Eigenvectors and Eigenvalues

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Proposition 1. *Eigenvectors corresponding to distinct eigenvalues are linearly independent.*

Proof. Easy to show for two eigenvectors, then use induction. $\square$

Conjecture 2. *A matrix A is diagonalizable iff the dimensions of its eigenspaces—i.e. the geometric multiplicities over all its eigenvalues—add up to the size of A. In this case the geometric multiplicity of each eigenvalue is equal to its algebraic multiplicity.*

Proposition 3. *Let T be a linear operator on a finite-dimensional vector space V, and let β be an ordered basis for V. Then λ is an eigenvalue of T iff it is an eigenvalue of $[T]_\beta$.*

Corollary 4. *Similar matrices have the same eigenvalues, but not necessarily the same eigenvectors.*

Proposition 5. *If v is an eigenvector of A corresponding to eigenvalue λ, and B is similar to A under change of coordinates matrix Q, then Qv is an eigenvector of B corresponding to the same eigenvalue λ. Another way of saying this is that change of coordinates preserves eigenvalues and eigenvectors.*

Proof. Let $A = Q^{-1}BQ$. Then

$$\begin{aligned} Av &= Q^{-1}BQv \\ QAv &= BQv \\ \lambda Qv &= BQv, \end{aligned}$$

so Qv is an eigenvector corresponding to λ of B. $\square$

Keywords. Eigenvectors, eigenvalues, differential operator, eigenfunctions, algebraic and geometric multiplicities, change of coordinates matrix.