MAT101 Programming – Homework 2

Deadline: Monday, 9.10.2023, 22:00 PM

Login to `https://w3.math.uzh.ch/my` with your UZH credentials to submit your solved exercises for grading. You can find more information on how to upload/submit your exercises on `https://wiki.math.uzh.ch/public/studentUpload`.

General advice: Remember to run your scripts with a few different inputs in order to find mistakes and to spot unexpected behaviours in edge cases.

Exercise 1. **15** P.

a) ✎ The *greatest common divisor* (gcd) of two integers a and b ($\gcd(a,b)$) is defined as the largest positive integer that divides a and b. This definition also applies when one argument a or b is zero. In that case, the gcd is the absolute value of the non-zero integer. Indeed, $\gcd(a,0)=\gcd(0,a)=|a|$ for $a \neq 0$, since any number is a divisor of 0 and the greatest divisor of a is $|a|$. By convention, and for the sake of simplicity, we define $\gcd(0,0)=0$.

Excursion: Two integers a and b are called *coprime* if the only positive integer that is a divisor of both of them is 1. Equivalently if $\gcd(a,b)=1$.

Write an algorithm such that for given integers a, b it returns the gcd of a and b. Use a flow chart to explain your algorithm or list the steps that it is supposed to do and explain them in words. (**5** P.)

Hints:

- You don't require the prime factorisation of a and b to solve this task.
- You might want to work with the modulo-operator %.
- If you're stuck, consult the Wikipedia article on the gcd.

b) 💻 Using an appropriate method of flow-control, implement your algorithm of part a) in a script `ex02_1.py` that gives the gcd of two given integer values a, b. Start your script by specifying the two integers a, b for appropriate values (e.g. $a = 6, b = 9$). Store the result in a variable named 'greatest_common_divisor'. (**5** P.)

c) 💻 The *least common multiple* (lcm) of two integers a and b ($\mathrm{lcm}(a,b)$) is the smallest positive integer that is divisible by both a and b. Since division of integers by zero is undefined, this definition has meaning only if a and b are both different from zero. However, here we define $\mathrm{lcm}(a,0)=0$ for all integer a.

Write a script that gives the lcm of two integers a and b, store the result in a variable named 'least_common_multiple'. Add said script to `ex02_1.py`. (**5** P.)

Hint: Use that $\gcd(a,b)*\mathrm{lcm}(a,b)=|a*b|$.

Exercise 2. **5** P.

💻 In this exercise you are given a list L (possibly empty, i.e. $L = []$) and a target value *target*. You can assume that *target* is either an integer or a string. Write a script that gives the first index of L where the value *target* occurs, if the target value *target* does not occur in L the script should give 'None'. Store your result in a variable named 'idx' and save your script as `ex02_2.py`. Use for example $L = [1, \text{'abc'}, 1, 2, 2]$ and the target values 2, 'abc' and 4 to test your script.

Exercise 3. **10** P.

💻 You are attending the MAT101 Programming exam and you are wondering if you have already solved enough exercises to achieve a grade that is satisfactory for you. You have a good idea on how many points s = score you have scored so far and you know the following:

- $90 < s \leq 100 \implies$ Grade: 6.0
- $80 < s \leq 90 \implies$ Grade: 5.5

- $70 < s \leq 80 \implies$ Grade: 5.0
- $60 < s \leq 70 \implies$ Grade: 4.5
- $50 < s \leq 60 \implies$ Grade: 4.0
- $0 \leq s \leq 50 \implies$ Grade: Failed attempt

Write a script `ex02_3.py` that given your score (possibly as a float) gives your grade according to the list above. In case you haven't passed the exam, return a string 'Failed attempt'. Store your result in a value named 'grade'.

Remark: Your script should also be able to handle an invalid input such as $s = -13$ (achieved negative points) or $s = 200$ (achieved more points than the maximum).

Exercise 4. **10** P.

💻 In number theory, *Euler's totient function* φ counts the positive integers up to a given integer n that are coprime to n. In other words, it is the number of integers k in the range $1 \leq k \leq n$ for which the greatest common divisor $\gcd(n, k)$ is equal to 1.

Write a script `ex02_4.py` that for a given $n \in \mathbb{N}$ gives $\varphi(n)$. Store your result in a value called 'euler_phi'.

Please start your script with the following line:

- from math import gcd

Once you have imported the gcd function from the math module, you can use its functionality freely to solve this exercise.