Chapter 3: PART B
EXPONENTIAL CARRIER WAVE (ANGLE) MODULATION

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Lectured by
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References: R. Ziemer, W.H. Tranter, Principles of Communications, HUT Finland, and A.B. Carlson's Communication Systems.

Analog Modulations

- An AM signal can be represented as

$$x_c(t) = A_c[1+m(t)]\ \cos\omega_c t \quad (\mu = 1)$$

- Information can be carried in the **angle** of the signal

$$x_c(t) = A_c \cos[\omega_c t + \phi(t)]$$

The amplitude A_c remains constant and the angle is modulated.

This Modulation Technique is called the ***Angle Modulation***

Angle modulation:

Vary either the **Phase** or the **Frequency** of the carrier signal namely **Phase Modulation** and **Frequency Modulation**

Linear and exponential modulation

$$x_c(t) = A(t)\,\mathrm{Re}[\exp(\omega_c t + \phi(t))]$$

- In linear CW (carrier wave) modulation:
  - transmitted spectra resembles modulating spectra
  - spectral width does not exceed twice the modulating spectral width
  - destination SNR can not be better than the baseband transmission SNR (lecture: Noise in CW systems)
- In exponential CW modulation FM/PM :
  - usually transmission BW>>baseband BW
  - bandwidth-power trade-off (channel adaptation): destination SNR can be much better than transmission SNR when transmission BW increased
  - baseband and transmitted spectra does not carry a simple relationship

Phase modulation (PM)

- Carrier Wave (CW) signal: $x_C(t) = A_C \cos(\underbrace{\omega_C t + \phi(t)}_{\theta_C(t)})$

- In exponential modulation the modulation is "in the exponent" or "in the angle"

$$x_c(t) = A_c \cos(\theta_c(t)) = A_c \operatorname{Re}[\exp(j\theta_c(t))]$$

- Note that in exponential modulation superposition does not apply:

$$x_c(t) = A\cos\{\omega_c t + k_f [a_1(t) + a_2(t)]\}$$

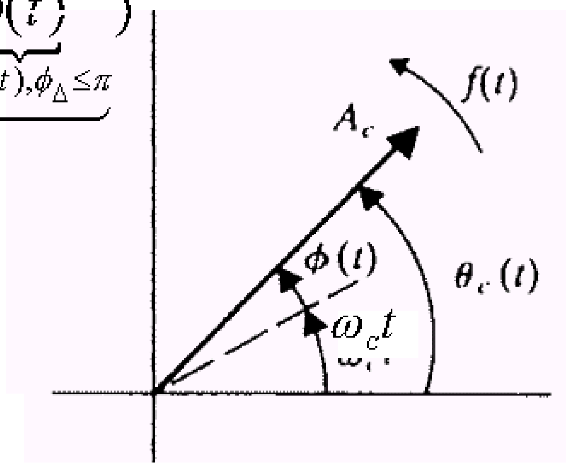
$$\neq A\cos\omega_c t + A\cos k_f [a_1(t) + a_2(t)]$$

- In phase modulation (PM) carrier phase is linearly proportional to the modulation amplitude:

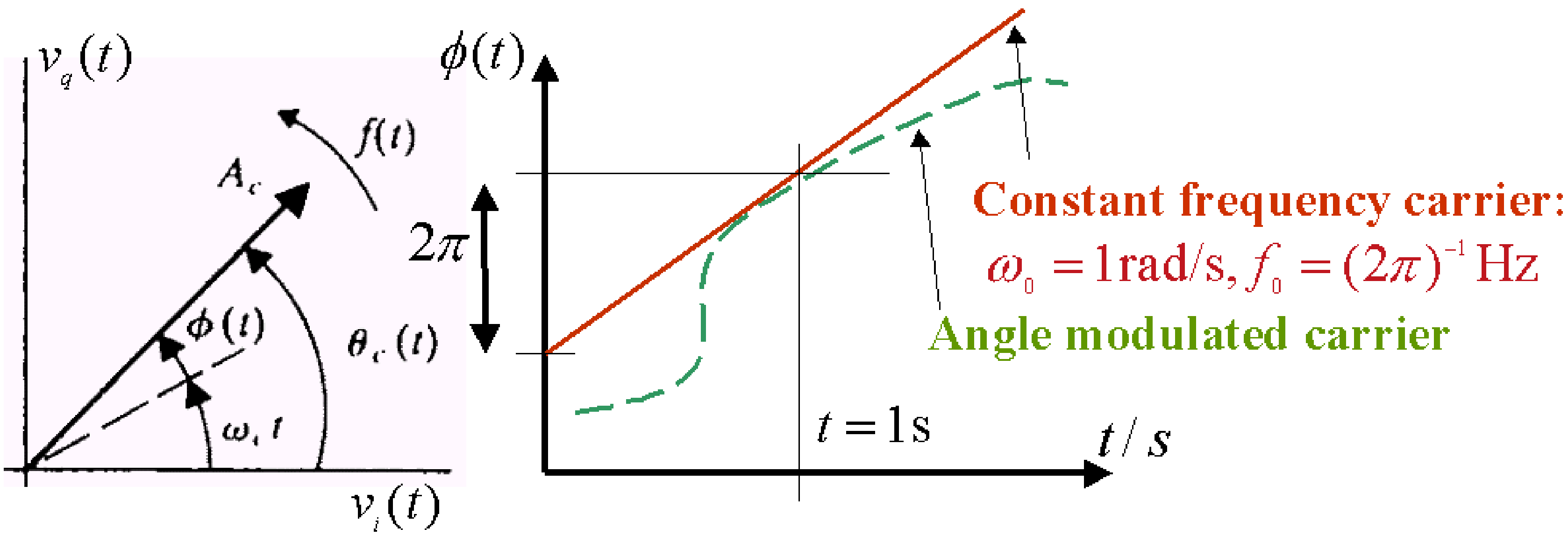
$$x_{PM}(t) = A_c \cos(\underbrace{\omega_c t + \underbrace{\phi(t)}_{\phi_\Delta x(t),\phi_\Delta \leq \pi}}_{\theta_C(t)})$$

- Angular phasor has the instantaneous frequency (*phasor rate*)

$$\omega = 2\pi f(t)$$



Instantaneous frequency



- Angular frequency ω (rate) is the derivative of the phase (the same way as the velocity *v(t)* is the derivative of distance *s(t)*)
- For continuously changing frequency instantaneous frequency is defined by differential changes:

$$\boxed{\omega(t) = \frac{d\phi(t)}{dt} \qquad \phi(t) = \int_{-\infty}^{t} \omega(\alpha)d\alpha}$$

Compare to linear motion: $v(t) = \dfrac{ds(t)}{dt}\left(\approx \dfrac{s_2(t) - s_1(t)}{t_2 - t_1}\right)$

Frequency modulation (FM)

- In frequency modulation carrier instantaneous frequency is linearly proportional to modulation amplitude

$$\omega = 2\pi f(t) = d\theta_C(t)/dt$$
$$= 2\pi[f_C + f_\Delta x(t)]$$

- Hence the FM waveform can be written as

$$x_C(t) = A_C \cos(\underbrace{\omega_C t + 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda}_{\theta_C(t)}), t \geq t_0$$

$$\phi(t) = \int_{-\infty}^{t} \omega(\alpha)d\alpha$$

integrate

- Note that for FM

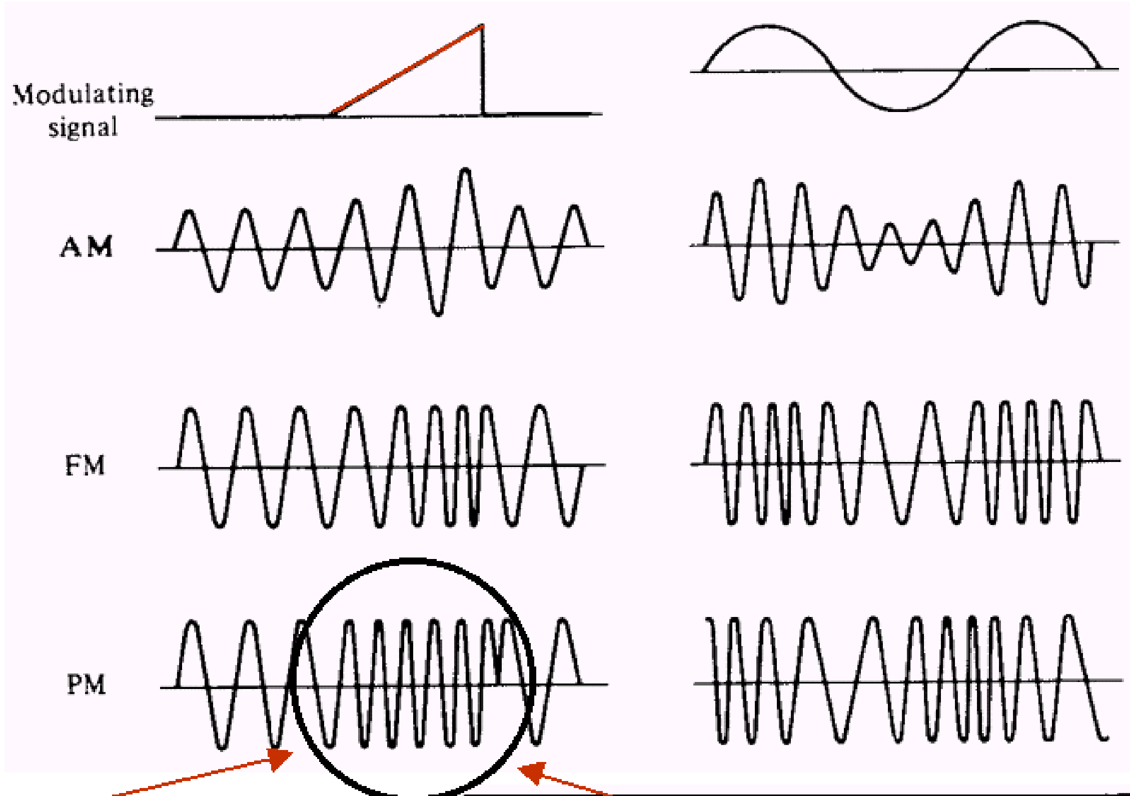
$$f(t) = f_C + f_\Delta x(t)$$

and for PM

$$\phi(t) = \phi_\Delta x(t)$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi}\phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int^{t} x(\lambda)\, d\lambda$	$f_c + f_\Delta x(t)$

AM, FM and PM waveforms



Constant frequency at slope: follows the derivative of the modulation waveform

$$x_{PM}(t) = A_c \cos(\omega_c t + \phi_\Delta x(t))$$

$$x_{FM}(t) = A_c \cos(\omega_c t + 2\pi f_\Delta \int_t x(\lambda) d\lambda)$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi} \phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int^t x(\lambda)\, d\lambda$	$f_c + f_\Delta x(t)$

Narrowband FM and PM (small modulation index, arbitrary modulation waveform)

- The CW presentation: $x_c(t) = A_c \cos[\omega_c t + \phi(t)]$
- The quadrature CW presentation:

$$x_c(t) = x_{ci}(t)\cos(\omega_c t) - x_{cq}(t)\sin(\omega_c t)$$

$$x_{ci}(t) = A_c \cos\phi(t) = A_c[1-(1/2!)\phi^2(t)+\ldots]$$

$$x_{cq}(t) = A_c \sin\phi(t) = A_c[\phi(t)-(1/3!)\phi^3(t)+\ldots]$$

- The narrow band condition: $|\phi(t)| << 1$rad

$$x_{ci}(t) \approx A_c \quad x_{cq}(t) \approx A_c\phi(t)$$

- Hence the Fourier transform of $X_C(t)$ is in this case

$$\mathbb{F}\left[x_C(t)\right] \approx \mathbb{F}\left[A_C\cos(\omega_C t) - A_C\phi(t)\sin(\omega_C t)\right]$$

$$\boxed{X_c(f) \approx \frac{1}{2}A_c\delta(f-f_c) + \frac{j}{2}A_c\Phi(f-f_c), f>0}$$

$$\mathbb{F}[\cos(2\pi f_0 t)] = \frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$$

$$\mathbb{F}[\cos(2\pi f_0 t+\theta)x(t)] = \frac{1}{2}[X(f-f_0)\exp(j\theta) + jX(f+f_0)\exp(-j\theta)]$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Narrow band FM and PM spectra

- Instantaneous phase in CW presentation:

$$x_c(t) = A_c \cos[\omega_c t + \phi(t)]$$

$$\phi_{PM}(t) = \phi_\Delta x(t)$$

$$\phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda, t \geq t_0$$

- The small angle assumption produces compact spectral presentation for both FM and AM:

$$\boxed{X_c(f) \approx \frac{1}{2}A_c\delta(f - f_c) + \frac{j}{2}A_c\Phi(f - f_c), f > 0}$$

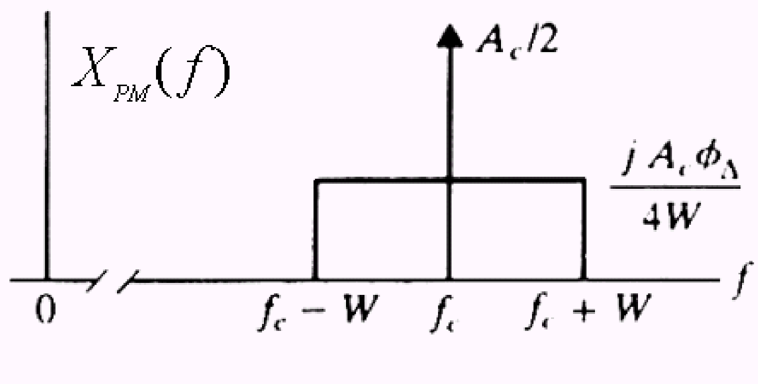
$$\Phi(f) = \mathbb{F}[\phi(t)]$$

$$= \begin{cases} \phi_\Delta X(f), \text{PM} \\ -jf_\Delta X(f)/f, \text{FM} \end{cases}$$

What does it mean to set this component to zero?

$$\int_{t_0}^{t} g(\tau)d\tau \Leftrightarrow \frac{G(\omega)}{j\omega} + \pi G(0)\delta(\omega)$$

Example



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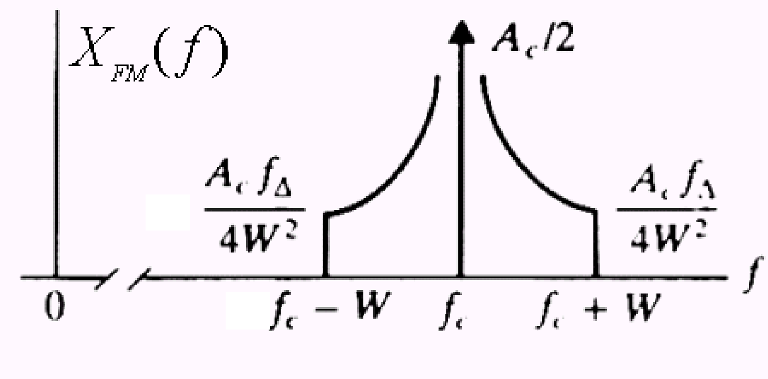


(b)

- Assume: $x(t) = \text{sinc}\,2Wt \Rightarrow X(f) = \frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$

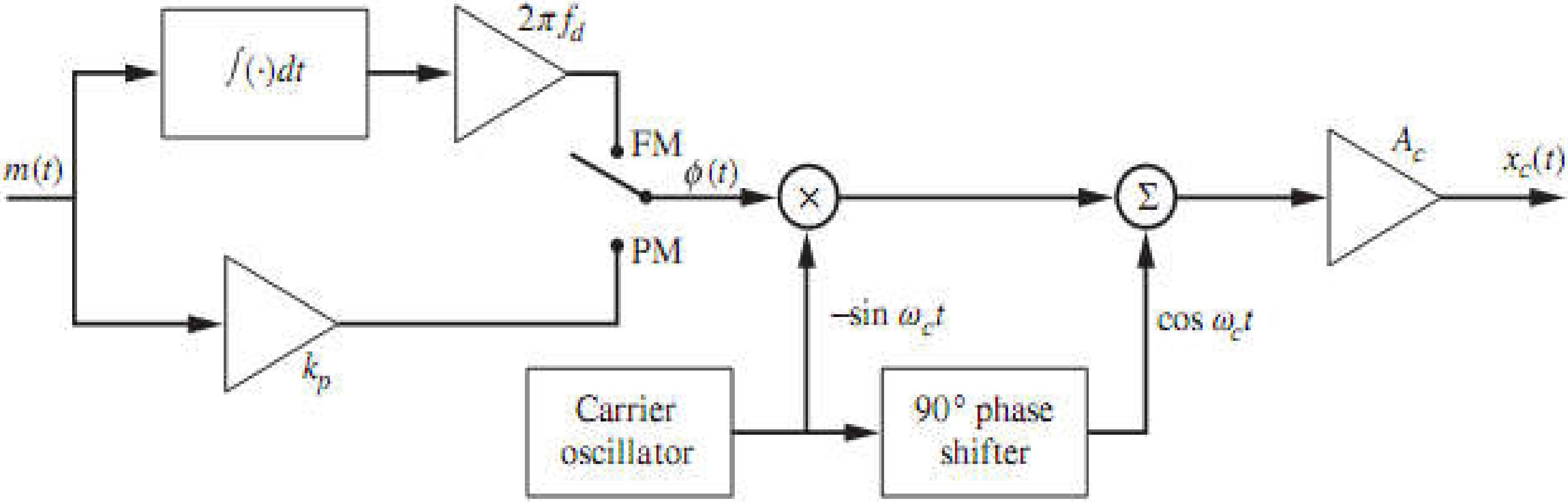
$$X_C(f) \approx \frac{1}{2}A_C\delta(f - f_C) + \frac{j}{2}A_C\Phi(f - f_C), f > 0$$

$$\Phi_{PM}(f) = F[\phi_{PM}(t)] = \phi_\Delta X(f) \qquad \Phi_{FM}(f) = F[\phi_{FM}(t)] = -jf_\Delta X(f)/f$$

$$X_{PM}(f) \approx \frac{1}{2}A_C\delta(f - f_C) + \frac{j}{4W}A_C\phi_\Delta\Pi\left(\frac{f - f_C}{2W}\right), f > 0$$

$$X_{FM}(f) \approx \frac{1}{2}A_C\delta(f - f_C) + \frac{f_\Delta}{4|f - f_C|W}A_C\Pi\left(\frac{f - f_C}{2W}\right), f > 0$$

Generation of narrowband angle modulation



Example: Consider FM system with the message signal

$$m(t) = A\cos(2\pi f_m t)$$

$$\phi(t) = k_f \int_0^t A\cos(2\pi f_m \alpha)d\alpha = \frac{Ak_f}{2\pi f_m}\sin(2\pi f_m t) = \frac{Af_d}{f_m}\sin(2\pi f_m t)$$

$$x_c(t) = A_c \cos\left[2\pi f_c t + \frac{Af_d}{f_m}\sin(2\pi f_m t)\right]$$

If $A f_d / f_m \ll 1$, the modulator output can be approximated as

$$x_c(t) = A_c \left[\cos(2\pi f_c t) - \frac{A f_d}{f_m} \sin(2\pi f_c t) \sin(2\pi f_m t) \right]$$

$$x_c(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{2} \frac{A f_d}{f_m} \{ \cos[2\pi (f_c + f_m) t] - \cos[2\pi (f_c - f_m) t] \}$$

$$x_c(t) = A_c \operatorname{Re} \left\{ \left[1 + \frac{A f_d}{2 f_m} \left(e^{j2\pi f_m t} - e^{-j2\pi f_m t} \right) \right] e^{j2\pi f_c t} \right\}$$

Compared to AM signal:

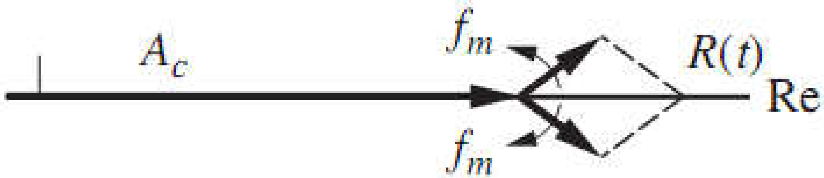
$$x_c(t) = A_c [1 + a \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

where $a = A f_d / f_m$ is the modulation index

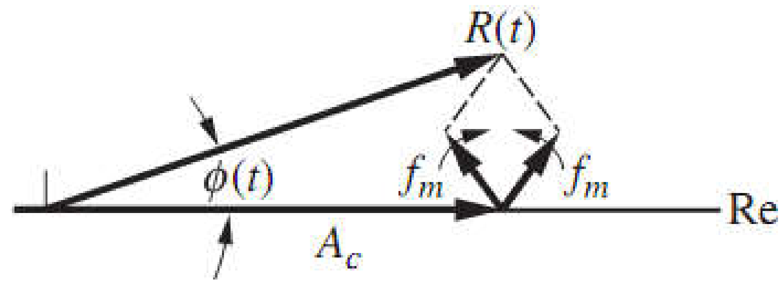
$$x_c(t) = A_c \cos(2\pi f_c t) + \frac{A_c a}{2} [\cos 2\pi (f_c + f_m) t + \cos 2\pi (f_c - f_m) t]$$

$$x_c(t) = A_c \operatorname{Re} \left\{ \left[1 + \frac{a}{2} (e^{j2\pi f_m t} + e^{-j2\pi f_m t}) \right] e^{j2\pi f_c t} \right\}$$

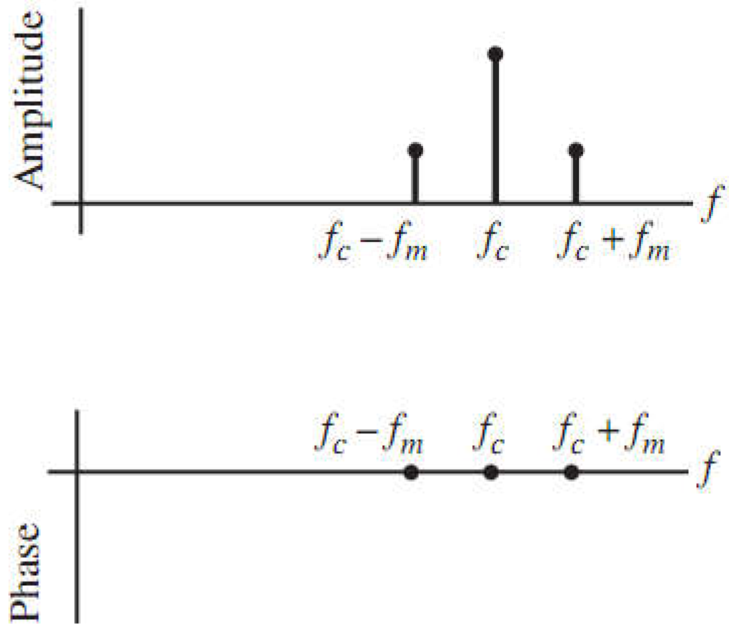
Comparison of AM and narrowband angle modulation

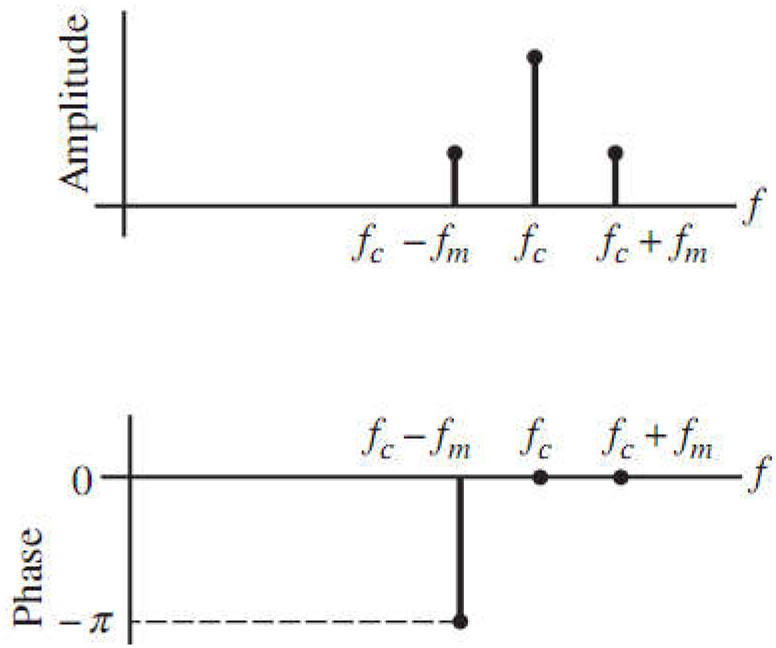


Amplitude Modulation



Narrowband Angle Modulation





Tone modulation with PM and FM: modulation index β

- Remember the FM and PM waveforms:

$$x_{PM}(t) = A_c \cos[\omega_c t + \underbrace{\phi_\Delta x(t)}_{\phi(t)}]$$

$$x_{FM}(t) = A_c \cos[\omega_c t + \underbrace{2\pi f_\Delta \int_t x(\lambda) d\lambda}_{\phi(t)}]$$

- Assume tone modulation

$$x(t) = \begin{cases} A_m \sin(\omega_m t), \mathrm{PM} \\ A_m \cos(\omega_m t), \mathrm{FM} \end{cases}$$

- Then

$$\phi(t) = \begin{cases} \phi_\Delta x(t) = \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), \mathrm{PM} \\ 2\pi f_\Delta \int_t x(\lambda) d\lambda = \underbrace{(A_m f_\Delta / f_m)}_{\beta} \sin(\omega_m t), \mathrm{FM} \end{cases}$$

FM and PM with tone modulation and arbitrary modulation index

- Time domain expression for FM and PM:

$$x_c(t) = A_c \cos[\omega_c t + \beta \sin(\omega_m t)]$$

$$\boxed{\begin{aligned} \beta_{PM} &= \phi_\Delta A_m \\ \beta_{FM} &= A_m f_\Delta / f_m \end{aligned}}$$

- Remember: $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

- Therefore:

$$x_c(t) = A_c \cos(\beta \sin(\omega_m t))\cos(\omega_c t) - A_c \sin(\beta \sin(\omega_m t))\sin(\omega_c t)$$

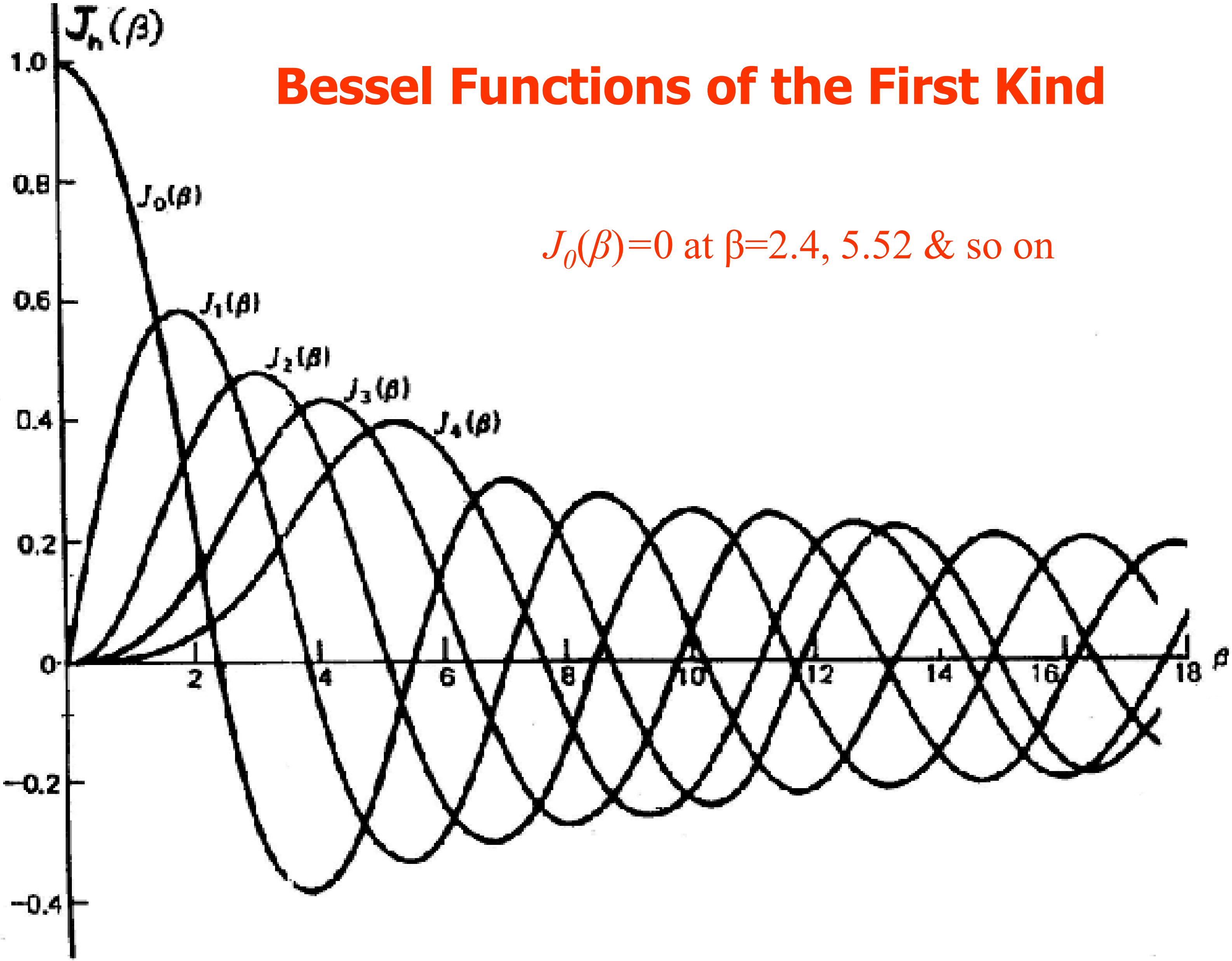
$$\cos(\beta \sin(\omega_m t)) = J_o(\beta) + \sum_{n\,\text{even}}^{\infty} 2J_n(\beta)\cos(n\omega_m t)$$

$$\sin(\beta \sin(\omega_m t)) = \sum_{n\,\text{odd}}^{\infty} 2J_n(\beta)\sin(n\omega_m t)$$

J_n is the first kind, order n Bessel function

Bessel Functions of the First Kind

$J_0(\beta)=0$ at β=2.4, 5.52 & so on



Bessel Functions of the First Kind

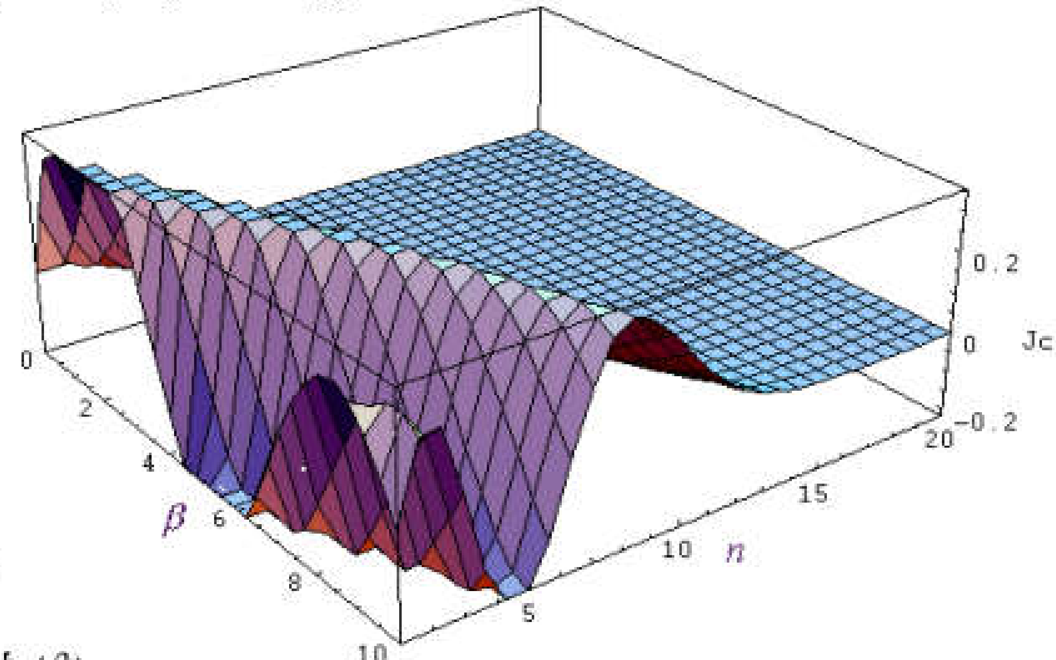
n	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.7$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 3.0$	$\beta = 5.0$	$\beta = 7.0$	$\beta = 8.0$	$\beta = 10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001

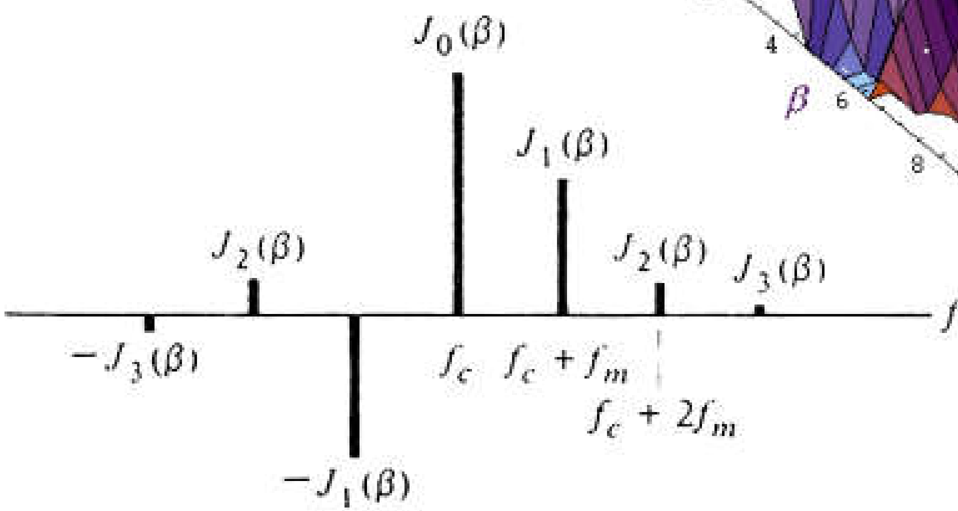
Wideband FM and PM spectra

- After simplifications we can write:

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

$$\beta = \begin{cases} \phi_\Delta A_m, \mathrm{PM} \\ A_m f_\Delta / f_m, \mathrm{FM} \end{cases}$$

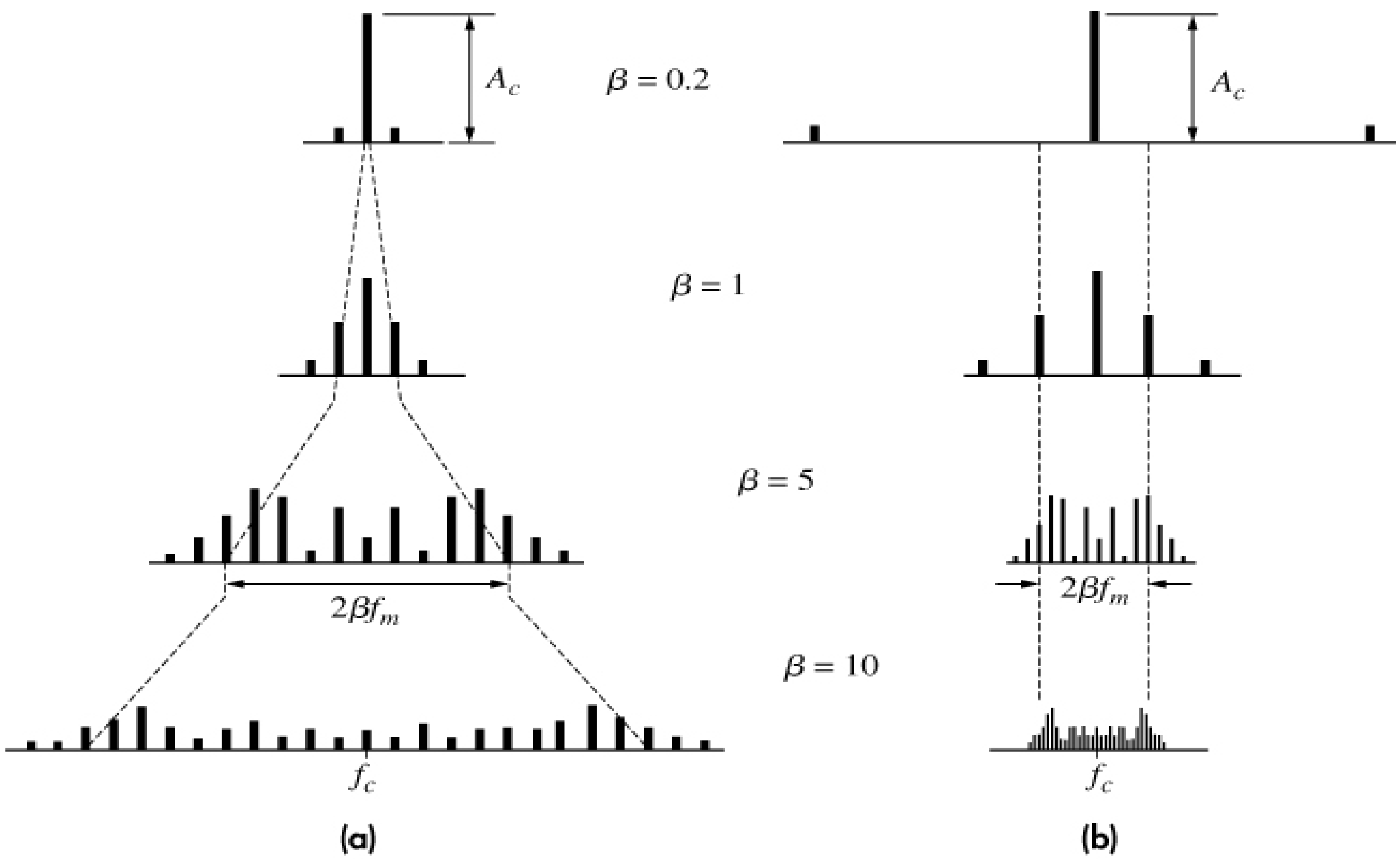




note: $J_{-n}(\beta) = (-1)^n J_n(\beta)$

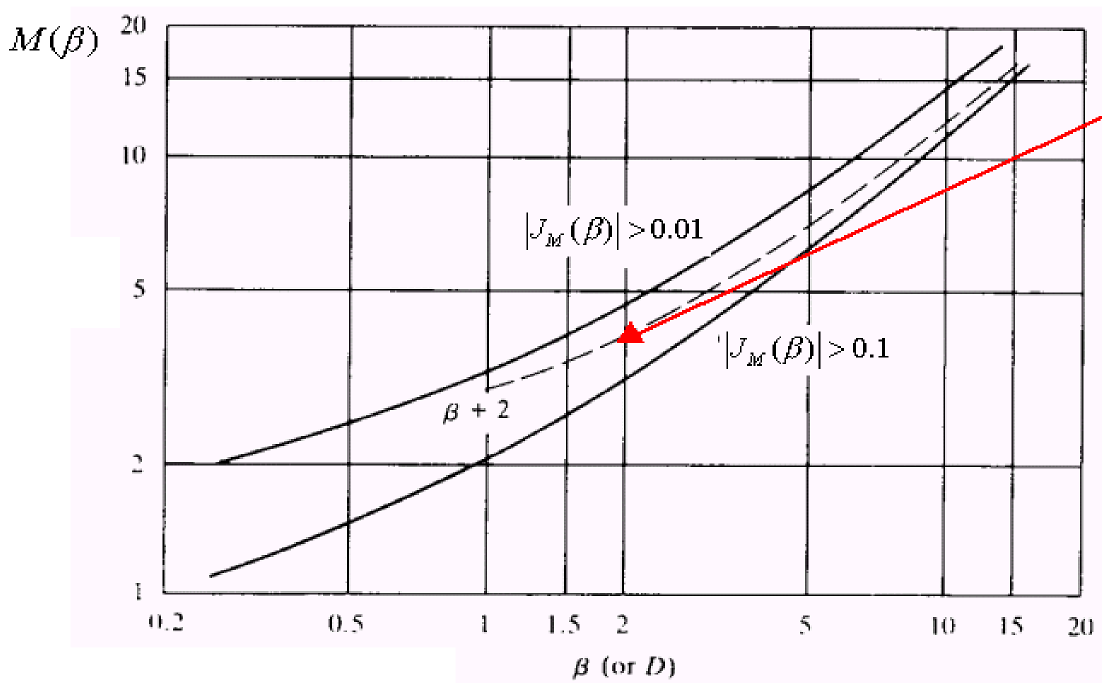
Tone-modulated line spectra

(a) FM or PM with fm fixed; (b) FM with Am$f\Delta$ fixed



Determination of transmission bandwidth

- The goal is to determine the number of significant sidebands
- Thus consider again how Bessel functions behave as the function of β, e.g. we consider $A_m \leq 1, f_m \leq W$
- Significant sidebands: $|J_n(\beta)| > \varepsilon$
- Minimum bandwidth includes 2 sidebands (why?): $B_{T,\min} = 2 f_m$
- Generally: $B_T = 2M(\beta) f_m, M(\beta) \geq 1$



$$M(\beta) \approx \beta + 2$$

$$\beta = \begin{cases} \phi_\Delta A_m, \text{PM} \\ A_m f_\Delta / f_m, \text{FM} \end{cases}$$

Transmission bandwidth and deviation D

- Tone modulation is extrapolated into arbitrary modulating signal by defining deviation by

$$\beta = A_m f_\Delta / f_m \Big|_{A_m=1, f_m=W} = f_\Delta / W \equiv D$$

- Therefore transmission BW is also a function of deviation

$$B_T = 2M(D)W$$

- For very large D and small D with

$$B_T \approx 2(D + \cancel{2}) f_m \Big|_{D>>1, f_m=W}$$

$$\approx 2DW \, , D >> 1$$

$$B_T = 2M(D)W$$

$$\approx 2W \, , D << 1 \text{ (a single pair of sidebands)}$$

- that can be combined into

$$\boxed{B_T = 2|D \mp 1| W \, , D >> 1, \text{and } D << 1}$$

$$\beta = \begin{cases} \phi_\Delta A_m, \text{PM} \\ A_m f_\Delta / f_m, \text{FM} \end{cases}$$

Example: Bandwidth of FM broadcasting

- Following commercial FM specifications

$$f_\Delta = 75\,\text{kHz}, W \approx 15\ \text{kHz}$$

$$\Rightarrow D = f_\Delta / W = 5$$

$$B_T = 2(D+2)W \approx 210\,\text{kHz}, (\text{D} > 2)$$

- High-quality FM radios RF bandwidth is about

$$B_T \geq 200\,\text{kHz}$$

- Note that

$$B_T = 2|D+1|W \approx 180\,\text{kHz},\ D >> 1$$

under estimates the bandwidth slightly

Generation of FM or PM by VCO

- Output signal of Voltage Controlled Oscillator (VCO) is

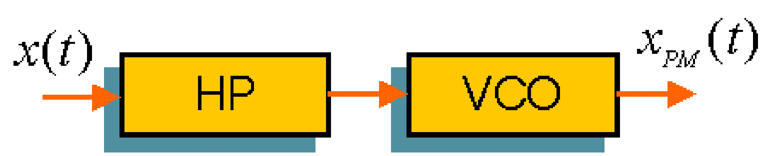
$$f_0(t) = f_c + K_D v_i(t)$$

- This is precisely how instantaneous frequency of FM was defined: $f(t) = f_c + f_\Delta x(t)$
- VCO can be used to produce also PM :

$$f(t) = f_c + \frac{1}{2\pi}\phi_\Delta x'(t)$$

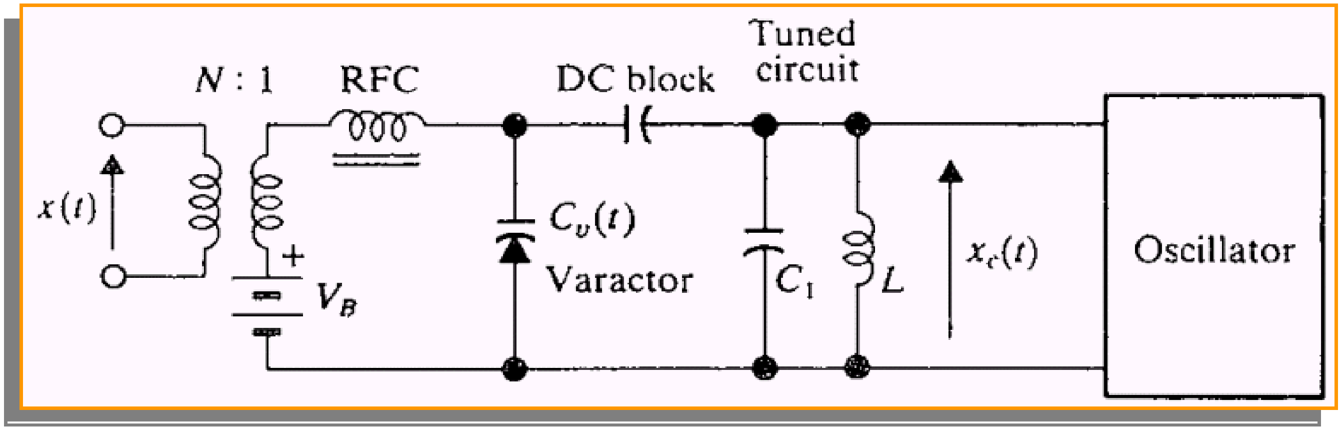
- Required differentiation can be realized by a high pass (HP) filter

FM-modulator



PM-modulator

Generating FM



- A de-tuned resonant circuit oscillator
  - biased varactor diode capacitance directly proportional to *x(t)*
  - other parts:
    - input transformer
    - RF-choke
    - DC-block
- See the detailed analysis in lecture supplementary material

FM-AM conversion based PM detector

- Differentiation of the PM-wave produces FM-AM conversion:

In[5]:= $D[\mathrm{Cos}[\omega_C\, t + A_m\, f[t]], t]$ // Expand

Out[5]= $-\mathrm{Sin}[f[t]\, A_m + t\, \omega_C]\, \omega_C -$
$\mathrm{Sin}[f[t]\, A_m + t\, \omega_C]\, \underline{A_m\, f'[t]}$

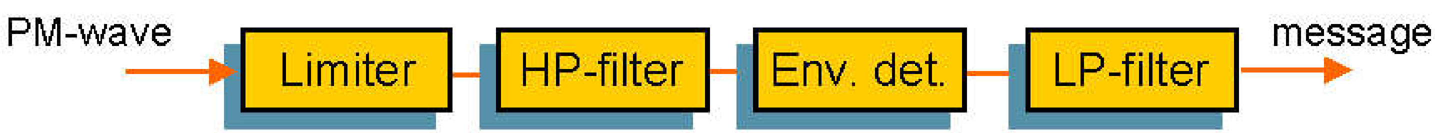
$$x_{PM}(t) = A_c \cos[\omega_c t + \underbrace{\phi_\Delta x(t)}_{\phi(t)}]$$

$$x_{FM}(t) = A_c \cos[\omega_c t + \underbrace{2\pi f_\Delta \int^t x(\lambda) d\lambda}_{\phi(t)}]$$

- Where after filtering the carrier, envelope detector yields

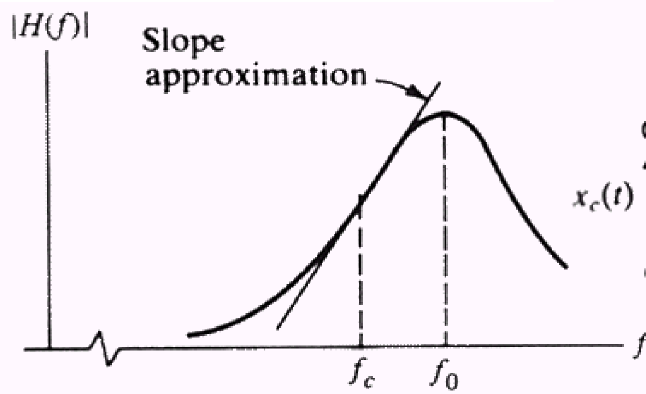
$A_m\, f'[t]$

whose integration (realized by an LP-filter) yields detected PM wave

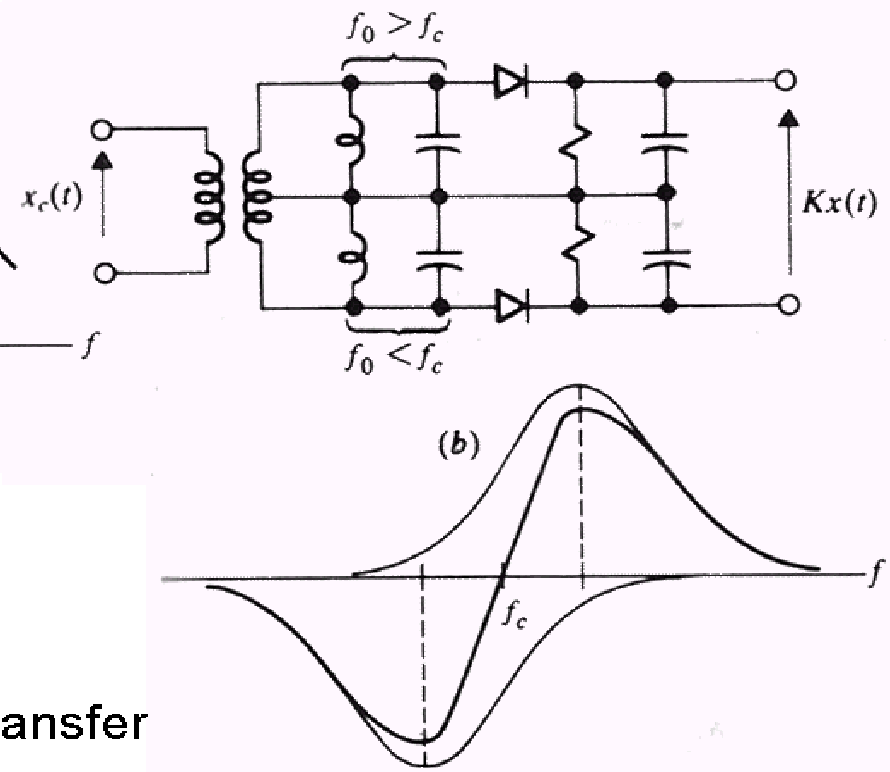


- How one should select the 3 dB corners of the LP-filters in this application?
- See supplementary material for a proof that integration can be approximated by LP filter

FM slope detector and balanced discriminator are based on FM-AM conversion



(a)



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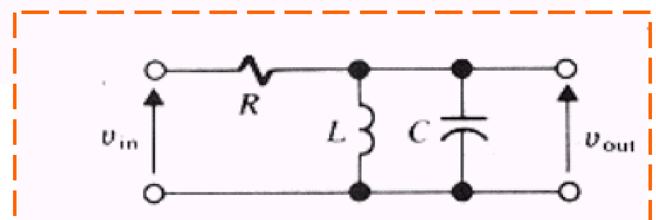


(c)

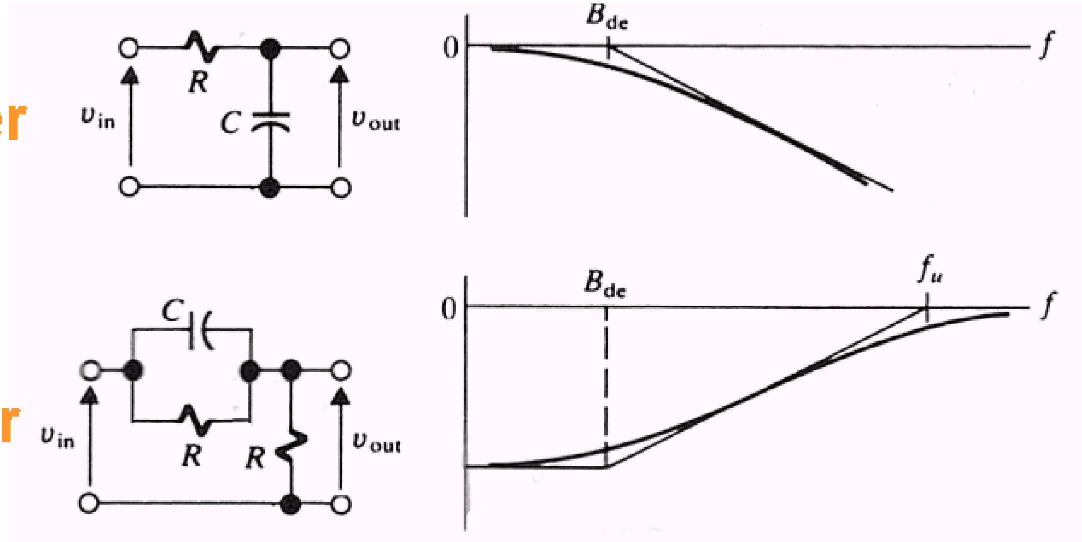
a) slope detector realized by tank-circuit
b) dual-slope detector and transfer characteristics
c) tank circuit

FM preemphases and deemphases filters

- FM related noise emphases can be suppressed by *pre-distortion* and post detection filters (preemphases and deemphases filters):

receiver filter

transmitter filter



$H_{de}(f)$

$H_{pe}(f)$

Q: What would happen if the filters would be reversed? (TX filter in receiver & vice versa)

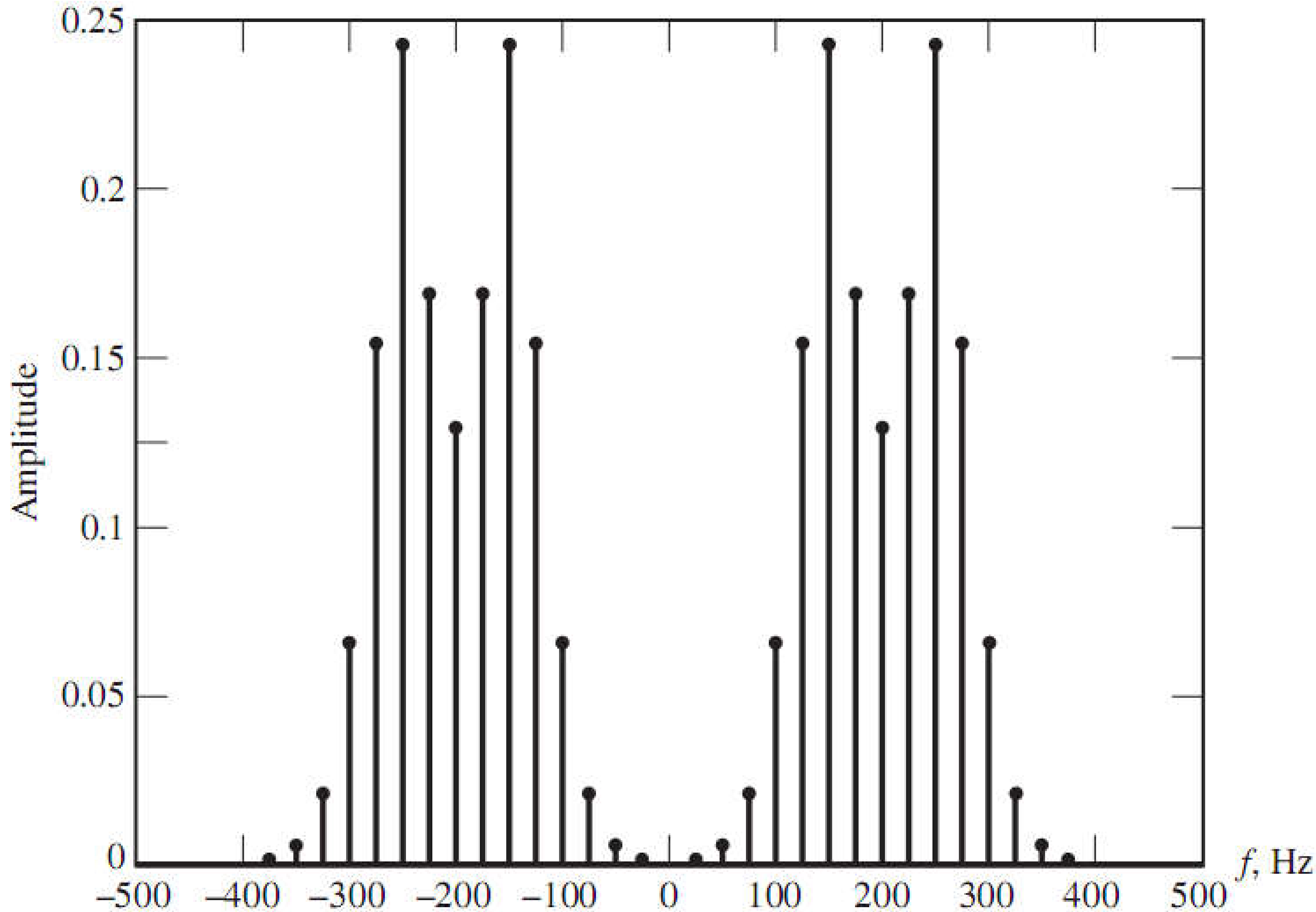
$$H_{de}(f) = \left[1 + j(f/B_{de})\right]^{-1} \approx \begin{cases} 1, |f| << B_{de} \\ B_{de}/(jf), |f| >> B_{de} \end{cases}$$ LPF

$$H_{pe}(f) = \left[1 + j(f/B_{de})\right] \approx \begin{cases} 1, |f| << B_{de} \\ j(f/B_{de}), |f| >> B_{de} \end{cases}$$ HPF

MATLAB Code Examples for Angle Modulation

Example 1: Two-sided Amplitude spectrum for FM (or PM) Signal using FFT algorithm

```
%File: c4ce2.m
fs=1000;                          %sampling frequency
delt=1/fs;                        %sampling increment
t=0:delt:1-delt;                  %time vector
npts=length(t);                   %number of points
fn=(0:npts)-(fs/2);               %frequency vector for plot
m=3*cos(2*pi*25*t);               %modulation
xc=sin(2*pi*200*t+m);             %modulated carrier
asxc=(1/npts)*abs(fft(xc));       %amplitude spectrum
evenf=[asxc((npts/2):npts)asxc(1:npts/2)];   %even amplitude spectrum
stem(fn,evenf,'.');
xlabel('Frequency-Hz')
ylabel('Amplitude')
%End of script.file.
```


0.25
0.2
0.15
0.1
0.05
0
Amplitude
–500
–400
–300
–200
–100
0
100
200
300
400
500
f, Hz

Power in an angle modulated signal

$$\langle x_c^2(t)\rangle = A_c^2\langle \cos^2[2\pi f_c t + \phi(t)]\rangle$$

$$\langle x_c^2(t)\rangle = \frac{1}{2}A_c^2 + \frac{1}{2}A_c^2\langle \cos\{2[2\pi f_c t + \phi(t)]\}\rangle$$

If the carrier frequency is large so that $x_c(t)$ has negligible Frequency content in the region of DC then

$$\langle x_c^2(t)\rangle = \frac{1}{2}A_c^2$$

The important notes for Angle Modulation:

- Power of Angle Modulator is independent of the message signal
- Constant transmitter power is one important difference between Angle modulation and linear modulation.

P_r is defined as the ratio of the power contained in the carrier component and the k components on the each side of the carrier to the total power in $x_c(t)$.

$$P_r = \frac{\frac{1}{2}A_c^2 \sum_{n=-k}^{k} J_n^2(\beta)}{\frac{1}{2}A_c^2} = \sum_{n=-k}^{k} J_n^2(\beta)$$

$$P_r = J_0^2(\beta) + 2\sum_{n=1}^{k} J_n^2(\beta)$$

Example an FM signal:

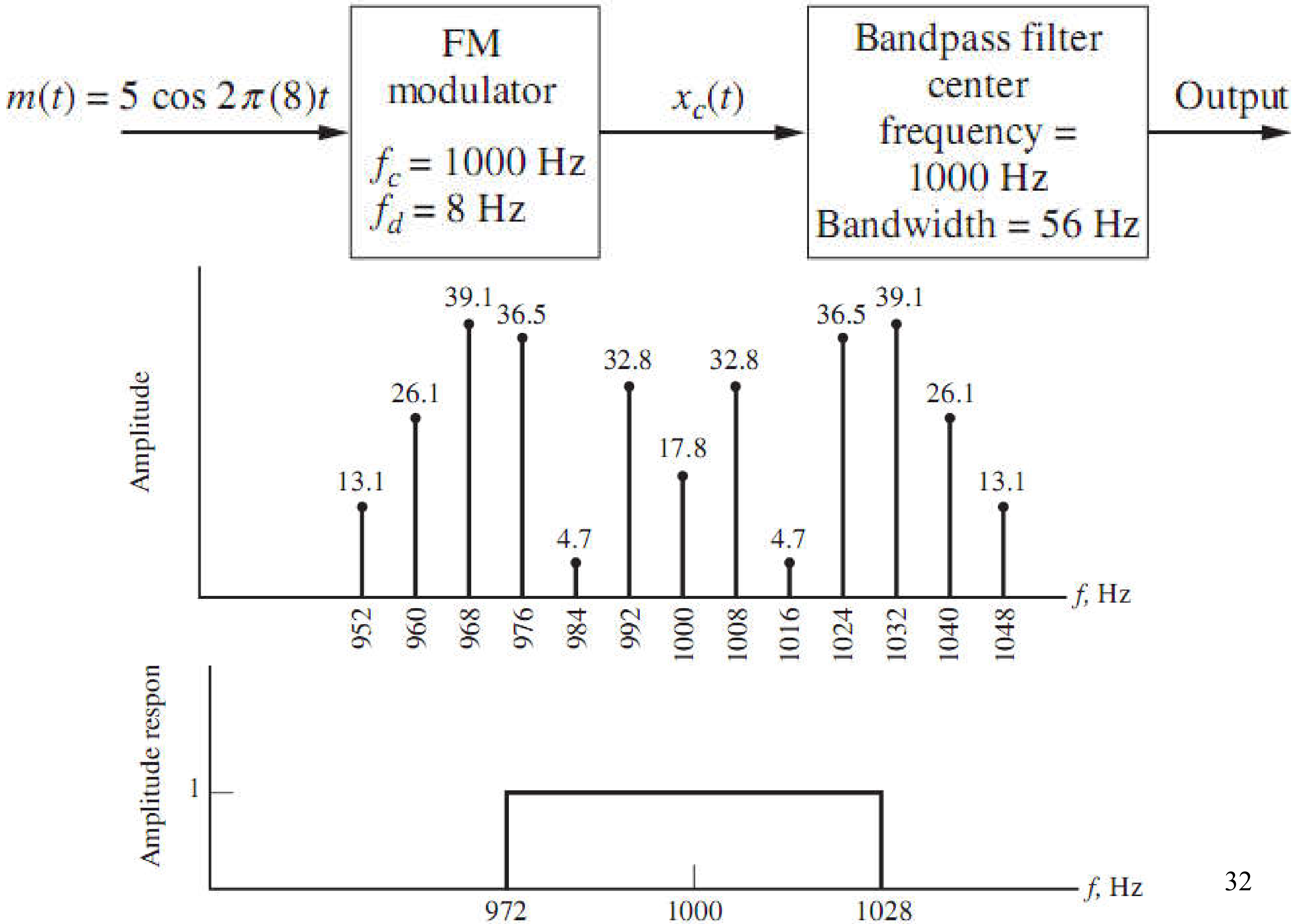
$$x_c(t) = 100\cos[2\pi(1000)t + \phi(t)]$$

The modulator operates with $f_d = 8$ and $m(t) = 5\cos 2\pi(8)t$

The peak deviation is $5f_d$ or 40 Hz, and $f_m = 8$ Hz

Thus, the modulation index is $40/5 = 8$

The power ration is: $$P_r = J_0^2(5) + 2[J_1^2(5) + J_2^2(5) + J_3^2(5)]$$



$$P_r = (0.178)^2 + 2\left[(0.328)^2 + (0.047)^2 + (0.365)^2\right]$$

$$P_r = 0.518$$

$$\overline{x_c^2} = \frac{1}{2}A_c^2 = \frac{1}{2}(100)^2 = 5000 \text{ W}$$

The power at the filter output $P_r\overline{x_c^2} = 2589$ W

Example 2:

$$m(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$

For FM modulation the phase deviation is:

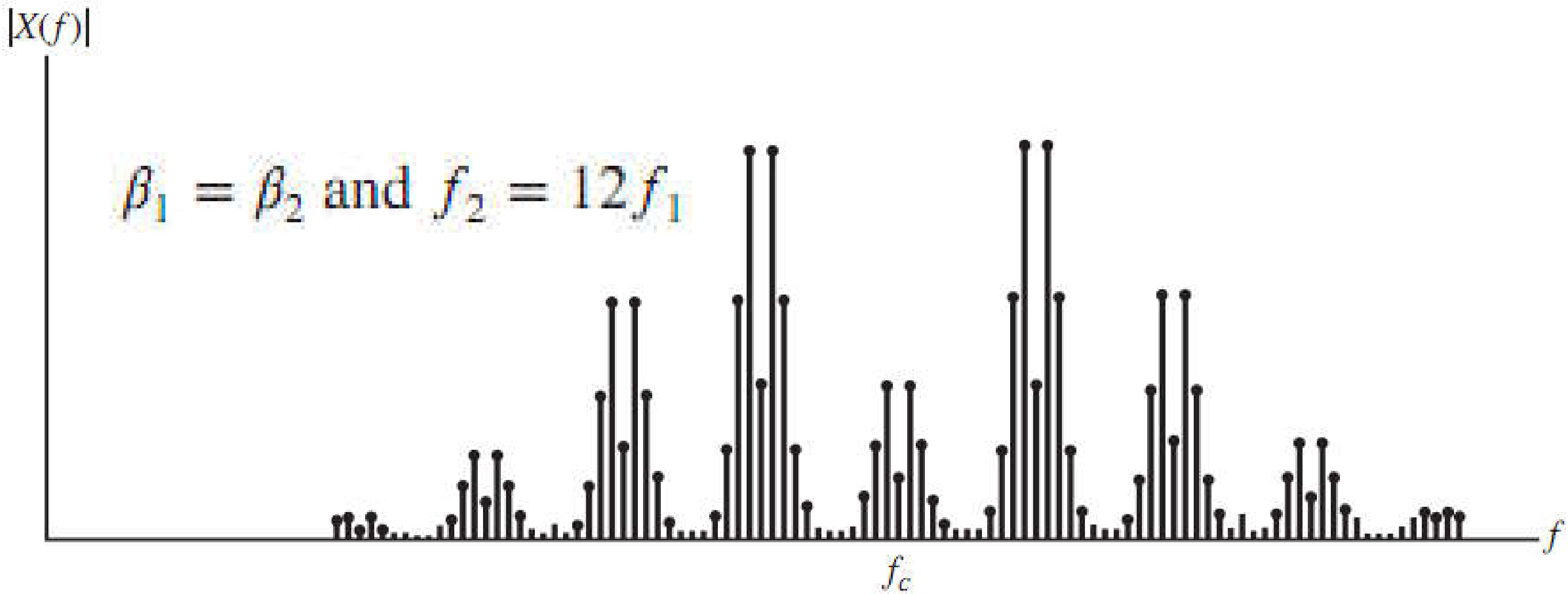
$$\phi(t) = \beta_1 \sin(2\pi f_1 t) + \beta_2 \sin(2\pi f_2 t)$$

where $\beta_1 = A f_d / f_1 > 1$ and $\beta_2 = B f_d / f_2$

$$x_c(t) = A_c \cos[2\pi f_c t + \beta_1 \sin(2\pi f_1 t) + \beta_2 \sin(2\pi f_2 t)]$$

$$x_c(t) = A_c \operatorname{Re}\left\{e^{j\beta_1 \sin(2\pi f_1 t)} e^{j\beta_2 \sin(2\pi f_2 t)} e^{j2\pi f_c t}\right\}$$

$$e^{j\beta_1 \sin(2\pi f_1 t)} = \sum_{n=-\infty}^{\infty} J_n(\beta_1) e^{j2\pi n f_1 t} \qquad e^{j\beta_2 \sin(2\pi f_2 t)} = \sum_{m=-\infty}^{\infty} J_m(\beta_2) e^{j2\pi n f_2 t}$$



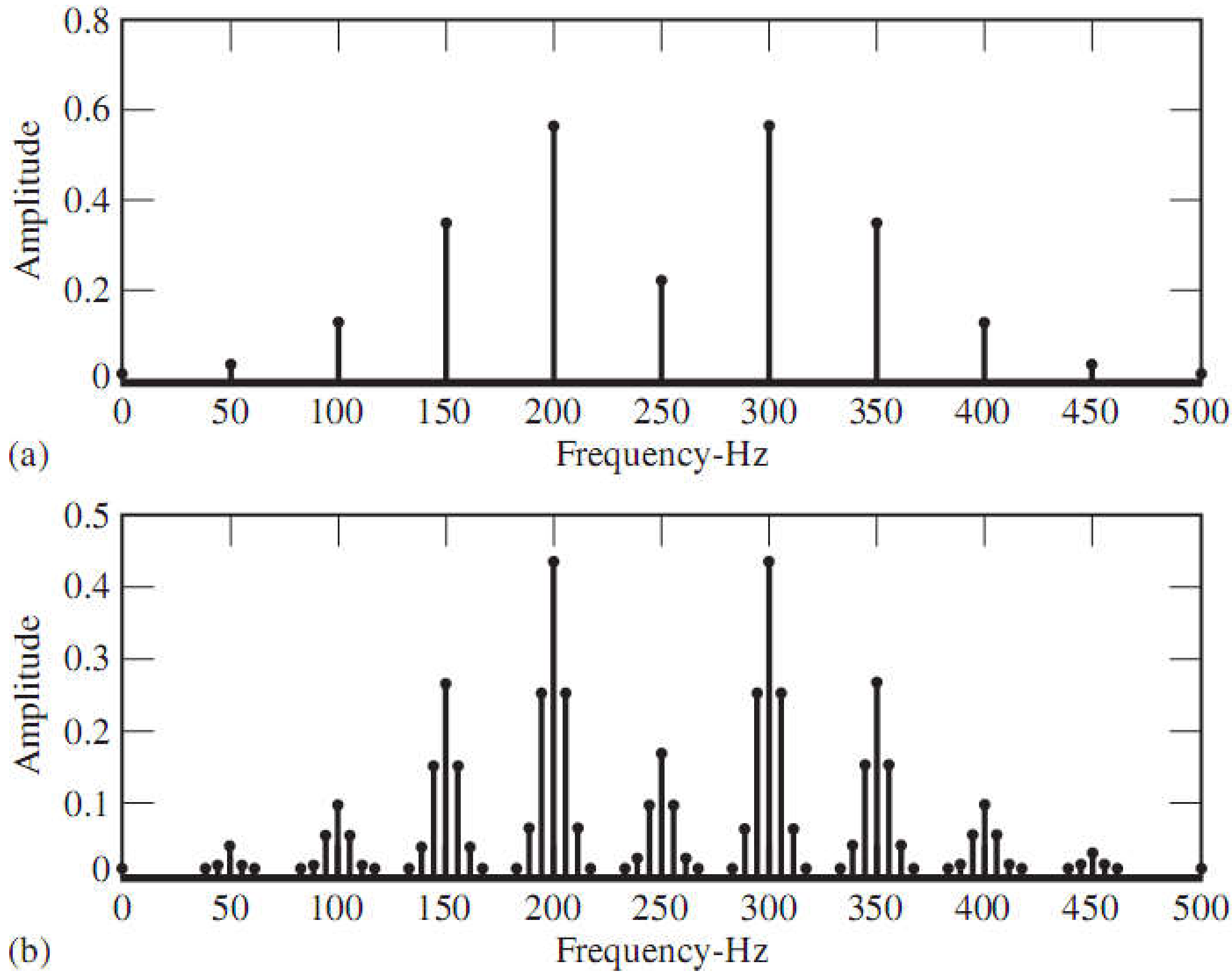
$$x_c(t) = A_c \operatorname{Re}\left\{\left[\sum_{n=-\infty}^{\infty} J_n(\beta_1) e^{j2\pi f_1 t} \sum_{m=-\infty}^{\infty} J_m(\beta_2) e^{j2\pi f_2 t}\right] e^{j2\pi f_c t}\right\}$$

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\beta_1) J_m(\beta_2) \cos[2\pi(f_c + n f_1 + m f_2)t]$$

Example 2:

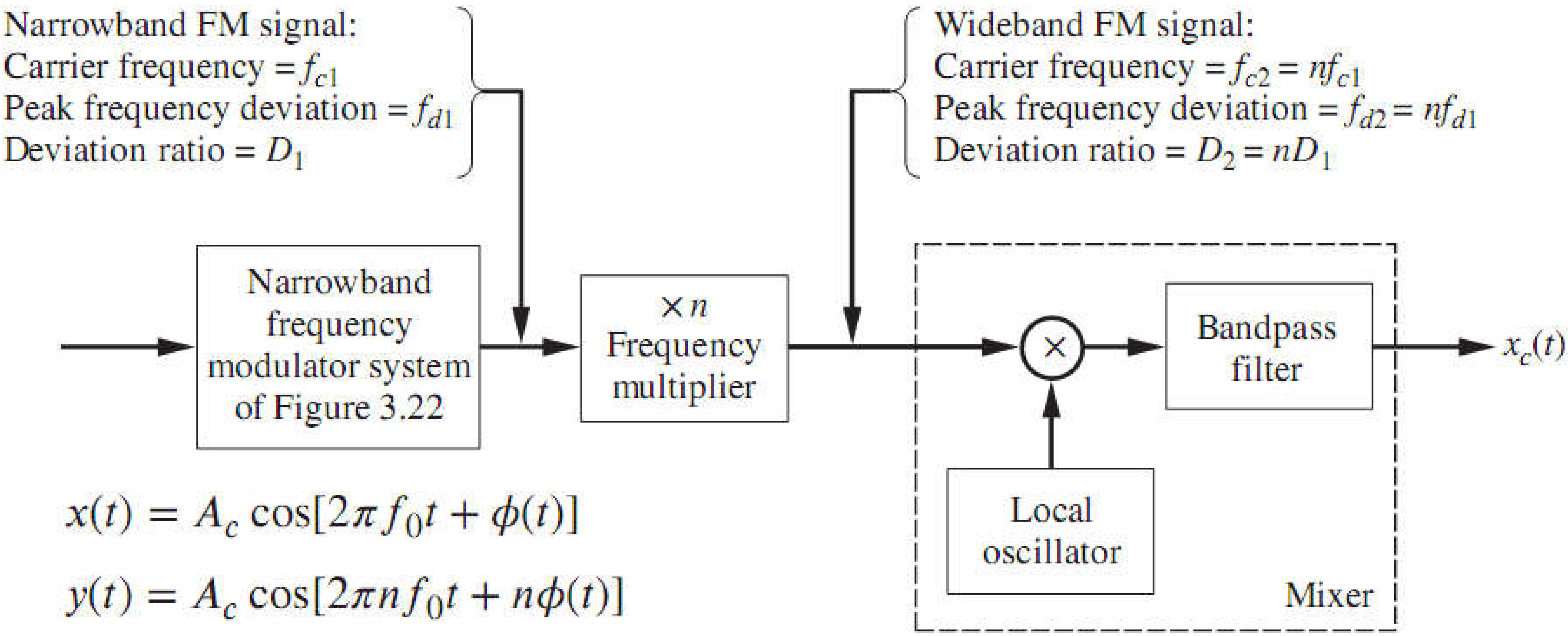
Signal 1 has frequency 50Hz, Signal 2 has 50Hz and 5Hz

```
fs=1000;                            %sampling frequency
delt=1/fs;                          %sampling increment
t=0:delt:1-delt;                    %time vector
npts=length(t);                     %number of points
fn=(0:(npts/2))*(fs/npts);              %frequency vector for plot
m1=2*cos(2*pi*50*t);                %modulation signal 1
m2=2*cos(2*pi*50*t)+1*cos(2*pi*5*t); %modulation signal 2
xc1=sin(2*pi*250*t+m1);             %modulated carrier 1
xc2=sin(2*pi*250*t+m2);             %modulated carrier 2
asxc1=(2/npts)*abs(fft(xc1));         %amplitude spectrum 1
asxc2=(2/npts)*abs(fft(xc2));         %amplitude spectrum 2
ampspec1=asxc1(1:((npts/2)+1));     %positive frequency portion 1
ampspec2=asxc2(1:((npts/2)+1));     %positive frequency portion 2
subplot(211)
stem(fn,ampspec1,'.k');
xlabel('Frequency-Hz')
ylabel('Amplitude')
subplot(212)
stem(fn,ampspec2,'.k');
xlabel('Frequency-Hz')
ylabel('Amplitude')
subplot(111)
```



Frequency Modulation Spectra: (a) Single tone; (b) Two-tone

FM utilizing narrowband-to-wideband conversion



$$x(t) = A_c \cos[2\pi f_0 t + \phi(t)]$$

$$y(t) = A_c \cos[2\pi n f_0 t + n\phi(t)]$$

$$e_{\mathrm{LO}}(t) = 2\cos(2\pi f_{\mathrm{LO}} t)$$

$$e(t) = A_c \cos[2\pi(n f_0 + f_{\mathrm{LO}})t + n\phi(t)]$$
$$+ A_c \cos[2\pi(n f_0 - f_{\mathrm{LO}})t + n\phi(t)]$$

$$f_c = n f_0 + f_{\mathrm{LO}} \quad \text{or} \quad f_c = n f_0 - f_{\mathrm{LO}}$$

$$x_c(t) = A_c \cos[2\pi f_c t + n\phi(t)]$$