MAT101 – Programming with Python

Exam - 13th September 2023

Lorenzo Micalizzi

Institute of Mathematics, University of Zurich

For each exercise produce a script named 'Exercise_n.py', with n number of the exercise. Appropriately comment your code to explain what it does, specifying inputs, outputs and algorithms.

Exercise 1
Consider the sequence $a_k = \sin\left(\frac{2\pi k}{100}\right)$ for $k \in \mathbb{N}$.

a) Write a function called `'my_sequence'`, which takes as input an integer $N \in \mathbb{N}$, a target value t and a string s and returns as output two lists called li and lv respectively. In particular,

- if s is equal to "geq", then li is the list of the indices $k \leq N$ for which $a_k \geq t$ and lv is the list of the corresponding values a_k;
- if s is equal to "leq", then li is the list of the indices $k \leq N$ for which $a_k \leq t$ and lv is the list of the corresponding values a_k;
- for any other value of s, an error message is displayed and the execution is stopped.

b) Use the function previously defined to produce two plots:

- the plot of the elements a_k of the sequence such that $a_k \geq 0$, with $k \leq 1000$, with respect to their indices.
- the plot of the elements a_k of the sequence such that $a_k \leq 0$, with $k \leq 1000$, with respect to their indices.

In particular,

- plot the values as discrete points (no lines between them);
- add a title on the top of the plots;
- add labels for the axes;
- add a grid;
- save the plots as `'plot_geq.pdf'` and `'plot_leq.pdf'` respectively.

Exercise 2
Write a script which

a) reads the abscissae x_n and the values y_n, for $n = 0, \ldots, 1000$, from the file `'input_data.dat'` and then closes the file;

b) plots the discrete values y_n with respect to the corresponding abscissae x_n together with the function $\cos(2\pi x)$ in the same plot with a title, labeled axes, a grid, a legend for the curves, different linestyles and colors for the two curves and no markers, and then saves the plot as `'my_plot.pdf'`;

c) computes the error err of the values y_n with respect to the function $\cos(2\pi x)$, defined as

$$err = \sum_{n=0}^{1000} |y_n - \cos(2\pi x_n)|, \tag{1}$$

writes it in a file `'my_error.dat'` and then closes the file.

Exercise 3

a) Write a function `'my_isolate'` that takes as input a matrix (bidimensional ndarray) M, a target value t, a tolerance ε and a string s and returns as output another matrix N with the same shape as M and with entries defined as follows:

- if s is equal to "close", then

$$N_{i,j} = \begin{cases} M_{i,j}, & \text{if} \quad |M_{i,j} - t| < \varepsilon \\ 0, & \text{otherwise;} \end{cases} \tag{2}$$

- if s is equal to "far", then

$$N_{i,j} = \begin{cases} M_{i,j}, & \text{if} \quad |M_{i,j} - t| > \varepsilon \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

For any other value of s, an error message is displayed and the execution is stopped.

b) Write a function `'my_replace'` that takes as input a list v of numbers, a target value t and a tolerance ε and returns as output a list w of numbers with length equal to the one of the input list v. The elements of w are the same as the elements of v, up to the replacement of the elements v_k, such that $|v_k - t| < \varepsilon$, by t.