}$$

$$0\ 0321 = \frac{0}{10}+\frac{3}{100}+\frac{2}{1{,}000}+\frac{1}{10{,}000} = \frac{321}{10{,}000}$$

$$1.56 = 1+\frac{5}{10}+\frac{6}{100} = \frac{156}{100}$$

Sometimes decimals are expressed as the product of a number with only one digit to the left of the decimal point and a power of 10 This is called *scientific notation* For example, 231 can be written as $2\ 31 \times 10^2$ and 0 0231 can be written as $2\ 31 \times 10^{-2}$ When a number is expressed in scientific notation, the exponent of the 10 indicates the number of places that the decimal point is to be moved in the number that is to be multiplied by a power of 10 in order to obtain the product The decimal point is moved to the right if the exponent is positive and to the left if the exponent is negative For example, $20\ 13 \times 10^3$ is equal to 20,130 and $1{\cdot}91 \times 10^{-4}$ is equal to 0 000191

Addition and subtraction of decimals To add or subtract two decimals, the decimal points of both numbers should be lined up If one of the numbers has fewer digits to the right of the decimal point than the other, zeros may be inserted to the right of the last digit For example, to add 17 6512 and 653 27, set up the numbers in a column and add

$$\begin{array}{r} 17\ 6512 \\ +\,653\ 2700 \\ \hline 670\ 9212 \end{array}$$

Likewise for 653 27 minus 17 6512

$$\begin{array}{r} 653\ 2700 \\ -\,17.6512 \\ \hline 635\ 6188 \end{array}$$

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Multiplication of decimals. To multiply decimals, multiply the numbers as if they were whole numbers and then insert the decimal point in the product so that the number of digits to the right of the decimal point is equal to the sum of the numbers of digits to the right of the decimal points in the numbers being multiplied. For example

$$\begin{array}{rl} 2.09 & \text{(2 digits to the right)} \\ \underline{\times\ 1.3} & \text{(1 digit to the right)} \\ 627 & \\ \underline{209} & \\ 2.717 & (2 + 1 = 3 \text{ digits to the right}) \end{array}$$

Division of decimals. To divide a number (the dividend) by a decimal (the divisor), move the decimal point of the divisor to the right until the divisor is a whole number. Then move the decimal point of the dividend the same number of places to the right, and divide as you would by a whole number. The decimal point in the quotient will be directly above the decimal point in the new dividend. For example, to divide 698.12 by 12.4

$$12.4\overline{)698.12}$$

will be replaced by

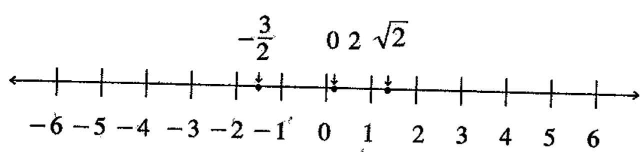
$$124\overline{)6981.2}$$

and the division would proceed as follows

$$\begin{array}{r} 56.3 \\ 124\overline{)6981.2} \\ \underline{620} \\ 781 \\ \underline{744} \\ 372 \\ \underline{372} \\ 0 \end{array}$$

4 REAL NUMBERS

All *real* numbers correspond to points on the number line and all points on the number line correspond to real numbers. All real numbers except zero are either positive or negative.



On a number line, numbers corresponding to points to the left of zero are negative and numbers corresponding to points to the right of zero are positive. For any two numbers on the number line, the number to the left is less than the number to the right, for example,

$-4 < -3$, $-\frac{3}{2} < -1$, and $1 < \sqrt{2} < 2$

To say that the number n is between 1 and 4 on the number line means that $n > 1$ and $n < 4$, that is, $1 < n < 4$. If n is "between 1 and 4, inclusive," then $1 \leq n \leq 4$.

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The distance between a number and zero on the number line is called the *absolute value* of the number. Thus 3 and −3 have the same absolute value, 3, since they are both three units from zero The absolute value of 3 is denoted $|3|$ Examples of absolute values of numbers are

$$|-5| = |5| = 5, \left|-\frac{7}{2}\right| = \frac{7}{2}, \text{ and } |0| = 0$$

Note that the absolute value of any nonzero number is positive

Here are some properties of real numbers that are used frequently If x, y, and z are real numbers, then

(1) $x + y = y + x$ and $xy = yx$
For example, $8 + 3 = 3 + 8 = 11$, and $(17)(5) = (5)(17) = 85$

(2) $(x + y) + z = x + (y + z)$ and $(xy)z = x(yz)$
For example, $(7 + 5) + 2 = 7 + (5 + 2) = 7 + (7) = 14$,
and $(5\sqrt{3})(\sqrt{3}) = (5\sqrt{3}\sqrt{3}) = (5)(3) = 15$

(3) $x(y + z) = xy + yz$
For example, $718(36) + 718(64) = 718(36 + 64) = 718(100) = 71{,}800$.

(4) If x and y are both positive, then $x + y$ and xy are positive

(5) If x and y are both negative, then $x + y$ is negative and xy is positive.

(6) If x is positive and y is negative, then xy is negative.

(7) If $xy = 0$, then $x = 0$ or $y = 0$. For example, $3y = 0$ implies $y = 0$

(8) $|x+y| \leq |x|+|y|$. For example, if $x = 10$ and $y = 2$,
then $|x+y| = |12| = 12 = |x|+|y|$, and if $x = 10$ and $y = -2$,
then $|x+y| = |8| = 8 < 12 = |x|+|y|$

5. RATIO AND PROPORTION

The *ratio* of the number a to the number b $(b \neq 0)$ is $\frac{a}{b}$

A ratio may be expressed or represented in several ways. For example, the ratio of 2 to 3 can be written as 2 to 3, 2 3, or $\frac{2}{3}$ The order of the terms of a ratio is important. For example, the ratio of the number of months with exactly 30 days to the number with exactly 31 days is $\frac{4}{7}$, not $\frac{7}{4}$

A *proportion* is a statement that two ratios are equal, for example, $\frac{2}{3} = \frac{8}{12}$ is a proportion One way to solve a proportion involving an unknown is to cross multiply, obtaining a new equality For example, to solve for n in the proportion $\frac{2}{3} = \frac{n}{12}$, cross multiply, obtaining $24 = 3n$, then divide both sides by 3, to get $n = 8$

6. PERCENTS

Percent means *per hundred* or *number out of 100* A percent can be represented as a fraction with a denominator of 100, or as a decimal For example, $37\% = \frac{37}{100} = 0\,37$.

To find a certain percent of a number, multiply the number by the percent expressed as a decimal or fraction For example

$$20\% \text{ of } 90 = 0\,2 \times 90 = 18$$

or

$$20\% \text{ of } 90 = \frac{20}{100} \times 90 = \frac{1}{5} \times 90 = 18$$

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Percents greater than 100%. Percents greater than 100% are represented by numbers greater than 1 For example

$$300\% = \frac{300}{100} = 3$$

$$250\% \text{ of } 80 = 2.5 \times 80 = 200$$

Percents less than 1% The percent 0.5% means $\frac{1}{2}$ of 1 percent For example, 0 5% of 12 is equal to $0.005 \times 12 = 0.06$

Percent change Often a problem will ask for the percent increase or decrease from one quantity to another quantity. For example, "If the price of an item increases from \$24 to \$30, what is the percent increase in price?" To find the percent increase, first find the amount of the increase, then divide this increase by the original amount, and express this quotient as a percent In the example above, the percent increase would be found in the following way: the amount of the increase is $(30 - 24) = 6$ Therefore, the percent increase is $\frac{6}{24} = 0.25 = 25\%$

Likewise, to find the percent decrease (for example, the price of an item is reduced from \$30 to \$24), first find the amount of the decrease, then divide this decrease by the original amount, and express this quotient as a percent In the example above, the amount of decrease is $(30 - 24) = 6$. Therefore, the percent decrease is $\frac{6}{30} = 0.20 = 20\%$.

Note that the percent increase from 24 to 30 is not the same as the percent decrease from 30 to 24

In the following example, the increase is greater than 100 percent: If the cost of a certain house in 1983 was 300 percent of its cost in 1970, by what percent did the cost increase?

If n is the cost in 1970, then the percent increase is equal to $\frac{3n-n}{n} = \frac{2n}{n} = 2$, or 200 percent.

7. POWERS AND ROOTS OF NUMBERS

When a number k is to be used n times as a factor in a product, it can be expressed as k^n, which means the nth power of k. For example, $2^2 = 2 \times 2 = 4$ and $2^3 = 2 \times 2 \times 2 = 8$ are powers of 2.

Squaring a number that is greater than 1, or raising it to a higher power, results in a larger number, squaring a number between 0 and 1 results in a smaller number For example

$3^2 = 9 \qquad (9 > 3)$

$\left(\frac{1}{3}\right)^2 = \frac{1}{9} \qquad \left(\frac{1}{9} < \frac{1}{3}\right)$

$(0.1)^2 = 0.01 \qquad (0.01 < 0.1)$

A *square root* of a number n is a number that, when squared, is equal to n. The square root of a negative number is not a real number Every positive number n has two square roots, one positive and the other negative, but $\sqrt{n}$ denotes the positive number whose square is n For example, $\sqrt{9}$ denotes 3 The two square roots of 9 are $\sqrt{9} = 3$ and $-\sqrt{9} = -3$

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Every real number r has exactly one real *cube root*, which is the number s such that $s^3 = r$ The real cube root of r is denoted by $\sqrt[3]{r}$ Since $2^3 = 8$, $\sqrt[3]{8} = 2$ Similarly, $\sqrt[3]{-8} = -2$, because $(-2)^3 = -8$.

8. DESCRIPTIVE STATISTICS

A list of numbers, or numerical data, can be described by various statistical measures One of the most common of these measures is the *average*, or *(arithmetic) mean*, which locates a type of "center" for the data. The average of n numbers is defined as the sum of the n numbers divided by n. For example, the average of 6, 4, 7, 10, and 4 is $\frac{6+4+7+10+4}{5} = \frac{31}{5} = 6.2$

The *median* is another type of center for a list of numbers To calculate the median of n numbers, first order the numbers from least to greatest, if n is odd, the median is defined as the middle number, while if n is even, the median is defined as the average of the two middle numbers In the example above, the numbers, in order, are 4, 4, 6, 7, 10, and the median is 6, the middle number For the numbers 4, 6, 6, 8, 9, 12, the median is $\frac{6+8}{2} = 7$. Note that the mean of these numbers is 7 5 The median of a set of data can be less than, equal to, or greater than the mean. Note that for a large set of data (for example, the salaries of 800 company employees), it is often true that about half of the data is less than the median and about half of the data is greater than the median, but this is not always the case, as the following data show

3, 5, 7, 7, 7, 7, 7, 7, 8, 9, 9, 9, 9, 10, 10

Here the median is 7, but only $\frac{2}{15}$ of the data is less than the median

The *mode* of a list of numbers is the number that occurs most frequently in the list For example, the mode of 1, 3, 6, 4, 3, 5 is 3 A list of numbers may have more than one mode For example, the list 1, 2, 3, 3, 3, 5, 7, 10, 10, 10, 20 has two modes, 3 and 10

The degree to which numerical data are spread out or dispersed can be measured in many ways The simplest measure of dispersion is the *range*, which is defined as the greatest value in the numerical data minus the least value For example, the range of $11, 10, 5, 13, 21$ is $21 - 5 = 16$ Note how the range depends on only two values in the data

One of the most common measures of dispersion is the *standard deviation* Generally speaking, the greater the data are spread away from the mean, the greater the standard deviation. The standard deviation of n numbers can be calculated as follows (1) find the arithmetic mean, (2) find the differences between the mean and each of the n numbers, (3) square each of the differences, (4) find the average of the squared differences, and (5) take the nonnegative square root of this average Shown below is this calculation for the data 0, 7, 8, 10, 10, which have arithmetic mean 7

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x	$x-7$	$(x-7)^2$
0	–7	49
7	0	0
8	1	1
10	3	9
10	3	9
	Total	68

Standard deviation $\sqrt{\dfrac{68}{5}} \approx 3\ 7$

Notice that the standard deviation depends on every data value, although it depends most on values that are farthest from the mean This is why a distribution with data grouped closely around the mean will have a smaller standard deviation than will data spread far from the mean To illustrate this, compare the data 6, 6, 6 5, 7 5, 9, which also have mean 7 Note that the numbers in the second set of data seem to be grouped more closely around the mean of 7 than the numbers in the first set This is reflected in the standard deviation, which is less for the second set (approximately 1 1) than for the first set (approximately 3 7)

There are many ways to display numerical data that show how the data are distributed One simple way is with a *frequency distribution*, which is useful for data that have values occurring with varying frequencies For example, the 20 numbers

$$\begin{array}{cccccccccc} -4 & 0 & 0 & -3 & -2 & -1 & -1 & 0 & -1 & -4 \\ -1 & -5 & 0 & -2 & 0 & -5 & -2 & 0 & 0 & -1 \end{array}$$

are displayed below in a frequency distribution by listing each different value x and the frequency f with which x occurs

Data Value x	Frequency f
–5	2
–4	2
–3	1
–2	3
–1	5
0	7
Total	20

From the frequency distribution, one can readily compute descriptive statistics

Mean $\dfrac{(-5)(2)+(-4)(2)+(-3)(1)+(-2)(3)+(-1)(5)+(0)(7)}{20} = -1\ 6$

Median –1 (the average of the 10th and 11th numbers)

Mode 0 (the number that occurs most frequently)

Range $0-(-5)=5$

Standard deviation

$$\sqrt{\frac{(-5+1\ 6)^2(2)+(-4+1\ 6)^2(2)+\quad +(0+1\ 6)^2(7)}{20}} \approx 1\ 7$$

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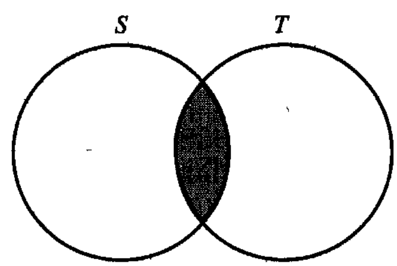
9. SETS

In mathematics a *set* is a collection of numbers or other objects The objects are called the *elements* of the set If S is a set having a finite number of elements, then the number of elements is denoted by $|S|$ Such a set is often defined by listing its elements, for example, $S = \{-5,0,1\}$ is a set with $|S| = 3$. The order in which the elements are listed in a set does not matter, thus $\{-5, 0, 1\} = \{0, 1, -5\}$ If all the elements of a set S are also elements of a set T, then S is a *subset* of T, for example, $S = \{-5, 0, 1\}$ is a subset of $T = \{-5, 0, 1, 4, 10\}$

For any two sets A and B, the *union* of A and B is the set of all elements that are in A *or* in B *or* in both The *intersection* of A and B is the set of all elements that are both in A *and* in B The union is denoted by $A \cup B$ and the intersection is denoted by $A \cap B$ As an example, if $A = \{3, 4\}$ and $B = \{4, 5, 6\}$, then $A \cup B = \{3, 4, 5, 6\}$ and $A \cap B = \{4\}$ Two sets that have no elements in common are said to be *disjoint* or *mutually exclusive*

The relationship between sets is often illustrated with a *Venn diagram* in which sets are represented by regions in a plane. For two sets S and T that are not disjoint and neither is a subset of the other, the intersection $S \cap T$ is represented by the shaded region of the diagram below



This diagram illustrates a fact about any two finite sets S and T: the number of elements in their union equals the sum of their individual numbers of elements minus the number of elements in their intersection (because the latter are counted twice in the sum), more concisely,

$$|S \cup T| = |S| + |T| - |S \cap T|$$

This counting method is called the general addition rule for two sets. As a special case, if S and T are disjoint, then

$$|S \cup T| = |S| + |T|$$

since $|S \cap T| = 0$

10. COUNTING METHODS

There are some useful methods for counting objects and sets of objects without actually listing the elements to be counted The following principle of multiplication is fundamental to these methods

If an object is to be chosen from a set of m objects and a second object is to be chosen from a different set of n objects, then there are mn ways of choosing both objects simultaneously

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As an example, suppose the objects are items on a menu If a meal consists of one entree and one dessert and there are 5 entrees and 3 desserts on the menu, then there are $5 \times 3 = 15$ different meals that can be ordered from the menu As another example, each time a coin is flipped, there are two possible outcomes, heads and tails If an experiment consists of 8 consecutive coin flips, then the experiment has 2^8 possible outcomes, where each of these outcomes is a list of heads and tails in some order

A symbol that is often used with the multiplication principle is the *factorial* If n is an integer greater than 1, then n factorial, denoted by the symbol $n!$, is defined as the product of all the integers from 1 to n Therefore,

$$\begin{aligned} 2! &= (1)(2) = 2, \\ 3! &= (1)(2)(3) = 6, \\ 4! &= (1)(2)(3)(4) = 24, \text{ etc} \end{aligned}$$

Also, by definition, $0! = 1! = 1$.

The factorial is useful for counting the number of ways that a set of objects can be ordered If a set of n objects is to be ordered from 1st to nth, then there are n choices for the 1st object, $n-1$ choices for the 2nd object, $n-2$ choices for the 3rd object, and so on, until there is only 1 choice for the nth object. Thus, by the multiplication principle, the number of ways of ordering the n objects is

$$n(n-1)(n-2) \quad (3)(2)(1) = n!$$

For example, the number of ways of ordering the letters A, B, and C is $3!$, or 6

ABC, ACB, BAC, BCA, CAB, and CBA

These orderings are called the *permutations* of the letters A, B, and C

A permutation can be thought of as a selection process in which objects are selected one by one in a certain order If the order of selection is not relevant and only k objects are to be selected from a larger set of n objects, a different counting method is employed Specifically, consider a set of n objects from which a complete selection of k objects is to be made without regard to order, where $0 \le k \le n$ Then the number of possible complete selections of k objects is called the number of *combinations* of n objects taken k at a time and is denoted by $\binom{n}{k}$ The value of $\binom{n}{k}$ is given by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Note that $\binom{n}{k}$ is the number of k-element subsets of a set with n elements For example, if $S = \{A, B, C, D, E\}$, then the number of 2-element subsets of S, or the number of combinations of 5 letters taken 2 at a time, is $\binom{5}{2} = \frac{5!}{2!3!} = \frac{120}{(2)(6)} = 10$

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The subsets are $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, E\}$, $\{B, C\}$, $\{B, D\}$, $\{B, E\}$, $\{C, D\}$, $\{C, E,\}$, and $\{D, E\}$ Note that $\binom{5}{2} = 10 = \binom{5}{3}$ since every 2-element subset chosen from a set of 5 elements corresponds to a unique 3-element subset consisting of the elements *not* chosen. In general,

$$\binom{n}{k} = \binom{n}{n-k}$$

11. DISCRETE PROBABILITY

Many of the ideas discussed in the preceding three topics are important to the study of discrete probability. Discrete probability is concerned with *experiments* that have a finite number of *outcomes* Given such an experiment, an *event* is a particular set of outcomes. For example, rolling a number cube with faces numbered 1 to 6 (similar to a 6-sided die) is an experiment with 6 possible outcomes 1, 2, 3, 4, 5, or 6 One event in this experiment is that the outcome is 4, denoted $\{4\}$, another event is that the outcome is an odd number $\{1, 3, 5\}$

The probability than an event E occurs, denoted by $P(E)$, is a number between 0 and 1, inclusive If E has no outcomes, then E is *impossible* and $P(E) = 0$, if E is the set of all possible outcomes of the experiment, then E is *certain* to occur and $P(E) = 1$ Otherwise, E is possible but uncertain, and $0 < P(E) < 1$ If F is a subset of E, then $P(F) \leq P(E)$ In the example above, if the probability of each of the 6 outcomes is the same, then the probability of each outcome is $\frac{1}{6}$, and the outcomes are said to be *equally likely* For experiments in which all of the individual outcomes are equally likely, the probability of an event E is

$$P(E) = \frac{\text{The number of outcomes in } E}{\text{The total number of possible outcomes}}$$

In the example, the probability that the outcome is an odd number is

$$P(\{1,3,5\}) = \frac{|\{1,3,5\}|}{6} = \frac{3}{6}$$

Given an experiment with events E and F, the following events are defined

"*not* E" is the set of outcomes that are not outcomes in E,

"E *or* F" is the set of outcomes in E or F or both, that is, $E \cup F$,

"E *and* F" is the set of outcomes in both E and F, that is, $E \cap F$

The probability that E does not occur is $P(\text{not } E) = 1 - P(E)$ The probability that "E or F" occurs is

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F),$$

using the general addition rule at the end of Section A 9 For the number cube, if E is the event that the outcome is an odd number, $\{1, 3, 5\}$, and F is the event that the outcome is a prime number, $\{2, 3, 5\}$, then $P(E \text{ and } F) = P(\{3,5\}) = \frac{2}{6}$, and so

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$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6}$$

Note that the event "E or F" is $E \cup F = \{1, 2, 3, 5\}$, and hence $P(E \text{ or } F) = \frac{|\{1,2,3,5\}|}{6} = \frac{4}{6}$

If the event "E and F" is impossible (that is, $E \cap F$ has no outcomes), then E and F are said to be *mutually exclusive* events, and $P(E \text{ and } F) = 0$. Then the general addition rule is reduced to

$$P(E \text{ or } F) = P(E) + P(F)$$

This is the special addition rule for the probability of two mutually exclusive events

Two events A and B are said to be *independent* if the occurrence of either event does not alter the probability that the other event occurs For one roll of the number cube, let $A = \{2, 4, 6\}$ and let $B = \{5, 6\}$ Then the probability that A occurs is $P(A) = \frac{|A|}{6} = \frac{3}{6} = \frac{1}{2}$, while, *presuming B occurs*, the probability that A occurs is

$$\frac{|A \cap B|}{|B|} = \frac{|\{6\}|}{|\{5,6\}|} = \frac{1}{2}$$

Similarly, the probability that B occurs is $P(B) = \frac{|B|}{6} = \frac{2}{6} = \frac{1}{3}$, while, *presuming A occurs*, the probability that B occurs is

$$\frac{|B \cap A|}{|A|} = \frac{|\{6\}|}{|\{2,4,6\}|} = \frac{1}{3}$$

Thus, the occurrence of either event does not affect the probability that the other event occurs Therefore, A and B are independent

The following multiplication rule holds for any independent events E and F

$$P(E \text{ and } F) = P(E)P(F)$$

For the independent events A and B above,

$$P(A \text{ and } B) = P(A)P(B)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \left(\frac{1}{6}\right)$$

Note that the event "A and B" is $A \cap B = \{6\}$, and hence $P(A \text{ and } B) = P(\{6\}) = \frac{1}{6}$

It follows from the general addition rule and the multiplication rule above that if E and F are independent, then

$$P(E \text{ or } F) = P(E) + P(F) - P(E)P(F)$$

For a final example of some of these rules, consider an experiment with events A, B, and C for which $P(A) = 0\ 23$, $P(B) = 0\ 40$, and $P(C) = 0\ 85$ Also, suppose that events A and B are mutually exclusive and events B and C are independent Then

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$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) \quad \text{(since } A \text{ and } B \text{ are mutually exclusive)} \\
&= 0.23 + 0.40 \\
&= 0.63 \\
P(B \text{ or } C) &= P(B) + P(C) - P(B)P(C) \quad \text{(by independence)} \\
&= 0.40 + 0.85 - (0.40)(0.85) \\
&= 0.91
\end{aligned}
$$

Note that $P(A \text{ or } C)$ and $P(A \text{ and } C)$ cannot be determined using the information given. But it can be determined that A and C are *not* mutually exclusive since $P(A) + P(C) = 1.08$, which is greater than 1, and therefore cannot equal $P(A \text{ or } C)$; from this it follows that $P(A \text{ and } C) \geq 0.08$. One can also deduce that $P(A \text{ and } C) \leq P(A) = 0.23$, since $A \cap C$ is a subset of A, and that $P(A \text{ or } C) \geq P(C) = 0.85$ since C is a subset of $A \cup C$. Thus, one can conclude that $0.85 \leq P(A \text{ or } C) \leq 1$ and $0.08 \leq P(A \text{ and } C) \leq 0.23$.

B. Algebra

Algebra is based on the operations of arithmetic and on the concept of an *unknown quantity*, or *variable*. Letters such as x or n are used to represent unknown quantities. For example, suppose Pam has 5 more pencils than Fred. If F represents the number of pencils that Fred has, then the number of pencils that Pam has is $F + 5$. As another example, if Jim's present salary S is increased by 7%, then his new salary is $1.07S$. A combination of letters and arithmetic operations, such as $F + 5$, $\frac{3x^2}{2x-5}$, and $19x^2 - 6x + 3$, is called an *algebraic expression*.

The expression $19x^2 - 6x + 3$ consists of the *terms* $19x^2$, $-6x$, and 3, where 19 is the *coefficient* of x^2, -6 is the coefficient of x^1, and 3 is a *constant term* (or coefficient of $x^0 = 1$). Such an expression is called a *second degree* (or *quadratic*) *polynomial in* x since the highest power of x is 2. The expression $F + 5$ is a *first degree* (or *linear*) *polynomial in* F since the highest power of F is 1. The expression $\frac{3x^2}{2x-5}$ is not a polynomial because it is not a sum of terms that are each powers of x multiplied by coefficients.

1. SIMPLIFYING ALGEBRAIC EXPRESSIONS

Often when working with algebraic expressions, it is necessary to simplify them by factoring or combining *like* terms. For example, the expression $6x + 5x$ is equivalent to $(6 + 5)x$, or $11x$. In the expression $9x - 3y$, 3 is a factor common to both terms: $9x - 3y = 3(3x - y)$. In the expression $5x^2 + 6y$, there are no like terms and no common factors.

If there are common factors in the numerator and denominator of an expression, they can be divided out, provided that they are not equal to zero.

For example, if $x \neq 3$, then $\frac{x-3}{x-3}$ is equal to 1; therefore,

$$
\begin{aligned}
\frac{3xy - 9y}{x-3} &= \frac{3y(x-3)}{x-3} \\
&= (3y)(1) \\
&= 3y
\end{aligned}
$$

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To multiply two algebraic expressions, each term of one expression is multiplied by each term of the other expression For example

$$\begin{aligned}(3x-4)(9y+x) &= 3x(9y+x)-4(9y+x)\\ &= (3x)(9y)+(3x)(x)+(-4)(9y)+(-4)(x)\\ &= 27xy+3x^2-36y-4x\end{aligned}$$

An algebraic expression can be evaluated by substituting values of the unknowns in the expression For example, if $x=3$ and $y=-2$, then $3xy-x^2+y$ can be evaluated as

$$3(3)(-2)-(3)^2+(-2) = -18-9-2 = -29$$

2. EQUATIONS

A major focus of algebra is to solve equations involving algebraic expressions Some examples of such equations are

$$5x-2=9-x \quad \text{(a linear equation with one unknown)}$$
$$3x+1=y-2 \quad \text{(a linear equation with two unknowns)}$$
$$5x^2+3x-2=7x \quad \text{(a quadratic equation with one unknown)}$$

$$\frac{x(x-3)(x^2+5)}{x-4}=0$$

(an equation that is factored on one side with 0 on the other)

The *solutions* of an equation with one or more unknowns are those values that make the equation true, or "satisfy the equation," when they are substituted for the unknowns of the equation An equation may have no solution or one or more solutions If two or more equations are to be solved together, the solutions must satisfy all of the equations simultaneously

Two equations having the same solution(s) are *equivalent equations* For example, the equations

$$\begin{aligned}2+x &= 3\\ 4+2x &= 6\end{aligned}$$

each have the unique solution $x=1$ Note that the second equation is the first equation multiplied by 2 Similarly, the equations

$$\begin{aligned}3x-y &= 6\\ 6x-2y &= 12\end{aligned}$$

have the same solutions, although in this case each equation has infinitely many solutions If any value is assigned to x, then $3x-6$ is a corresponding value for y that will satisfy both equations, for example, $x=2$ and $y=0$ is a solution to both equations, as is $x=5$ and $y=9$

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3. SOLVING LINEAR EQUATIONS WITH ONE UNKNOWN

To solve a linear equation with one unknown (that is, to find the value of the unknown that satisfies the equation), the unknown should be isolated on one side of the equation This can be done by performing the same mathematical operations on both sides of the equation Remember that if the same number is added to or subtracted from both sides of the equation, this does not change the equality, likewise, multiplying or dividing both sides by the same nonzero number does not change the equality For example, to solve the equation $\frac{5x-6}{3} = 4$ for x, the variable x can be isolated using the following steps

$$\frac{5x-6}{3} = 4$$

$$5x - 6 = 12 \quad \text{(multiplying by 3)}$$

$$5x = 12 + 6 = 18 \quad \text{(adding 6)}$$

$$x = \frac{18}{5} \quad \text{(dividing by 5)}$$

The solution, $\frac{18}{5}$, can be checked by substituting it for x in the original equation to determine whether it satisfies that equation

$$\frac{5\left(\frac{18}{5}\right)-6}{3} = \frac{18-6}{3} = \frac{12}{3} = 4$$

Therefore, $x = \frac{18}{5}$ is the solution

4 SOLVING TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

For two linear equations with two unknowns, if the equations are equivalent, then there are infinitely many solutions to the equations, as illustrated at the end of Section B 2 If the equations are not equivalent, then they have either one unique solution or no solution The latter case is illustrated by the two equations

$$3x + 4y = 17$$
$$6x + 8y = 35$$

Note that $3x + 4y = 17$ implies $6x + 8y = 34$, which contradicts the second equation Thus, no values of x and y can simultaneously satisfy both equations

There are several methods of solving two linear equations in two unknowns With any method, if a contradiction is reached, then the equations have no solution, if a trivial equation such as $0 = 0$ is reached, then the equations are equivalent and have infinitely many solutions Otherwise, a unique solution can be found

One way to solve for the two unknowns is to express one of the unknowns in terms of the other using one of the equations, and then substitute the expression into the remaining equation to obtain an equation with one unknown This equation can be solved and the value of the unknown substituted into either of the original equations to find the value of the other unknown For example, the following two equations can be solved for x and y

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$$(1)\quad 3x + 2y = 11$$
$$(2)\quad x - y = 2$$

In equation (2), $x = 2 + y$ Substitute $2 + y$ in equation (1) for x

$$3(2+y) + 2y = 11$$
$$6+3y + 2y = 11$$
$$6+5y = 11$$
$$5y = 5$$
$$y = 1$$

If $y = 1$, then $x = 2 + 1 = 3$

There is another way to solve for x and y by eliminating one of the unknowns This can be done by making the coefficients of one of the unknowns the same (disregarding the sign) in both equations and either adding the equations or subtracting one equation from the other For example, to solve the equations

$$(1)\quad 6x + 5y = 29$$
$$(2)\quad 4x - 3y = -6$$

by this method, multiply equation (1) by 3 and equation (2) by 5 to get

$$18x + 15y = 87$$
$$20x - 15y = -30$$

Adding the two equations eliminates y, yielding $38x = 57$, or $x = \frac{3}{2}$ Finally, substituting $\frac{3}{2}$ for x in one of the equations gives $y = 4$ These answers can be checked by substituting both values into both of the original equations

5. SOLVING EQUATIONS BY FACTORING

Some equations can be solved by factoring To do this, first add or subtract expressions to bring all the expressions to one side of the equation, with 0 on the other side Then try to factor the nonzero side into a product of expressions If this is possible, then using property (7) in Section A 4 each of the factors can be set equal to 0, yielding several simpler equations that possibly can be solved The solutions of the simpler equations will be solutions of the factored equation As an example, consider the equation $x^3 - 2x^2 + x = -5\,(x-1)^2$

$$x^3 - 2x^2 + x + 5\,(x-1)^2 = 0$$
$$x(x^2 - 2x + 1) + 5\,(x-1)^2 = 0$$
$$x(x-1)^2 + 5\,(x-1)^2 = 0$$
$$(x+5)\,(x-1)^2 = 0$$
$$x + 5 = 0 \text{ or } (x-1)^2 = 0$$
$$x = -5 \text{ or } x = 1$$

For another example, consider $\dfrac{x(x-3)(x^2+5)}{x-4} = 0$ A fraction equals 0 if and only if its numerator equals 0 Thus, $x(x-3)(x^2+5) = 0$

$$x = 0 \text{ or } x - 3 = 0 \text{ or } x^2 + 5 = 0$$
$$x = 0 \text{ or } x = 3 \text{ or } x^2 + 5 = 0$$

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But $x^2 + 5 = 0$ has no real solution since $x^2 + 5 > 0$ for every real number Thus, the solutions are 0 and 3

The solutions of an equation are also called the *roots* of the equation These roots can be checked by substituting them into the original equation to determine whether they satisfy the equation

6. SOLVING QUADRATIC EQUATIONS

The standard form for a *quadratic equation* is

$$ax^2 + bx + c = 0,$$

where $a, b,$ and c are real numbers and $a \neq 0$, for example

$$x^2 + 6x + 5 = 0,$$
$$3x^2 - 2x = 0, \text{ and}$$
$$x^2 + 4 = 0$$

Some quadratic equations can easily be solved by factoring For example

(1) $$x^2 + 6x + 5 = 0$$
$$(x+5)(x+1) = 0$$
$$x + 5 = 0 \text{ or } x + 1 = 0$$
$$x = -5 \text{ or } x = -1$$

(2) $$3x^2 - 3 = 8x$$
$$3x^2 - 8x - 3 = 0$$
$$(3x+1)(x-3) = 0$$
$$3x + 1 = 0 \text{ or } x - 3 = 0$$
$$x = -\frac{1}{3} \text{ or } x = 3$$

A quadratic equation has at most two real roots and may have just one or even no real root For example, the equation $x^2 - 6x + 9 = 0$ can be expressed as $(x-3)^2 = 0$, or $(x-3)(x-3) = 0$, thus the only root is 3 The equation $x^2 + 4 = 0$ has no real root; since the square of any real number is greater than or equal to zero, $x^2 + 4$ must be greater than zero

An expression of the form $a^2 - b^2$ can be factored as $(a-b)(a+b)$

For example, the quadratic equation $9x^2 - 25 = 0$ can be solved as follows

$$(3x-5)(3x+5) = 0$$
$$3x - 5 = 0 \text{ or } 3x + 5 = 0$$
$$x = \frac{5}{3} \text{ or } x = -\frac{5}{3}$$

If a quadratic expression is not easily factored, then its roots can always be found using the *quadratic formula*. If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the roots are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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These are two distinct real numbers unless $b^2 - 4ac \leq 0$. If $b^2 - 4ac = 0$, then these two expressions for x are equal to $-\frac{b}{2a}$, and the equation has only one root If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number and the equation has no real roots.

7 EXPONENTS

A positive integer exponent of a number or a variable indicates a product, and the positive integer is the number of times that the number or variable is a factor in the product For example, x^5 means $(x)(x)(x)(x)(x)$, that is, x is a factor in the product 5 times

Some rules about exponents follow

Let x and y be any positive numbers, and let r and s be any positive integers

(1) $(x^r)(x^s) = x^{(r+s)}$, for example, $(2^2)(2^3) = 2^{(2+3)} = 2^5 = 32$

(2) $\frac{x^r}{x^s} = x^{r-s}$, for example, $\frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64$

(3) $(x^r)(y^r) = (xy)^r$, for example, $(3^3)(4^3) = 12^3 = 1{,}728$

(4) $\left(\frac{x}{y}\right)^r = \frac{x^r}{y^r}$, for example, $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

(5) $(x^r)^s = x^{rs} = (x^s)^r$, for example, $(x^3)^4 = x^{12} = (x^4)^3$

(6) $x^{-r} = \frac{1}{x^r}$, for example, $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(7) $x^0 = 1$, for example, $6^0 = 1$

(8) $x^{\frac{r}{s}} = \left(x^{\frac{1}{s}}\right)^r = \left(x^r\right)^{\frac{1}{s}} = \sqrt[s]{x^r}$, for example, $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(8^2\right)^{\frac{1}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64}$

$= 4$ and $9^{\frac{1}{2}} = \sqrt{9} = 3$

It can be shown that rules 1-6 also apply when r and s are not integers and are not positive, that is, when r and s are any real numbers

8. INEQUALITIES

An *inequality* is a statement that uses one of the following symbols

$\neq$	not equal to
$>$	greater than
$\geq$	greater than or equal to
$<$	less than
$\leq$	less than or equal to

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Some examples of inequalities are $5x-3<9$, $6x \geq y$, and $\frac{1}{2}<\frac{3}{4}$. Solving a linear inequality with one unknown is similar to solving an equation; the unknown is isolated on one side of the inequality. As in solving an equation, the same number can be added to or subtracted from both sides of the inequality, or both sides of an inequality can be multiplied or divided by a positive number without changing the truth of the inequality. However, multiplying or dividing an inequality by a negative number reverses the order of the inequality. For example, $6>2$, but $(-1)(6)<(-1)(2)$.

To solve the inequality $3x-2>5$ for x, isolate x by using the following steps:

$$3x-2>5$$
$$3x>7 \quad \text{(adding 2 to both sides)}$$
$$x>\frac{7}{3} \quad \text{(dividing both sides by 3)}$$

To solve the inequality $\frac{5x-1}{-2}<3$ for x, isolate x by using the following steps:

$$\frac{5x-1}{-2}<3$$
$$5x-1>-6 \quad \text{(multiplying both sides by } -2\text{)}$$
$$5x>-5 \quad \text{(adding 1 to both sides)}$$
$$x>-1 \quad \text{(dividing both sides by 5)}$$

9. ABSOLUTE VALUE

The absolute value of x, denoted $|x|$, is defined to be x if $x \geq 0$ and $-x$ if $x<0$. Note that $\sqrt{x^2}$ denotes the nonnegative square root of x^2, and so $\sqrt{x^2}=|x|$.

10. FUNCTIONS

An algebraic expression in one variable can be used to define a *function* of that variable. A function is denoted by a letter such as f or g along with the variable in the expression. For example, the expression x^3-5x^2+2 defines a function f that can be denoted by

$$f(x)=x^3-5x^2+2$$

The expression $\frac{2z+7}{\sqrt{z+1}}$ defines a function g that can be denoted by

$$g(z)=\frac{2z+7}{\sqrt{z+1}}$$

The symbols "$f(x)$" or "$g(z)$" do not represent products; each is merely the symbol for an expression, and is read "f of x" or "g of z."

Function notation provides a short way of writing the result of substituting a value for a variable. If $x=1$ is substituted in the first expression, the result can be written $f(1)=-2$, and $f(1)$ is called the "value of f at $x=1$." Similarly, if $z=0$ is substituted in the second expression, then the value of g at $z=0$ is $g(0)=7$.

Once a function $f(x)$ is defined, it is useful to think of the variable x as an input and $f(x)$ as the corresponding output. In any function there can be no more than one output for a given input. However, more than one input can give the same output; for example, if $h(x)=|x+3|$, then $h(-4)=1=h(-2)$.

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The set of all allowable inputs for a function is called the *domain* of the function For f and g defined above, the domain of f is the set of all real numbers and the domain of g is the set of all numbers greater than -1 The domain of any function can be arbitrarily specified, as in the function defined by "$h(x) = 9x - 5$ for $0 \le x \le 10$" Without such a restriction, the domain is assumed to be all values of x that result in a real number when substituted into the function

The domain of a function can consist of only the positive integers and possibly 0 For example,

$$a(n) = n^2 + \frac{n}{5} \text{ for } n = 0, 1, 2, 3,$$

Such a function is called a *sequence* and $a(n)$ is denoted by a_n The value of the sequence a_n at $n = 3$ is $a_3 = 3^2 + \frac{3}{5} = 9\ 6$ As another example, consider the sequence defined by $b_n = (-1)^n (n!)$ for $n = 1, 2, 3,$ A sequence like this is often indicated by listing its values in the order $b_1, b_2, b_3,$ $, b_n,$ as follows

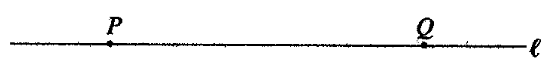
$$-1, 2, -6, \quad , (-1)^n(n!), \quad ,$$

and $(-1)^n(n!)$ is called the nth term of the sequence

C. Geometry

1. LINES

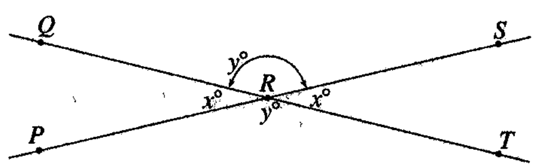
In geometry, the word "line" refers to a straight line that extends without end in both directions



The line above can be referred to as line PQ or line ℓ. The part of the line from P to Q is called a *line segment*. P and Q are the *endpoints* of the segment The notation PQ is used to denote both the segment and the length of the segment The intention of the notation can be determined from the context

2. INTERSECTING LINES AND ANGLES

If two lines intersect, the opposite angles are called *vertical angles* and have the same measure In the figure



$\angle PRQ$ and $\angle SRT$ are vertical angles and $\angle QRS$ and $\angle PRT$ are vertical angles Also, $x + y = 180$ since PRS is a straight line

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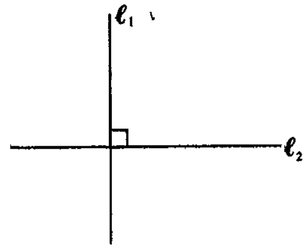
3. PERPENDICULAR LINES

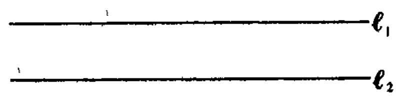
An angle that has a measure of 90° is a *right angle.* If two lines intersect at right angles, the lines are *perpendicular.* For example.



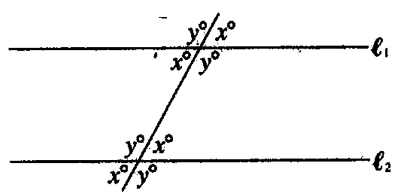
ℓ_1 and ℓ_2 above are perpendicular, denoted by $\ell_1 \perp \ell_2$. A right angle symbol in an angle of intersection indicates that the lines are perpendicular

4. PARALLEL LINES

If two lines that are in the same plane do not intersect, the two lines are *parallel* In the figure



lines ℓ_1 and ℓ_2 are parallel, denoted by $\ell_1 \parallel \ell_2$. If two parallel lines are intersected by a third line, as shown below, then the angle measures are related as indicated, where $x + y = 180$



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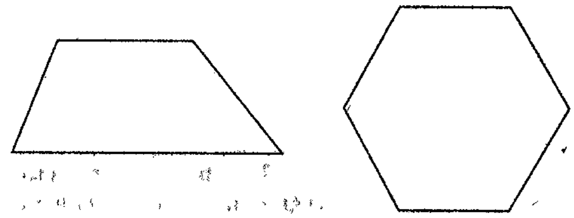
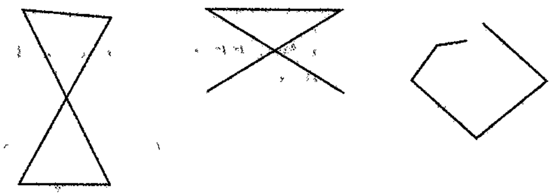
5 POLYGONS (CONVEX)

A *polygon* is a closed plane figure formed by three or more line segments, called the *sides* of the polygon Each side intersects exactly two other sides at their endpoints The points of intersection of the sides are *vertices* The term "polygon" will be used to mean a convex polygon, that is, a polygon in which each interior angle has a measure of less than 180°

The following figures are polygons

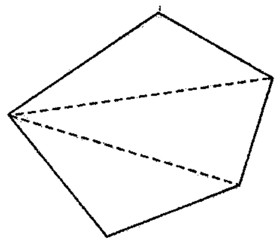


The following figures are not polygons



A polygon with three sides is a *triangle*, with four sides, a *quadrilateral*, with five sides, a *pentagon*, and with six sides, a *hexagon*.

The sum of the interior angle measures of a triangle is 180° In general, the sum of the interior angle measures of a polygon with n sides is equal to $(n-2)180°$ For example, this sum for a pentagon is $(5-2)180 = (3)180 = 540$ degrees



Note that a pentagon can be partitioned into three triangles and therefore the sum of the angle measures can be found by adding the sum of the angle measures of three triangles

The *perimeter* of a polygon is the sum of the lengths of its sides

The commonly used phrase "area of a triangle" (or any other plane figure) is used to mean the area of the region enclosed by that figure

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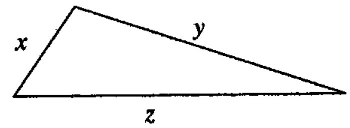
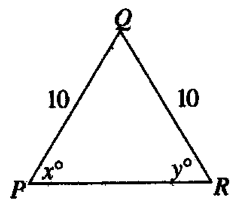
6. TRIANGLES

There are several special types of triangles with important properties But one property that all triangles share is that the sum of the lengths of any two of the sides is greater than the length of the third side, as illustrated below

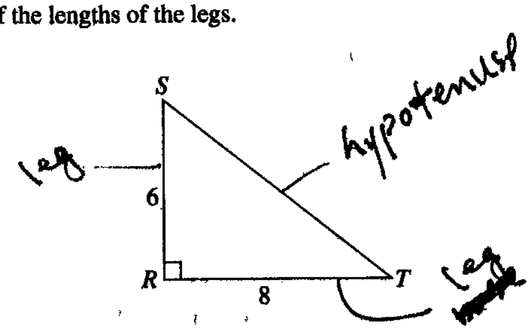


$$\boxed{x + y > z, \quad x + z > y, \text{ and } y + z > x}$$

An *equilateral* triangle has all sides of equal length All angles of an equilateral triangle have equal measure An *isosceles* triangle has at least two sides of the same length If two sides of a triangle have the same length, then the two angles opposite those sides have the same measure Conversely, if two angles of a triangle have the same measure, then the sides opposite those angles have the same length. In isosceles triangle PQR below, $x = y$ since $PQ = QR$



A triangle that has a right angle is a *right* triangle In a right triangle, the side opposite the right angle is the *hypotenuse*, and the other two sides are the *legs* An important theorem concerning right triangles is the *Pythagorean theorem*, which states In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



In the figure above, ΔRST is a right triangle, so $(RS)^2 + (RT)^2 = (ST)^2$ Here, $RS = 6$ and $RT = 8$, so $ST = 10$, since $6^2 + 8^2 = 36 + 64 = 100 = (ST)^2$ and $ST = \sqrt{100}$. Any triangle in which the lengths of the sides are in the ratio 3:4:5 is a right triangle In general, if a, b, and c are the lengths of the sides of a triangle and $a^2 + b^2 = c^2$, then the triangle is a right triangle

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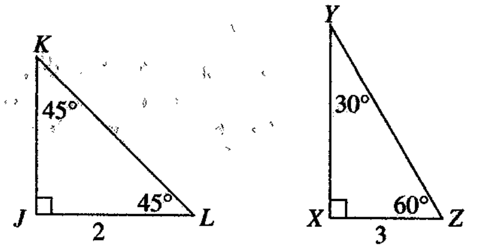

In 45°-45°-90° triangles, the lengths of the sides are in the ratio $\boxed{1:1:\sqrt{2}}$. For example, in ΔJKL, if $JL = 2$, then $JK = 2$ and $KL = 2\sqrt{2}$. In 30°-60°-90° triangles, the lengths of the sides are in the ratio $1:\sqrt{3}:2$. For example, in ΔXYZ, if $XZ = 3$, then $XY = 3\sqrt{3}$ and $YZ = 6$.

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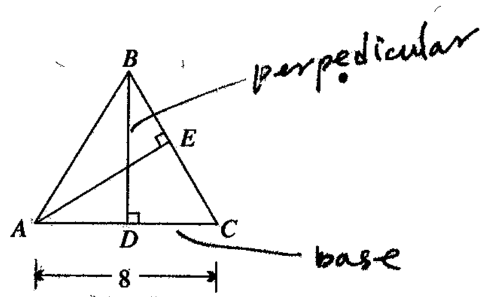
The *altitude* of a triangle is the segment drawn from a vertex perpendicular to the side opposite that vertex. Relative to that vertex and altitude, the opposite side is called the *base*.

The area of a triangle is equal to

$$\frac{\text{(the length of the altitude)} \times \text{(the length of the base)}}{2}$$

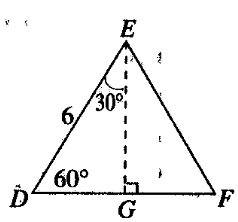


$BD = 5$

In ΔABC, BD is the altitude to base AC and AE is the altitude to base BC. The area of ΔABC is equal to

$$\frac{BD \times AC}{2} = \frac{5 \times 8}{2} = 20$$

The area is also equal to $\frac{AE \times BC}{2}$. If ΔABC above is isosceles and $AB = BC$, then altitude BD bisects the base, that is, $AD = DC = 4$. Similarly, any altitude of an equilateral triangle bisects the side to which it is drawn.



In equilateral triangle DEF, if $DE = 6$, then $DG = 3$ and $EG = 3\sqrt{3}$. The area of ΔDEF is equal to $\frac{3\sqrt{3} \times 6}{2} = 9\sqrt{3}$.

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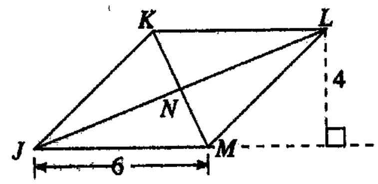
7. QUADRILATERALS

A polygon with four sides is a *quadrilateral* A quadrilateral in which both pairs of opposite sides are parallel is a *parallelogram*. The opposite sides of a parallelogram also have equal length.



In parallelogram $JKLM$, $JK \parallel LM$ and $JK = LM$, $KL \parallel JM$ and $KL = JM$.

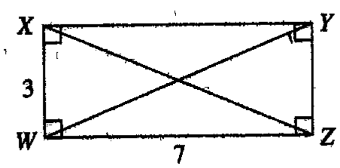
The diagonals of a parallelogram bisect each other (that is, $KN = NM$ and $JN = NL$)

The area of a parallelogram is equal to

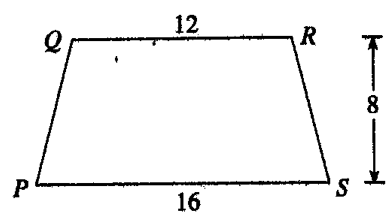
(the length of the altitude) × (the length of the base).

The area of $JKLM$ is equal to $4 \times 6 = 24$

A parallelogram with right angles is a *rectangle*, and a rectangle with all sides of equal length is a *square*



The perimeter of $WXYZ = 2(3) + 2(7) = 20$ and the area of $WXYZ$ is equal to $3 \times 7 = 21$ The diagonals of a rectangle are equal, therefore $WY = XZ = \sqrt{9+49} = \sqrt{58}$.



A quadrilateral with two sides that are parallel, as shown above, is a *trapezoid*. The area of trapezoid $PQRS$ may be calculated as follows.

$$\frac{1}{2}\text{ (sum of bases)(height)} = \frac{1}{2}(QR + PS)(8) = \frac{1}{2}(28 \times 8) = 112.$$

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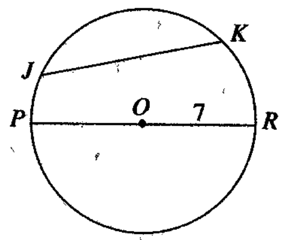
8. CIRCLES

A *circle* is a set of points in a plane that are all located the same distance from a fixed point (the *center* of the circle)

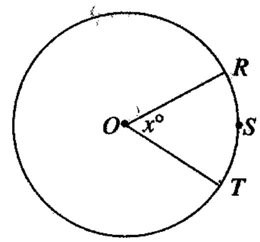
A *chord* of a circle is a line segment that has its endpoints on the circle A chord that passes through the center of the circle is a *diameter* of the circle A *radius* of a circle is a segment from the center of the circle to a point on the circle The words "diameter" and "radius" are also used to refer to the lengths of these segments

The *circumference* of a circle is the distance around the circle If r is the radius of the circle, then the circumference is equal to $2\pi r$, where π is approximately $\frac{22}{7}$ or 3 14 The *area* of a circle of radius r is equal to πr^2



In the circle above, O is the center of the circle and JK and PR are chords. PR is a diameter and OR is a radius. If $OR = 7$, then the circumference of the circle is $2\pi(7) = 14\pi$ and the area of the circle is $\pi(7)^2 = 49\pi$

The number of degrees of arc in a circle (or the number of degrees in a complete revolution) is 360



In the circle with center O above, the length of arc RST is $\frac{x}{360}$ of the circumference of the circle; for example, if $x = 60$, then arc RST has length $\frac{1}{6}$ of the circumference of the circle

A line that has exactly one point in common with a circle is said to be *tangent* to the circle, and that common point is called the *point of tangency* A radius or diameter with an endpoint at the point of tangency is perpendicular to the tangent line, and, conversely, a line that is perpendicular to a diameter at one of its endpoints is tangent to the circle at that endpoint

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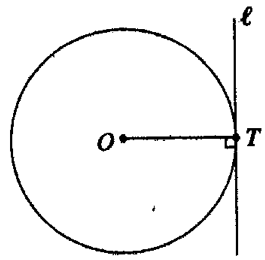
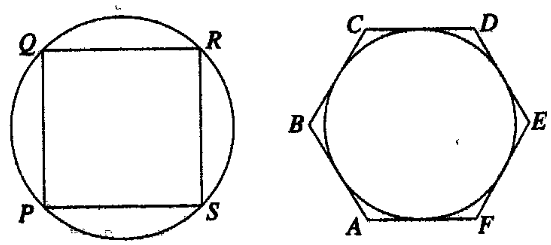


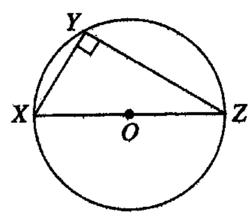
The line ℓ above is tangent to the circle and radius OT is perpendicular to ℓ.

If each vertex of a polygon lies on a circle, then the polygon is *inscribed* in the circle and the circle is *circumscribed* about the polygon If each side of a polygon is tangent to a circle, then the polygon is *circumscribed* about the circle and the circle is *inscribed* in the polygon



In the figure above, quadrilateral $PQRS$ is inscribed in a circle and hexagon $ABCDEF$ is circumscribed about a circle.

If a triangle is inscribed in a circle so that one of its sides is a diameter of the circle, then the triangle is a right triangle



In the circle above, XZ is a diameter and the measure of $\angle XYZ$ is 90°

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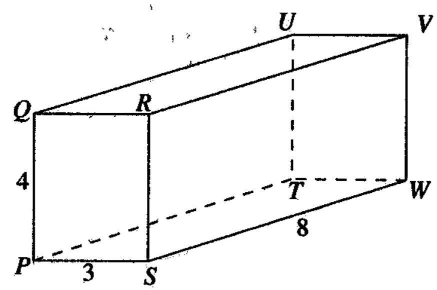
9. RECTANGULAR SOLIDS AND CYLINDERS

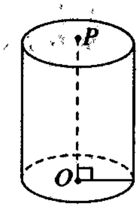
A *rectangular solid* is a three-dimensional figure formed by six rectangular surfaces, as shown below. Each rectangular surface is a *face*. Each solid or dotted line segment is an *edge*, and each point at which the edges meet is a *vertex*. A rectangular solid has six faces, twelve edges, and eight vertices. Opposite faces are parallel rectangles that have the same dimensions. A rectangular solid in which all edges are of equal length is a *cube*.

The *surface area* of a rectangular solid is equal to the sum of the areas of all the faces. The *volume* is equal to

(length) × (width) × (height),
in other words, (area of base) × (height).



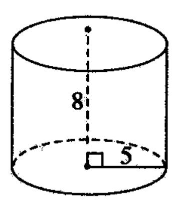
In the rectangular solid above, the dimensions are 3, 4, and 8. The surface area is equal to $2(3 \times 4) + 2(3 \times 8) + 2(4 \times 8) = 136$. The volume is equal to $3 \times 4 \times 8 = 96$.



The figure above is a right circular *cylinder*. The two bases are circles of the same size with centers O and P, respectively, and altitude (height) OP is perpendicular to the bases. The surface area of a right circular cylinder with a base of radius r and height h is equal to $2(\pi r^2) + 2\pi rh$ (the sum of the areas of the two bases plus the area of the curved surface).

The volume of a cylinder is equal to $\pi r^2 h$, that is,

(area of base) × (height).



In the cylinder above, the surface area is equal to

$$2(25\pi) + 2\pi(5)(8) = 130\pi,$$

and the volume is equal to

$$25\pi(8) = 200\pi$$

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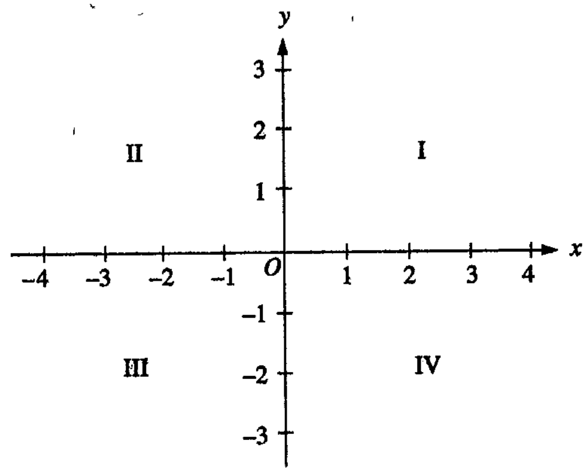
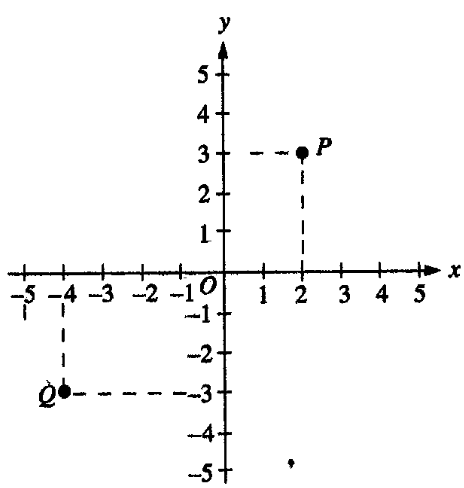
10. COORDINATE GEOMETRY



The figure above shows the (rectangular) *coordinate plane* The horizontal line is called the *x-axis* and the perpendicular vertical line is called the *y-axis* The point at which these two axes intersect, designated O, is called the *origin*. The axes divide the plane into four quadrants, I, II, III, and IV, as shown.

Each point in the plane has an *x-coordinate* and a *y-coordinate*. A point is identified by an ordered pair (x, y) of numbers in which the x-coordinate is the first number and the y-coordinate is the second number



In the graph above, the (x, y) coordinates of point P are (2, 3) since P is 2 units to the right of the y-axis (that is, $x = 2$) and 3 units above the x-axis (that is, $y = 3$) Similarly, the (x, y) coordinates of point Q are $(-4, -3)$ The origin O has coordinates (0, 0).

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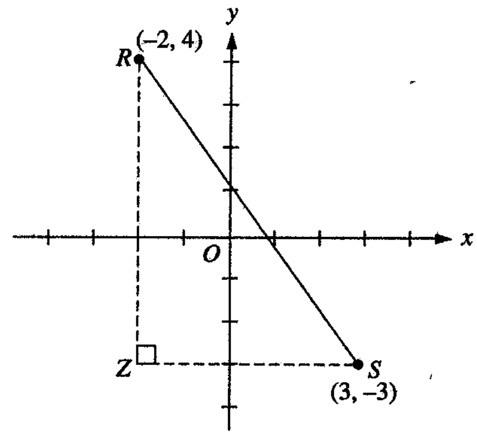
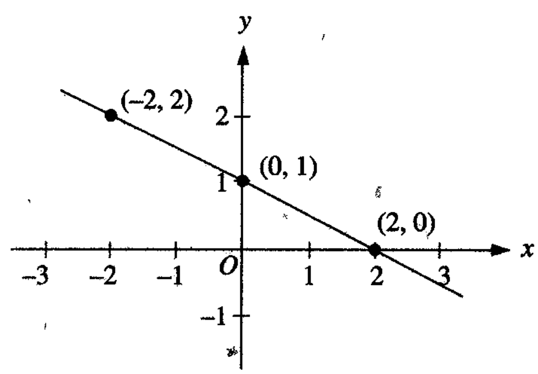
One way to find the distance between two points in the coordinate plane is to use the Pythagorean theorem



To find the distance between points R and S using the Pythagorean theorem, draw the triangle as shown Note that Z has (x, y) coordinates $(-2, -3)$, $RZ = 7$, and $ZS = 5$ Therefore, the distance between R and S is equal to

$$\sqrt{7^2 + 5^2} = \sqrt{74}$$

For a line in the coordinate plane, the coordinates of each point on the line satisfy a linear equation of the form $y = mx + b$ (or the form $x = a$ if the line is vertical) For example, each point on the line below satisfies the equation $y = -\frac{1}{2}x + 1$ One can verify this for the points $(-2, 2)$, $(2, 0)$, and $(0, 1)$ by substituting the respective coordinates for x and y in the equation



In the equation $y = mx + b$ of a line, the coefficient m is the *slope* of the line and the constant term b is the *y-intercept* of the line For any two points on the line, the slope is defined to be the ratio of the difference in the y-coordinates to the difference in the x-coordinates Using $(-2, 2)$ and $(2, 0)$ above, the slope is

$$\frac{\text{The difference in the y-coordinates}}{\text{The difference in the x-coordinates}} = \frac{0-2}{2-(-2)} = \frac{-2}{4} = -\frac{1}{2}$$

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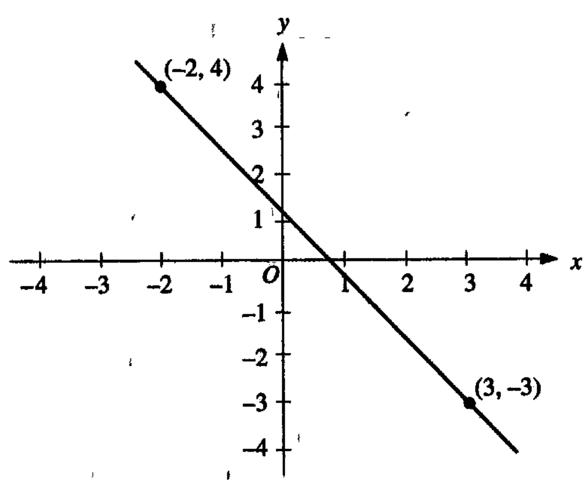
The y-intercept is the y-coordinate of the point at which the line intersects the y-axis For the line above, the y-intercept is 1, and this is the resulting value of y when x is set equal to 0 in the equation $y = -\frac{1}{2}x+1$ The *x-intercept* is the x-coordinate of the point at which the line intersects the x-axis The x-intercept can be found by setting $y=0$ and solving for x For the line $y = -\frac{1}{2}x+1$, this gives

$$\begin{aligned} -\frac{1}{2}x+1 &= 0 \\ -\frac{1}{2}x &= -1 \\ x &= 2. \end{aligned}$$

Thus, the x-intercept is 2

Given any two points (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$, the equation of the line passing through these points can be found by applying the definition of slope Since the slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$, then using a point known to be on the line, say (x_1, y_1), any point (x, y) on the line must satisfy $\frac{y-y_1}{x-x_1} = m$, or $y - y_1 = m(x - x_1)$ (Using (x_2, y_2) as the known point would yield an equivalent equation) For example, consider the points $(-2, 4)$ and $(3, -3)$ on the line below



The slope of this line is $\frac{-3-4}{3-(-2)} = \frac{-7}{5}$, so an equation of this line can be found using the point $(3, -3)$ as follows

$$\begin{aligned} y-(-3) &= -\frac{7}{5}(x-3) \\ y+3 &= -\frac{7}{5}x+\frac{21}{5} \\ y &= -\frac{7}{5}x+\frac{6}{5} \end{aligned}$$

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The y-intercept is $\frac{6}{5}$ The *x-intercept* can be found as follows

$$0 = -\frac{7}{5}x + \frac{6}{5}$$
$$\frac{7}{5}x = \frac{6}{5}$$
$$x = \frac{6}{7}$$

Both of these intercepts can be seen on the graph

If the slope of a line is negative, the line slants downward from left to right, if the slope is positive, the line slants upward If the slope is 0, the line is horizontal, the equation of such a line is of the form $y = b$ since $m = 0$ For a vertical line, slope is not defined, and the equation is of the form $x = a$, where a is the x-intercept

There is a connection between graphs of lines in the coordinate plane and solutions of two linear equations with two unknowns. If two linear equations with unknowns x and y have a unique solution, then the graphs of the equations are two lines that intersect in one point, which is the solution If the equations are equivalent, then they represent the same line with infinitely many points or solutions If the equations have no solution, then they represent parallel lines, which do not intersect

There is also a connection between functions (see Section B 10) and the coordinate plane If a function is graphed in the coordinate plane, the function can be understood in different and useful ways Consider the function defined by

$$f(x) = -\frac{7}{5}x + \frac{6}{5}$$

If the value of the function, $f(x)$, is equated with the variable y, then the graph of the function in the xy-coordinate plane is simply the graph of the equation

$$y = -\frac{7}{5}x + \frac{6}{5}$$

shown above Similarly, any function $f(x)$ can be graphed by equating y with the value of the function

$$y = f(x)$$

So for any x in the domain of the function f, the point with coordinates $(x, f(x))$ is on the graph of f, and the graph consists entirely of these points

As another example, consider a quadratic polynomial function defined by $f(x) = x^2 - 1$ One can plot several points $(x, f(x))$ on the graph to understand the connection between a function and its graph

x	$f(x)$
–2	3
–1	0
0	–1
1	0
2	3

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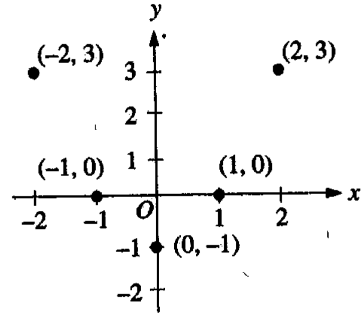
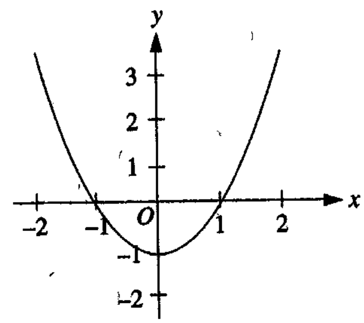


If all of the points were graphed for $-2 \le x \le 2$, then the graph would appear as follows



The graph of a quadratic function is called a *parabola* and always has the shape of the curve above, although it may be upside down or have a greater or lesser width Note that the roots of the equation $f(x) = x^2 - 1 = 0$ are $x = 1$ and $x = -1$, these coincide with the x-intercepts since x-intercepts are found by setting $y = 0$ and solving for x. Also, the y-intercept is $f(0) = -1$ since this is the value of y corresponding to $x = 0$ For any function f, the x-intercepts are the solutions of the equation $f(x) = 0$ and the y-intercept is the value $f(0)$

D. Word Problems

Many of the principles discussed in this chapter are used to solve word problems The following discussion of word problems illustrates some of the techniques and concepts used in solving such problems

1. RATE PROBLEMS

The distance that an object travels is equal to the product of the average speed at which it travels and the amount of time it takes to travel that distance, that is,

$$\text{Rate} \times \text{Time} = \text{Distance}$$

Example 1 If a car travels at an average speed of 70 kilometers per hour for 4 hours, how many kilometers does it travel?

Solution Since rate × time = distance, simply multiply 70 km/hour × 4 hours Thus, the car travels 280 kilometers in 4 hours

To determine the average rate at which an object travels, divide the total distance traveled by the total amount of traveling time

Example 2 On a 400-mile trip, car X traveled half the distance at 40 miles per hour and the other half at 50 miles per hour What was the average speed of car X?

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Solution First it is necessary to determine the amount of traveling time During the first 200 miles, the car traveled at 40 mph, therefore, it took $\frac{200}{40} = 5$ hours to travel the first 200 miles During the second 200 miles, the car traveled at 50 mph, therefore, it took $\frac{200}{50} = 4$ hours to travel the second 200 miles Thus, the average speed of car X was $\frac{400}{9} = 44\frac{4}{9}$ mph Note that the average speed is *not* $\frac{40+50}{2} = 45$

Some rate problems can be solved by using ratios

Example 3 If 5 shirts cost $44, then, at this rate, what is the cost of 8 shirts?

Solution If c is the cost of the 8 shirts, then $\frac{5}{44} = \frac{8}{c}$ Cross multiplication results in the equation

$$5c = 8 \times 44 = 352$$

$$c = \frac{352}{5} = 70\ 40$$

The 8 shirts cost $70.40

2. WORK PROBLEMS

In a work problem, the rates at which certain persons or machines work alone are usually given, and it is necessary to compute the rate at which they work together (or vice versa)

The basic formula for solving work problems is $\frac{1}{r} + \frac{1}{s} = \frac{1}{h}$, where r and s are, for example, the number of hours it takes Rae and Sam, respectively, to complete a job when working alone, and h is the number of hours it takes Rae and Sam to do the job when working together The reasoning is that in 1 hour Rae does $\frac{1}{r}$ of the job, Sam does $\frac{1}{s}$ of the job, and Rae and Sam together do $\frac{1}{h}$ of the job

Example 1 If machine X can produce 1,000 bolts in 4 hours and machine Y can produce 1,000 bolts in 5 hours, in how many hours can machines X and Y, working together at these constant rates, produce 1,000 bolts?

Solution

$$\frac{1}{4} + \frac{1}{5} = \frac{1}{h}$$

$$\frac{5}{20} + \frac{4}{20} = \frac{1}{h}$$

$$\frac{9}{20} = \frac{1}{h}$$

$$9h = 20$$

$$h = \frac{20}{9} = 2\frac{2}{9}$$

Working together, machines X and Y can produce 1,000 bolts in $2\frac{2}{9}$ hours

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Example 2: If Art and Rita can do a job in 4 hours when working together at their respective constant rates and Art can do the job alone in 6 hours, in how many hours can Rita do the job alone?

Solution

$$\frac{1}{6}+\frac{1}{R}=\frac{1}{4}$$

$$\frac{R+6}{6R}=\frac{1}{4}$$

$$\begin{aligned} 4R + 24 &= 6R \\ 24 &= 2R \\ 12 &= R \end{aligned}$$

Working alone, Rita can do the job in 12 hours.

3. MIXTURE PROBLEMS

In mixture problems, substances with different characteristics are combined, and it is necessary to determine the characteristics of the resulting mixture.

Example 1 If 6 pounds of nuts that cost \$1.20 per pound are mixed with 2 pounds of nuts that cost \$1.60 per pound, what is the cost per pound of the mixture?

Solution The total cost of the 8 pounds of nuts is

$$6(\$1.20) + 2(\$1.60) = \$10.40$$

The cost per pound is $\frac{\$10.40}{8} = \$1.30$.

Example 2: How many liters of a solution that is 15 percent salt must be added to 5 liters of a solution that is 8 percent salt so that the resulting solution is 10 percent salt?

Solution Let n represent the number of liters of the 15% solution. The amount of salt in the 15% solution [$0.15n$] plus the amount of salt in the 8% solution [$(0.08)(5)$] must be equal to the amount of salt in the 10% mixture [$0.10(n+5)$]. Therefore,

$$\begin{aligned} 0.15n + 0.08(5) &= 0.10(n+5) \\ 15n + 40 &= 10n + 50 \\ 5n &= 10 \\ n &= 2 \text{ liters.} \end{aligned}$$

Two liters of the 15% salt solution must be added to the 8% solution to obtain the 10% solution.

4. INTEREST PROBLEMS

Interest can be computed in two basic ways. With simple annual interest, the interest is computed on the principal only and is equal to (principal) × (interest rate) × (time). If interest is compounded, then interest is computed on the principal as well as on any interest already earned.

Example 1 If \$8,000 is invested at 6 percent simple annual interest, how much interest is earned after 3 months?

Solution. Since the annual interest rate is 6%, the interest for 1 year is

$(0.06)(\$8,000) = \$480$. The interest earned in 3 months is $\frac{3}{12}(\$480) = \$120$.

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Example 2 If \$10,000 is invested at 10 percent annual interest, compounded semiannually, what is the balance after 1 year?

Solution. The balance after the first 6 months would be

10,000 + (10,000)(0.05) = 10,500 dollars. The balance after one year would be 10,500 + (10,500)(0.05) = 11,025 dollars.

Note that the interest rate for each 6-month period is 5%, which is half of the 10% annual rate. The balance after one year can also be expressed as

$$10{,}000\left(1+\frac{0.10}{2}\right)^2 \text{ dollars}$$

5. DISCOUNT

If a price is discounted by n percent, then the price becomes $(100 - n)$ percent of the original price.

Example 1 A certain customer paid \$24 for a dress. If that price represented a 25 percent discount on the original price of the dress, what was the original price of the dress?

Solution: If p is the original price of the dress, then $0.75p$ is the discounted price and $0.75p = \$24$, or $p = \$32$. The original price of the dress was \$32.

Example 2 The price of an item is discounted by 20 percent and then this reduced price is discounted by an additional 30 percent. These two discounts are equal to an overall discount of what percent?

Solution: If p is the original price of the item, then $0.8p$ is the price after the first discount. The price after the second discount is $(0.7)(0.8)p = 0.56p$. This represents an overall discount of 44 percent (100% – 56%).

6. PROFIT

Gross profit is equal to revenues minus expenses, or selling price minus cost.

Example A certain appliance costs a merchant \$30. At what price should the merchant sell the appliance in order to make a gross profit of 50 percent of the cost of the appliance?

Solution If s is the selling price of the appliance, then $s - 30 = (0.5)(30)$, or $s = \$45$. The merchant should sell the appliance for \$45.

7. SETS

If S is the set of numbers 1, 2, 3, and 4, you can write $S = \{1, 2, 3, 4\}$. Sets can also be represented by Venn diagrams. That is, the relationship among the members of sets can be represented by circles.

Example 1 Each of 25 people is enrolled in history, mathematics, or both. If 20 are enrolled in history and 18 are enrolled in mathematics, how many are enrolled in both history and mathematics?

Solution The 25 people can be divided into three sets: those who study history only, those who study mathematics only, and those who study history and mathematics. Thus a Venn diagram may be drawn as follows, where n is the number of people enrolled in both courses, $20 - n$ is the number enrolled in history only, and $18 - n$ is the number enrolled in mathematics only.

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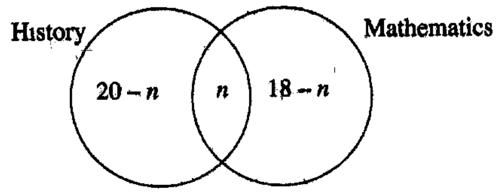

Since there is a total of 25 people, $(20 - n) + n + (18 - n) = 25$, or $n = 13$. Thirteen people are enrolled in both history and mathematics. Note that $20 + 18 - 13 = 25$, which is the general addition rule for two sets.
(See Section A.9.)

Example 2: In a certain production lot, 40 percent of the toys are red and the remaining toys are green. Half of the toys are small and half are large. If 10 percent of the toys are red and small, and 40 toys are green and large, how many of the toys are red and large?

Solution: For this kind of problem, it is helpful to organize the information in a table.

	Red	Green	Total
Small	10%		50%
Large			50%
Total	40%	60%	100%

The numbers in the table are the percents given. The following percents can be computed on the basis of what is given:

	Red	Green	Total
Small	10%	40%	50%
Large	30%	20%	50%
Total	40%	60%	100%

Since 20% of the number of toys (n) are green and large, $0.20n = 40$ (40 toys are green and large), or $n = 200$. Therefore, 30% of the 200 toys, or $(0.3)(200) = 60$, are red and large.

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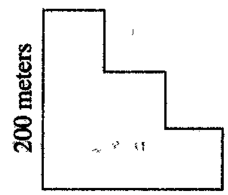
8 GEOMETRY PROBLEMS

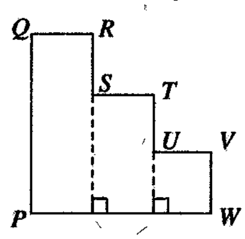
The following is an example of a word problem involving geometry

Example



The figure above shows an aerial view of a piece of land If all angles shown are right angles, what is the perimeter of the piece of land?

Solution. For reference, label the figure as



If all the angles are right angles, then $QR + ST + UV = PW$, and $RS + TU + VW = PQ$ Hence, the perimeter of the land is $2PW + 2PQ = 2 \times 200 + 2 \times 200 = 800$ meters

9 MEASUREMENT PROBLEMS

Some questions on the GMAT involve metric units of measure, whereas others involve English units of measure However, except for units of time, if a question requires conversion from one unit of measure to another, the relationship between those units will be given

Example A train travels at a constant rate of 25 meters per second How many kilometers does it travel in 5 minutes? (1 kilometer = 1,000 meters)

Solution In 1 minute the train travels (25)(60) = 1,500 meters, so in 5 minutes it travels 7,500 meters Since 1 kilometer = 1,000 meters, it follows that 7,500 meters equals $\frac{7,500}{1,000}$, or 7 5 kilometers

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10. DATA INTERPRETATION

Occasionally a question or set of questions will be based on data provided in a table or graph. Some examples of tables and graphs are given below.

Example 1

POPULATION BY AGE GROUP
(in thousands)

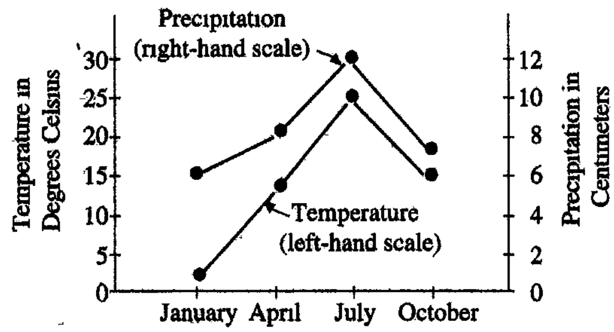
Age	Population
17 years and under	63,376
18-44 years	86,738
45-64 years	43,845
65 years and over	24,054

How many people are 44 years old or younger?

Solution: The figures in the table are given in thousands The answer in thousands can be obtained by adding 63,376 thousand and 86,738 thousand The result is 150,114 thousand, which is 150,114,000

Example 2

AVERAGE TEMPERATURE AND PRECIPITATION IN CITY *X*



What are the average temperature and precipitation in City *X* during April?

Solution Note that the scale on the left applies to the temperature line graph and the one on the right applies to the precipitation line graph. According to the graph, during April the average temperature is approximately 14° Celsius and the average precipitation is 8 centimeters.

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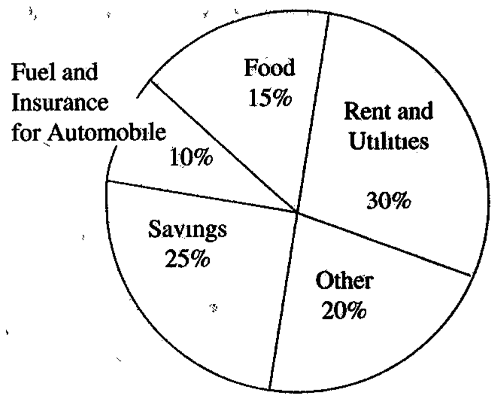
Example 3

DISTRIBUTION OF AL'S WEEKLY NET SALARY



Weekly Net Salary. $350

To how many of the categories listed was at least $80 of Al's weekly net salary allocated?

Solution In the circle graph, the relative sizes of the sectors are proportional to their corresponding values and the sum of the percents given is 100% Note that $\frac{80}{350}$ is approximately 23%, so at least $80 was allocated to each of 2 categories — Rent and Utilities, and Savings — since their allocations are each greater than 23%

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