rithmomachia

Rithmomachia

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Introduction

Welcome to the world of rithmomachia, one of the most obscure and intriguing board games in history! The earliest recorded mention is around 1000 CE, but the roots of the philosopher's game run deeper.

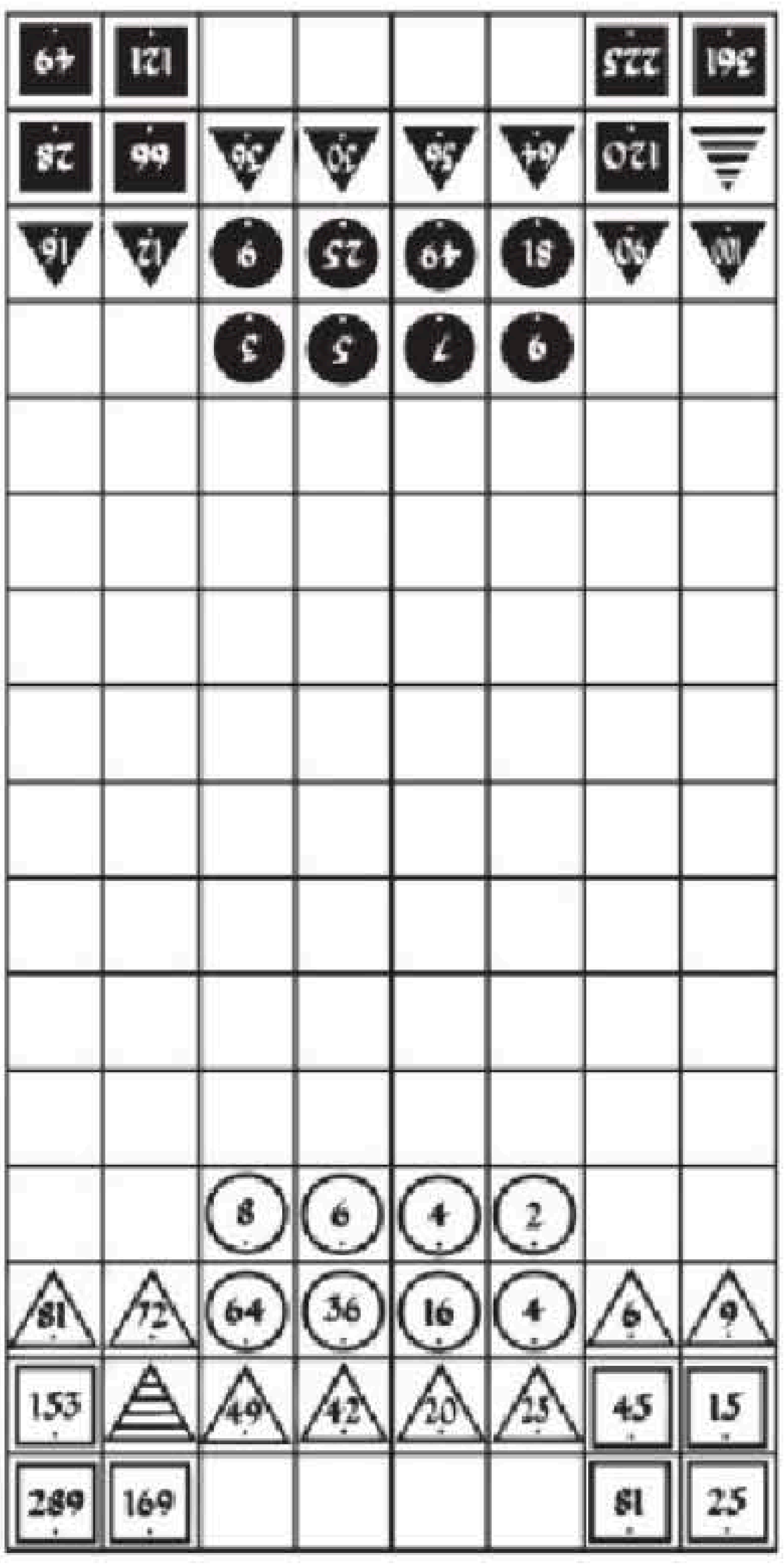
Rithmomachia, or the Battle of Numbers, was developed as a teaching tool for monastery schools, not as a board game for divertissement in the modern sense. It propounds the philosophical and harmonic principles of Anicius Manlius Severinus Boëthius (480–525 CE), son of a Roman patrician and author of the Consolation of Philosophy.

The works of Boëthius formed much of the backbone of the educational system, such as it was, in the medieval era, and the game of rithmomachia traveled through the centers of learning.

As the teachings of Boëthius fell from favor in the Renaissance, the game seems to have been simplified and stripped of its academic nature, early sets were marked in ancient Greek numerals and required the use of an abacus or rote memorization of tables.

The rapid popularization of chess in Europe all but extinguished rithmomachia by 1700. Only recently has interest in this quirky game has been reignited.

I hope that you will enjoy it!

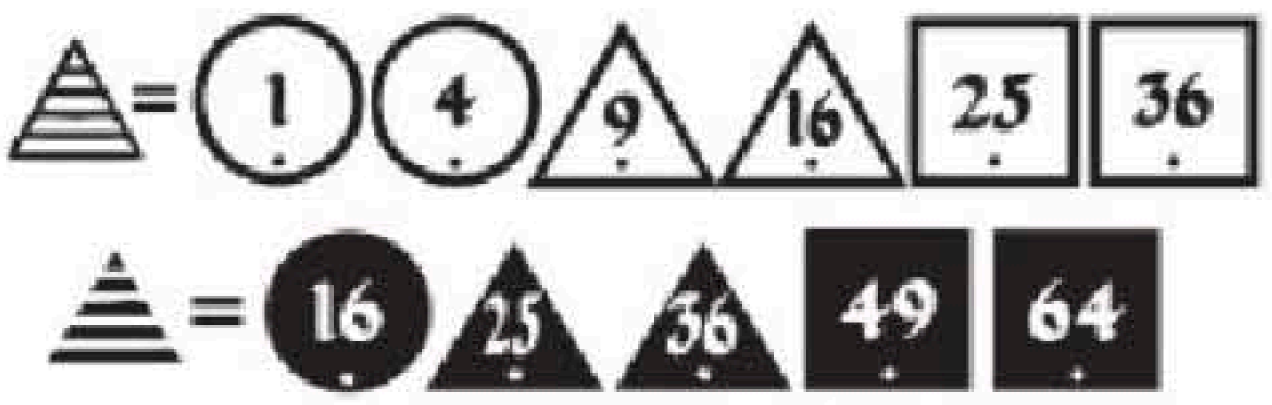


Board and Pieces

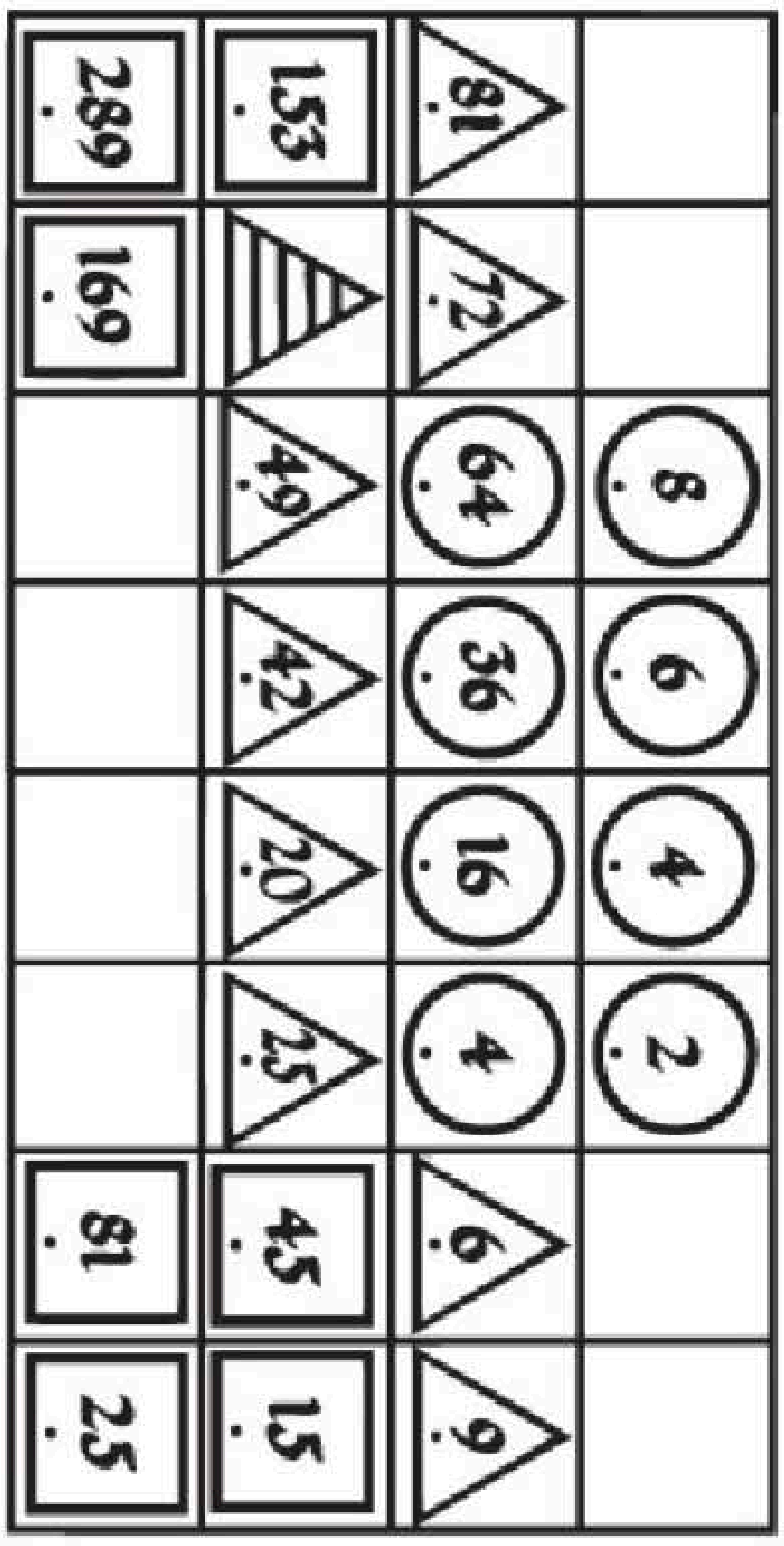
The board is an 8 by 16 grid, the same size as two chess boards. At the start of play each side occupies 24 spaces with an assortment of round, triangle and square pieces with various numerical values.

White has 29 pieces that occupy 24 spaces, 23 held by single pieces and one space occupied by a “pyramid” stack of square 36 and 25, triangle 9 and 16, and round 4 and 1. The sum of the White pyramid, known as the perfecta, is 91 at the start of play.

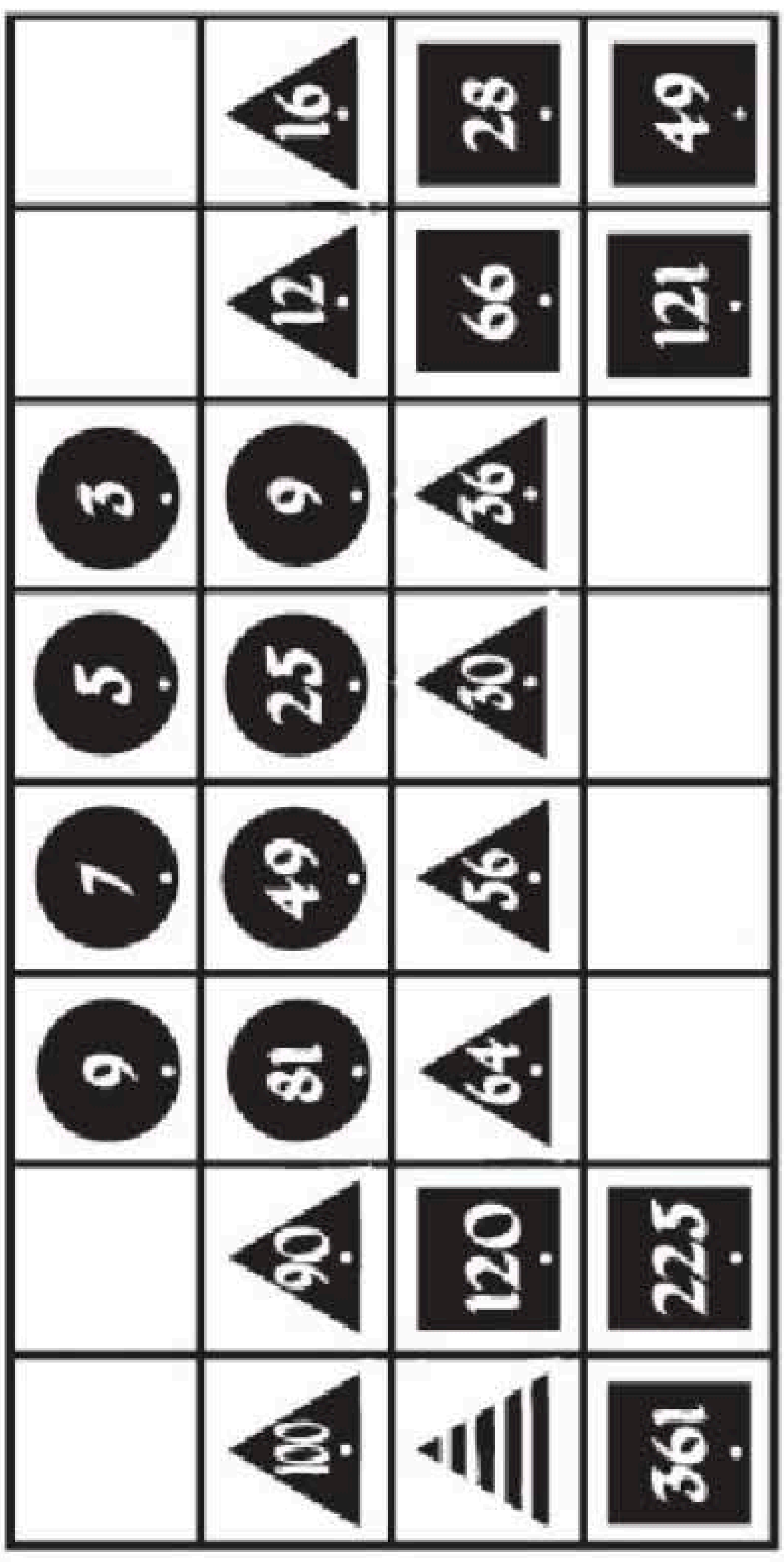
Black has 28 pieces that occupy 24 spaces, 23 held by single pieces and one space occupied by a “pyramid” stack of square 64 and 49, triangle 25 and 36, and round 16. The sum of the Black pyramid, known as the tricutera, is 190 at the start of play.


1 4 9 16 25 36
16 25 36 49 64

White starting position



Black starting position



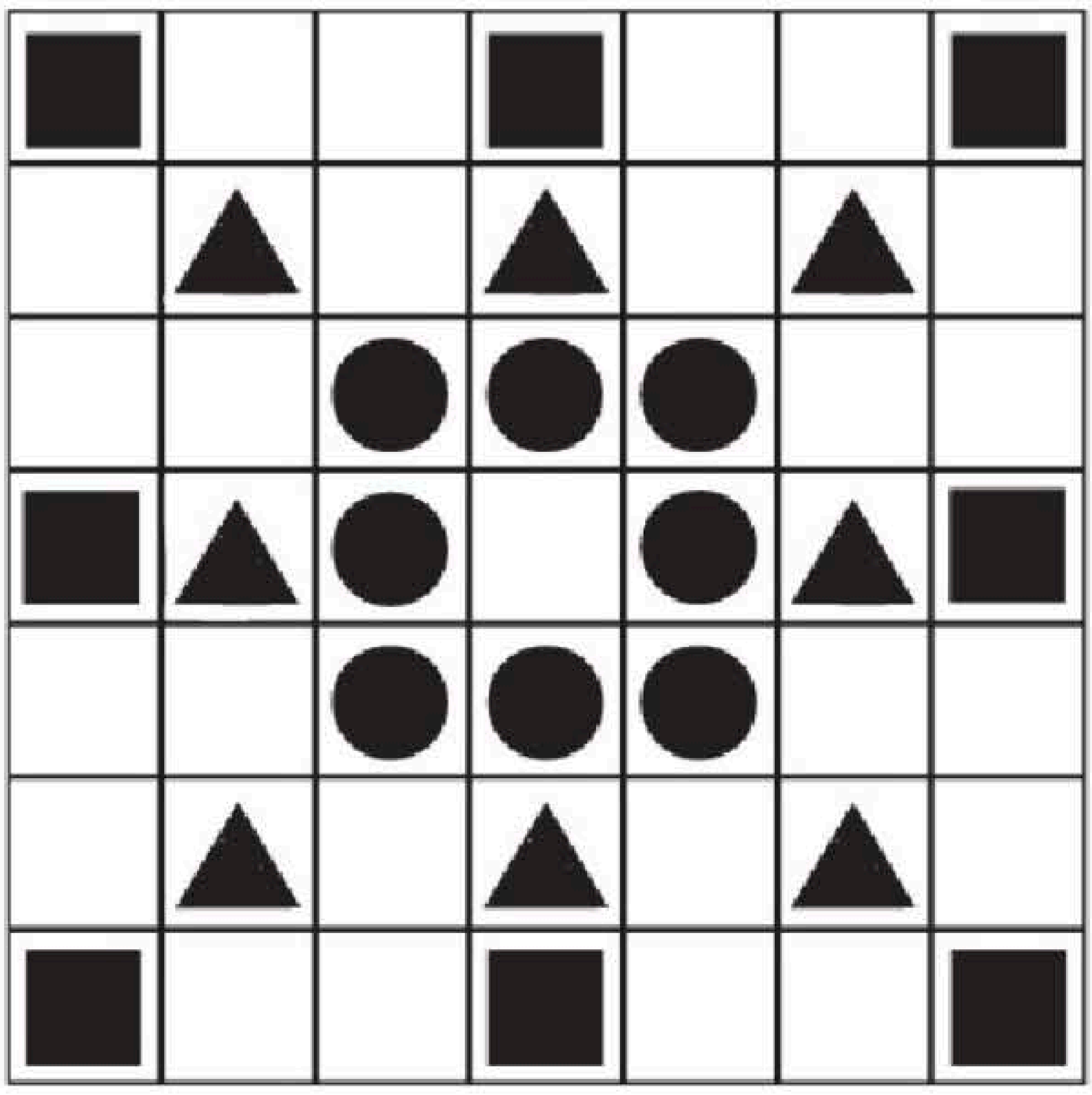
Movement

Black moves first. Each piece may move orthogonally or diagonally, but the distance the piece may travel depends on its shape. Round pieces may move one space, triangles two, squares three - no more, no less. This is easy to remember, as the more complex a shape is, the more movement it is capable of.

A pyramid may move any distance based on its component pieces. At the start of the game each pyramid contains at least one round, triangle and square, and as such, may move one, two or three spaces. If the pyramid loses components to capture during the course of play, it may only call on the movement of the pieces that it currently comprises. For example, a pyramid made up of only round and square pieces would be able to move one or three spaces, but not two as it no longer contains a triangle.

Each space on the board can only contain one piece. Pyramid stacks count as one piece for the purposes of occupying a space. Pieces may not jump over other pieces, enemy or allied. For example, a square must have a clear path of three empty spaces in the desired direction in order to move.

Table of movement



(assuming a piece starts in the center for this illustration, the table shows the distance and direction each type of piece may move)

Capture

Rithmomachia differs from chess in that the attacking piece does not move into the enemy space when capturing. Instead, players must chose to either use a piece's movement into a free space, or call out a valid attack to their opponent's attention – leaving the attacking piece in its current space and removing the enemy from the board, thus ending their turn.

There are four basic categories of capture, by siege, encounter, eruption, and deceit.

Siege:

Capturing by siege requires skill in maneuvering, but no mathematical calculations. To capture an enemy piece by siege, the player must maneuver his pieces to surround it on four side. Either all four orthogonal sides, or all four diagonal sides, but not a combination thereof. The edge of the board may also be used, so that a piece at the edge would only need to be surrounded on three sides, or by two pieces if it was besieged in a corner.

Encounter:

If the attacking piece can lawfully move to a space occupied by an enemy piece of equal value, the enemy piece is captured and removed from the board. Please recall that the attacking piece does not actually move into the vacated space, but remains at its original space. Taking by equality is not possible for every piece on the board, as the only numbers each force share are the perfect squares 9 through 81.

Eruption:

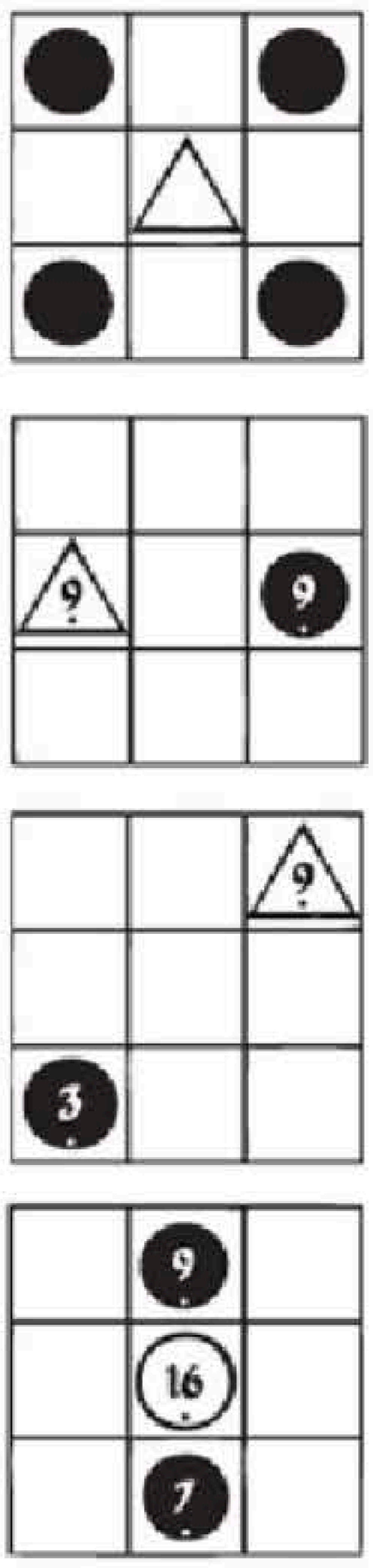
Taking by eruption is based on distance multiplication (or division). The value of the attacking piece multiplied by the number of spaces between its quarry must be equal to the enemy's value. The distance between the pieces includes the spaces that each occupies, so that two pieces meeting on a mutual edge will still have a minimum distance of two.

The same principle of capture by eruption allows for a piece of higher value to capture an enemy if the value of the attacking piece divided by the distance is equal to the enemy's value.

The attack can extend any distance as long as the path is clear.

Deceit:

To take by deceit, also called ambush or laying in wait, requires maneuvering and addition. A piece has been ambushed if it is surrounded on two sides, orthogonally or diagonally in a straight line, but is only removed if its value is equal to the sum of the two attackers.



Siege, Encounter, Eruption, Deceit

Pyramids and Ransom:

The value of the pyramids start as 91 for White and 190 for Black. As long as a pyramid retains all of its starting pieces it may use that value or chose to use the value of any single component piece in a capturing action. If a pyramid loses a piece to capture, it may only chose to use a single remaining value from those components it retains, not the new sum.

Pyramids may be captured as a whole by siege, or their component pieces may be singled out for attack until the pyramid has been whittled away to nothing.

The pyramid itself has no special powers beyond the flexibility of movement and capturing actions that it may avail the player. Its loss is not an endgame as the checkmate of a king in chess. Play

continues until a victory condition has been achieved.

If the pyramid is in danger of losing a component to enemy attack, the player may offer any other remaining active piece as ransom. There is no obligation to accept a ransom, but negotiation may help as some victory conditions require specific sequences of pieces that will be more attractive.

Multiple Captures:

If, through strategy or luck, a player finds that they may use a single piece to capture multiple enemies in a single turn, by any combination of lawful means, they are allowed to remove them all. The value of the pyramid is determined as above, and can only be used for multiple captures if the single selected value for that turn is valid for all simultaneous attacks.

Variant Captures:

The base rules as laid out above provide for only four simple methods of capturing the enemy. For more complexity, please see the

variants at the end of the rule book. There are provisions for numerous other arithmetical methods of capture that can further enrich, or bog down, the game depending on your proclivity.

Victories

There are five "common" victories and three "glorious" victories. Players agree beforehand which one of the five common victory conditions they will be playing for. The game ends when the conditions are met, however a glorious victory may be secured en route to, or indeed in spite of, that goal. Such glorious victories are rare and require some serious maneuvering, but this is what makes them all the more triumphant.

Common Victories:

Victory of Bodies is achieved when a player captures a predetermined number of pieces. This is the ideal victory condition for beginners as the value of the pieces is not important, only the quantity captured. Before starting a game, the players agree on a number of men, say 4, or 6, or 10, etc. and the first player to capture this many enemy pieces wins. The players must also agree if the pyramid counts as a single piece or as components, White having 29 pieces to Black's 28 in that case.

Victory of Goods is obtained when a predetermined sum of the value of pieces is met. Players may capture any number of pieces, but the first to reach or exceed the decided sum of these pieces is the victor. If, for instance, the players name an arbitrary goal of 100, White could achieve this by capturing the Black pieces 3, 16 and 81. Or simply by taking Black's 225 piece. It should be noted again that Black and White have asymmetrical forces.

Victory of Quarrel is concerned not only with the value of men, but also with the digits of the numbers that make up the captured pieces. The players agree on how many digits to play to, say 8 for example, and the value to be reached in those digits, say 160.

In this example White would be victorious if it captured Black's pieces 16, 28, 56 and 64. The numbers on the men contain 8 total digits, and the value of the pieces exceed the goal of 160. The value can be exceeded, and any number of men can be captured, but the combination of digits and value must be valid.

Victory of Honor is won when a predetermined value is met in a set number of men. Players agree on a value and how many men it will take to meet that value.

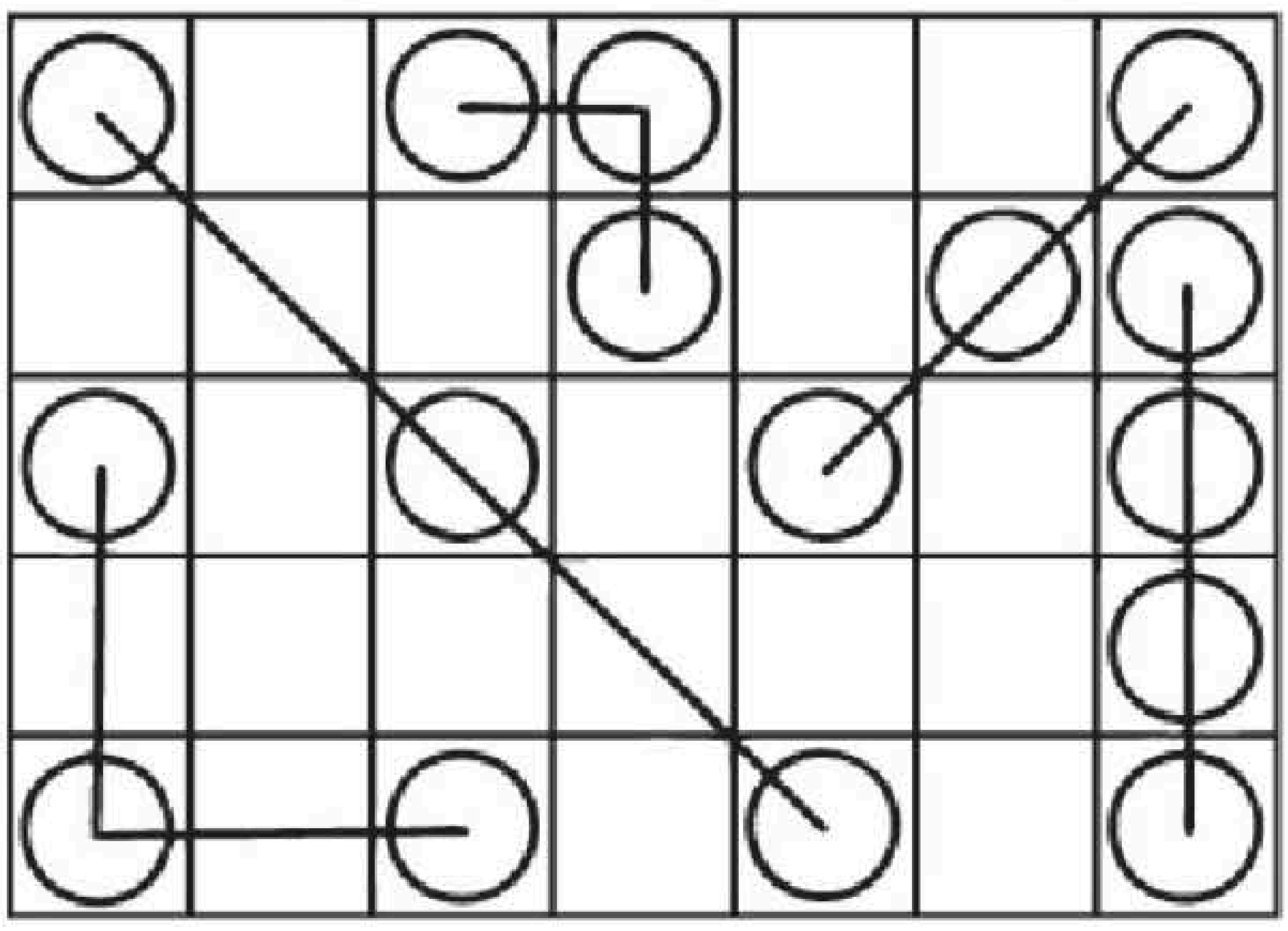
Say in this example the players choose the value of 100 in 8 men. Black would be victorious if it captured the White pieces 2, 4, 4, 6, 8, 15, 16 and 45. The value of 100 is exceeded and the minimum of 8 captured pieces is met.

Victory of Honor and Quarrel is the final common victory and requires the most computation. In this victory, the winner must satisfy an agreed upon value, digits and number of men. Let's use the same example from the victory of honor, namely a value of 100 comprised of 8 men, and add to it the goal of being obtained in 9 digits. Black would be victorious if it captured 2, 4, 4, 6, 6, 8, 9, 64. That is 8 pieces with 9 total digits and a value exceeding the goal of 100.

Glorious Victories:

As may be apparent, players can start with simple victory goals and scale up the complexity as they become familiar with the game. The glorious victories require even more intense maneuvering and harken back to the game's early history as a Boëthian teaching aide, rather than a mere amusement. These triumphs are known as Victoria Magna, Victoria Mayor, and Victoria Excelentisima.

A common victory is determined by the men that are removed from the board by capture, while a glorious victory is obtained by making an arrangement of pieces still in play on the board. The series will be three or four pieces, your own piece bringing up at least one end, but it may also incorporate enemy pieces. The series may be arranged in a straight line row, column, diagonally, or even as an angle or box of any size as long as the distance between the men is in proportion and unobstructed. The series must also be positioned in enemy territory, that is, the spaces of the grid forward from the first row of your own starting position.

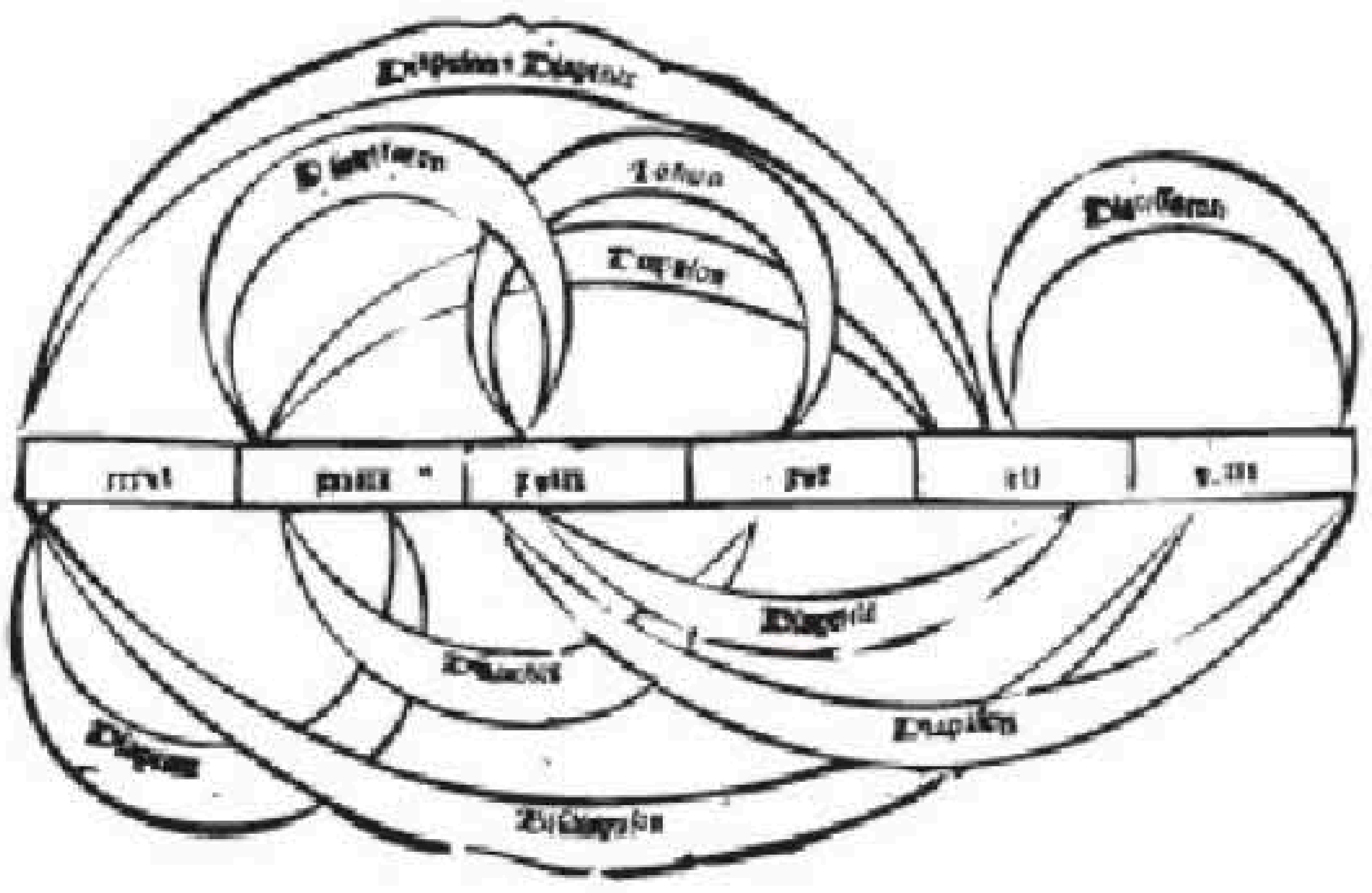


Most importantly, the sequence must form a mathematical relationship, either arithmetical, geometric, or harmonic. More on these relationships later, but first the conditions of the three types of glorious victory.

Victoria Magna, the simplest, requires only three pieces to be placed in a sequence that forms either an arithmetical, geometric, or harmonic progression.

Victoria Mayor. For this victory, the player must form two different progressions of three pieces within a total of a four piece sequence. An example would be the series of pieces 2, 3, 4, 8 which contains the arithmetical progression 2,3,4 and the geometrical progression 2,4,8.

Victoria Excelentisima. The final victory requires that the player form a four piece sequence that contains sets of all three progressions. There are only a handful of possible solutions for this arrangement, so great care must be taken not to lose the required pieces to capture.



Mathematical Relationships

White's starting force is based on the even numbers 2, 4, 6, 8, while Black's are based on the odd numbers 3, 5, 7, 9. These numbers represented the feminine and masculine aspects of nature and were logically derived from unity. In the game these are represented by each side's first order of rounds. The orders are four wide and six deep. For the purposes of the game the second half of the orders have been split out to the sides. The starting array of the pieces on the board can be seen as analogous to the formation of ancient armies in the field. The center is held by a slow moving phalanx of infantry, rounds in our game, while the sides and rear are composed of flanking cavalry and chariots, the more mobile triangles and squares.

Each successive order is found by using a series of formulas that were significant to Boëthian adepts. If a piece in the first order is n, the second order is n squared. The following order is the first triangle, found by the formula n(n+1). The next order of triangles is (n+1) squared. The first order of squares is (2n+1)(n+1), and finally the last order of squares is found by (2n+1) squared.

The pyramids substitute for the pieces 91 and 190 and are composed only of squared numbers; for White the squares of 1 through 6 (1, 4, 8, 16, 25, 36 = 91) and of 4 through 8 for Black (16, 25, 36, 49, 64 = 190)

$n=2$

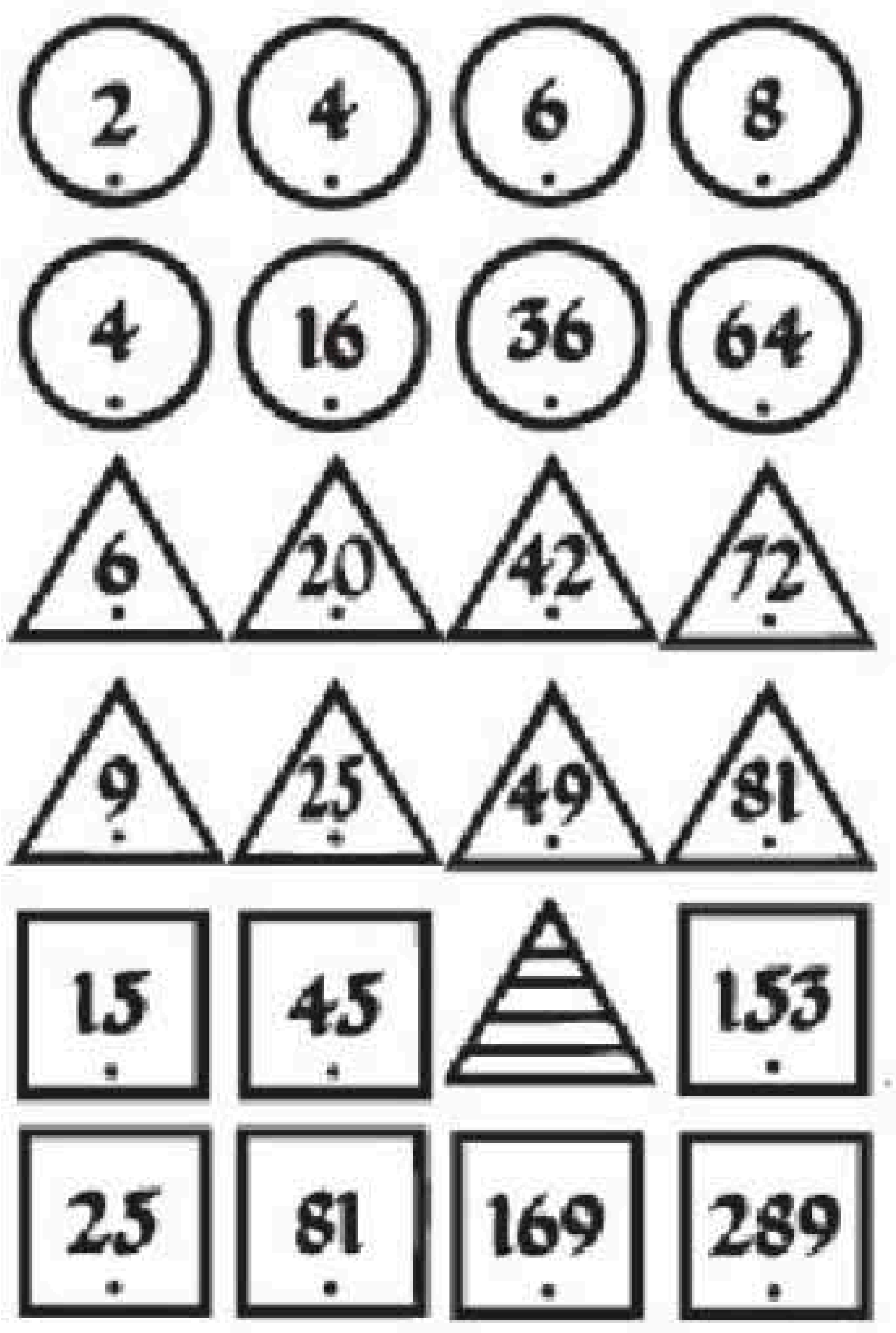
$n(n)=4$

$n(n+1)=6$

$(n+1)(n+1)=9$

$(2n+1)(n+1)=15$

$(2n+1)(2n+1)=25$



David Parlett describes the progressions required for the glorious victories as follows:

“Given three integers a (x) b (y) c, such that a<b<c, x=b-1, and y=c-b, the progression a,b,c is arithmetic if x=y, geometric if b/a=c/b, and harmonic if y/x=c/a.”

This description is succinct but tells players everything they need to know to understand the progressions involved in play, and how to plan for the victories.

Fulke offers some simple advice for finding proportions among the pieces in play. To find an arithmetical proportion when you have the first and last pieces in a possible sequence, add the first number to the last and divide the sum by 2. He gives the example of 5 as the first and 25 as the last number. To find the piece you need to complete the sequence add 5 to 25, that is 30; divided by 2 is 15. So the progression of pieces 5, 15, 25 would satisfy the condition for an arithmetical proportion.

Table of arithmetical proportions

2	3	4
2	4	6
2	5	8
2	7	12
2	9	16
2	15	28
2	16	30
3	4	5
3	5	7
3	6	9
3	9	15
3	42	81
4	5	6
4	6	8
4	8	12
4	12	20
4	16	28
4	20	36

4 30 56

5 6 7

5 7 9

5 15 25

5 25 45

6 7 8

6 9 12

6 36 66

7 8 9

7 16 25

7 28 49

7 49 91

7 64 121

8 12 16

8 25 42

8 36 64

8 49 90

8 64 120

9 12 15

9 45 81

9 81 153

12 16 20

12 20 28

12 42 72

12 56 100

12 66 120

15 20 25

15 30 45

15 120 225

16 36 56

20 25 30

20 28 36

20 42 64

28 42 56

28 64 100

30 36 42

42 49 56

42 66 90

42 81 120

49 169 289

56 64 72

72 81 90

81 153 225

91 190 289

If you have the first and last pieces you can also find a geometric proportion by multiplying the first and last, then finding the square root as a whole number. For example, 5 multiplied by 45 is 225, the square root of which is 15. So the sequence 5, 15, 45 is a valid geometrical progression.

Harmonic, or musical, progression requires a little more math but can be found by multiplying the first and last pieces, taking that product and multiplying by 2, then divide that number by the sum of the original first and last number. For example, if you have the pieces 5 and 20, their product is 100; doubled is 200, then divided by 25 (the sum of 5 and 20) is 8. So 5, 8, 20 is a valid harmonic progression.

The ultimate glorious victory must satisfy all three of these progressions. Please consult the Victoria Excelentisima solutions table.

Table of geometrical proportions

2	4	8
2	12	72
3	6	12
4	6	9
4	8	16
4	12	36
4	16	64
4	20	100
4	30	225
5	15	45
9	12	16
9	15	25
9	30	100
9	45	225
16	20	25
16	28	49
16	36	81
20	30	45

25 30 36

25 45 81

36 42 49

36 66 121

36 90 225

49 56 64

49 91 169

64 72 81

64 120 225

81 90 100

81 153 289

100 190 361

Table of musical proportions

2 3 6

3 4 6

3 5 15

4 6 12

4 7 28

5 8 20

5 9 45

6 8 12

7 12 42

8 15 120

9 15 45

9 16 72

12 15 20

15 20 30

25 45 225

30 36 45

30 45 90

72 90 120

Table of Victoria Excelentisima solutions

2	3	4	6
2	3	6	9
2	4	6	12
2	5	8	20
2	7	12	42
2	9	16	72
3	4	6	8
3	4	6	9
3	5	9	15
3	5	15	25
3	9	15	45
4	6	8	12
4	6	9	12
4	7	16	28
4	7	28	49
5	9	25	45
5	9	45	81

5 25 45 225

6 8 9 12

6 8 12 16

7 12 42 72

8 15 64 120

8 15 120 225

9 12 15 20

12 15 16 20

12 15 20 25

15 20 30 45

15 30 45 90

When a modern game player first sees the numbered pieces of rithmomachia laid out, it may seem like an arbitrary jumble of values. However, one must remember the game was originally created to serve as a teaching aide for the numerical theories of Boëthius, not as a board game in the modern sense. Boëthius' musical principles were thought to be a natural insight and playing rithmomachia would not only bring peace of mind, but a better understanding of the very workings of the world. As the curriculum of Boëthius fell out of favor, so did the game.

Rithmomachia was modified as it traveled through the various regions of Europe and common victory conditions were added that deemphasized computation. Of course in the game's 1000 year history many variations have been recorded. It is important that two foes meeting for the first time over a rithmomachia board discuss what victories and movement rules will be abided. One interesting aspect of playing the game of rithmomachia after it has been dormant since the Renaissance, instead of chess, is that you are very unlikely to find an opponent who will overwhelm you with years of experience, as we are all approaching this peculiar game from the same shared lack of exposure.

Variations

The following are some optional variants for play. The numerous sources over the years have many conflated and often conflicting rules, but the game is very flexible. Try some of these out, or create your own!

Allow jumps over allied pieces, during movement and/or attack.

Restrict movement for pieces. Rounds may move one space in any direction. Triangles may only move diagonally two spaces. Squares may move only orthogonally three spaces.

Increase movement to match the number of surfaces for each shape. Rounds still move one, triangles three and squares four spaces.

Pyramid may only be attacked from the top down, or the bottom up (chose before play begins) so only one piece is exposed to attack at a time.

Pyramid must be destroyed before the common or glorious victories are attained.

Allow flying movement during capture instead of the attacking piece remaining at its original space.

Allow the knight movement from chess for squares.

More complex captures using two attacking pieces (a1 and a2) that form relationships with an enemy piece (b) to remove it.

Addition: a1+a2=b

Subtraction: a1-a2=b or a2-a1=b

Multiplication: a1(a2)=b

Division: a1/a2=b or a2/a1=b

Progression: a1, a2 and b are harmonic.

Pieces a1 and a2 must each have valid control of the space b occupies for the attack, either via free movement or a combination of the attacks listed above, such as eruption or deceit.

Modify the length or width of the board and starting positions (refer to Fulke's first and second variants).

If you find the pyramid stack hard to manipulate or forget what pieces it comprises, set them in a row off to the side of the board and replace it with a coin or token. Surrender pieces as lost to your opponent.

Further Reading

The following sources compile and analyze the original Medieval and Renaissance rithmomachia texts, anecdotal references, number tables, retranslations, etc. from approximately 1000-1700. Players who crave more variants or are interested in the complete history of the game are encouraged to research here:

Arno Borst, Das mittelalterliche Zahlenkampfspiel. Supplemente zu den Sitzungsberichten der Heidelberger Akademie der Wissenschaften, Philosophisch-historische Klasse 5, Heidelberg, 1986. Leading modern researcher of the game. German language.

Jack Botermans, The Book of Games, Sterling Publishing, 2008. Brief overview of the game including an example of play.

Alfred Holl, Spiel mit Zahlen - Kampf mit Zahlen?, Växjö University, 2005. Another German language account of the history and play of the play.

Detlef Illmer et al., Rhythmomachia. Ein uraltes Zahlenspiel neu entdeckt, Hugendubel, 1987. German language account of the history and play.

Ann E. Moyer, The Philosophers' Game, The University of Michigan Press, 2001. Complete history of the game and it's context in Medieval and Renaissance Europe. Includes an edition of The Most Noble, Auncient, and Learned Playe by Lever and Fulke (1563)

David Parlett, The Oxford History of Board Games, Oxford University Press, 1999. Excellent overview including a comparison of the rules of play from the various original sources.

David Eugene Smith and Clara C. Eaton, Rithmomachia, the great mediaeval number game, The American Mathematical Monthly 18, 1911. Early modern account of the game in English.