MAT101 – Programming with Python

Exam - 7th February 2023

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For each exercise produce a script named 'Exercise_n.py', with n number of the exercise. Appropriately comment your code to explain what it does, specifying inputs, outputs and algorithms.

Exercise 1

For a general $q \in \mathbb{R}$, consider the series $\sum_{k=0}^{\infty} q^k$ which converges to $S(q) = \frac{1}{1-q}$ if and only if $|q| < 1$. Its partial sums are defined as $S_n(q) = \sum_{k=0}^{n} q^k$ for each $n \in \mathbb{N}$.

a) Write a function called `'convergence_vector'` which takes as input a list v of numbers and gives as output another list of booleans w with the same length. The generic element w_n of w with index n is "True" if the series converges for $q = v_n$, with v_n element of v with index n, it is "False" otherwise.

b) Write a function called `'convergence_limit'` that takes in input an integer $N \in \mathbb{N}$ and gives as output the list of the values of the limit $S(q)$ of the series for $N+1$ values of q equispaced in $[-0.5, 0.5]$. Use it to produce a plot of $S(q)$ with respect to q in $[-0.5, 0.5]$, for $N = 100$. In particular,

- plot the $S(q)$ values with a continuous line and without markers;
- add a title on the top of the plot;
- add labels for the axes;
- add a grid;
- save the plot as `'plot_convergence.pdf'`.

c) Write a function called `'convergence_even_odd'` which takes as input a value for q, an integer m and a string s and returns as output the list of the partial sums S_n with even indices $n \leq m$ if $s =$ "*even*", with odd indices $n \leq m$ if $s =$ "*odd*". The execution is stopped if s has any other value with a displayed message for the user.

d) Write a function called `'convergence_up_to_tolerance'` which takes as input a value for q, a tolerance ε and an integer m and returns as output a boolean b. If the series does not converge, i.e., if $|q| \geq 1$, then b is "False". If the series converges , i.e., if $|q| < 1$, then b is "True" if $|S_n(q) - S(q)| \leq \varepsilon$ occurs within $n \leq m$, it is "False" otherwise.

Exercise 2

Write a script which

a) reads the times t_n and the measurements $f_n = f(t_n)$ of a given quantity from the file `'time_evolution.dat'` and then closes the file;

b) plots the discrete values of $f(t)$ with respect to the time t together with an horizontal line $y = 5$ in the same plot with a title, labeled axes, a grid, a legend for the curves, different linestyles and colors for the two curves and no markers, and then saves the plot as `'my_plot.pdf'`;

c) writes the minimum and the maximum f_n in a file `'my_min_max.dat'` in a single line separated by a semicolon.

Exercise 3

a) Write a function `'my_select'` that takes as input a tolerance δ, a list of numbers w and a scalar number y and returns as output the list of the the elements w_n of w which are such that $|w_n - y| > \delta$, i.e., which are distant from y more than the given tolerance.

b) Write a function `'my_columns'` that takes as input a matrix (bidimensional ndarray) A and a scalar y and returns as output a list l, with size equal to the number of columns of the matrix A, containing boolean elements. The element l_n of l with index n is "True" if y is present in the n-th column of the matrix A, it is False otherwise.