Machine Learning Notes

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Note 1. Refer to assignment PDF's. We'll use the usual subscript indexing notation instead of superscript like the lecture.

Part I

Ex 8. Anomaly Detection and Recommender Systems

1 Collaborative Filtering Learning Algorithm

Let n_m be the number of movies, n_u be the number of users. Given rating matrix Y and a number n, we want to find a feature matrix X of size $n_m \times n$ and parameter matrix Θ of size $n_u \times n$, where the i-th row of X represents the feature vector for the i-th movie, and the j-th row of Θ represents the parameter vector for the j-th user. In this context, n represents the number of hidden dimensions of a movie, e.g. X_{ik} could refer to say how much action movie i has, X_{il} could refer to how much romance it has, and so on. Similarly, Θ_{jk} would refer to how much user j likes action, Θ_{jl} how much they like romance.

Note 2. These are only example features, since in fact we don't know what features the algorithm will pick up given rating matrix Y. The features learned might have nothing to do with common movie genres, for example.

Question 3. *Can we cross validate to choose the best value n for the number of hidden features?*

2 Cost Function and Gradient

Definition 4. *Define the collaborative filtering cost function to be the squared error over all parameters* Θ *and features* X*:*

$$J(X, \Theta) = \frac{1}{2} \sum_{i,j:R_{ij}=1} (\Theta_j \cdot X_i - Y_{ij})^2.$$

Then the partial derivatives of J with respect to X and Θ are:

$$\begin{aligned} \frac{\partial J}{\partial X_{ik}} &= \sum_{j:R_{ij}=1} (\Theta_j \cdot X_i - Y_{ij})\Theta_{jk} \\ \frac{\partial J}{\partial \Theta_{jk}} &= \sum_{i:R_{ij}=1} (\Theta_j \cdot X_i - Y_{ij})X_{ik}. \end{aligned}$$

The vectorized forms are surprisingly simple:

$$\begin{aligned} D &\stackrel{\text{def}}{=} R * (X\Theta^T - Y) \\ J &= \frac{1}{2} D \cdot D \\ \frac{\partial J}{\partial X} &= D\Theta \\ \frac{\partial J}{\partial \Theta} &= D^T X, \end{aligned}$$

where $\cdot$ denotes the Frobenius inner product (just a natural extension of the vector dot product to matrices), and $*$ denotes element-wise multiplication. We need to multiply element-wise by R to reduce $X\Theta^T - Y$ to elements where the corresponding entries in Y are nonzero, because the summation is only over i, j such that $R_{ij} = 1$, i.e. where Y_{ij} is nonzero.

3 Cost Function and Gradient with Regularization

With regularization, the cost function and partials are:

$$
\begin{aligned}
D &\stackrel{\text{def}}{=} R * (X\Theta^T - Y) \\
J &= \frac{1}{2} D \cdot D + \frac{\lambda}{2} X \cdot X + \frac{\lambda}{2} \Theta \cdot \Theta \\
\frac{\partial J}{\partial X} &= D\Theta + \lambda X \\
\frac{\partial J}{\partial \Theta} &= D^T X + \lambda \Theta.
\end{aligned}
$$

4 Some useful matrix derivative formulae

Proposition 5. *If D is a matrix and $J = \frac{1}{2} D \cdot D$, then $\frac{\partial J}{\partial D} = D$.*

Keywords. Collaborative filtering, cost function, gradient, regularization, null reduction, Frobenius inner product.