Guideline to Simplex Method

Step1. Check if the linear programming problem is a **standard maximization problem in standard form**, i.e., if all the following conditions are satisfied:

- It's to **maximize** an objective function;
- All variables should be non-negative (i.e. ≥ 0).
- Constraints should all be $\leq$ a non-negative.

Step 2. Create **slack variables** to convert the inequalities to equations.

Step 3. Write the *objective* function as an equation in the form "left hand side"= 0 where terms involving variables are negative. *Example*: $Z = 3x + 4y$ *becomes* $-3x - 4y + Z = 0$.

Step 4. Place the system of equations with slack variables, into a matrix. Place the revised objective equation in the bottom row.

Step 5. Select **pivot column** by finding the most negative indicator. (**Indicators** are those elements in bottom row except last two elements in that row)

Step 6. Select **pivot row**. (Divide the last column by pivot column for each corresponding entry except bottom entry and negative entries. Choose the smallest positive result. The corresponding row is the pivot row. In case there is no positive entry in pivot column above dashed line, there is no optimal solutions)

Step 7. Find **pivot**: Circle the pivot entry at the intersection of the pivot column and the pivot row, and identify entering variable and exit variable at mean time. Divide pivot by itself in that row to obtain 1. (NEVER SWAP TWO ROWS in Simplex Method!) Also obtain zeros for all rest entries in pivot column by row operations.

Step 8. Do we get all **nonnegative** indicators? If yes, we may stop. Otherwise repeat step 5 to step 7.

Step 9. Read the results: Correspond the last column entries to the variables in front of the first column. The variables not showing are automatically equal to 0.

Example. Maximize $P = 3x_1 + x_2$ Subject to: $$\begin{aligned} 2x_1 + x_2 &\le 8 \\ 2x_1 + 3x_2 &\le 12 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Solution

Step 1. This is of course a standard maximization problem in standard form.

Step 2. Rewrite the two problem constraints as equations by using slack variables:

$$\begin{aligned} 2x_1 + x_2 + s_1 \quad &= 8 \\ 2x_1 + 3x_2 \quad + s_2 &= 12 \end{aligned}$$

Step 3. Rewrite the objective function in the form $-3x_1 - x_2 + P = 0$. Put it together with the problem constraints: $$\begin{aligned} 2x_1 + x_2 + s_1 \qquad &= 8 \\ 2x_1 + 3x_2 \quad + s_2 \quad &= 12 \\ -3x_1 - x_2 \qquad + \mathrm{P} &= 12 \end{aligned}$$ we get a linear system with 5 variables and 3 equations, which is called **initial system**.

Step 4. Write the initial system in matrix form (**initial simplex tableau**). See below.

Step 5 to step 9:

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 2 \text{ pivot} & 1 & 1 & 0 & 0 & 8 \\ 2 & 3 & 0 & 1 & 0 & 12 \\ \hdashline -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \frac{8}{2} = 4 \text{ smallest positive pivot row} \\ \frac{12}{2} = 6 \\ \\ \end{array}$$

most negative indicator pivot column

$\frac{1}{2}R_1 \to R_1$
$$\begin{array}{c} x_1 \\ s_2 \\ P \end{array} \left[\begin{array}{ccccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 4 \\ 2 & 3 & 0 & 1 & 0 & 12 \\ \hdashline -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$-2R_1 + R_2 \to R_2$
$3R_1 + R_3 \to R_3$
$$\begin{array}{c} x_1 \\ s_2 \\ P \end{array} \left[\begin{array}{ccccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 4 \\ 0 & 2 & -1 & 1 & 0 & 4 \\ \hdashline \underbrace{0 \quad \frac{1}{2} \quad \frac{3}{2} \quad 0}_{\text{no negative indicators, stop!}} & & & & 1 & 12 \end{array}\right]$$

max. value of P
@ $x_1 = 4$, $x_2 = 0$

Check: compare to the method we did in 5-3, we got same answer!